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Tests of Quantum Mechanics at a ϕ–Factory.*

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Abstract

Unique tests of quantum mechanics, which can only be performed at a ϕ–factory, are proposed for Daϕne. Each of these tests consists of measuring the difference between the predicted and the actual amount of interference between two processes leading from a single pure initial state to a single pure final state of a kaon system.

Estimates are made of the upper limits that will be set for the amount of violation if the predictions of quantum mechanics turn out to be correct. They are of the order a fraction of one percent. For the case where, on the contrary, a significant violation is found, several decoherence mechanisms are considered.

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1 Introduction.

Opportunities for testing quantum mechanics are unique at a ϕ–factory. Some of the tests that can be performed at Daϕne are described here. In the past, few checks of quantum mechanics have been made in neutral kaon physics and none with a two-kaon system. At a ϕ–factory, several such tests are possible. The quantity we proposed to check is the amount of interference predicted between two different physical processes.

When there are two different mechanisms, 1 and 2, by which a system can transit from one state to another, the resulting probability of transition P in classical physics is the sum of the transition probabilities, P_1 and P_2, due to the individual processes,

$$P = P_1 + P_2 \quad . \tag{1}$$

In quantum mechanics, when the initial and the final states are pure single states and when the system does not interact with the environment, what one is supposed to add are not probabilities, but probability amplitudes, ψ_1 and ψ_2

$$\psi = \psi_1 + \psi_2 \quad . \tag{2}$$

The interference between the two processes results from that property.

For the tests of concern here, we express the transition probability P as

$$P = |\psi_1|^2 + |\psi_2|^2 + 2(1-\zeta)Re\{\psi_1^*\psi_2\} \quad . \tag{3}$$

where $|\psi_1|^2$ and $|\psi_2|^2$ are the individual transition probabilities of each process, the last term represents the amplitude of the interference effect, and ζ is a parameter which we call the "decoherence parameter". ζ can be measured by measuring P. If there is no decoherence mechanism, quantum mechanics predicts $\zeta = 0$, thus ζ measures the amount of violation of the theoretical prediction. The tests proposed here are designed to yield a low upper limit for ζ if the predictions of quantum mechanics are 100% correct.

1–ζ is analogous to what is called "visibility" in optics. If the initial or the final state included in the sample of events is not pure, the interference effect is reduced. Background can fake a violation. The advantage of a ϕ–factory is the possibility to have initial kaon systems in quite pure states, final decay modes that are theoretically equivalent to pure states, and practically no known decoherence mechanism.

2 Test Performed in the Past.

Tests of quantum mechanics using $K_L \to K_S$ regeneration have been proposed long ago, [1]. They consist of transporting a neutral kaon beam over a distance long enough for all its K_S–component to decay; letting it impinge on a piece of material called regenerator; and recording the number of decays into two charged pions as a function of distance to that regenerator.

There are two processes by which a kaon emerging from the regenerator in the very forward direction can decay into $\pi^+\pi^-$:
1) plain transmission of a K_L and decay via the CP violating mode;
2) $K_L \to K_S$ regeneration followed by a CP conserving decay.

The initial K_L can be considered as a pure state in each momentum interval, if that interval is small enough. Because the kaon has spin zero, all configurations of its $\pi^+\pi^-$–decays can be considered as two pions in an S–wave state, which is a pure state. Thus, at each momentum, the two-pion state from the decay of a kaon in the very forward direction is a pure state. Whether the kaon is transmitted or coherently regenerated, the state of the regenerator is the same, after as well as before the kaon traversed it. Therefore the two processes interfere between one another. As a function of proper time t, the decay rate can be written as

$$\Gamma(t) = c_S e^{-\Gamma_S t} + c_L e^{-\Gamma_L t} + 2 c_I e^{-(\Gamma_S+\Gamma_L)\frac{t}{2}} \cos(\Delta m\; t - \phi) \quad , \tag{4}$$

where Γ_S and Γ_L are the total decay rates of the K_S and of the K_L mesons, respectively; Δm is the K_L–K_S mass difference; and c_S, c_L, c_I, and ϕ are parameters that can be measured by the shape of the time distribution $\Gamma(t)$.

The third term in Eq. (4) is the interference term between the two processes. Quantum theory predicts $c_i = \sqrt{c_S c_L}$. In accordance with Eq. (3), we define

$$\zeta_{\text{ regen}} = 1 - \frac{c_I}{\sqrt{c_S c_L}} \quad , \tag{5}$$

which measures the amount of violation in the regeneration process.

A test of this kind was performed with neutral kaons of momentum ranging from 4 to 10 GeV/c and a carbon regenerator, [2]. The result was

$$\zeta_{\text{ regen}} = 3\% \pm 2\% \quad . \tag{6}$$

The error was essentially statistical.

The same type of test could be performed at Daϕne. The accuracy would be improved. For instance, one may surround one of the two intersection points with a regenerator of a cylindric form, with its axis along the e^+e^- beam line, with a radius of 20 cm and a height of 20 cm too. The regenerator would intercept K_L emitted within a solid angle of about 5 sr. The thickness of the cylinder should be adjusted to produce, just behind the regenerator, more $\pi^+\pi^-$–decays by regenerated K_S than by transmitted K_L. Then one makes sure that the K_S–K_L interference pattern visible in the $\pi^+\pi^-$–mode is as long as possible. For instance, a beryllium cylinder 2.4 cm thick would be an adequate regenerator. It would regenerate one K_L in 10^4 according to Ref. [3], i.e. an appropriate amount of regenerated K_S.

At a luminosity $\mathcal{L} = 10^{33}$, in one month calendar time with an efficiency of 30%, i.e. an integrated luminosity of 1000 pb^{-1}, one may expect more than 10^9 K_L through the regenerator according to Refs. [4] and [5]. Then the decoherence parameter ζ would be measured with a statistical error of less 1%. If the angle of the kaon emerging from the regenerator can be determined to better than 20 mr, the effect of the incoherent regeneration and elastic scattering should introduce an uncertainty of less than 1%. Finally one should to be able to eliminate background in $\pi^+\pi^-$–decays down to less than 10^{-3} times the K_L decay rate into $\pi^+\pi^-$ at a machine like Daϕne designed to measure the CP violation parameter called ϵ'/ϵ. Therefore combining statistical and systematic uncertainties, the final error should of the order of 1%.

Since the value of ζ of Ref. [2] is 1.5 standard deviation away from zero, it would be significant to repeat that test with an error of, let us say, 1%. Of course, this test is not one that can be considered as unique at a ϕ–factory. However it would establish a limit for spontaneous decoherence in regeneration of kaons comparable to the limit of 0.6% obtained for decoherence in neutron interferometry, [6].

3 Two-Kaon Systems.

The really unique tests of quantum mechanics at Daϕne are tests with quantum states made of two kaons. In e^+e^-–collisions of this kind of energy, only objects odd under charge conjugation are formed. The two-neutral-kaon system created via ϕ production and two-body decay in a given direction is in a pure quantum state. Its state vector can be expressed as

$$\Psi_0 = \frac{1}{\sqrt{2}}(K^0\bar{K}^0 - \bar{K}^0K^0) \tag{7}$$

$$\approx \frac{1}{\sqrt{2}}(K_LK_S - K_SK_L) \ . \tag{8}$$

In the terms $K^0\bar{K}^0$, $\bar{K}^0K^0$, K_LK_S, and K_SK_L, the first symbol refers to the state of the particle emitted on the left and the second symbol to the particle on the right. The sign $\approx$ is used in Eq. (8) instead of the sign $=$ because of a slight inaccuracy in the normalization factor.

As stated above, kaons living long enough to be reduced to their K_L component are pure states and $\pi^+\pi^-$–decay states are equivalent to pure $\pi^+\pi^-$ S–wave states. If we accept the $\Delta S/\Delta Q$ rule, each of the semileptonic decay states $\ell^+\nu\,\pi^-$ and $\ell^-\,\overline{\nu}\,\pi^+$ can also be considered as a pure state in the context of this paper. As to multiple-particle states where the kaon on the left has decayed into a pure state f_a and the kaon on the right into a pure state f_b, they are pure states themselves. All processes leading from the initial

ϕ–meson to any one of these pure multiparticle final states $f_a f_b$ interfere between each other.

In particular, consider the possibility of having the same decay state f, for instance $\pi^+\pi^-$, or $\ell^+\nu\,\pi^-$, or $\ell^-\,\overline{\nu}\,\pi^+$, as the final decay product on the left as well as on the right. This may happen via two processes,
1) a K_L emitted to the left and a K_S to the right; or
2) a K_S to the left and a K_L to the right;
and, in either case, both the K_L and the K_S decaying into the same state f. These two processes interfere between one another.

Let B_L and B_S be the branching ratios of K_L and K_S, respectively, into the decay state f . From Eq. (8), one gets the probability for the kaon on the left to decay into f at the proper time t_a and the one on the right to decay into f at proper time t_b, per unit of t_a and unit of t_b :

$$\frac{\partial^2 P}{\partial t_a \partial t_b} \approx \frac{B_L\Gamma_L B_S\Gamma_S}{2}\left(e^{-\Gamma_S t_a} + e^{-\Gamma_S t_b} - 2e^{-\Gamma_S\frac{t_a+t_b}{2}}\cos\left(\Delta m(t_a - t_b)\right)\right) \ . \tag{9}$$

The third term in the largest bracket of Eq. (9) is the interference term. For $t_a = t_b$, that term is negative and large enough to make $\frac{\partial^2 P}{\partial t_a \partial t_b} = 0$. However, if, for any reason, the interference term is reduced by a factor $1 - \zeta$, the rate for $t_a = t_b$ becomes non-zero. These conditions are particularly favorable for a test of quantum mechanics because, then, the test for $\zeta = 0$ is a null experiment.

3.1 Predictions and Errors.

Let us use the variable z, which we define in each event as the difference between t_a and t_b measured in units of the K_S average lifetime,

$$z = \Gamma_S(t_a - t_b) \ \ . \tag{10}$$

Considering those events where both kaons of the pair decay via the same decay mode, one can integrate all those with the same $t_a - t_b$, (i.e. with the same z) over the variable $t_a + t_b$ and histogram them as a function of the variable z. In each bin of width dz, the number of events $\frac{dn}{dz}dz$ is a function of z that can be developed up to second order in z around the point $z = 0$. Because of the symmetry between the kaon on the left and the kaon on the right, the term in z to the first power is zero.

$$\tfrac{dn}{dz} = C_0 + C_2\ z^2 \ \ . \tag{11}$$

C_0 and C_2 are two parameters that can be obtained from a fit to the data.

Let N be the number of pairs emitted in the fiducial volume, B_L and B_S the K_L- and the K_S-branching ratios, Γ_L and Γ_S the K_L- and the K_S-decay rates, as above. Let us define the constants A and γ :

$$A \ = \ N B_L B_S \frac{\Gamma_L}{\Gamma_S} \ \ , \tag{12}$$

$$\gamma \;=\; \frac{1+4\left(\frac{\Delta m}{\Gamma_S}\right)^2}{8} = 0.239 \quad , \tag{13}$$

using Ref. [7]. In absence of background, quantum mechanics predicts

$$C_0 \;=\; 0 \quad , \tag{14}$$
$$C_2 \;=\; A\gamma \quad . \tag{15}$$

If there is background and if, because of a possible violation of quantum mechanics, the interference term is reduced by a factor $1-\zeta$,

$$C_0 = A\zeta + \frac{dn}{dz}_{\text{(Background)}} \quad . \tag{16}$$

Measuring C_0 permits us to determine the decoherence parameter

$$\zeta = \frac{1}{A}\left(C_0 - \frac{dn}{dz}_{\text{(Background)}}\right) \quad . \tag{17}$$

At the limit of large N, the particular form of Eq. (11) permits us to derive special expressions for the error δC_0 on C_0 :

$$\delta C_0 \;=\; \left(\frac{C_0 C_2}{\pi^2}\right)^{\frac{1}{4}} \qquad \text{if} \;\; C_0 > \left(\frac{C_2}{\pi^2}\right)^{\frac{1}{3}} \quad , \tag{18}$$
$$\delta C_0 \;=\; \left(\frac{C_2}{\pi^2}\right)^{\frac{1}{3}} \qquad \text{if} \;\; C_0 < \left(\frac{C_2}{\pi^2}\right)^{\frac{1}{3}} \quad . \tag{19}$$

The proper form to be used depends on the size of C_0. The appropriate formula is always the one that gives the largest error. In any case, the error $\delta\zeta$ on ζ is given by

$$\delta\zeta = \frac{\delta C_0}{A} \quad . \tag{20}$$

Among the possible sources of background, special attention will be paid to $\phi \to K^0\overline{K^0}\gamma$ events misidentified as two-body $K^0\overline{K^0}$'s, because there are predictions for this effect, [8]. In these background events, the $K^0\overline{K^0}$ system is $CP=+$,

$$\Psi_0 \approx \frac{1}{\sqrt{2}}\left(K_S K_S - K_L K_L\right) \quad , \tag{21}$$

and both kaons can decay into the same mode for $t_a = t_b$. We call r_γ the ratio of the number of these background events to the number of genuine $K^0\overline{K^0}$ events and, in the numerical applications, we use the worst (highest) estimate given in the conclusion of Ref. [8]: $r_\gamma = 2\ 10^{-8}$. Because of the particular dependance of that background as a function of $t_a + t_b$, we assume that r_γ can be measured to better than 10% of $2\ 10^{-8}$.

3.2 Performance at Daϕne.

To predict how good the tests could be at Daϕne, we consider an experiment lasting one year with 30% efficiency, (i.e. effective duration 10^7 s.) at the luminosity $\mathcal{L} = 10^{33}$ cm^{-2}s^{-1}. The integrated luminosity is 10 000 pb^{-1}. In that time, $N \approx 10^{10}$ neutral-kaon pairs are expected to be produced.

For both kaons decaying into $\pi^+\pi^-$, using the kaon data of Ref. [7], we compute

$$\begin{aligned} A &= 24000 \quad , && (22)\\ C_2 &= 5700 \quad . && (23) \end{aligned}$$

We assume the background due to misidentified events in the $\pi^+\pi^-$ sample to be no more than what can be tolerated in a measurement of ϵ'/ϵ, i.e. let us say 10^{-3} of the K_L rate into $\pi^+\pi^-$. To that we add the $\phi \to K^0\overline{K}^0\gamma$-background and yet another contribution, equivalent to background, due to the measurement error on the vertex positions of the two kaon decays. The latter effect generates uncertainties δt_a and δt_b in the determinations of the proper times t_a and t_b, thus δz in z, and populates the region around $z = 0$. We will assume that the sample of events can be limited to events where the vertex uncertainty is 1 mm in average. Then the uncertainty in z is $\delta z = 0.2$.

$$\frac{dn}{dz}_{(\text{Background})} = A\ 10^{-3} + \frac{r_\gamma N B_S^2}{4} + C_2(\delta z)^2 \approx 300 \quad . \tag{24}$$

Assuming that the contribution of these backgrounds is known within 10% and can be subtracted off with that 10% uncertainty, we find, in the case of no violation, that the systematic error is about equal to the statistical error, given by Eqs. (18) and (20).

$$\delta\zeta \approx 10^{-3} \quad . \tag{25}$$

It follows that this test would explore possible values of $\zeta_{\pi^+\pi^-}$ as small as one or two 10^{-3}. This value is better than the one given in Ref. [6] in the context of neutron interferometry. Even smaller values of ζ can be detected if the vertex accuracy can be improved.

A similar calculation can be made for the case where both kaons decay into $\ell^-\overline{\nu}\pi^+$ or both into $\ell^+\nu\pi^-$. Predictions of the performance at Daϕne are less reliable than for the $\pi^+\pi^-$–decays because the background due to misidentified events is more difficult to estimate at this point in time. The $\phi \to K^0\overline{K}^0\gamma$ background contributes to $\frac{dn}{dz}_{(\text{Background})}$ for much less than $r_\gamma N B_L^2 \Gamma_L/(4\Gamma_S)$ because of the limited acceptance of the detector for K_L-decays. That background is totally negligible. Assuming that, as for the $\pi^+\pi^-$–mode, the background is dominated by the effect of the vertex position uncertainty, we get

$$\begin{aligned} A &= 3300 \quad , && (26)\\ C_2 &= 800 \quad , && (27)\\ C_0 &\approx 30 \quad , && (28)\\ \delta\zeta &= 2\ 10^{-3} \qquad \text{for } \zeta = 0 \quad , && (29) \end{aligned}$$

for each of the two modes $\ell^- \overline{\nu} \pi^+$ and $\ell^+ \nu \pi^-$. This value of $\delta\zeta$ would permit exploring values of $\zeta_{\ell^- \overline{\nu} \pi^+}$ and of $\zeta_{\ell^+ \nu \pi^-}$ comparable to the value estimated for the $\pi^+\pi^-$–mode, i.e. a few 10^{-3}. Of course, if a violation is found, questions will be raised about the $\Delta S/\Delta Q$ rule that was assumed to justify the test with leptons in the final state. However studies of correlations between regions of the $\ell^\pm \nu \pi^\mp$–Dalitz plot will permit us to distinguish a violation of quantum mechanics from a violation of the $\Delta S/\Delta Q$ rule.

3.3 $K_L K_L$ events

Another possible test consists of measuring the number of pairs of K_L in the same event. As obviously shown by Eq. (8), quantum mechanics predicts no such pair in any $\phi \to K^0 \overline{K}^0$ event. That same prediction can be obtained using Eq. (7) instead, as the result of a 100% destructive interference effect between the process where the ϕ decays into K^0 on the left and $\overline{K}^0$ on the right and the process where it is the other way around. If the interference term were reduced by a factor $1 - \zeta_{LL}$, there would be some $K_L K_L$ events and their number would be

$$n_{LL} = \frac{N\zeta_{LL}}{4} \quad . \tag{30}$$

To identify two-K_L events in the KLOE detector, the signature can be two particles decaying or interacting at a large distance from the interaction point. Some background is going to be generated by $K_L K_S$ or $K_S K_L$ events where the K_S decays via the two-π^0 mode and the γ's from π^0 decays get confused with a K_L decay or interaction, but it is beyond the scope of this paper to estimate this effect. The $\phi \to K^0 \overline{K}^0 \gamma$ background yields :

$$n_{LL(\mathrm{Background})} = \frac{r_\gamma N}{2} \underset{\sim}{<} 100 \tag{31}$$

If that background and the statistical error were the only limitations, one could detect values of ζ_{LL} as low as 10^{-8}.

4 Alternate Theories.

The tests above are designed to detect effects described by alternate theories to quantum mechanics, more specifically by those which do not use hidden variables. The maximum information that can characterize a quantum system is still given by a density matrix ρ. In quantum mechanics, the time evolution of ρ is described by a unitary transformation,

$$\rho(t) = U(t)\rho(0)U^\dagger(t) \quad , \tag{32}$$

from which one can derive a hamiltonian,

$$H(t) = i\frac{dU}{dt}U^\dagger \quad . \tag{33}$$

Eq. (32) conserves the rank of ρ, thus insures that a pure state cannot evolve into a mixed state. That is the property tested by our tests of interferences. In these tests, the initial state is one that quantum mechanics considers pure and the measurements check that, within error, the final density matrix is of rank 1, i.e. has the mathematical property that describes a pure state.

4.1 Completeness.

The test of Sect. 2 involves a one-kaon system, consisting of a pure K_L initially. The density matrix is a two-by-two matrix with only two eigenvectors ψ and ψ'. The time distribution of $\pi^+\pi^-$–decays is of the form of Eq. (4) with

$$c_S = B_S\Gamma_S\left(\left|{K_S}^\dagger\psi\right|^2 + \left|{K_S}^\dagger\psi'\right|^2\right) \quad , \tag{34}$$

$$c_L = B_L\Gamma_L\left(\left|{K_L}^\dagger\psi\right|^2 + \left|{K_L}^\dagger\psi'\right|^2\right) \quad , \tag{35}$$

$$c_I = \sqrt{B_S\Gamma_S B_L\Gamma_L}\left(\left|({K_S}^\dagger\psi)(\psi^\dagger K_L) + ({K_S}^\dagger\psi')(\psi'^\dagger K_L)\right|\right) \quad . \tag{36}$$

Using Schwarz' inequality, one proves that, to have $c_I = \sqrt{c_S c_L}$, vectors ψ and ψ' have to be collinear. Then the rank of ρ is 1. If the test had infinite precision and if it was found that $\zeta_{\text{ regen}}$ of Eq. (5) was equal to zero, the density matrix would have been shown to correspond to a pure state. In that sense, the test is complete.

The tests of Sect. 3 involve a two-kaon system, made initially of a pure state of two neutral kaons from ϕ–decay in one direction. The density matrix ρ is a four-by-four matrix, with four eigenvectors. To prove that ρ is of rank 1 (pure state), it is enough to show that the product of ρ by three linearly independent vectors give zero. This can be shown experimentally by proving that decay rates in three different modes corresponding to three amplitudes represented by three linearly independent vectors in the two-kaon Hilbert space are zero. The tests of Sect. 3.2 consist of measuring the rates of decay of the two-kaon system with the kaon on the left decaying into the same thing at the same time as the kaon on the right; and of checking that these rates are zero. There are three of these tests, involving either the $\pi^+\pi^-$, $\ell^-\overline{\nu}\pi^+$, or $\ell^+\nu\pi^-$decay mode. The test of Sect. 3.3 is a fourth test which measures the probability of the system evolving into two K_L. The vectors corresponding to any three of the four amplitudes measured in these four tests are linearly independent. Therefore the set of any three of these tests is complete.

4.2 Examples of Decoherence Mechanisms.

1) <u>$K_L \to K_S$ evolution.</u> Processes that imply such an evolution affect the $\pi^+\pi^-$–decay time distribution. Let $\Gamma_{K_L\to K_S}$ be the rate of incoherent evolution of K_L into K_S. In the experimental setup of Sect. 2, a fraction $\Gamma_{K_L\to K_S}/\Gamma_S$ of incoherent K_S would be present all the time alongside with the K_L mesons, in equilibrium, i.e. with as many K_S being formed and decaying per unit of time. Each one of these K_S would decay into $\pi^+\pi^-$ at

a rate equal to $B_S\Gamma_S$ instead of $B_L\Gamma_L$ for K_L. Thus the coefficient c_L of $e^{-\Gamma_L t}$ in Eq. (4) would be increased, without c_S and c_I being appreciably changed. Thus $\zeta_{\rm regen}$ of Eq. (5) is not zero anymore, and one can deduce

$$\Gamma_{K_L \to K_S} = 2\zeta_{\rm regen}\frac{B_L\Gamma_L}{B_S} = \frac{\zeta_{\rm regen}}{10\ \mu{\rm s}} \quad . \tag{37}$$

The test of Ref. [2] yields

$$\Gamma_{K_L \to K_S} = (2 \pm 1.5)\ 10^{-21}\ {\rm GeV} \quad . \tag{38}$$

Note that $\Gamma_{K_L \to K_S}$ is equivalent to the parameter γ of Refs. [9] and [10] in the context of their theory.[1]

In the test of Sect. 3.2 that relies on the $\pi^+\pi^-$–decay distribution, the incoherent $K_L \to K_S$ process would act on the K_L of the K_LK_S and K_SK_L systems and produce K_SK_S states after some time. As an average in the sample of events

$$\tfrac{dn}{dz} = \frac{N}{2}B_S^2\frac{\Gamma_{K_L \to K_S}}{\Gamma_S} \quad . \tag{39}$$

$\Gamma_{K_L \to K_S}$ can be derived from $\zeta_{\pi^+\pi^-}$ by the same relation as Eq. (37) for $\zeta_{\rm regen}$. At Daϕne, we can hope to get an upper limit of the order of 1/10 ms, i.e. 10^{-22} GeV. [2]

2) <u>$K_S \to K_L$ evolution.</u> Such process would produce K_LK_L events in the test of Sect. 3.3.

$$n_{LL} = N\ \frac{\Gamma_{K_S \to K_L}}{\Gamma_S} \quad , \tag{40}$$

$$\Gamma_{K_S \to K_L} = \frac{\zeta_{LL}\ \Gamma_S}{4} \quad . \tag{41}$$

Suppose that the uncertainty about the background in the test of Sect. 3.3 is, let us say, 100 times the contribution of the $\phi \to K^0\bar{K}^0\gamma$ events, i.e. 10^{-6}, then one could still detect values of $\Gamma_{K_S \to K_L}$ as low as 1/100 μs or 10^{-20} GeV.

3) <u>Evolution K_LK_S–K_SK_L into K_LK_S+K_SK_L.</u> This evolution can be expressed also as

$$\left(K^0\bar{K}^0 - \bar{K}^0K^0\right) \to \left(K^0K^0 - \bar{K}^0\bar{K}^0\right) \quad . \tag{42}$$

It will produce states where both kaons can decay into $\ell^-\,\bar{\nu}\,\pi^+$ or both into $\ell^+\nu\,\pi^-$ at the same time. Let us call $\Gamma_{(-)\to(+)}$ the rate of this process. For the tests of Sect. 3.2 involving $\ell^-\,\bar{\nu}\,\pi^+$ or $\ell^+\nu\,\pi^-$,

$$\tfrac{dn}{dz} = 2\ NB_S^2\ \frac{\Gamma_{(-)\to(+)}}{\Gamma_S} \tag{43}$$

$$\Gamma_{(-)\to(+)} = \zeta_{\ell^-\bar{\nu}\pi^+}\ \frac{\Gamma_S}{2} = 2\ \zeta_{\ell^-\nu\pi^+}\ \frac{\Gamma_S}{2} \quad . \tag{44}$$

[1] The error on $\Gamma_{K_L \to K_S}$ given by the measurement of Ref. [2] is twice as small as the one quoted in Refs. [9] and [10] because, in those references, only the CPT-violating effect of that parameter is taken into consideration. Another advantage of the test of Ref. [2] is that one can be sure that no constraint derived from quantum coherence was assumed anywhere in the analysis of the events.

[2] This is not yet comparable to the result obtained in neutron interferometry, 10^{-25} GeV, [6], but it would be the best result relevant to kaon physics.

Using the estimate of Eq. (29), we could explore values of $\Gamma_{(-)\to(+)}$ lower than 1/0.1 ms, i.e. 10^{-17} GeV.

If a violation of quantum mechanics is ever revealed by one of the tests above, comparing the amount of violation found in each test will bring information about the decoherence mechanism responsible for the violation. Also questions will be raised about the background due to $K_S K_S$ and $K_L K_L$ events produced by ϕ–decay into $K^0\overline{K}^0\gamma$, [11]. However, the effect of this background can be distinguished from the decoherence effects described in this section by dividing the sample of events into those where kaons decay near the intersection point and those where they decay far from it. In the test where the two kaons decay via the $\pi^+\pi^-$ mode, the $\phi\to K^0\overline{K}^0\gamma$ background populates the bin $\frac{dn}{dz}$ at $z = 0$ with more events when $t_a + t_b$ is small than when $t_a + t_b$ is large, actually like $e^{-\Gamma_S(t_a+t_b)}$. On the contrary, the decoherence mechanisms take time to accumulate events for $z = 0$; thus $\frac{dn}{dz}$ at $z = 0$ grows as a function of $t_a + t_b$.

5 Conclusions.

Significant tests of quantum mechanics that can be performed at Daϕne are proposed. They consist of measuring the amplitude of interference effects and comparing it to the predicted value. They may reach the relative accuracy of 10^{-3}. The tests of Sect. 3.2 and, maybe, the one of Sect. 3.3 can be conducted with the KLOE detector as is. They only require a specific analysis of data that will be collected anyhow. These tests of Sect. 3.2 are valid regardless of the Weisskopf-Wigner approximation, [12], because the probability for the two kaons to decay at the same time into the same mode can be shown to be zero just regardless of the time evolution operator $U(t)$, [13].

Tests of quantum mechanics in kaon physics were proposed already in Ref. [1]. One of the motivations was to look for possible spontaneous wave-function collapses not involving an observer, because such effects would induce decoherence. That motivation is still valid. Differential equations for the time evolution of the density matrix in presence of a decoherence mechanism were developed in Ref. [14]. In Ref. [15], that kind of evolution was considered relevant to particles in the vicinity of black holes, therefore to string theory.[3] Guesses at how strong this effect could be in kaon physics are given in Ref. [16].[4] A more detailed analysis is made in Ref. [9] and expressions relevant to two-kaon states are given in Ref. [10]. The latest theory assumes strangeness conservation in the decoherence mechanism and does not require "completeness". However, it is our opinion that it is better to keep as broad a perspective as possible and to perform all the tests proposed here, which test all the above theories, rather than only those that have the best chance to reveal a

[3] Because, when particles disappear beyond the horizon of a black-hole, information about their quantum numbers disappears too.

[4] Let m_K be the kaon mass and M_P the Planck mass. Ref [16] claims there are diagrams leading to rates $\Gamma_{K_L\to K_S}$ of the order of m_K^2/M_P, i.e. 2 10^{-20} GeV. The ratio of the measured value of $\Gamma_{K_L\to K_S}$ to 2 10^{-20} GeV is a measurement of the coefficient in that diagram. According to this paper, a measurement at Daϕne can detect the alleged effect even if that coefficient is much smaller than 1. See Ref [17].

violation of quantum mechanics in the context of the theory of Refs. [9], [10], and [16].

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