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Non-Critical-String-Inspired Modifications of Quantum Mechanics

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Abstract

Recently, together with J. Ellis and D.V. Nanopoulos, we have proposed a new approach to the concept of time in non-critical (Liouville) string theory associated with singular space-time backgrounds of quantum gravity. Among the most important physical aspects of this approach is the emergence of a microscopic arrow of time, CPT violation, conservation of energy on the average, and the possible apparent non-conservation of angular momentum in the effective (observable) low-energy sector of the theory. In this work we review the application of a phenomenological parametrization of this formalism to the neutral kaon system.

1 Introduction and Summary

One of the most profound issues in microphysics is a consistent formulation of gravity. The only candidate we have for resolving this issue is string theory, which is known [1] to be free of the "trivial" perturbative divergences that beset quantum calculations in a fixed smooth space-time background. Potentially far more profound problems arise when one considers curved backgrounds with horizons or singularities. Semiclassical calculations in such backgrounds indicate that a pure quantum-mechanical description cannot be maintained, as reflected in the thermodynamic description of macroscopic black holes, with non-zero temperature and entropy [2].

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The question arises whether such a mixed quantum description is also necessary at the microscopic level, once quantum fluctuations in the space-time background and back-reaction of particles on the metric are taken into account. Hawking [3] has argued that asymptotic scattering must be described in terms of a linear operator $\not{S}$ relating "in" and "out" density matrices :

$$\rho_{out} = \not{S}\rho_{in} \tag{1}$$

that does not factorize as a product of S and $S^{\dagger}$ matrix elements. It has been argued [4] that, if this is the case, there should be a corresponding modification of the quantum Liouville equation describing the time-evolution of the density matrix:

$$\dot{\rho} = i[\rho, H] + \not{\delta} H \rho \tag{2}$$

This would cause pure states to evolve into mixed states, yielding a loss of coherence associated with the loss of information accross microscopic event horizons, that would be enhanced for macroscopic systems [5].

Ordinary local quantum field theory is an incomplete guide to these issues, but string theory appears to resolve them. The scattering of particles off a black hole in string theory is described by a well-defined S matrix, which reflects the existence of an infinite set of local symmetries (and associated conserved charges) that interelate (and characterize) different string states in the presence of a black hole, as long as quantum fluctuations in the space-time background and back-reaction of particles on the metric are ignored. However, we have argued [6, 7] that non-trivial modifications (1) of the effective field theory description of scattering and (2) of the quantum Liouville equation appear once these effects are taken into account. We describe such effects using non-critical string theory [8], with time introduced as a renormalization group flow variable [6], associated with a covariant world-sheet scale introduced via the Liouville field [9, 10, 11]. We only mention here the basics of this approach, which is described in some detail elsewhere [7, 12, 13]. We give emphasis on the physical aspects of this approach that are particularly relevant to experiment.

One is a possible microscopic arrow of time [6, 7]. We have argued in ref. [6, 12, 13] that string theory in singular space-time backgrounds [14], with the target time identified with the Liouville mode, satisfies the general condition for the existence of an irreversible "ageing" variable, as a corollary of the existence of the Zamolodchikov metric [15] in the space of couplings of two-dimensional field theories on the world sheet. The arrow of time emerges as a result of the stringy symmetry-induced couplings of light particles to massive, non-propagating solitonic states of the string in a black-hole background [16]. In this respect, we see some similarity with ideas advocated by Penrose in the context of local field theories in highly-curved space-times [17]. Moreover the coupling of propagating to topological degrees of freedom makes the former an 'open' system. The formulation of time arrow in open systems, in the modern approach of refs. [18, 19], can be used as a nice dictionary to describe temporal evolution of the string density matrix. The problem is then reduced to a dissipative statistical mechanics problem [19] in a phase space with coordinates and momenta which are the background fields of the σ-model and their conjugate momenta. Details are provided in [12, 20]. Below we state for completeness the basic equation derived in this approach for the temporal evolution of the quantum density matrix ρ of the low-energy modes :

$$\partial_t \rho = -i[H,\rho] - i\beta^i G_{ij}[g^j,\rho] \tag{3}$$

where g^i are (low-energy) space-time fields, viewed as coordinates in a generalized field space, $\beta^i \equiv \dot{g}^i$ denote their temporal evolution, and the quantity G_{ij} is an appropriate functional of g^i, computable in principle in string theory, that serves as a 'metric' in the field space. For string models that are unitary on the world sheet G_{ij} is positive definite. Entropy S is defined in terms of the density matrix ρ as usual, by

$$S = -Tr\rho ln\rho \tag{4}$$

and it increases with time t if $G_{ij} > 0$,

$$\partial_t S = \beta^i G_{ij} \beta^j S \geq 0 \tag{5}$$

This implies non-equilibrium time evolution for the non-critical matter theory. The latter approaches asymptotically its critical state.

A second issue, of phenomenological importance, is the violation of CPT [21] and other conservation laws in the effective low-energy theory. CPT is expected to be violated in any theory, such as ours, which allows pure states to evolve into mixed states [25]. As we have discussed elsewhere [6, 21] energy and probability are conserved in our string approach to density matrix mechanics as a result of the *renormalizability* of the theory[21, 12]. However, the renormalizability does not guarantee the conservation of the angular momentum. Unlike string contributions to the increase in entropy, which cannot cancel, the apparent non-conservation of angular momentum may vanish in some backgrounds, though not in one cosmological background that we studied [20].

The above considerations motivate searches for violations of quantum mechanics in physical systems, such as neutral kaons, that could be sensitive to quantum gravity effects [22]. We note that if the non-quantum mechanical terms in the evolution equation (3) are suppressed by one power of Planck mass, $O(10^{-19})$ GeV, then there is a chance of their being observed in CPLEAR [23] and or Daϕne experiments [24].

2 Generic CPT Violation

One of the most profound tests of quantum mechanics is the test of CPT conservation. The evolution of pure states into mixed ones, as a result of the entropy increase (5) along the positive direction of time, implies the violation of CPT symmetry as we now review in the context of a general analysis [25]. Let us make the assumption that a CPT operator $\hat{\Theta}$ exists in target space, such that

$$
\begin{aligned}
\rho'_{in} &= \hat{\Theta}^{-1}\rho_{out} \\
\rho'_{out} &= \hat{\Theta}\rho_{in}
\end{aligned}
\tag{6}
$$

and the *in* and *out* density matrices are related through the superscattering operator $\not{\$}$ [3]

$$
\begin{aligned}
\rho_{out} &= \not{\$}\rho_{in} \\
\rho'_{out} &= \not{\$}\rho'_{in}
\end{aligned}
\tag{7}
$$

The following relation is a trivial consequence of the above equations

$$
\hat{\Theta} = \not{\$}\hat{\Theta}^{-1}\not{\$} \tag{8}
$$

which implies that $\not{\$}$ has an inverse. Clearly this cannot happen if there is evolution of pure states into mixed ones and not vice versa, as implied by the monotonic increase of the entropy (5). This proves the breaking of CPT symmetry in the above framework.

In ref. [7] we have described non-factorisable (i.e. $\not{\$} \neq SS^{\dagger}$, where S is the conventional S-matrix operator) contributions to the string $\not{\$}$-matrix, coming from valleys between topological defects on the world sheet [2]. This provides an explicit demonstration of the non-existence of an inverse $\not{\$}^{-1}$, and hence of induced CPT violation in the target space of this effective string theory. It should be stressed, however, that the above considerations cannot exclude the possibility of a some weaker form of CPT invariance [25] which might cause violations of CPT symmetry to be unobservable in an experimental apparatus. Such a situation falls beyond the scope of the present analysis, and in what follows we simply explore the possibility that a detectable violation of CPT occurs, which we parametrize in a way suitable for present experiments with neutral kaons and at future ϕ factories [23, 24], that constitute the most sensitive probes in a search for violations of quantum mechanics at the microscopic level. For more details we refer the reader to the literature [21, 28, 29].

[2]We note for the interested reader that, in this picture, the creation and annihilation of a target-space black hole is represented as a world-sheet monopole-anti-monopole pair [26]. Instantons induce transitions among such configurations of different charge, the latter being proportional to the black hole mass. We note that there is a formal analogy [7, 12] with the Quantum Hall fluids: in that case, instantons in the respective Wess-Zumino models, describing the effective theories in 'conductivity space', induce transitions among the transverse-conductivity plateaux [27]. In the presence of instantons, certain world-sheet charges cease to be conserved, as a result of logarithmic renormalization scale dependences. Such a situation implies the non-commutativity of the Hamiltonian operator with the respective charge operators on the world sheet. If one defines a generalized CPT symmetry in such a way that this symmetry leaves the mass of a string state invariant, but changes the sign of the charge, then it is straightforward to argue that in our case the elevation of CPT symmetry to target space fails in general. This is a heuristic argument, and one should really construct a rigorous proof of such a stituation, which is not yet available.

3 Application to the Neutral Kaon System

We are now well equipped to discuss the phenomenology of violations of quantum mechanics in the above framework. The formalism of [4] will be adopted. Below, we describe briefly the formalism [4, 21] for a discussion of the possible modification of quantum mechanics and violation of CPT in the neutral kaon system, which is among the most sensitive microscopic laboratories for studying these possibilities. In the normal quantum-mechanical formalism, the time-evolution of a neutral kaon density matrix is given by

$$\partial_t \rho = -i(H\rho - \rho H^\dagger) \tag{9}$$

where the Hamiltonian takes the following form in the $(K^0, \overline{K}^0)$ basis:

$$H = \begin{pmatrix} (M + \frac{1}{2}\delta M) - \frac{1}{2}i(\Gamma + \frac{1}{2}\delta\Gamma) & M_{12}^* - \frac{1}{2}i\Gamma_{12}^* \\ M_{12} - \frac{1}{2}i\Gamma_{12} & (M - \frac{1}{2}\delta M) - \frac{1}{2}i(\Gamma - \frac{1}{2}\delta\Gamma) \end{pmatrix} \tag{10}$$

The δM and $\delta\Gamma$ violate CPT, but this is an 'apparent' CPT violation, within ordinary quantum mechanics. It will not interest us and from now on we ignore it. The non-hermiticity of H reflects the process of K decay: an initially-pure state evolving according to (9) and (10) remains pure, even when $\delta M, \delta\Gamma \neq 0$.

In order to discuss the possible modification of this normal quantum-mechanical evolution, and allow for the possibility of genuine CPT violation, it is convenient to rewrite [21] (9) and (10) in a Pauli σ-matrix basis [4], introducing components ρ_α of the density matrix:

$$\rho = 1/2 \rho_\alpha \sigma_\alpha \tag{11}$$

which evolves according to

$$\partial_t \rho_\alpha = h_{\alpha\beta}\rho_\beta \tag{12}$$

with

$$h_{\alpha\beta} \equiv \begin{pmatrix} Imh_0 & Imh_1 & Imh_2 & Imh_3 \\ Imh_1 & Imh_0 & -Reh_3 & Reh_2 \\ Imh_2 & Reh_3 & Imh_0 & -Reh_1 \\ Imh_3 & -Reh_2 & Reh_1 & Imh_0 \end{pmatrix} \tag{13}$$

It is easy to check that at large times ρ takes the form

$$\rho \simeq e^{-\Gamma_L t} \begin{pmatrix} 1 & \epsilon^* \\ \epsilon & |\epsilon|^2 \end{pmatrix} \tag{14}$$

where ϵ is given by

$$\epsilon = \frac{\frac{1}{2}iIm\Gamma_{12} - ImM_{12}}{\frac{1}{2}\Delta\Gamma - i\Delta M} \tag{15}$$

in the usual way.

A modification of quantum mechanics of the form discussed in section 3 can be introduced by modifying equation (12) to become

$$\partial_t \rho_\alpha = h_{\alpha\beta}\rho_\beta + \not{h}_{\alpha\beta}\rho_\beta \tag{16}$$

The form of $\not{h}_{\alpha\beta}$ is determined if we assume probability and energy conservation, as proved in the string context in section 3, and that the leading modification conserves strangeness:

$$\not{h}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2\alpha & -2\beta \\ 0 & 0 & -2\beta & -2\gamma \end{pmatrix} \tag{17}$$

It is easy to solve the 4×4 linear matrix equation (16) in the limits of large time:

$$\rho_L \propto \begin{pmatrix} 1 & \frac{-\frac{1}{2}i(Im\Gamma_{12}+2\beta)-ImM_{12}}{\frac{1}{2}\Delta\Gamma+i\Delta M} \\ \frac{\frac{1}{2}i(Im\Gamma_{12}+2\beta)-ImM_{12}}{\frac{1}{2}\Delta\Gamma-i\Delta M} & |\epsilon|^2 + \frac{\gamma}{\Delta\Gamma} - \frac{4\beta ImM_{12}(\Delta M/\Delta\Gamma)+\beta^2}{\frac{1}{4}\Delta\Gamma^2+\Delta M^2} \end{pmatrix} \tag{18}$$

and of short time:

$$\rho_S \propto \begin{pmatrix} |\epsilon|^2 + \frac{\gamma}{|\Delta\Gamma|} - \frac{-4\beta ImM_{12}(\Delta M/\Delta\Gamma)+\beta^2}{\frac{1}{4}\Delta\Gamma^2+\Delta M^2} & \epsilon - \frac{i\beta}{\frac{\Delta\Gamma}{2}-i\Delta M} \\ \epsilon^* + \frac{i\beta}{\frac{\Delta\Gamma}{2}+i\Delta M} & 1 \end{pmatrix} \tag{19}$$

We note that the density matrix (18) for K_L is mixed to the extent that the parameters β and γ are non-zero. It is also easy to check [21] that the parameters α, β and γ all violate CPT, in accord with the general argument of [25], and consistent with the string analysis mentioned earlier in this section.

Experimental observables O can be introduced [4, 21] into this framework as matrices, with their measured values being given by

$$< O >= Tr(O\rho) \tag{20}$$

Examples are the K to 2π and 3π decay observables

$$O_{2\pi} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad ; \quad O_{3\pi} = (0.22)\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{21}$$

and the semileptonic decay observables

$$O_{\pi^- l^+ \nu} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$O_{\pi^+ l^- \overline{\nu}} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \tag{22}$$

A quantity of interest is the difference between the K_L to 2π and K_S to 3π decay rates [21]:

$$\delta R \equiv R_{2\pi} - R_{3\pi} = \frac{8\beta}{|\Delta\Gamma|}|\epsilon| sin\phi_\epsilon \tag{23}$$

where $R^L_{2\pi} \equiv Tr(O_{2\pi}\rho_L)$, and $R^S_{3\pi} \equiv Tr(O_{3\pi}\rho_S)/0.22$, and the prefactors are determined by the measured [30] branching ratio for $K_L \to 3\pi^0$. (Strictly speaking, there should be a corresponding prefactor of 0.998 in the formula (21) for the $O_{2\pi}$ observable.)

Using (22), one can calculate the semileptonic decay asymmetry [21]

$$\delta \equiv \frac{\Gamma(\pi^- l^+ \nu) - \Gamma(\pi^+ l^- \overline{\nu})}{\Gamma(\pi^- l^+ \nu) + \Gamma(\pi^+ l^- \overline{\nu})} \tag{24}$$

in the long- and short-lifetime limits:

$$\begin{aligned} \delta_L &= 2Re[\epsilon(1 - \frac{i\beta}{ImM_{12}})] \\ \delta_S &= 2Re[\epsilon(1 + \frac{i\beta}{ImM_{12}})] \end{aligned} \tag{25}$$

The difference between these two values

$$\delta\delta \equiv \delta_L - \delta_S = -\frac{8\beta}{|\Delta\Gamma|}\frac{sin\phi_\epsilon}{\sqrt{1+\tan^2\phi_\epsilon}} = -\frac{8\beta}{|\Delta\Gamma|} sin\phi_\epsilon cos\phi_\epsilon \tag{26}$$

with $tan\phi_\epsilon = (2\Delta M)/\Delta\Gamma$, is a signature of CPT violation that can be explored at the CPLEAR and DAϕNE facilities [23, 24].

We have used [21] the latest experimental values [30] of $R_{2\pi}$ and $R_{3\pi}$ to bound δR, and the latest experimental values of $\delta_{L,S}$ to bound $\delta\delta$, expressing the results as contours in the (β, γ) plane [21]. In our formalism, the usual CP-violating parameter ϵ is given by [21]

$$|\epsilon| = -\frac{2\beta}{|\Delta\Gamma|} sin\phi_\epsilon + \sqrt{\frac{4\beta^2}{|\Delta\Gamma|^2} - \frac{\gamma}{|\Delta\Gamma|} + R^L_{2\pi}} \tag{27}$$

On the basis of this preliminary analysis, it is safe to conclude that

$$|\frac{\beta}{\Delta\Gamma}| \lesssim 10^{-4} \ to \ 10^{-3} \qquad ; \qquad |\frac{\gamma}{\Delta\Gamma}| \lesssim 10^{-6} \ to \ 10^{-5} \tag{28}$$

In addition to more precise experimental data, what is also needed is a more complete global fit to all the available experimental data, including those at intermediate times, which are essential for bounding α, and may improve our bounds (28) on β and γ [21, 28, 29]. We now give a brief account of the intermediate-time formalism. As a first step, we consider a perturbative ansatz for the density matrix elements ρ_{ij}, $i, j = 1, 2$ that appear in the system of equations (16) after a (convenient) change of basis to $K_{1,2} \equiv \sqrt{1}\sqrt{2}(K^0 \mp \overline{K}^0)$ [4, 21]. We write [28]

$$\rho_{ij}(t) = \rho_{ij}^{(0)} + \rho_{ij}^{(1)} + \ldots \tag{29}$$

where $\rho_{ij}^{(k)}$ are polynomials in α, β, γ and $|\epsilon|$ of degree k :

$$\rho_{ij}^{(k)} \equiv \alpha^{P_\alpha}\beta^{P_\beta}\gamma^{P_\gamma}|\epsilon|^{P_\epsilon} \qquad ; \qquad P_\alpha + P_\beta + P_\gamma + P_\epsilon = k \tag{30}$$

with the initial condition of having a pure K^0 state, i.e., $\rho_{ij}^0(0) = \frac{1}{2}$, $\rho_{ij}^k(0) = 0$, $k \geq 1$. The ansatz (29) leads to the following iterative system of differential equations, describing the time evolution of the density matrix of the neutral-kaon system at arbitrary time intervals [28] :

$$\frac{d}{dt}[e^{At}\rho_{ij}^{(k)}(t)] = e^{At}\sum_{kl\neq ij}\rho_{kl}^{(k-1)}(t) \tag{31}$$

where A is a generic factor that can be expressed in terms of known data of the neutral-kaon system [28]. In the long and short time limits one recovers the bounds (28) of β and γ . On the other hand, a fit to presently available intermediate time data from two-pion decays [31] can place more stringent bounds on these quantities [28], confirming that the standard CP-violation (27), observed so far, is mainly quantum mechanical in origin. Moreover, an upper bound on the quantity α can also be placed by such fits,

$$|\frac{\alpha}{\Delta\Gamma}| \lesssim 2 \times 10^{-3} \tag{32}$$

although more stringent bounds can be placed by a study of ϕ-decays at a ϕ-factory [24, 29]. In connection with this latter system, we should point out that the energy conservation on the average, which appears as a distinctive feature of our non-critical string approach [21, 12], provides a highly non-trivial constraint on the correct form of the effective parametrizations of the density matrix of two-kaon systems produced in a ϕ-factory, which can be used to analyze the tests of quantum mechanics suggested in [22]. In ref. [29] a parametrization of the density matrix of two particles was used, based on the formalism of ref. [4] and [21] for single-particle states. Although for the case of single-particle states energy is conserved in the formalism of ref. [4, 21], however in the formalism of ref. [29] violations of energy and angular momentum appear generic for processes involving two-particle states. Our stringy formulation requires a more specific parametrization that takes into account the energy conservation in multi-particle states. Such considerations are currently under investigation.

A concrete phenomenological consequence of the CPT-violation will be a shift $\delta\phi$ in the minimum of the time-dependent semileptonic decay asymmetry $\delta(t)$ (24) as a function of time t. A preliminary estimate of this shift, using the bound (32) yields [28]

$$\delta\phi \lesssim 6 \times 10^{-3} \tag{33}$$

and we expect this range to be probed in the foreseeable future.

We cannot resist pointing out that the bounds (28) are quite close to

$$O(\Lambda_{QCD}/M_P)m_K \simeq 10^{-19} GeV \tag{34}$$

which is perhaps the largest magnitude that any such CPT- and quantum-mechanics-violating parameters could conceivably have. Since any such effects are associated with topological string states that have masses of order M_P, we expect them to be suppressed by some power of $1/M_P$. This expectation is supported by the analogy with the Feynman-Vernon model of quantum friction [32], in which coherence is suppressed by some power of the unobserved oscillator mass or frequency. If the CPT- and quantum-mechanics-violating parameters discussed in this section are suppressed by just one power of M_P, they may be accessible to the next round of experiments with CPLEAR and/or DAϕNE [23, 24]. Unfortunately, at present, the string theoretic part of our analysis is too crude to allow for a precise estimate of the non-quantum mechanical terms in the evolution equation (3). Some upper bounds can be put, however, based on the analogy of the problem to that of quantum statistical mechanics of open systems and the theory of measurement in string theory space [33]. It worths pointing out that such upper bounds are in agreement with the estimate (34), suppresed by one power of Planck Mass, which is obtained by naive dimensional analysis.

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