

Introduction to Extended Nambu-Jona-Lasinio Models

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Abstract

A short introduction to quark models with four fermion terms and their predictions for the parameters of low-energy effective Lagrangians is given. Special attention is paid to predictions that are as general as possible.

1 Introduction

The original Nambu-Jona-Lasinio model[1] was an extension of the BCS-theory of superconductivity to the domain of spontaneous symmetry breaking in the strong interaction. The original model was phrased in terms of nucleons but has been rephrased in terms of quarks in the seventies by the work of Kleinert and others. In the mid-1980's it was revived once more as a phenomenological model[2]. Recent extensive reviews can be found in [3].

The description given here will more or less present the model from the point of view of the work that I have been involved in [4, 5, 6, 7]. This method tries to get as much as possible out of the underlying structure of this class of models before putting in actual values of the parameters and choosing a specific regularization.

The main aim of this general class of models is an attempt to understand the low energy parameters of the Lagrangians described in chapter 2 from a model that is somewhat closer to QCD with a minimal amount of extra free parameters. The framework presented here can be easily extended to include more nonlocality. A more general treatment of the nonlocality can be found in the Quark Resonance model[8] or via attempts to approximately solve the Schwinger-Dyson equations in QCD (see ref. [9] for a review). It also in some

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sense includes a lot of the popular models like the chiral quark model(CQM) [10] and the QCD-effective action approach[11].

I will first give a short description of the model and a few arguments about its connection with QCD. Then the occurrence of spontaneous symmetry breakdown will be discussed. In the next part the low-energy expansion and a few of the relations between low-energy parameters that follow in general from it are given. Last I will discuss a little how (Vector) Meson Dominance(VMD) finds a basis in this way of looking at the low energy hadronic world.

2 The model and its connection with QCD

The QCD Lagrangian is given by

$$\begin{aligned}\mathcal{L}_{\text{QCD}} &= \mathcal{L}_{\text{QCD}}^0 - \frac{1}{4}G_{\mu\nu}G^{\mu\nu}, \\ \mathcal{L}_{\text{QCD}}^0 &= \bar{q}\{i\gamma^\mu(\partial_\mu - iv_\mu - ia_\mu\gamma_5 - iG_\mu) - (\mathcal{M} + s - ip\gamma_5)\}q.\end{aligned}\quad (1)$$

Here summation over colour degrees of freedom is understood and we have used the following short-hand notations: $\bar{q} \equiv (\bar{u}, \bar{d}, \bar{s})$; G_μ is the gluon field in the fundamental $\text{SU}(N_c)$ (N_c =number of colours) representation; $G_{\mu\nu}$ is the gluon field strength tensor in the adjoint $\text{SU}(N_c)$ representation; v_μ , a_μ , s and p are external vector, axial-vector, scalar and pseudoscalar field matrix sources; \mathcal{M} is the quark-mass matrix.

All indications are that in the purely gluonic sector there is a mass-gap. Therefore there seems to be a kind of cut-off mass in the gluon propagator (see the discussion in ref. [12]). Alternatively one can think of integrating out the high-frequency (higher than Λ_χ , a cut-off of the order of the spontaneous symmetry breaking scale) gluon and quark degrees of freedom and then expand the resulting effective action in terms of local fields. We then stop this expansion after the dimension six terms. This leads to the following effective action at leading order in the $1/N_c$ expansion

$$\begin{aligned}\mathcal{L}_{\text{QCD}} &\rightarrow \mathcal{L}_{\text{QCD}}^{\Lambda_\chi} + \mathcal{L}_{\text{NJL}}^{\text{S,P}} + \mathcal{L}_{\text{NJL}}^{\text{V,A}} + \mathcal{O}(1/\Lambda_\chi^4), \\ \text{with} \quad \mathcal{L}_{\text{NJL}}^{\text{S,P}} &= \frac{8\pi^2 G_S(\Lambda_\chi)}{N_c \Lambda_\chi^2} \sum_{i,j} (\bar{q}_R^i q_L^j) (\bar{q}_L^j q_R^i) \\ \text{and} \quad \mathcal{L}_{\text{NJL}}^{\text{V,P}} &= -\frac{8\pi^2 G_V(\Lambda_\chi)}{N_c \Lambda_\chi^2} \sum_{i,j} [(\bar{q}_L^i \gamma^\mu q_L^j) (\bar{q}_L^j \gamma_\mu q_L^i) + (L \rightarrow R)].\end{aligned}\quad (2)$$

Here i, j are flavour indices and $\Psi_{R,L} \equiv (1/2)(1 \pm \gamma_5)\Psi$. The couplings G_S and G_V are dimensionless and $\mathcal{O}(1)$ in the $1/N_c$ expansion and summation over colours between brackets is understood. The Lagrangian $\mathcal{L}_{\text{QCD}}^{\Lambda_\chi}$ includes only low-frequency modes of quark and gluon fields. The remaining gluon fields can be assumed to be fully absorbed in the coefficients of the local quark field operators or alternatively also described by vacuum expectation values of gluonic operators (see the discussions in refs. [4, 5]).

So at this level we have two different pictures of this model. One is where we have integrated out all the gluonic degrees of freedom and then expanded the resulting effective action in a set of **local** operators keeping only the first nontrivial terms in the expansion. In addition to this we can make additional assumptions. If we simply assume that these operators are produced by the short-range part of the gluon propagator we obtain $G_S = 4G_V = N_c \alpha_S / \pi$. The two extra terms in (2) have however different anomalous dimensions so at the strong interaction regime where these should be generated there is no reason to believe this relation to be valid. In fact the best fit is for $G_S \approx G_V$. We report however also the fit with the constraint $G_S = 4G_V$ included.

The other picture is the one where we only integrate out the short distance part of the gluons and quarks. We then again expand the resulting effective action in terms of low-energy gluons and quarks in terms of local operators. Here we make the additional assumption that gluons only exists as a perturbation on the quarks. The quarks feel only the interaction with background gluons. This is worked out by only keeping the vacuum expectation values of gluonic operators and not including propagating gluonic interchanges. Most fits are in fact best with the gluonic expectation value equal to zero (see table 1).

This model has the same symmetry structure as the QCD action at leading order in $1/N_c$ [13] (notice that the $U(1)_A$ problem is absent at this order [14]). (For explicit symmetry properties under $SU(3)_L \times SU(3)_R$ of the fields in this model see reference [4].) The QCD anomaly can also be consistently reproduced[6].

There has been a recent suggestion that the four-quark operators could result from QCD ultraviolet renormalon effects [22]. There it is suggested that G_V should be very small. The picture there is in fact quite different from the one usually pictured in this model. They argue that the vector sector follows more or less from the standard QCD picture but that the scalar and pseudoscalar sector is strongly perturbed by ultraviolet renormalons. These are local effects and can thus be described via a local four-quark operator like the term proportional to G_S . Therefore I have included a fit to the low-energy data with $G_V = 0$. Other arguments for this model can be given by looking at the local effects of purely gluonic operators. The most prominent gluonic operator beyond $\langle G^2 \rangle$ are in fact gluonic correlators of $D^\alpha G_{\alpha\beta}$ which is related to four-quark operators. This again leads to a NJL-like model at low-energies[23].

3 Spontaneous Symmetry Breaking

We can self-consistently solve the Schwinger-Dyson equation for the fermion propagator in terms of the bare propagator and a one-loop diagram (see figure 1). In the case where the current quark masses are set to zero this equation allows for two solutions for $G_S > 1$, one with constituent quark mass $M = 0$ and the other with $M \neq 0$ and the model shows spontaneous chiral symmetry breaking. In the presence of explicit chiral symmetry breaking only the second solution is allowed. In the leading $1/N_c$ limit the solution of the Schwinger-Dyson equation is a flavour diagonal matrix for the constituent quark masses



Figure 1: The Schwinger Dyson equation for the propagator. A thin (thick) line is the bare (full) fermion propagator.

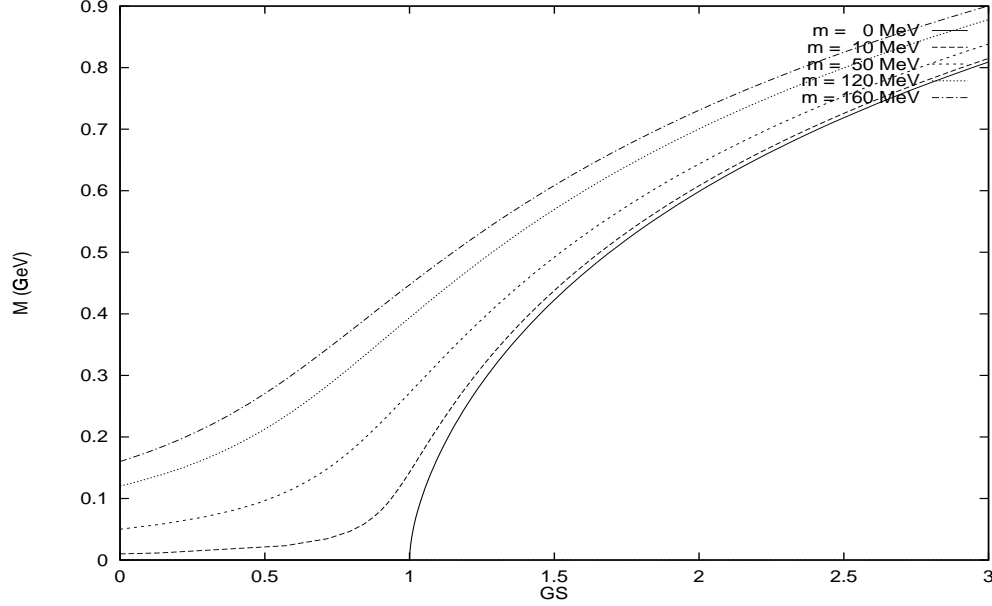


Figure 2: Plot of the dependence of the constituent quark mass M_i as a function of G_S for several values of m_i

with elements $M_{u,d,s}$. The gap-equation now becomes

$$M_i = m_i - g_S \langle 0 | : \bar{q}_i q_i : | 0 \rangle, \quad (3)$$

$$\langle 0 | : \bar{q}_i q_i : | 0 \rangle \equiv \langle \bar{q}_i q_i \rangle = -N_c 4 M_i \int \frac{d^4 p}{(2\pi)^4} \frac{i}{p^2 - M_i^2} = -\frac{N_c}{16\pi^2} 4 M_i^3 \Gamma(-1, \epsilon_i), \quad (4)$$

$$g_S \equiv \frac{4\pi^2 G_S}{N_c \Lambda_\chi^2}. \quad (5)$$

Therefore, in this model the scalar quark-antiquark one-point function (quark condensate) obtains a non-trivial nonzero value. The dependence on the current quark-mass is somewhat obscured in eq. (4). We use here a cut-off in proper time as the regulator. The quantity ϵ_i appearing in (4) is M_i^2/Λ_χ^2 . In figure 2 we have plotted the dependence of M_i on G_S for various values of m_i and $\Lambda_\chi = 1.160$ GeV. It can be seen that the value of M_i for small m_i converges smoothly towards the value in the chiral limit for the spontaneously broken phase. This is an indication that an expansion in the quark masses as Chiral Perturbation Theory assumes for QCD is also valid in this model. However, it can also be

seen that the validity of this expansion breaks down quickly and for $m_i \simeq 200$ MeV we already have $2M_i \simeq \Lambda_\chi$. We note that the ratio of vacuum expectation values for light quark flavours increases with increasing current quark mass at $p^2 = 0$ in this model and starts to saturate for $m_i > 200$ MeV.

In the mean-field approximation, we can introduce the vacuum expectation value into the Lagrangian, via an auxiliary field, and then keep only the terms quadratic in quark fields. The $\mathcal{L}_{\text{NJL}}^{\text{S,P,V,A}}$ above are then equivalent to a constituent chiral quark-mass term [11] of the form $-M_Q(\bar{q}_L U^\dagger q_R + \bar{q}_R U q_L)$.

4 The low-energy expansion

This section is essentially a very abbreviated version of [4]. The Lagrangian (2) can be made into a form bilinear in quark fields by introducing a set of auxiliary fields L_μ, R_μ and M . The first two transform respectively as a left (right) handed vector field (3 by 3 hermitian matrices) and the last one as $M \rightarrow g_R M g_L^\dagger$, M is a 3 by 3 complex matrix. Once the Lagrangian has been brought into the bilinear form we can then assume a vacuum expectation value for M and make an expansion in its inverse. This expansion can be done using the heat kernel expansion (see [15] for a review). The vacuum expectation value is determined selfconsistently from this expansion. This leads to the same gap equation (in the chiral limit) as the one discussed in the previous section. Current quark mass dependence is of course treated perturbatively with this method.

This procedure in fact generates kinetic terms for all the auxiliary fields. So the model reproduces the low lying hadronic spectrum of pseudoscalars, vectors, scalars and axial-vectors. The combination $L_\mu + R_\mu$ becomes after a wave function renormalization essentially the vector field. The different vector representations can be reached by making a redefinition of the vector field here. Since the underlying model does not depend on the form of the vector field chosen it is obvious that the resulting physics also does not depend on it. (See [16] for a discussion in the general case). An added feature that appears here is that the axial-vector auxiliary field, $R_\mu - L_\mu$ mixes with the derivative of the pseudoscalar field U . The U field is obtained as the polar decomposition of M , $M = \xi H \xi$. H is hermitian and contains the vacuum expectation value of M and the scalar excitation part. This mixing introduces a coupling of the pion field to the quarks which is different from 1, the axial coupling of the (constituent) quark, g_A . This already occurs at leading order in $1/N_c$. The latter point is discussed exhaustively in [17]. This coupling is smaller than 1 for $G_V \neq 0$. As an example of a successful prediction we have the value for L_9 ,

$$L_9 = \frac{N_c}{16\pi^2} \frac{1}{6} \left[(1 - g_A^2) \Gamma(0, x) + 2g_A^2 \Gamma(1, x) \right] . \quad (6)$$

$x = M_Q^2 / \Lambda_\chi^2$ and $\Gamma(0, x)$ the incomplete gamma-function. In fact the main improvement is obtaining a better value for L_5 and L_8 where the inclusion of the vectors and scalar degrees leads to an improvement over the QCD effective action model. The expressions are in fact

rather simple.

$$\begin{aligned} L_5 &= \frac{N_c}{16\pi^2} \frac{1}{4} g_A^3 [\Gamma(0, x) - \Gamma(1, x)] , \\ L_8 &= \frac{N_c}{16\pi^2} \frac{1}{16} g_A^2 \left[\Gamma(0, x) - \frac{2}{3} \Gamma(1, x) \right] . \end{aligned} \quad (7)$$

It is the presence of the extra g_A factors and the cancellation between both terms that allow for phenomenologically good values for these latter two constants.

The main feature of this analysis is that even leaving the coefficients in the heat kernel expansion completely free, leaves a number of interesting relations. In this respect it should be mentioned that as a consequence these relations survive in extensions of the model that do not change the structure of the heat kernel terms. In particular, background gluonic contributions do obey this criterion. These relations correspond to a set of well-known phenomenological relations that were previously derived using Meson dominance arguments and QCD short distance relations. Some of them are

$$\begin{aligned} f_\pi^2 &= f_V^2 M_V^2 - f_A^2 M_A^2 && \text{First Weinberg sum rule} \\ L_9 &= \frac{1}{2} f_V g_V && \text{VMD of } \pi \text{ form factor} \\ L_{10}(2H_1) &= -\frac{1}{4} f_V^2 + (-)\frac{1}{4} f_A^2 && \text{VMD of VV and AA 2-point functions} \\ g_A &= 1 - \frac{f_\pi^2}{f_V^2 M_V^2} . \end{aligned} \quad (8)$$

The full list of relations can be found in [4]. This approach also works very well numerically, witness table 1. The input parameters there are M_Q , $x = M_Q^2/\Lambda_\chi^2$, g_A and the combination $g = \pi\alpha_S\langle G^2\rangle/(6N_c M_Q^4)$. Fit 2 is the fit with only low-energy parameters as input and everything free. The meaning of the other fits can be found in [4] while the last columns are with the low-energy parameters and the constraints $G_V = G_S/4$ (fit 6) and the renormalon picture with $G_V = 0$. We have also enforced the gluon condensate to vanish in these two fits. Notice that in both cases the cut-off has decreased significantly compared to the full case where G_V was left free.

In general this model seems to interpolate well between VMD type of predictions and chiral quark model type of predictions. This can be seen in eq. (6), the first term is the one coming from the vector exchange while the second one is the “chiral quark loop” contribution. By changing g_A one can have either a full VMD or a full CQM picture. A major improvement compared to previous attempts in quark models is the correct value for L_5 and L_8 that are obtained here.

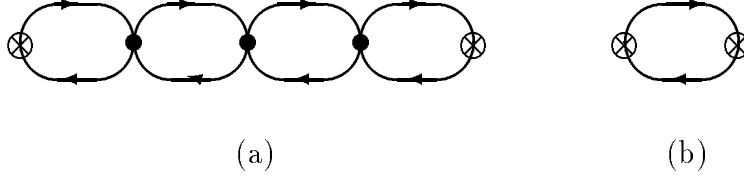


Figure 3: The graphs contributing to the two point-functions in the large N_c limit. a) The class of all strings of constituent quark loops. The four-fermion vertices are either $\mathcal{L}_{\text{NJL}}^{\text{S,P}}$ or $\mathcal{L}_{\text{NJL}}^{\text{V,A}}$ in eq. (2). The crosses at both ends are the insertion of the external sources. b) The one-loop case.

5 Beyond the low-energy expansion and Meson Dominance

This section is a very short summary of references [5] and [7]. Similar work can be found in [18, 19, 20]. For 2-point functions the general graph is depicted in fig 3. The sum of all diagrams is essentially a geometric series that can be easily summed. The full two-point function then becomes

$$\Pi = \frac{\overline{\Pi}}{1 - K\overline{\Pi}}. \quad (9)$$

Here $\overline{\Pi}$ denotes the 2-point function at one-loop and K the relevant four-fermi coupling. When the 2-point functions mix a similar formula exists except that Π and $\overline{\Pi}$ become vectors and K a matrix. The rest can be dealt with as a matrix inverse.

Here we see how the resummation has produced a pole and thus a bound state. In the previous section this was dealt with by getting the kinetic term and then using the equations of motion for the auxiliary fields. The 2-point functions can usually be rewritten in a meson-dominance form but with slowly varying parameters rather than constants. E.g.(in the equal mass case) for the transverse vector case:

$$\begin{aligned} \Pi_V^{(1)}(Q^2) &= \frac{2f_V^2(Q^2)M_V^2(Q^2)}{M_V^2(Q^2) - Q^2} \\ 2f_V^2(Q^2)M_V^2(Q^2) &= N_c\Lambda_\chi^2/(8\pi^2 G_V) \\ 2f_V^2(Q^2) &= \frac{8N_c}{16\pi^2} \int_0^1 dx \, x(1-x)\Gamma(0, x_Q) \end{aligned} \quad (10)$$

with $x_Q = (M_Q^2 + x(1-x)Q^2)/\Lambda_\chi^2$, and $Q^2 = -q^2$.

In fact some of the relations referred to in the previous section remain true even after resummation to all orders. This is because the symmetries impose certain identities on

the one-loop functions and these have still some consequences after resummation. Examples are the first and second Weinberg sum rules, the famous $M_S = 2M_Q$ relation and generalizations of these away from the chiral limit [5, 7].

A general argument for meson dominance appears here. An n -point function consists of a set of graphs consisting of one-loop vertices joined by 2-point functions. These two-point functions can now be rewritten in a form that looks very much like meson dominance. Then as far as the "vertices" are slowly varying we will find the VMD predictions within this model. In general it is not so simple but still numerically results look very much like VMD, see section 5 in [7].

6 Conclusions

I have presented a short representation of the extended Nambu-Jona-Lasinio model somewhat biased towards my own view of this set of models. Further references can be found in the reviews cited. There is also work on more vector meson phenomenology within the same approach [21], both for anomalous and non-anomalous decays. The general conclusion is that within its limitations the ENJL-type models do include a reasonable amount of the expected physics from QCD, its symmetries, their spontaneous breakdown and even some of its short distance information (as embodied in the Weinberg sum rules). It is also quantitatively successful. Its major drawback is the lack of a confinement mechanism. A general understanding of the interplay between meson dominance and the chiral quark model is understood within this framework and a rather good description of all low-energy parameters is obtained.

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Table 1: Experimental values and predictions of the ENJL model for various low-energy parameters from the heat kernel expansion. All dimensionful quantities are in MeV. The difference between the predictions is for slightly different choices of parameters. (*) means that there are in addition uncertainties due to higher order chiral corrections. The meaning of the different fits are explained in [4], fit 2 only uses f_π and the L_i while fit 6 and 7 include constraints on G_V , see text. The numerical error in [4] for H_2 has been corrected. All masses are determined from the low-energy expansion, not the pole position of the 2-point functions.

	exp. value	exp. error	fit error	fit 1	fit 2	fit 3	fit 4	fit 5	fit 6	fit 7
f_π	86([†])	—	10	89	86	86	87	83	86	86
$\sqrt[3]{- < \bar{q}q >}$	194(#)	8(#)	—	281	260	255	178	254	210	170
$10^3 \cdot L_2$	1.2	0.4	0.5	1.7	1.6	1.6	1.6	1.7	1.5	1.6
$10^3 \cdot L_3$	-3.6	1.3	1.3	-4.2	-4.1	-4.4	-5.3	-4.7	-3.1	-3.0
$10^3 \cdot L_5$	1.4	0.5	0.5	1.6	1.5	1.1	1.7	1.6	2.1	1.9
$10^3 \cdot L_8$	0.9	0.3	0.5	0.8	0.8	0.7	1.1	1.0	0.9	0.8
$10^3 \cdot L_9$	6.9	0.7	0.7	7.1	6.7	6.6	5.8	7.1	5.7	5.2
$10^3 \cdot L_{10}$	-5.5	0.7	0.7	-5.9	-5.5	-5.8	-5.1	-6.6	-3.9	-2.6
$10^3 \cdot H_1$	—	—	—	-4.7	-4.4	-4.0	-2.4	-4.6	-3.7	-2.6
$10^3 \cdot H_2$	—	—	—	1.4	1.2	1.2	1.0	2.3	-0.2	0.8
M_V	768.3	0.5	100	811	830	831	—	802	1260	—
M_A	1260	30	300	1331	1376	1609	—	1610	2010	—
f_V	0.20	(*)	0.02	0.18	0.17	0.17	—	0.18	0.15	—
g_V	0.090	(*)	0.009	0.081	0.079	0.079	—	0.080	0.076	—
f_A	0.097	0.022(*)	0.022	0.083	0.080	0.068	—	0.072	0.084	—
M_S	983.3	2.6	200	617	620	709	989	657	643	760
c_m	—	—	—	20	18	20	24	25	16	6
c_d	34	(*)	10	21	21	18	23	19	26	27
x				0.052	0.063	0.057	0.089	0.035	0.1	0.2
g_A				0.61	0.62	0.62	1.0	0.66	0.79	1.0
M_Q				265	263	246	199	204	262	282
g				0.0	0.0	0.25	0.58	0.5	0.0	0.0