INFN - Laboratori Nazionali di Frascati, May 1995
From: **The Second DaΦne Physics Handbook***
Chapter 4: Other Approaches to Low Energy Dynamics
Eds. L. Maiani, G. Pancheri, N. Paver

Quark - Resonance Model

E. Pallante
INFN, Laboratori Nazionali di Frascati, Via E. Fermi, I-00044 Frascati
and
R. Petronzio
Dipartimento di Fisica, Università di Roma "Tor Vergata", Via della Ricerca Scientifica, and INFN, Sezione di Roma "Tor Vergata"I-00133 Roma

Abstract

An effective model for low energy hadronic interactions is formulated arising form the bosonization of a Nambu-Jona Lasinio type Lagrangian including all chiral invariant multiquark effective interactions. The bosonized Lagrangian includes non-renormalizable quark-meson vertices which are next-to-leading in the low energy inverse cutoff expansion. Their relevance is explicitely discussed for the two point vector currents Green's function.

1 Introduction

In the framework of effective fermion models *à la* Nambu-Jona Lasinio for low energy hadronic interactions, the Quark-Resonance model [1] can be thought of as a generalized ENJL [2] (see also chapter 1) model. While the ENJL model only includes the lowest dimensional non-renormalizable four fermion interaction terms, the Quark-Resonance (QR)

*Supported by the INFN, by the EC under the HCM contract number CHRX-CT920026 and by the authors home institutions

Lagrangian results from the bosonization of the infinite tower of chiral invariant multiquark effective interactions ordered by an expansion in inverse powers of the ultraviolet cutoff Λ_χ ($\simeq 1$ GeV). The addition of higher dimensional multifermion interactions with increasing powers of derivatives takes into account in a perturbative way the nonlocality of the effective low energy action.

In section 2. the quark-resonance Lagrangian is constructed up to including $1/\Lambda_\chi^2$ terms, which are next-to-leading in the inverse cutoff expansion and leading in the $1/N_c$ expansion. All meson SU(3) flavour octet quantum numbers are included: the pseudoscalar mesons π, K, η, the vector, axial, scalar and pseudoscalar resonances. In section 3. two parameters of the leading vector resonance chiral effective Lagrangian are derived, including $(Q^2/\Lambda_\chi^2)\ln(\Lambda_\chi^2/Q^2)$ corrections to the leading logarithmic ENJL contribution: the coupling of the vector resonance to the external vector current and the vector mass. They enter the calculation of the vector two point function, which is studied in section 4 in the chiral limit; the numerical relevance of $(Q^2/\Lambda_\chi^2)\ln(\Lambda_\chi^2/Q^2)$ in the intermediate Q^2 region is shown through the comparison with experimental data.

2 The model

The effective action of the quark-resonance model, coming from the bosonization of the most general NJL model, in the *constituent* quark base and in the presence of external vector, axial, scalar and pseudoscalar sources is given by:

$$
\begin{aligned}
e^{-\Gamma_{eff}[R;v,a,s,p]} &= \frac{1}{Z}\int \mathcal{D}G_\mu \exp\left(-\int d^4x \frac{1}{4} G^{(a)}_{\mu\nu} G^{(a)\mu\nu}\right) e^{-f[R]} \int \mathcal{D}Q\mathcal{D}\bar{Q} \\
&\quad \exp\left[\int d^4x \left(\bar{Q}\gamma^\mu(\partial_\mu + iG_\mu)Q + \sum_0^\infty \left(\frac{1}{\Lambda_\chi}\right)^n \bar{Q}RQ\right)\right],
\end{aligned}
\tag{1}
$$

where the functional $f[R]$ contains the terms with auxiliary boson fields which are not coupled to fermions. The most general structure of the R operator can be represented by:

$$
R = \beta(\Lambda_\chi) \times \{\gamma_{Dirac}\} \times \{W_\mu^+, W_\mu^-, \tilde{H}\} \times \{\nabla_\mu^n, (\nabla_\mu^{C^T})^n\},
\tag{2}
$$

where the generic coupling $\beta(\Lambda_\chi)$ is not deducible from symmetry principles. ∇_μ and $\nabla_\mu^{C^T}$ are the covariant derivatives of the *constituent* quarks defined as

$$
\vec{\nabla}_\mu \equiv \vec{\partial}_\mu + \Gamma_\mu - \frac{i}{2}\gamma_5\xi_\mu
$$

$$\overleftarrow{\nabla}^{CT}_{\mu} \equiv \overleftarrow{\partial}_{\mu} - \Gamma_{\mu} - \frac{i}{2}\gamma_5 \xi_{\mu}, \tag{3}$$

with the vector current Γ_μ and the axial current ξ_μ defined in terms of the pseudoscalar field $\xi = \sqrt{U} = \exp(i/f\Phi)$ which is the square root of the usual exponential representation of the pseudoscalar meson octet Φ

$$\begin{aligned} \Gamma_\mu &= \frac{1}{2}\{\xi^\dagger[\partial_\mu - i(v_\mu + a_\mu)]\xi + \xi[\partial_\mu - i(v_\mu - a_\mu)]\xi^\dagger)\} \\ \xi_\mu &= i\{\xi^\dagger[\partial_\mu - i(v_\mu + a_\mu)]\xi - \xi[\partial_\mu - i(v_\mu - a_\mu)]\xi^\dagger)\}. \end{aligned} \tag{4}$$

The set $\{W^+_\mu, W^-_\mu, \tilde{H}\}$ contains all possible fields introduced by the bosonization which can couple to the *constituent* quark bilinears and which can be identified with the physical degrees of freedom of the low energy effective theory: the vector field W^+_μ, the axial-vector field W^-_μ and the scalar field $\tilde{H}$.

The equivalence of the most general chiral invariant *constituent* quark-resonance Lagrangian with the most general chiral invariant *current* quark-resonance Lagrangian holds with two *caveats*:

i) The presence of ξ_μ and Γ_μ currents defined by (4) in the *constituent* quark Lagrangian is entirely due to the transformation from *current* to *constituent* quarks

$$\begin{aligned} Q_L &= \xi q_L & Q_R &= \xi^\dagger q_R \\ \bar{Q}_L &= \bar{q}_L \xi^\dagger & \bar{Q}_R &= \bar{q}_R \xi, \end{aligned} \tag{5}$$

and the following identities hold:

$$\begin{aligned} \overrightarrow{\nabla}_\mu Q_L &= \xi \overrightarrow{d}_\mu q_L & \overrightarrow{\nabla}_\mu Q_R &= \xi^\dagger \overrightarrow{d}_\mu q_R \\ \bar{Q}_L \overleftarrow{\nabla}^{CT}_\mu &= \bar{q}_L \overleftarrow{d}_\mu \xi^\dagger & \bar{Q}_R \overleftarrow{\nabla}^{CT}_\mu &= \bar{q}_R \overleftarrow{d}_\mu \xi, \end{aligned} \tag{6}$$

where d_μ is the covariant derivative of the *current* quark field $d_\mu q_{L,(R)} = \partial_\mu q_{L,(R)} - il(r)_\mu q_{L,(R)}$, $\overrightarrow{\nabla}_\mu$ and $\overleftarrow{\nabla}^{CT}_\mu$ are the covariant derivatives defined in eq. (3).

As a consequence, the currents ξ_μ and Γ_μ can only appear in the combinations (3) through the covariant derivatives on *constituent* quarks.

ii) The vector field W_μ^+ and the axial-vector field W_μ^- can only appear in the combination $W_\mu^+ - \gamma_5 W_\mu^-$ and its charge conjugate $W_\mu^+ + \gamma_5 W_\mu^-$, i.e. in the combination of the leading ENJL Lagrangian.

The equivalence of the *current* quark-resonance Lagrangian to the original *current* multi-quark Lagrangian is guaranteed by a multi-step bosonization procedure [1] that generates interactions among the auxiliary boson fields which are the physical meson degrees of freedom and with their excited states.

The QR Lagrangian at leading order in the $1/\Lambda_\chi$ expansion and in the $1/N_c$ expansion, in the *constituent* quark base, coincides with the bosonization of the ENJL model discussed in the previous section in this Handbook.

We proceed now to the classification in the chiral limit of all the *constituent* quark-resonance bilinears which appear up to $\frac{1}{\Lambda_\chi^2}$ order, i.e. suppressed by at most two powers of Λ_χ with respect to the leading quark-resonance bilinears. They are all the quark-meson bilinears which are locally chiral invariant and with the *caveats* already discussed. They can be generally represented by:

$$\left(\frac{1}{\Lambda_\chi}\right)^n \times \mathcal{R}^k \times (\nabla, \nabla^C)^{n-k+1}, \tag{7}$$

with $n \leq 2$. k ranges from 0 to 3 and identifies four possible classes. $\mathcal{R}$ is a resonance from the set $\{W^+ \pm \gamma_5 W^-, \tilde{H}\}$.

At order $\frac{1}{\Lambda_\chi}$ there are no invariants.

In terms of the linear combinations of the axial and vector fields $\hat{W}_\mu^\pm = W_\mu^+ \pm \gamma_5 W_\mu^-$ all possible invariants at $1/\Lambda_\chi^2$ order are:

1. $\bar{Q}\gamma_\mu[\overrightarrow{\nabla}^\lambda, [\overrightarrow{\nabla}_\mu, \overrightarrow{\nabla}_\lambda]]Q$
2. $\bar{Q}\gamma_\mu\{\overrightarrow{\nabla}_\mu, \overrightarrow{\nabla}^2\}Q$

1. $\bar{Q}\gamma_\mu\Big\{[\overrightarrow{\nabla}_\nu, \hat{W}_\mu^-]\,\overrightarrow{\nabla}^\nu - \overleftarrow{\nabla}_\nu\,[\overrightarrow{\nabla}_\nu, \hat{W}_\mu^-]\Big\}Q$
2. $\bar{Q}\gamma_\mu\{\hat{W}_\mu^-\,\overrightarrow{\nabla}^2\}Q$
3. $\bar{Q}\gamma_\mu\{\{\overrightarrow{\nabla}_\mu, \overrightarrow{\nabla}_\nu\}\hat{W}_\nu^-\}Q$
4. $\bar{Q}\gamma_\mu(\overleftarrow{\nabla}_\mu\,\hat{W}_\nu^-\,\overrightarrow{\nabla}_\nu + \overleftarrow{\nabla}_\nu\,\hat{W}_\nu^-\,\overrightarrow{\nabla}_\mu)Q$
5. $\bar{Q}\gamma_\mu[[\overrightarrow{\nabla}_\mu, \overrightarrow{\nabla}_\nu], \hat{W}_\nu^-]Q$
6. $\bar{Q}\gamma_\mu\{\overrightarrow{\nabla}_\mu, \hat{W}^{-2}\}Q$

$$
\begin{array}{ll}
7. & \bar{Q}\gamma_\mu([\overrightarrow{\nabla}_\mu, \hat{W}^-_\nu]\hat{W}^-_\nu + \hat{W}^-_\nu[\hat{W}^-_\nu, \overrightarrow{\nabla}_\mu])Q \\
8. & \bar{Q}\gamma_\mu(\hat{W}^-_\mu \hat{W}^-_\nu \overrightarrow{\nabla}_\nu + \overrightarrow{\nabla}_\nu \hat{W}^-_\nu \hat{W}^-_\mu)Q \\
9. & \bar{Q}\gamma_\mu([\overrightarrow{\nabla}_\nu, \hat{W}^-_\mu \hat{W}^-_\nu] - [\overrightarrow{\nabla}_\nu, \hat{W}^-_\nu \hat{W}^-_\mu])Q \\
10. & \bar{Q}\gamma_\mu([\overrightarrow{\nabla}_\nu, \hat{W}^-_\mu]\hat{W}^-_\nu + \hat{W}^-_\nu[\hat{W}^-_\mu, \overrightarrow{\nabla}_\nu])Q \\
11. & \bar{Q}\gamma_\mu\{\hat{W}^{-2}, \hat{W}^-_\mu\}Q \\
\\
1. & \bar{Q}_L \tilde{H}^3 Q_R + h.c. \\
2. & \bar{Q}\gamma_\mu\{\overrightarrow{\nabla}_\mu, \tilde{H}^2\}Q \\
3. & \bar{Q}\gamma_\mu \tilde{H} \overrightarrow{\nabla}_\mu \tilde{H} Q \\
4. & \bar{Q}(\tilde{H} \overrightarrow{\nabla}^2 + \overleftarrow{\nabla}^{C^T 2} \tilde{H})Q \\
5. & \bar{Q} \overleftarrow{\nabla}^{C^T}_\mu \tilde{H} \overrightarrow{\nabla}_\mu Q \\
\\
1. & \bar{Q}(\tilde{H}\hat{W}^{-2} + \hat{W}^{+2}\tilde{H})Q \\
2. & \bar{Q}\hat{W}^+_\mu \tilde{H}\hat{W}^-_\mu Q \\
3. & \bar{Q}\gamma_\mu\{\hat{W}^-_\mu, \tilde{H}^2\}Q \\
4. & \bar{Q}\gamma_\mu \tilde{H}\hat{W}^+_\mu \tilde{H} Q \\
5. & \bar{Q}(\hat{W}^+_\mu \tilde{H} \overrightarrow{\nabla}_\mu - \overleftarrow{\nabla}^{C^T}_\mu \tilde{H}\hat{W}^-_\mu)Q \\
6. & \bar{Q}(\tilde{H}\hat{W}^-_\mu \overrightarrow{\nabla}_\mu - \overleftarrow{\nabla}^{C^T}_\mu \hat{W}^+_\mu \tilde{H})Q \\
7. & \bar{Q}[\hat{W}^+_\mu(\overleftarrow{\nabla}^{C^T}_\mu \tilde{H} + \tilde{H} \overrightarrow{\nabla}_\mu) - (\overleftarrow{\nabla}^{C^T}_\mu \tilde{H} + \tilde{H} \overrightarrow{\nabla}_\mu)\hat{W}^-_\mu]Q,
\end{array}
\tag{8}
$$

where we have used the hermiticity of the scalar field $\tilde{H} = \tilde{H}^\dagger$. We have grouped the terms into four classes according to the types of interactions among resonances.

The first class contains two independent terms with higher derivatives: the first term is totally antisymmetric and is proportional to the field strenghts of Γ_μ and ξ_μ currents defined in (4) through the identity

$$
[\overrightarrow{\nabla}_\mu, \overrightarrow{\nabla}_\lambda] = iG_{\mu\lambda} + \Gamma_{\mu\lambda} - \frac{i}{2}\gamma_5\xi_{\mu\lambda} - \frac{1}{4}[\xi_\mu, \xi_\lambda], \tag{9}
$$

where $G_{\mu\lambda} = \partial_\mu G_\lambda - \partial_\lambda G_\mu + i[G_\mu, G_\lambda]$, $\Gamma_{\mu\lambda} = \partial_\mu\Gamma_\lambda - \partial_\lambda\Gamma_\mu + [\Gamma_\mu, \Gamma_\lambda]$, $\xi_{\mu\lambda} = d_\mu\xi_\lambda - d_\lambda\xi_\mu$, with $d_\mu\xi_\lambda = \partial_\mu\xi_\lambda + [\Gamma_\mu, \xi_\lambda]$. The second term acts as a renormalization of the fermion propagator $\partial_\mu \to \partial_\mu(1+\partial^2/\Lambda^2_\chi)$. The second class is the Vector set and contains interactions among vector and axial vector fields $W_\mu^\pm$ with pseudoscalar mesons through the covariant derivatives ∇_μ, ∇_μ^C. The first three terms of this set enter the calculation of the two-point vector Green's function of section 4. The third class is the Scalar set which contains interactions among scalars and interactions among scalars and pseudoscalar mesons. The last set is the mixed Vector-Scalar sector. These terms contribute to the vector Lagrangian parameters proportionally to M_Q, where M_Q is the vev of the scalar field $\tilde{H}$. We have neglected corrections of order M_Q^2/Λ_χ^2 in the calculation of the two-point vector Greens' function.

$1/\Lambda_\chi^2$ terms generate two types of corrections to the parameters of the effective meson Lagrangian:

- next-to-leading power corrections to the leading logarithms (NPLL): $(Q^2/\Lambda_\chi^2)\ln(\Lambda_\chi^2/Q^2)$,
- next-to-leading power corrections: $Q^2/\Lambda_\chi^2 \cdot 1$.

It is crucial for the predictivity of the model the fact that NPLL corrections arise always from a finite class of $1/\Lambda_\chi^2$ terms, while genuine power corrections arise from an infinite tower of higher dimensional terms. In the next sections the two-point vector correlation fucntion is analyzed within this framework.

3 The vector meson Lagrangian

The leading non anomalous Lagrangian with one vector meson (i.e. of order p^3) is:

$$
\begin{aligned}
\mathcal{L}_V \;=\; & -\frac{1}{4} < V_{\mu\nu}V^{\mu\nu} > + \frac{1}{2}M_V^2 < V_\mu V^\mu > - \frac{f_V}{2\sqrt{2}} < V_{\mu\nu} f_+^{\mu\nu} > \\
& - i\frac{g_V}{2\sqrt{2}} < V_{\mu\nu}[\xi^\mu, \xi^\nu] > + H_V < V_\mu[\xi_\nu, f_-^{\mu\nu}] > + iI_V < V_\mu[\xi^\mu, \chi_-] >
\end{aligned}
\tag{10}
$$

and corresponds to the so called Conventional Vector model [3, 4].

The parameters which enter the calculation of the vector two-point function are the vector mass M_V and the coupling f_V to the external vector source. The full Lagrangian up to $1/\Lambda_\chi^2$ order which gives contribution to f_V and Z_V, the vector wave function renormalization constant, (or equivalently to M_V) is:

$$
\mathcal{L} \;=\; \bar{Q}(\hat{\partial} - M_Q)Q + \bar{Q}\gamma_\mu\Gamma_\mu Q - \frac{i}{2}\bar{Q}\gamma_\mu W_\mu^+ Q + \frac{\beta_\Gamma^1}{\Lambda_\chi^2}\bar{Q}\gamma_\mu d^\lambda \Gamma_{\mu\lambda} Q + \frac{\beta_\Gamma^2}{\Lambda_\chi^2}\bar{Q}\gamma_\mu\{\overrightarrow{d}_\mu, \overrightarrow{d}^2\}Q
$$

$$+\frac{i}{2}\frac{\beta_V^1}{\Lambda_\chi^2}\bar{Q}\gamma_\mu d^2 W_\mu^+ Q+\frac{i}{2}\frac{\beta_V^2}{\Lambda_\chi^2}\bar{Q}\gamma_\mu\{d^2,W_\mu^+\}Q+\frac{i}{2}\frac{\beta_V^3}{\Lambda_\chi^2}\bar{Q}\gamma_\mu\{W_\nu^+,\{d_\mu,d_\nu\}\}Q. \tag{11}$$

The first term defines the inverse free fermion propagator $D_0=\hat{\partial}-M_Q$. The remaining part defines the local perturbation to the free Lagrangian up to order $1/\Lambda_\chi^2$. There are five $1/\Lambda_\chi^2$ terms with new coefficients β_i. Each term can be traced back to the corresponding term in the list (8) where the covariant derivative d_μ is defined in terms of the covariant derivative ∇_μ as follows:

$$\nabla_\mu=\partial_\mu+\Gamma_\mu-\frac{i}{2}\gamma_5\xi_\mu\equiv d_\mu-\frac{i}{2}\gamma_5\xi_\mu. \tag{12}$$

The covariant derivative on the vector-like fields W_μ^+,Γ_μ is defined as:

$$d_\mu W_\nu^+=\partial_\mu W_\nu^+ +[\Gamma_\mu,W_\nu^+]. \tag{13}$$

The general formula resulting for f_V and M_V^2 and including the leading contribution from the ENJL model can be written as follows:

$$\begin{aligned}
f_V &= \sqrt{2Z_V}+\frac{N_c}{16\pi^2}\frac{\sqrt{2}}{3}\frac{1}{\sqrt{Z_V}}\frac{Q^2}{\Lambda_\chi^2}\Big[\sum_{i=1}^{2}\frac{\beta_\Gamma^i}{2}\int_0^1 d\alpha\ P_i^\Gamma(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\\
&\quad -\frac{1}{2}\sum_{i=1}^{3}\ \beta_V^i\int_0^1 d\alpha\ P_i^V(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\Big]\\
M_V^2 &= \frac{N_c}{16\pi^2}\Big(\frac{\Lambda_\chi^2}{2G_V}\Big)\frac{1}{Z_V},
\end{aligned} \tag{14}$$

where the wave function renormalization constant Z_V is given by:

$$\begin{aligned}
Z_V &= \frac{N_c}{16\pi^2}\frac{1}{3}\Big[6\int_0^1 d\alpha\ \alpha(1-\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}+\sum_{i=1}^{3}\ \beta_V^i\frac{Q^2}{\Lambda_\chi^2}\int_0^1 d\alpha\ P_i^V(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\\
&\quad +\frac{3}{2}\beta_\Gamma^2\frac{Q^2}{\Lambda_\chi^2}\int_0^1 d\alpha\ P_2^\Gamma(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\Big]\\
&\equiv Z_V^l+\frac{N_c}{16\pi^2}\frac{1}{3}\Big[\sum_{i=1}^{3}\ \beta_V^i\frac{Q^2}{\Lambda_\chi^2}\int_0^1 d\alpha\ P_i^V(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\\
&\quad +\frac{3}{2}\beta_\Gamma^2\frac{Q^2}{\Lambda_\chi^2}\int_0^1 d\alpha\ P_2^\Gamma(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\Big].
\end{aligned} \tag{15}$$

The $\beta_{V,\Gamma}^i$ coefficients must be determined from experimental data. The function $S(\alpha)$ is equal to $M_Q^2+\alpha(1-\alpha)Q^2$. The $P_i^{V,\Gamma}(\alpha)$ are polynomials in the Feynman parameter α. Their explicit expression is given by:

$$
\begin{aligned}
P_1^V(\alpha) &= P_1^\Gamma(\alpha) = 12\alpha(1-\alpha) \\
P_2^V(\alpha) &= \frac{3}{2}[8\alpha(1-\alpha) - 16\alpha^2(1-\alpha) - 36\alpha^2(1-\alpha)^2 + 24\alpha^3(1-\alpha)] \\
P_3^V(\alpha) &= 6[3\alpha^2(1-\alpha)^2 - 2\alpha^3(1-\alpha)] \\
P_2^\Gamma(\alpha) &= -\frac{2}{3}[36\alpha^3(1-\alpha)^2 - 18\alpha^4(1-\alpha)].
\end{aligned} \tag{16}
$$

From eq.(16) one obtains that the purely divergent contribution (i.e. $\ln(\Lambda_\chi^2/M_Q^2)\int_0^1 d\alpha\, P_i(\alpha)$) to $\beta_V^2, \beta_V^3, \beta_\Gamma^2$ terms is identically zero.
We are left with two independent coefficients: $\beta_\Gamma^1, \beta_V^1$.

4 Phenomenology of the Vector-Vector correlation function

To estimate the values of the $\beta_\Gamma^1, \beta_V^1$ coefficients we study the Q^2 behaviour of the vector-vector correlation function where one can compare the theoretical predictions with experimental results.

The two-point vector correlation function is defined as

$$
\Pi_{\mu\nu}^{V(ab)}(q^2) = i\int d^4x\ e^{iqx} < 0|T(V_\mu^a(x)V_\nu^b(0)|0>, \tag{17}
$$

where $V_\mu^a(x)$ is the flavoured vector quark current:

$$
V_\mu^a(x) = \bar{q}(x)\gamma_\mu \frac{\lambda^a}{\sqrt{2}} q(x), \tag{18}
$$

with λ^a the Gell-Mann matrices normalized as $tr(\lambda^a\lambda^b) = 2\delta^{ab}$. Lorentz covariance and $SU(3)$ invariance imply for the $\Pi_{\mu\nu}^V$ the following structure:

$$
\Pi_{\mu\nu}^{V(ab)}(q^2) = (q_\mu q_\nu - g_{\mu\nu}q^2)\ \Pi_V^1(Q^2)\delta^{ab} + q_\mu q_\nu\ \Pi_V^0(Q^2)\delta^{ab}, \tag{19}
$$

where $Q^2 = -q^2$, with q^2 euclidean. $\Pi_V^0(Q^2)$ is zero at all orders in the chiral limit. For $\Pi_V^1(Q^2)$ the following expression holds at NPLL order [1]:

$$
\Pi_V^1(Q^2) = \frac{2f_V^2(Q^2)M_V^2(Q^2)}{M_V^2(Q^2) + Q^2}, \tag{20}
$$

where the running of f_V^2 and M_V^2 at NPLL order can be extracted from eq. (14) with $\beta_\Gamma^2, \beta_V^2, \beta_V^3$ terms set to zero.

The real part of the invariant $\Pi^1_V(Q^2)$ function is related to its imaginary part through a standard dispersion relation

$$Re\Pi^1_V(Q^2) = \int_0^\infty ds \, \frac{\frac{1}{\pi} Im\Pi^1_V(s)}{s+Q^2}. \tag{21}$$

For a review on QCD spectral Sum rules and the calculation of QCD two-point Green's functions see [5]. The imaginary part is given in terms of the experimentally known total hadronic ratio of the e^+e^- annihilation in the isovector channel

$$Im\Pi^1_V(s) = \frac{1}{6\pi} R^{I=1}(s), \tag{22}$$

with

$$R^{I=1}(s) = \frac{\sigma^{I=1}(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}. \tag{23}$$

We have performed a comparison between the QR model parametrization (20), valid in the energy region $0 < Q^2 < \Lambda^2_\chi$, and the prediction obtained from a modelization of the experimental data on $e^+e^- \to hadrons$ [6] in the channel with the ρ meson quantum numbers ($I = 1, J = 1$). For a determination of the function $\Pi^1_V(Q^2)$ in the high Q^2 region (i.e. beyond the cutoff Λ_χ) see [7].

We adopted the following parametrization of the experimental hadronic isovector ratio:

$$R^{I=1}(s) = \frac{9}{4\alpha^2} \frac{\Gamma_{ee}\Gamma_\rho}{(\sqrt{s} - m_\rho)^2 + \frac{\Gamma^2_\rho}{4}} + \frac{3}{2}\Big(1 + \frac{\alpha_s(s)}{\pi}\Big)\theta(s - s_0). \tag{24}$$

This is a generalization of the one proposed in ref. [8], where corrections due to the finite width of the rho meson have not been included. $\Gamma_{ee} = 6.7 \pm 0.4$ KeV is the $\rho \to e^+e^-$ width and $\Gamma_\rho = 150.9 \pm 3.0$ is the total widht of the neutral ρ [9]. We used the leading logarithmic approximation for $\alpha_s(s)$:

$$\alpha_s(s) = \frac{12\pi}{33 - 2n_f} \, \frac{1}{\log(s/\Lambda^2_{QCD})}. \tag{25}$$

Expression (24) includes a dependence of the ρ channel upon the ρ width and the contribution from the continuum starting at a threshold $s_0 = 1.5 \; GeV^2$ [8]. For the running of α_s we used a value of 260 MeV for Λ_{QCD}, according to the average experimental value $\Lambda^{(4)}_{QCD} = 260^{+54}_{-46}$ MeV [9] and with $n_f = 4$ flavours.

The results are practically insensitive to the α_s running corrections and our leading log approximation turns out to be adequate.

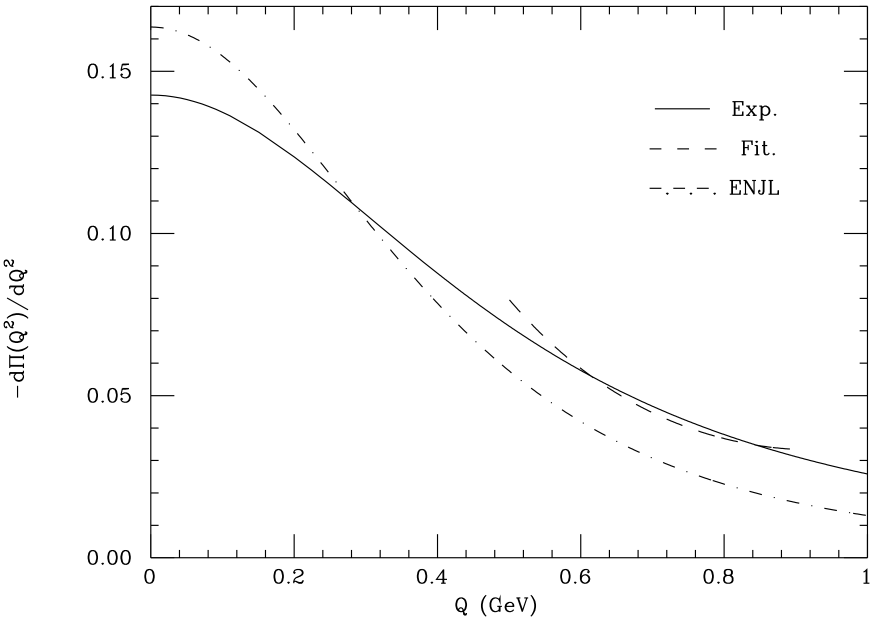

Figure 1: The derivative of the experimental vector invariant function $\Pi^1_V(Q^2)$ (solid line) the curve from the best fit in the region $0.5 < Q < 0.9$ GeV (dashed line) and the ENJL prediction (dot-dashed line).

To extract information on $\beta^1_\Gamma, \beta^1_V$ coefficients of the NTL logarithmic corrections we made a best fit of the first derivative of the 2-point function coming from the parametrization (24) of the experimental data:

$$\Pi'(Q^2)_{exp} = -\frac{2}{12\pi^2}\int_0^\infty ds \frac{R^{I=1}(s)}{(s+Q^2)^2}, \tag{26}$$

where the derivative of the VV function in the QR model is given by:

$$\Pi'(Q^2)_{QR} = \frac{\left[2f_V^{2\prime}\left(1+\frac{Q^2}{M_V^2}\right) - 2\frac{f_V^2}{M_V^2}\left(1 - Q^2\frac{M_V^{2\prime}}{M_V^2}\right)\right]}{\left(1+\frac{Q^2}{M_V^2}\right)^2}. \tag{27}$$

We have used $M_Q = 265$ MeV for the IR cutoff and $\Lambda_\chi = 1.165$ GeV for the UV cutoff, determined by a global fit in ref. [2].
In fig.(1) we show the Q^2 behaviour of the derivative of the experimental 2-point function, the curve from the best fit, which has been done in the region: $0.5 < Q < 0.9$ GeV, and the derivative of the ENJL prediction with quark-bubbles resummation.

The best values of the two free coefficients are

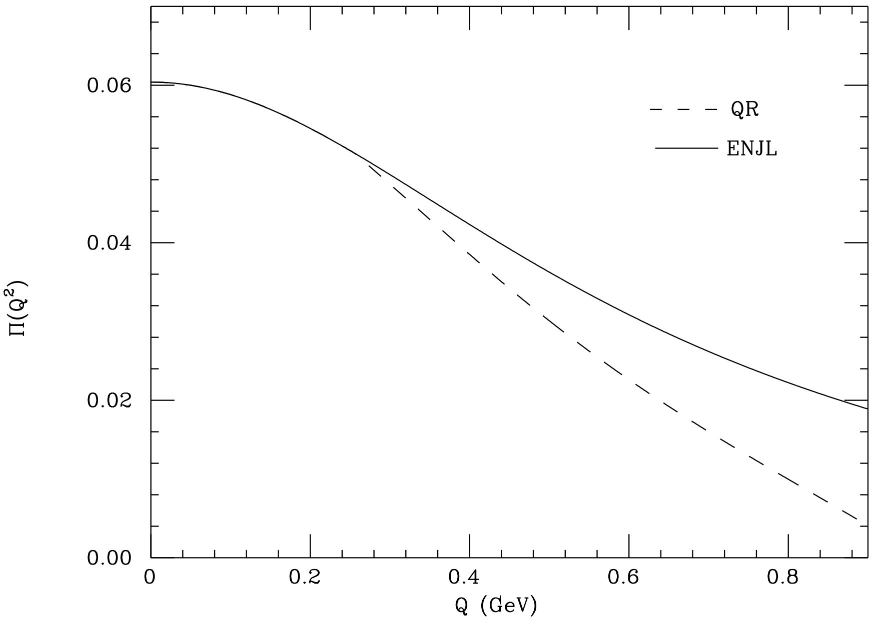

Figure 2: $\Pi_V^1(Q^2)$ from the QR model (dashed line), obtained from the fitted derivative of fig. (1) by imposing the matching with the ENJL function at $Q = M_Q$, versus $\Pi_V^1(Q^2)$ from the ENJL model (solid line).

$$\beta_\Gamma^1 = -0.75 \pm 0.01 \qquad\qquad \beta_V^1 = -0.79 \pm 0.01 \tag{28}$$

The χ^2 of the fit has been defined as $\sum_i(\Pi_i' - \Pi_i'^{exp})^2/\sigma_i^2$ and the σ_i are defined assuming a 10% of uncertainty on the experimental data. A $\chi^2/n.d.f. = 0.2$ has been obtained. The ENJL prediction differs by roughly a 40% from the experimental curve at 0.8 GeV. Most of this discrepancy can be accounted for with the corrections that we have calculated.

The invariant function $\Pi_V^1(Q^2)$ obtained from the best fit automatically match the ENJL function at $Q = M_Q$, because we have normalized the corrections to vanish at $Q^2 = M_Q^2$:

$$\Pi_V^1(Q^2) = \Pi_V^{ENJL}(Q^2)\ \theta(M_Q^2 - Q^2) + \int_{M_Q^2}^{Q^2} \frac{d\Pi_V^{Fit}}{dQ'^2} dQ'^2\ \theta(Q^2 - M_Q^2). \tag{29}$$

The $\Pi_V^1(Q^2)$ function obtained with the values (28) and with the matching of eq. (29) is plotted in fig.(2) and compared with the ENJL prediction (i.e. including the resummation of linear chains of quark bubbles and including only logarithmic corrections).

The difference between the two curves reaches a 30% at 0.7 GeV.

5 Conclusions

The extension of effective quark models *à la* Nambu-Jona Lasinio to hadronic interactions is a powerful tool to predict the hadron parameters in the low energy region, i.e. $0 < Q^2 < 1$ GeV. Within this framework the quark-resonance model fully takes into account the nonlocality of the effective interactions through a derivative expansion in inverse powers of the ultraviolet cutoff Λ_χ.

The bosonization of the lowest dimensional four-quark operators leads to the ENJL model, while the inclusion of higher dimensional multiquark operators generates next-to-leading corrections to the leading ENJL contributions to the parameters of the effective low energy meson Lagrangian.

Next-to-leading power corrections to the leading logarithms proportional to Q^2 have been explicitely calculated for the two-point vector correlation function. They can be written in terms of two new coefficients which are fixed by experimental data. Their numerical relevance in the energy region $500 \div 900$ MeV has been shown in this case.

The type of corrections we have analyzed are not the only ones: next-to-leading corrections in the $1/N_c$ expansion and gluonic corrections can be present to bridge the remaining gap with experimental data.

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