

# Weak Decays of $\eta$ Mesons

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## Abstract

At the DAΦNE facilities, an observation of the weak decays of  $\eta$  mesons would be an evidence of new physics beyond the Standard Model.

As it was pointed in ref. [1] a study of the  $\eta \rightarrow 3\pi$  and electromagnetic decays of  $\eta$  mesons at the DAΦNE  $\Phi$ -factory will improve a precision in determination of  $C$ -violating part of the amplitudes by one-two orders. A comparatively big number of  $\eta$  mesons  $N_\eta \approx 3 \cdot 10^8$ /year expected at the DAΦNE machine makes also pertinent the question on possibility to observe the weak decays of  $\eta$  mesons.

These decays possess some specific properties which would be not harm to verify. Namely, because of  $G$ -parity conservation [2] the decays

$$\eta \rightarrow \pi^\pm l^\mp \nu \quad (1)$$

$$\eta \rightarrow \pi^0 \pi^\pm l^\mp \nu \quad (2)$$

are suppressed in comparison with the analogous decays of  $K_L^0$  mesons possessing the same CP-properties as  $\eta$ .

The decays

$$\eta \rightarrow K^\pm e^\mp \nu \quad (3)$$

are allowed, but they are suppressed compare to  $K_L^0 \rightarrow \pi^\pm e^\mp \nu$  decay by the momentum-space factor  $\sim 10^{-3}$ .

Of the nonleptonic weak decays the decay

$$\eta \rightarrow \pi \pi \quad (4)$$

is the most interesting as it needs CP violation and it can not be masked by strong interaction contribution as in the case of  $\eta \rightarrow 3\pi$  decay.

Let's consider the above mentioned processes in details.

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# 1 $\eta \rightarrow \pi^\pm l^\mp \nu$

The hadronic part of the matrix element is

$$\langle \pi^\pm | J_\mu^1 + iJ_\mu^2 | \eta \rangle = f_+^{(\eta)}(q^2)(p_\eta + p_\pi)_\mu + f_-^{(\eta)}(q^2)(p_\eta - p_\pi)_\mu \quad (5)$$

In the Standard Model, where the second-class currents [2] are absent, the form factors  $f_\pm$  can be different from zero only due to isospin breakdown occurring through the virtual electromagnetic interactions or due to  $(m_d - m_u)$  mass difference. As the last mechanism gives the largest effect one can estimate a probability of the decay using a Chiral Theory of low-energy mesonic processes. The Effective Lagrangian approach taking into account the scalar mesons [4] gives the result <sup>2</sup>

$$f_+^{(\eta)}(q^2) = -\sqrt{\frac{3}{8}} \frac{m_d - m_u}{m_s - \frac{1}{2}(m_d + m_u)} [1 + q^2/(M_\rho^2 - q^2)] \quad (6)$$

$$f_-^{(\eta)}(q^2) = \sqrt{\frac{3}{8}} \frac{(m_d - m_u)(m_\eta^2 - m_\pi^2)}{m_s - \frac{1}{2}(m_d + m_u)} [(M_\rho^2 - q^2)^{-1} - (M_{a_0(980)}^2 - q^2)^{-1}] \quad (7)$$

Therefore, the form factors of the decay (1) are suppressed by the factor

$$\beta = \sqrt{\frac{3}{8}} \frac{m_d - m_u}{m_s - \frac{1}{2}(m_d + m_u)} \cdot \text{ctg} \theta_C \quad (8)$$

in comparison with the form factors of  $K_L^0 \rightarrow \pi^\pm l^\mp \nu$  decay.

Using the most conservative estimate [5] for the quantity (8) we come to the result

$$\beta \lesssim 0.1 \quad (9)$$

Then

$$B.r.(\eta \rightarrow \pi^\pm l^\mp \nu) \cong 2\beta^2 \left(\frac{m_\eta}{m_K}\right)^5 \frac{\Gamma(K_L^0 \rightarrow \pi^\pm l^\mp \nu)}{\Gamma_{tot}(\eta)} \lesssim 2 \cdot 10^{-13} \quad (10)$$

Therefore, an observation of the decay (1) with the rate considerably larger than  $10^{-13}$  would be an evidence of existence of the some new physics beyond SM. The estimates of possible contribution of the second-class currents to this process are contained in refs. [3] and [6].

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<sup>2</sup>An appearance of  $m_s - \frac{1}{2}(m_u + m_d)$  in denominator of the expression (6) is explained by contribution of the successive transitions  $\eta \rightarrow \pi^0 \rightarrow \pi^\pm l^\mp \nu$  to  $f_+^{(\eta)}$  proportional to  $(m_\eta^2 - m_\pi^2)^{-1}$ . For  $f_-^{(\eta)}$ , the dependence on SU(3) breaking parameter is cancelled.

## 2 $\eta \rightarrow \pi^0 \pi^\pm l^\mp \nu$

The hadronic part of the matrix element is

$$\langle \pi^0 \pi^+ | A_\mu | \eta \rangle = f_1(q^2)(p_\pi + p_{\pi'})_\mu + f_2(q^2)(p_\pi - p_{\pi'})_\mu + f_3(q^2)(p_\eta - p_\pi - p_{\pi'})_\mu \quad (11)$$

$$\langle \pi^0 \pi^+ | V_\mu | \eta \rangle = f_4 \frac{i\varepsilon_{\mu\nu\alpha\beta}}{M_K^2} (p_\pi + p_{\pi'})_\alpha (p_\pi - p_{\pi'})_\beta (p_\eta)_\nu \quad (12)$$

Again, as in the case of the decay (1) the form factors  $f_{1,2,3}$  are suppressed by  $G$ -parity conservation and they are smaller than the corresponding form factors of  $K_L^0 \rightarrow \pi^0 \pi^\pm l^\mp \nu$  decay by the factor  $\beta$ . This is not the case for the form factor  $f_4$  which is allowed. But the contribution of this form factor to probability of  $K_L^0 \rightarrow \pi^\pm e^\mp \nu$  decay is approximately 0.5%.

Then the estimate for B.r. of  $\eta_{e4}$  decay is

$$\begin{aligned} B.r.(\eta \rightarrow \pi^0 \pi^\pm l^\mp \nu) &\cong 2(\beta^2 \text{ or } \frac{2}{3} ctg^2 \theta_c \cdot 0.005) \left( \frac{m_\eta}{m_K} \right)^7 \frac{\Gamma(K_L^0 \rightarrow \pi^0 \pi^\pm l^\mp \nu)}{\Gamma_{tot}(\eta)} \lesssim \\ &\lesssim 4 \cdot 10^{-16} \end{aligned} \quad (13)$$

## 3 $\eta \rightarrow K^\pm e^\mp \nu$

The hadronic part of the matrix element is of the form of (5). But

$$f_+(\eta \rightarrow K^- e^+ \nu) = \sqrt{\frac{3}{2}} f_+(K^0 \rightarrow \pi^- e^+ \nu) \quad (14)$$

and

$$f_-(\eta \rightarrow K^- e^+ \nu) = \sqrt{\frac{3}{2}} \left( \frac{m_\eta^2 - m_K^2}{m_K^2 - m_\pi^2} \right) \cdot f_-(K^0 \rightarrow \pi^- e^+ \nu) \quad (15)$$

At these values of the form factors

$$\frac{\Gamma(\eta \rightarrow K^\pm e^\mp \nu)}{\Gamma(K_L^0 \rightarrow \pi^\pm e^\mp \nu)} \approx 0.867 \cdot 10^{-3} \quad (16)$$

or

$$B.r.(\eta \rightarrow K^\pm e^\mp \nu) = 4 \cdot 10^{-15} \quad (17)$$

## 4 $\eta \rightarrow \pi\pi$

Like  $K_L^0 \rightarrow 2\pi$  decay, this decay violates CP invariance, but the strangeness does not change in  $\eta \rightarrow 2\pi$  transition.

For this reason, in the Standard Model, the amplitude of  $\eta \rightarrow 2\pi$  decay must be suppressed at least by the factor  $G_F \Lambda^2 \sin \theta_C$  with  $\Lambda \lesssim 1$  GeV in comparison with  $K_L^0 \rightarrow 2\pi$

amplitude. This estimate follows from the fact, that CP violation in SM occurs due to imaginary parts of the Yukawa couplings and to have the observable effect of CP violation one needs to admit the flavour-changing transition like  $\eta \rightarrow K^0, \bar{K}^0$  containing non-self-conjugated product of the Yukawa couplings. Considering then the decay of  $\{K^0, \bar{K}^0\}$  system into  $2\pi$  state we come to the above estimate.

## 5 Conclusion

The estimates obtained in the framework of Standard Model

$$B.r.(\eta \rightarrow \pi^\pm l^\mp \nu) \lesssim 2 \cdot 10^{-13}$$

$$B.r.(\eta \rightarrow \pi^0 \pi^\pm l^\mp \nu) \approx 4 \cdot 10^{-16}$$

$$B.r.(\eta \rightarrow K^\pm l^\mp \nu) \approx 4 \cdot 10^{-15}$$

$$B.r.(\eta \rightarrow 2\pi) \lesssim 4[G_F \Lambda^2 \sin\theta_C]^2 \cdot 10^{-14}$$

show that the observation of these decays at the DAΦNE factory with  $N_\eta \approx 3 \cdot 10^8/\text{year}$  would be an evidence of some new physics beyond the Standard Model.

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## References

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