The $\pi\pi$ scattering amplitude in chiral perturbation theory

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Abstract

We discuss $\pi\pi$ scattering in the framework of chiral perturbation theory. In particular, we recall the predictions [1] for the threshold parameters and for the phase shift difference $\delta_0^0 - \delta_1^1$.

1 Notation

The scattering amplitude for $\pi\pi$ scattering,

$$\pi^i(p_1)\pi^k(p_2) \to \pi^l(p_3)\pi^m(p_4) \quad ,$$

reads

$$\begin{aligned} \langle \pi^m(p_4)\pi^l(p_3)\mathrm{out}|\pi^i(p_1)\pi^k(p_2)\mathrm{in}\rangle &= \langle \pi^m(p_4)\pi^l(p_3)\mathrm{in}|\pi^i(p_1)\pi^k(p_2)\mathrm{in}\rangle \\ &+ \; i(2\pi)^4\delta^{(4)}(P_f - P_i)T^{ik;lm}(s,t,u) \quad , \end{aligned}$$

where $T^{ik;lm}(s,t,u)$ is a Lorentz invariant function of the standard Mandelstam variables

$$\begin{aligned} s &= (p_1+p_2)^2 = 4(M_\pi^2 + q^2) \quad , \\ t &= (p_3-p_1)^2 = -2q^2(1-\cos\theta) \quad , \\ u &= (p_4-p_1)^2 = -2q^2(1+\cos\theta) \quad . \end{aligned}$$

q (θ) is the center-of-mass momentum (center-of-mass scattering angle). On account of isospin symmetry, the amplitude $T^{ik;lm}(s,t,u)$ may be expressed in terms of a single amplitude $A(s,t,u) = A(s,u,t)$,

$$T^{ik;lm}(s,t,u) = \delta^{ik}\delta^{lm}A(s,t,u) + \delta^{il}\delta^{km}A(t,s,u) + \delta^{im}\delta^{kl}A(u,t,s) \quad .$$

To compare the calculated amplitude with data on $\pi\pi$ scattering [2], one expands the combinations with definite isospin in the s-channel

$$\begin{aligned} T^0(s,t) &= 3A(s,t,u) + A(t,u,s) + A(u,s,t) \\ T^1(s,t) &= A(t,u,s) - A(u,s,t) \\ T^2(s,t) &= A(t,u,s) + A(u,s,t) \end{aligned}$$