INFN - Laboratori Nazionali di Frascati, May 1995
From: **The Second DaΦne Physics Handbook**[1]
Chapter 1: CP and CPT Violation
Eds. L. Maiani, G. Pancheri, N. Paver

CP and CPT Measurements at DAΦNE

G. D'Ambrosio
INFN - Sezione di Napoli, Naples, Italy

G. Isidori and A. Pugliese
Dipartimento di Fisica, Universitá di Roma "La Sapienza"
and
INFN - Sezione di Roma, Rome, Italy

Abstract

Starting from the time evolution of the C-odd $\bar{K}^0 K^0$ system, we analyze the asymmetries measurable at DAΦNE and their implications on CP violation and on the possible tests of T and CPT symmetries. In particular the ratio $\frac{\epsilon'}{\epsilon}$ can be measured with high precision (up to about 10^{-4} for the real part). The CP-, T- and CPT-violating parameters can be explored in K_S semileptonic decays with an accuracy of the order of 10^{-3}. The possibility to detect $K_S \to 3\pi$ and $K_L \to \pi\pi\gamma$ decays is also discussed.

1 Introduction

The $\bar{K}^0$ K^0 state produced in the decay of the ϕ resonance is odd under charge conjugation and is therefore an antisymmetric K_L K_S state. This characteristic makes a ϕ factory very suitable to study CP violation and to test CPT symmetry in K meson decays [1, 2].

For a long time it has been stressed that the presence in the same detector of K_L and K_S beams, produced without regeneration and thus with the relative fluxes perfectly known, will allow a very clean determination of the ratio $\frac{\epsilon'}{\epsilon}$ [3]. A non-zero value for $\frac{\epsilon'}{\epsilon}$ is an unambiguous signal of the existence of direct CP violation, which is naturally expected

[1]Supported by the INFN, by the EC under the HCM contract number CHRX-CT920026 and by the authors home institutions

in the Standard Model. The present experimental situation is:

$$\Re\left(\frac{\epsilon'}{\epsilon}\right) = (2.3 \pm 0.7) \times 10^{-3} \qquad \Im\left(\frac{\epsilon'}{\epsilon}\right) = (-1.2 \pm 17) \times 10^{-3} \qquad \text{NA31[4]}$$
$$\Re\left(\frac{\epsilon'}{\epsilon}\right) = (0.74 \pm 0.60) \times 10^{-3} \qquad \Im\left(\frac{\epsilon'}{\epsilon}\right) = (+4.7 \pm 3.5) \times 10^{-3}, \qquad \text{E731[5]}$$

still consistent with $\epsilon' = 0$. The theoretical calculation of $\frac{\epsilon'}{\epsilon}$ in the Standard Model is strongly affected by QCD corrections and, for large values of the top mass, large cancellations are expected [6]. The present estimate is $\Re(\frac{\epsilon'}{\epsilon}) = (2.8 \pm 2.4) \times 10^{-4}$, [7] thus a fundamental goal of DAΦNE is certainly to reach the sensitivity of 10^{-4} in the measurement of this ratio. Independent information about direct CP violation could be obtained also by charge asymmetries in $K^{\pm} \to 3\pi$ [8] and $K^{\pm} \to 2\pi\gamma$ [9] decays.

Beyond the study of direct CP violation, the presence of a pure K_S beam will allow the observation at DAΦNE of some suppressed K_S decays, such as the semileptonic and the three-pion ones. The theoretical predictions for semileptonic decays are not strongly affected by QCD corrections and, as we will discuss later, the measurement of the semileptonic rates and charge asymmetries in $K_{S,L}$ decays can give many interesting tests of CPT and of the $\Delta S = \Delta Q$ rule. Moreover, due to the coherence of the initial state, T and CPT symmetries can be directly tested in events with two leptons in the final state [10, 11].

Some time ago it was pointed out that the radiative decay $\phi \to \gamma f_0 \to \gamma (K^0\bar{K}^0)_{C=+}$ could have a non-negligible branching ratio [12] and therefore a dangerous background, namely a $K^0\bar{K}^0$ component even under charge conjugation, could be present. New determinations of the $\phi \to \gamma (K^0\bar{K}^0)_{C=+}$ branching ratio [13] turn out to be much smaller, then, as we will show, the inclusion of the C-even background does not sensibly affect the measurements of $\frac{\epsilon'}{\epsilon}$ and K_S suppressed decays.

It has been suggested that quantum mechanics violations may be generated by non-local theories at the Planck scale [14]. As a consequence CP- and CPT-violating effects of non quantum-mechanical origin could be induced [15, 16]. In [16] it was proposed to investigate such effects in quantum correlated particle systems, such as the $\bar{K}^0$ K^0 system. The coherence of the ϕ-factory initial state will help in disentangling these effects, and quite stringent bounds could be obtained for the quantum mechanics violating parameters [17, 18]. It is worth while to note that the quantum mechanics violation induces a loss of the initial state coherence which can somehow simulate the effect of a C-even background.

The plan of the paper is the following: in section 2 we recall the time evolution of the initial state. In section 3 we report its implications on the determination of real and imaginary parts of $\frac{\epsilon'}{\epsilon}$. In section 4 the semileptonic decays, with possible direct tests of T and CPT symmetry, are discussed. Sections 5 and 6 are devoted, respectively, to $K_S \to 3\pi$ and $K_L \to \pi\pi\gamma$ decays. In section 7 we study the effect of the C-even background. In section 8 we discuss the implications of possible quantum mechanics violations. Finally, in the appendix, the relation between time and distance measurements are discussed.

2 Time evolution

The antisymmetric $\bar{K}^0 K^0$ state produced in the ϕ decay can be written as

$$\phi \to \frac{1}{\sqrt{2}}\left[\bar{K}^{0(q)}K^{0(-q)} - K^{0(q)}\bar{K}^{0(-q)}\right] = \frac{h}{\sqrt{2}}\left[K_S^{(q)}K_L^{(-q)} - K_L^{(q)}K_S^{(-q)}\right], \tag{1}$$

where $\mathbf{q}$ and $-\mathbf{q}$ indicate the spatial momenta of the two-kaons and the normalization factor h is

$$h = \frac{1+|\epsilon|^2}{1-\epsilon^2} \simeq 1. \tag{2}$$

The decay amplitude of the two-kaon system into the final state $|a^{(q)}(t_1), b^{(-q)}(t_2)\rangle$ is given by:

$$\begin{aligned} A\left(a^{(q)}(t_1), b^{(-q)}(t_2)\right) &= \frac{h}{\sqrt{2}}\Big[A(K_S \to a)e^{-i\lambda_S t_1}A(K_L \to b)e^{-i\lambda_L t_2} \\ &\qquad -A(K_L \to a)e^{-i\lambda_L t_1}A(K_S \to b)e^{-i\lambda_S t_2}\Big], \end{aligned} \tag{3}$$

where $\lambda_{S(L)} = m_{S(L)} - i\Gamma_{S(L)}/2$. As usual we define also:

$$\Gamma = \frac{\Gamma_S + \Gamma_L}{2}, \qquad \Delta\Gamma = \Gamma_S - \Gamma_L \quad \text{and} \quad \Delta m = m_L - m_S. \tag{4}$$

If $|a\rangle \neq |b\rangle$ the two states $|a^{(q)}(t_1), b^{(-q)}(t_2)\rangle$ and $|a^{(-q)}(t_1), b^{(q)}(t_2)\rangle$ are physically different for any value of t_1 and t_2, therefore the double differential rate is:

$$\begin{aligned} \Gamma(a(t_1), b(t_2)) &= |h|^2 \int \Big\{ |A_S^a|^2 |A_L^b|^2 e^{-(\Gamma_S t_1 + \Gamma_L t_2)} + |A_L^a|^2 |A_S^b|^2 e^{-(\Gamma_L t_1 + \Gamma_S t_2)} \\ &\qquad -2\Re\left[A_S^a A_L^{a*} A_L^b A_S^{b*} e^{-\Gamma(t_1+t_2)+i\Delta m(t_1-t_2)}\right]\Big\} d\phi_a d\phi_b, \end{aligned} \tag{5}$$

where ϕ_a and ϕ_b are the phase spaces of the final states. Integrating eq. (5) on t_1 and t_2 one obtains the probability for the decay into the $|a,b\rangle$ state with both the decay vertices inside the detector:

$$P(a,b) = \frac{|h|^2}{\Gamma_S\Gamma_L}\left[\left(\Gamma_S^a\Gamma_L^b + \Gamma_L^a\Gamma_S^b\right)S_L - 2\frac{\Gamma_S\Gamma_L}{\Gamma^2+\Delta m^2}\int \Re\left(A_S^a A_L^{a*} A_L^b A_S^{b*}\right) d\phi_a d\phi_b\right] \tag{6}$$

where $S_L = (1 - e^{-D/d_L})$ is the K_L acceptance of the detector: the KLOE project quotes for the fiducial length $D \simeq 120$ cm [3] ($d_L = 340$ cm is the K_L mean decay path), thus $S_L \simeq 0.3$.

For $|a\rangle = |b\rangle$ the interchange of $\mathbf{q} \leftrightarrow -\mathbf{q}$ is equivalent to $t_1 \leftrightarrow t_2$, thus:

$$P(a,a) = |h|^2 \frac{\Gamma_S^a \Gamma_L^a}{\Gamma_S \Gamma_L}\left[S_L - \frac{\Gamma_S\Gamma_L}{\Gamma^2 + \Delta m^2}\right]. \tag{7}$$

As will be discussed in the following, the choice of appropriate time integration intervals supplies a powerful tagging of K_L or K_S decays.

Finally we define also the so-called "time difference distribution":

$$I(a,b;t) = \int dt_1 dt_2 |A(a(t_1), b(t_2))|^2 \delta(t_1 - t_2 - t)$$
$$= \frac{|h|^2 e^{-\Gamma|t|}}{2\Gamma} \Big\{ |A_S^a|^2 |A_L^b|^2 e^{-\frac{\Delta\Gamma}{2}t} + |A_L^a|^2 |A_S^b|^2 e^{+\frac{\Delta\Gamma}{2}t} - 2\Re \left[A_S^a A_L^{a*} A_L^b A_S^{b*} e^{+i\Delta m t} \right] \Big\}. \quad (8)$$

3 Real and imaginary parts of $\frac{\epsilon'}{\epsilon}$

As extensively discussed, for example in Refs. [3, 19, 20], the study of the time difference distribution, for $\pi^+\pi^-$, $\pi^0\pi^0$ final states, leads to the determination of both $\Re\left(\frac{\epsilon'}{\epsilon}\right)$ and $\Im\left(\frac{\epsilon'}{\epsilon}\right)$.

Introducing as usual the amplitudes

$$\eta^{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \epsilon + \epsilon' \qquad \text{and} \qquad \eta^{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \epsilon - 2\epsilon', \quad (9)$$

eq. (8), integrated over the pion phase space, gives:

$$F(t) = \int d\phi_{+-}\, d\phi_{00}\, I(\pi^+\pi^-, \pi^0\pi^0; t)$$
$$= \frac{\Gamma_S^{+-}\Gamma_S^{00}}{2\Gamma} e^{-\Gamma|t|} \left[|\, \eta^{+-}\, |^2\, e^{+\frac{\Delta\Gamma}{2}t} + |\, \eta^{00}\, |^2\, e^{-\frac{\Delta\Gamma}{2}t} - 2\Re\left(\eta^{+-} (\eta^{00})^* e^{-i\Delta m t} \right) \right]. \quad (10)$$

If $\epsilon' \neq 0$ there is an asymmetry between the events with positive and negative values of t:

$$A(t) = \frac{F(|t|) - F(-|t|)}{F(|t|) + F(-|t|)} = A_R(t) \times \Re\left(\frac{\epsilon'}{\epsilon}\right) - A_I(t) \times \Im\left(\frac{\epsilon'}{\epsilon}\right), \quad (11)$$

neglecting in eq. (11) terms proportional to $\left(\frac{\epsilon'}{\epsilon}\right)^2$, the $A_R(t)$ and $A_I(t)$ coefficients, shown in Fig.1, are given by:

$$A_R(t) = 3\frac{e^{-|t|\Gamma_L} - e^{-|t|\Gamma_S}}{e^{-|t|\Gamma_L} + e^{-|t|\Gamma_S} - 2\cos(\Delta m|t|)\, e^{-\Gamma|t|}}$$
$$A_I(t) = 3\frac{2\sin(\Delta m|t|)\, e^{-\Gamma|t|}}{e^{-|t|\Gamma_L} + e^{-|t|\Gamma_S} - 2\cos(\Delta m|t|)\, e^{-\Gamma|t|}}. \quad (12)$$

It can be seen that $A_R(t)$ becomes nearly independent of t, and equal to 3, for $t >> \tau_S$; on the other hand $A_I(t)$ is strongly dependent on t and vanishes for $t >> \tau_S$.

Therefore a measurement of the asymptotic value of $A(t)$ or of the value of the integrated asymmetry

$$A = \frac{F(t>0) - F(t<0)}{F(t>0) + F(t<0)} \simeq 3\Re\left(\frac{\epsilon'}{\epsilon}\right) \quad (13)$$

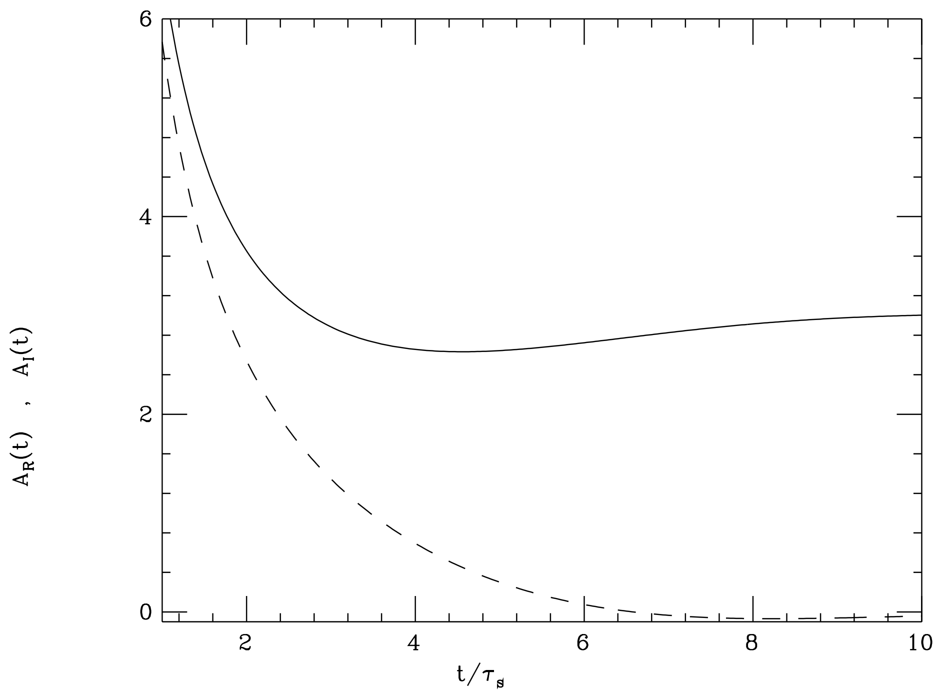

Figure 1: Coefficients of $\Re\left(\frac{\epsilon'}{\epsilon}\right)$ (full line) and $\Im\left(\frac{\epsilon'}{\epsilon}\right)$ (dashed line) defined in eq.(11).

allows a clean determination of $\Re\left(\frac{\epsilon'}{\epsilon}\right)$. The statistical error on A is given by:

$$\sigma_A = \sqrt{\frac{(1+A)\times(1-A)}{N}}, \tag{14}$$

where N is the number of $\phi \to \pi^+\pi^-, \pi^0\pi^0$ events. At the reference DAΦNE luminosity the statistical error on $\Re\left(\frac{\epsilon'}{\epsilon}\right)$ is then:

$$\sigma_{\Re\left(\frac{\epsilon'}{\epsilon}\right)} \simeq \frac{\sigma_A}{3} \simeq \frac{1}{3\sqrt{N}} \simeq 1.7\times 10^{-4}. \tag{15}$$

The integrated asymmetry A allows a precise determination of $\Re\left(\frac{\epsilon'}{\epsilon}\right)$ but gives no information on the imaginary part of $\frac{\epsilon'}{\epsilon}$. To overcome this problem a further method can be exploited to measure both $\Re\left(\frac{\epsilon'}{\epsilon}\right)$ and $\Im\left(\frac{\epsilon'}{\epsilon}\right)$ from the $K_L K_S \to \pi^0\pi^0, \pi^+\pi^-$ decay time difference: the experimental distribution $F(d)$[2] can be fitted by the theoretical distribution of eq. (10), and $\Re\left(\frac{\epsilon'}{\epsilon}\right)$ and $\Im\left(\frac{\epsilon'}{\epsilon}\right)$ can be used as free parameters of the fit.

It must be stressed that this procedure is very sensitive to the experimental resolution on the measurement of d. The information contained in the shape of the $F(d)$ distribution can

[2]The decay times are measured through the decay paths, and the time difference t is given by $t = (d_1 - d_1)\tau_S/d_S = d\,\tau_S/d_S$, where d_1 (d_2) is the decay path into charged (neutral) pions and $d_S \simeq 0.6$ cm is the mean decay path of the K_S (see the appendix).

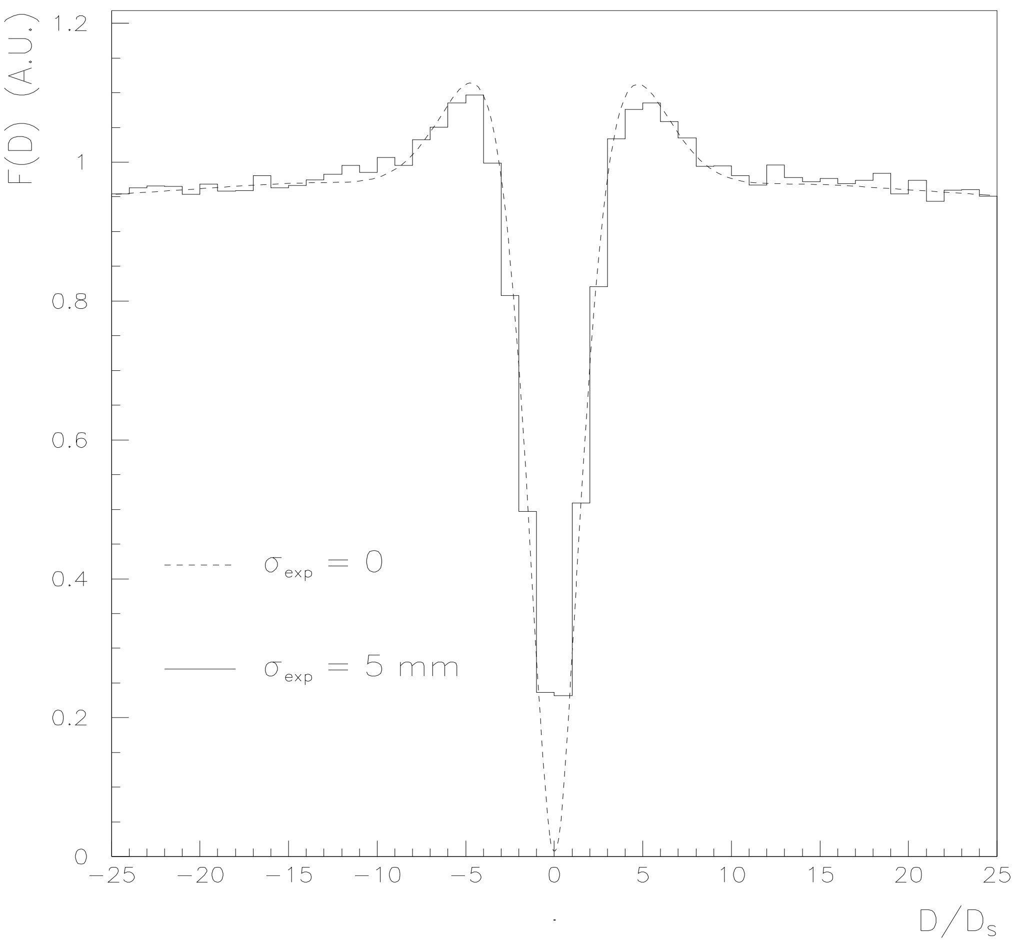

Figure 2: Comparison between the theoretical F(d) distribution for $\Re\left(\frac{\epsilon'}{\epsilon}\right) = 2.8 \times 10^{-4}$ and that obtained with an experimental vertex resolution $\sigma = 5$mm.

be easily washed out, in particular in the region of interest for the determination of $\Im\left(\frac{\epsilon'}{\epsilon}\right)$, where $d \simeq d_S$. In fact only in this range of d values $A_I(d)$ is different from zero and the strongly varying behaviour of $A_I(d)$ can be smeared out by a bad vertex reconstruction. This effect is shown in Fig. 2, where the theoretical distribution is compared with a simulated experimental distribution with a Gaussian error on the d measurement equal to 5 mm.[3]

The effects of the finite experimental resolution have been discussed, for example in [20], to which we refer. The results of the quoted analysis are that the determination of $\Re\left(\frac{\epsilon'}{\epsilon}\right)$ is practically unaffected by the experimental resolution, while the statistical error on $\Im\left(\frac{\epsilon'}{\epsilon}\right)$ increases by more than a factor 2. This analysis estimates that the accuracy achievable for a realistic detector is:

$$\sigma_{\Re\left(\frac{\epsilon'}{\epsilon}\right)} = 1.8 \times 10^{-4} \qquad ; \qquad \sigma_{\Im\left(\frac{\epsilon'}{\epsilon}\right)} = 3.4 \times 10^{-3}. \tag{16}$$

[3]We acknowledge V. Patera for providing us with the simulation of Fig. 2.

These numbers have to be compared with the present experimental situation shown in the introduction.

4 Semileptonic Decays

4.1 Theoretical introduction

We will discuss the semileptonic decays of neutral kaons in a very general framework, without assuming the $\Delta S = \Delta Q$ rule and the CPT symmetry.

The $\Delta S = \Delta Q$ rule is well supported by experimental data and is naturally accounted for by the Standard Model, where the $\Delta S = -\Delta Q$ transitions are possible only with two effective weak vertices. Explicit calculations give a suppression factor of about 10^{-6}–10^{-7} [21]. Furthermore in any quark model, $\Delta S = -\Delta Q$ transitions can be induced only by operators with dimension higher than 6 and therefore are suppressed [22].

Although it is very unlikely to have a theory with a large violation of the $\Delta S = \Delta Q$ rule, this does not conflict with any general principle. On the contrary CPT symmetry must hold in any Lorentz-invariant local field theory. The problem of possible sources of CPT violation has recently received much attention. Attempts to include also gravitation in the unification of fundamental interactions lead to non-local theories, like superstrings, which suggest possible CPT violation above the Planck mass, which turns out to be the natural suppression scale [23].

We neglect for the moment quantum mechanics violating effects [16], which will be discussed later, introducing CPT violation through an "ad hoc" parametrization of the decay amplitudes and the mass matrix elements.

Following the notations of Ref. [22] we define:

$$\begin{aligned} A(K^0 \to l^+ \nu \pi^-) &= a + b \\ A(K^0 \to l^- \nu \pi^+) &= c + d \\ A(\bar{K}^0 \to l^- \nu \pi^+) &= a^* - b^* \\ A(\bar{K}^0 \to l^+ \nu \pi^-) &= c^* - d^* \end{aligned} \tag{17}$$

CPT implies $b = d = 0$, CP implies $\Im(a) = \Im(c) = \Re(b) = \Re(d) = 0$, T requires real amplitudes and $\Delta S = \Delta Q$ implies $c = d = 0$.

Writing the mass matrix for the $K^0 \bar{K}^0$ system in the form:

$$\begin{pmatrix} M_{11} - i\Gamma_{11}/2 & M_{12} - i\Gamma_{12}/2 \\ & \\ M_{21} - i\Gamma_{21}/2 & M_{22} - i\Gamma_{22}/2 \end{pmatrix}, \tag{18}$$

the eigenstates are given by:

$$\begin{aligned} K_S &= \frac{1}{\sqrt{2\left(1+|\epsilon_S|^2\right)}}\left[\left(1+\epsilon_S\right)K^0+\left(1-\epsilon_S\right)\bar{K}^0\right] \\ K_L &= \frac{1}{\sqrt{2\left(1+|\epsilon_L|^2\right)}}\left[\left(1+\epsilon_L\right)K^0-\left(1-\epsilon_L\right)\bar{K}^0\right], \end{aligned} \tag{19}$$

where the ϵ_i parameters are:

$$\begin{aligned} \epsilon_S &= \frac{-i\Im\left(M_{12}\right)-\frac{1}{2}\Im\left(\Gamma_{12}\right)-\frac{1}{2}\left[M_{11}-M_{22}-\frac{i}{2}\left(\Gamma_{11}-\Gamma_{22}\right)\right]}{m_L-m_S+i(\Gamma_S-\Gamma_L)/2}=\epsilon_M+\Delta \\ \epsilon_L &= \frac{-i\Im\left(M_{12}\right)-\frac{1}{2}\Im\left(\Gamma_{12}\right)+\frac{1}{2}\left[M_{11}-M_{22}-\frac{i}{2}\left(\Gamma_{11}-\Gamma_{22}\right)\right]}{m_L-m_S+i(\Gamma_S-\Gamma_L)/2}=\epsilon_M-\Delta. \end{aligned} \tag{20}$$

Then the masses and widths are:

$$m_{S(L)}=\frac{M_{11}+M_{22}}{2}\pm\Re(M_{12}), \qquad \Gamma_{S(L)}=\frac{\Gamma_{11}+\Gamma_{22}}{2}\pm\Re(\Gamma_{12}). \tag{21}$$

CPT symmetry would require $M_{11}=M_{22}$ and $\Gamma_{11}=\Gamma_{22}$, recovering the relation $\epsilon_S=\epsilon_L=\epsilon_M$.

Using eqs. (17) and (19) the semileptonic partial rates are given by:

$$\begin{aligned} \Gamma_S^{l\pm} &= \frac{|a|^2}{2}\left[1\pm 2\Re(\epsilon_S)\pm 2\Re\left(\frac{b}{a}\right)+2\Re\left(\frac{c^*}{a}\right)\mp 2\Re\left(\frac{d^*}{a}\right)\right] \\ \Gamma_L^{l\pm} &= \frac{|a|^2}{2}\left[1\pm 2\Re(\epsilon_L)\pm 2\Re\left(\frac{b}{a}\right)-2\Re\left(\frac{c^*}{a}\right)\pm 2\Re\left(\frac{d^*}{a}\right)\right]; \end{aligned} \tag{22}$$

thus the charge asymmetries for K_S and K_L are:

$$\begin{aligned} \delta_S &= \frac{\Gamma_S^{l+}-\Gamma_S^{l-}}{\Gamma_S^{l+}+\Gamma_S^{l-}}=2\Re(\epsilon_S)+2\Re\left(\frac{b}{a}\right)-2\Re\left(\frac{d^*}{a}\right) \\ \delta_L &= \frac{\Gamma_L^{l+}-\Gamma_L^{l-}}{\Gamma_L^{l+}+\Gamma_L^{l-}}=2\Re(\epsilon_L)+2\Re\left(\frac{b}{a}\right)+2\Re\left(\frac{d^*}{a}\right). \end{aligned} \tag{23}$$

A non-vanishing value of the difference $\delta_S-\delta_L$ would be an evidence of CPT violation, either in the mass matrix or in the $\Delta S=-\Delta Q$ amplitudes (Δ and d^*/a cannot be disentangled by semileptonic decays alone). The sum $\delta_S+\delta_L$ has CPT-conserving ($\Re(\epsilon_M)$) and CPT-violating ($\Re(a/b)$) contributions that cannot be disentangled.

The ratio of K_S and K_L semileptonic widths

$$\eta = \frac{\Gamma^l_S}{\Gamma^l_L} = 1 + 4\Re\left(\frac{c^*}{a}\right), \tag{24}$$

where $\Gamma^l_{S(L)} = \Gamma^{l+}_{S(L)} + \Gamma^{l-}_{S(L)}$, allows us to determine the CPT-conserving part of the amplitudes with $\Delta S = -\Delta Q$.

All imaginary parts disappear from the rates and only the time evolution can potentially give some information on them.

4.2 Determination of semileptonic branching ratios at DAΦNE

The semileptonic branching ratios of the K_S can be measured at DAΦNE by selecting the following final states $|l^\pm\pi^\mp\nu(t_1), x_L(t_2)\rangle$, where $t_1 \leq 10\tau_S$, $t_2 \geq 10\tau_S$ and $|x_L\rangle$ is one of the allowed final states in K_L decays ($|\pi^+\pi^-\pi^0\rangle$ or $|l^\pm\pi^\mp\nu\rangle$). The probability of such events is obtained by integrating eq. (5)[4] in the appropriate time intervals[5]. Therefore the number of events for N_0 initial K_SK_L pairs is given by:

$$\begin{aligned} N_S(l^\pm) \quad &= \; N_0\Big\{\mathrm{Br}(K_S \to l^\pm\pi^\mp\nu)\mathrm{Br}(K_L \to x_L)S_1 \\ &+\mathrm{Br}(K_L \to l^\pm\pi^\mp\nu)\mathrm{Br}(K_S \to x_L)S_2 - \Re\Big[S_3 \int A(K_S \to l^\pm\pi^\mp\nu) \times \\ &\times A^*(K_L \to l^\pm\pi^\mp\nu)A(K_L \to x_L)A^*(K_S \to x_L)\frac{d\phi_{\pi l\nu}d\phi_{x_L}}{\Gamma_S\Gamma_L}\Big]\Big\}, \end{aligned} \tag{25}$$

where

$$\begin{aligned} S_1 = &\; (1-e^{-10})\left(e^{-10\frac{\Gamma_L}{\Gamma_S}} - e^{-\frac{D}{d_L}}\right) &&= 0.28 \simeq S_L \\ S_2 = &\; \left(1 - e^{-10\frac{\Gamma_L}{\Gamma_S}}\right)e^{-10} &&= 7.8 \times 10^{-7} \\ S_3 = &\; \frac{2\Gamma_S\Gamma_L}{|\Gamma|^2 + |\Delta m|^2}e^{-10\frac{(\Gamma + i\Delta m)}{\Gamma_S}} &&= (0.3 - i4.8) \times 10^{-5}. \end{aligned} \tag{26}$$

As can be seen, S_1 is by far the dominant contribution; the branching ratio products in eq. (25) are predicted to be of the same order, while the interference term should be further suppressed by large cancellations. Therefore inserting the experimental value [24] $\mathrm{Br}(K_L \to x_L) = (78.1 \pm 0.7)\%$ eq. (25) becomes:

$$N_S(l^\pm) = 0.22 \times N_0 \times \mathrm{Br}(K_S \to l^\pm\pi^\mp\nu). \tag{27}$$

The project luminosity of DAΦNE ($\mathcal{L} = 5 \times 10^{32}$ cm^{2}s^{-1}) gives about $8.6 \times 10^9 K_LK_S$/year. Using eq. (22) and the present upper limit on the violation of the $\Delta S = \Delta Q$ rule [24], we

[4]If CPT is not conserved the only change in time evolution equations concerns the normalization factor, which becomes $h = \sqrt{(1+|\epsilon_L|^2)(1+|\epsilon_S|^2)}/(1-\epsilon_L\epsilon_S) \simeq 1$.

[5]The general constraints on the time intervals for K_S tagging are: $t_1 \leq t_1^{max}$ and $t_2 \geq t_2^{min}$, with $\tau_S \ll t_1^{max} \leq t_2^{min} \ll \tau_L$. A good choice is given by $t_1^{max} = t_2^{min} = 10\tau_S$.

Parameter	PDG	CPLEAR		DAΦNE
		$\sigma('93)$	$\sigma('95)$	$\sigma(1\ \mathrm{yr})$
δ_L	$(3.27 \pm 0.12) \times 10^{-3}$	–		0.04×10^{-3}
δ_S	–	–		0.9×10^{-3}
$\Re\epsilon_S$	–	0.7×10^{-3}	0.4×10^{-3}	–
$\Re(c^*/a) = \Re x$	$(6 \pm 18) \times 10^{-3}$	8×10^{-3}	5×10^{-3}	3×10^{-3}
A_T	–	2×10^{-3}	1×10^{-3}	2×10^{-3}
A_{CPT}	–	2×10^{-3}	1×10^{-3}	2×10^{-3}

Table 1: Comparison between the present experimental data [24], CPLEAR present and expected sensitivity [25] and the achievable sensitivity in 1 year at DAΦNE, for the semileptonic parameters. For both CPLEAR and DAΦNE only the statistical error has been reported. Note that A_T and A_{CPT} asymmetries have different theoretical expressions, for CPLEAR and DAΦNE, if one considers CPT violation in the decay amplitudes.

estimate $\mathrm{Br}(K_S \to \mu^\pm\pi^\mp\nu) = 4.66 \times 10^{-4}$ and $\mathrm{Br}(K_S \to e^\pm\pi^\mp\nu) = 6.68 \times 10^{-4}$, therefore 2.1×10^6 events/year are expected.

With these numbers we can estimate the sensitivity of DAΦNE to CP, CPT and the $\Delta S = \Delta Q$ rule violating parameters defined in eqs. (23) and (24). Since the $\mu^+\pi^-\nu$ final state can hardly be distinguished from the $\mu^-\pi^+\bar{\nu}$ one, we conservatively assume that only electrons can be used to derive K_S charge asymmetry. In this case the number of event is 1.2×10^6/year and the statistical error on δ_S turns out to be $\sigma_{\delta_S} = 9.0 \times 10^{-4}$. Since the experimental value of K_L charge asymmetry is $\delta_L = (3.27 \pm 0.12) \times 10^{-3}$ [24], we expect $\sigma_{\delta_S - \delta_L}/\delta_L \simeq 0.28$, testing the CPT prediction $\delta_S = \delta_L$ at a significant level.

Eq. (24) (test of $\Delta S = \Delta Q$ rule) involves the semileptonic rates of K_S and K_L; thus to estimate the error on η one has to take into account also the experimental errors on tagging branching ratios and on $K_{S,L}$ widths. Using the values in Ref. [24], these effects give a large contribution to the total error, which turns out to be $\sigma_\eta = 1.1 \times 10^{-2}$, whereas the pure statistical contribution would give only $= 1.4 \times 10^{-3}$. This large value for σ_η will perhaps be lowered by measuring all the quantities involved in the same experimental set-up.

In Table 1 we report the predicted sensitivity of DAΦNE in comparison with other experiments. As one can see, DAΦNE is very powerful to test $\Delta S = \Delta Q$ rule.

4.3 Direct tests of T and CPT symmetries

The dilepton events allow direct tests of T and CPT symmetries [2, 11]. A long time ago Kabir [26] showed that T violation implies different probabilities for $K^0 \to \bar{K}^0$ and $\bar{K}^0 \to K^0$ transitions, while CPT requires equal probabilities for $K^0 \to K^0$ and $\bar{K}^0 \to \bar{K}^0$

transitions. Then a T-violating asymmetry:

$$A_T = \frac{P\left(\bar{K^0} \to K^0\right) - P\left(K^0 \to \bar{K^0}\right)}{P\left(\bar{K^0} \to K^0\right) + P\left(K^0 \to \bar{K^0}\right)} \tag{28}$$

and a CPT-violating one:

$$A_{CPT} = \frac{P\left(K^0 \to K^0\right) - P\left(\bar{K^0} \to \bar{K^0}\right)}{P\left(\bar{K^0} \to \bar{K^0}\right) + P\left(K^0 \to K^0\right)} \tag{29}$$

can be defined.

Both these tests can be done at a ϕ factory, where the initial state is an antisymmetric $K^0\bar{K^0}$ state, if the $\Delta S = \Delta Q$ rule holds.

If a neutral kaon decays into a positive lepton at a time t, the other neutral kaon is at the same time a $\bar{K^0}$ and the sign of the lepton, emitted in a subsequent semileptonic decay, signals if the $\bar{K^0}$ has changed or conserved its own flavour. Therefore, if $|A(K^0 \to l^+x)|^2 = |A(\bar{K^0} \to l^-x)|^2$, the charge asymmetry in equal-sign dilepton pairs measured at the ϕ factory will be equal to A_T. On the other hand, time asymmetry in opposite-sign dilepton pairs signals CPT violation.

In the more general case, taking into account also possible violations of the $\Delta S = \Delta Q$ rule one gets[6]:

$$A_T = \frac{L^{++} - L^{--}}{L^{++} + L^{--}} = 2\Re\left(\epsilon_L + \epsilon_S\right) + 4\Re\left(\frac{b}{a}\right) = \delta_L + \delta_S, \tag{30}$$

and

$$\begin{aligned} A_{CPT} &= \frac{L^{-+} - L^{+-}}{L^{-+} + L^{+-}} = 2\Re\left(\epsilon_L - \epsilon_S\right) + 4\Re\left(\frac{d^*}{a}\right) + 4\Re\left(\epsilon_L + \epsilon_S\right)\Re\left(\frac{c^*}{a}\right) \\ &+ \frac{4}{S_L}\left[\Im\left(\epsilon_L - \epsilon_S\right) - 2\Im\left(\frac{c^*}{a}\right) + 2\Im\left(\epsilon_L + \epsilon_S\right)\Re\left(\frac{c^*}{a}\right)\right]\frac{\Delta m \Gamma_L}{\Gamma^2 + \Delta m^2}, \end{aligned} \tag{31}$$

where L^{+-} (L^{-+}) is the number of dilepton pairs with the positive lepton emitted before (after) the negative one. The number of equal-sign electron pairs ($L^{++} + L^{--}$) and that of opposite-sign ($L^{+-} + L^{-+}$) expected at DAΦNE is about 3.3×10^5 events/year, therefore the T- and CPT-violating asymmetries can be measured with a statistical error of about 1.7×10^{-3}.

Violation of the $\Delta S = \Delta Q$ rule does not affect eq. (30) but the CPT violation in the decay amplitude contributes together with the true T-violating term $(\epsilon_L + \epsilon_S)$. On the contrary in eq. (31) the effects of CPT violation and $\Delta S = -\Delta Q$ transitions cannot be disentangled. In the CPT limit the time asymmetry can be written as:

$$\bar{A}_{CPT} = 8\Re(\epsilon_M)\Re\left(\frac{c^*}{a}\right) - 8\frac{\Im\left(\frac{c^*}{a}\right) - 2\Im\epsilon_M\Re\left(\frac{c^*}{a}\right)}{S_L}\frac{\Delta m \Gamma_L}{\Gamma^2 + \Delta m^2} \tag{32}$$

[6]In the following equations we include also the CPT-conserving higher-order terms, namely the ones proportional to $\epsilon_M \frac{c^*}{a}$.

and inserting the experimental limits on c^*/a [24] one has:

$$|\bar{A}_{CPT}| < 1.1 \times 10^{-3}. \tag{33}$$

Thus a value of A_{CPT} larger than 10^{-3} indicates an actual CPT violation either in the kaon mass matrix or in $\Delta S = -\Delta Q$ transition amplitudes.

More information can be obtained by the study of the time dependence of opposite sign dilepton events. Choosing for the final states of eq. (8) $|a\rangle = |e^+\pi^-\nu\rangle$ and $|b\rangle = |e^-\pi^+\bar{\nu}\rangle$ and integrating over the phase space one gets:

$$\begin{aligned} I(e^+, e^-; t) = \frac{\Gamma_L^e \Gamma_S^e}{8\Gamma} e^{-\Gamma|t|} \Bigg\{ & \left[1 + 4\Re\Delta - 4\Re\left(\frac{d^*}{a}\right) - 8\Re\epsilon_M \Re\left(\frac{c^*}{a}\right)\right] e^{-\frac{\Delta\Gamma}{2}t} \\ & + \left[1 - 4\Re\Delta + 4\Re\left(\frac{d^*}{a}\right) + 8\Re\epsilon_M \Re\left(\frac{c^*}{a}\right)\right] e^{+\frac{\Delta\Gamma}{2}t} + 2\cos(\Delta m t) \\ & -8\left[\Im\Delta + \Im\left(\frac{c^*}{a}\right) - 2\Im\epsilon_M \Re\left(\frac{c^*}{a}\right)\right] \sin(\Delta m t) \Bigg\} \\ = \frac{\Gamma_L^e \Gamma_S^e}{4\Gamma} e^{-\Gamma|t|} & \left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) + \cos(\Delta m t) - 4\Delta_R \sinh\left(\frac{\Delta\Gamma t}{2}\right) - 4\Delta_I \sin(\Delta m t)\right] \end{aligned} \tag{34}$$

where

$$\Delta_R = \Re\Delta - \Re\left(\frac{d^*}{a}\right) - 2\Re\epsilon_M \Re\left(\frac{c^*}{a}\right), \quad \Delta_I = \Im\Delta + \Im\left(\frac{c^*}{a}\right) - 2\Im\epsilon_M \Re\left(\frac{c^*}{a}\right). \tag{35}$$

The difference in the asymptotic limits ($|t| \gg \tau_S$) leads to the determination of Δ_R, while the interference term singles out Δ_I. The higher-order terms can be neglected (their upper bound is about 0.6×10^{-4}, smaller than the DAΦNE sensitivity), but the CPT-violating parameter Δ and the $\Delta S = -\Delta Q$ contributions are still mixed. An exact determination of the statistical error on Δ_R and Δ_I would require a simulation of the experimental apparatus, which is beyond the purpose of this work. To give an idea of the DAΦNE sensitivity we report in Fig. 3 the asymmetry in opposite-sign dileptons as a function of the time difference

$$A_{CPT}(t) = \frac{I(e^+, e^-; |t|) - I(e^+, e^-; -|t|)}{I(e^+, e^-; |t|) + I(e^+, e^-; -|t|)}, \tag{36}$$

for $\Delta_I = 0$ and $\Delta_I = \pm 5\Delta_R$. As can be seen the asymptotic value is reached very soon and the three curves are clearly distinct. Therefore we estimate $\sigma_{\Delta_R} \simeq \sigma_{A_{CPT}}/4 \simeq 5 \times 10^{-4}$. The value of σ_{Δ_I} depends critically on the experimental resolution. We estimate that, as happens for the real and the imaginary parts of $\frac{\epsilon'}{\epsilon}$, σ_{Δ_I} will be about 20 times larger than σ_{Δ_R}.

As shown in Ref. [22], the inclusion in the analysis of the $K \to \pi\pi$ decays allows us to disentangle almost all the amplitudes. Indeed, in the Wu-Yang phase convention,[7] unitarity

[7] The $\pi\pi$ decay amplitudes are parametrized in each isospin channel like the semileptonic ones, A_I is the CPT-conserving part and B_I the CPT-violating one. The Wu-Yang convention is $\Im(A_0) = 0$.

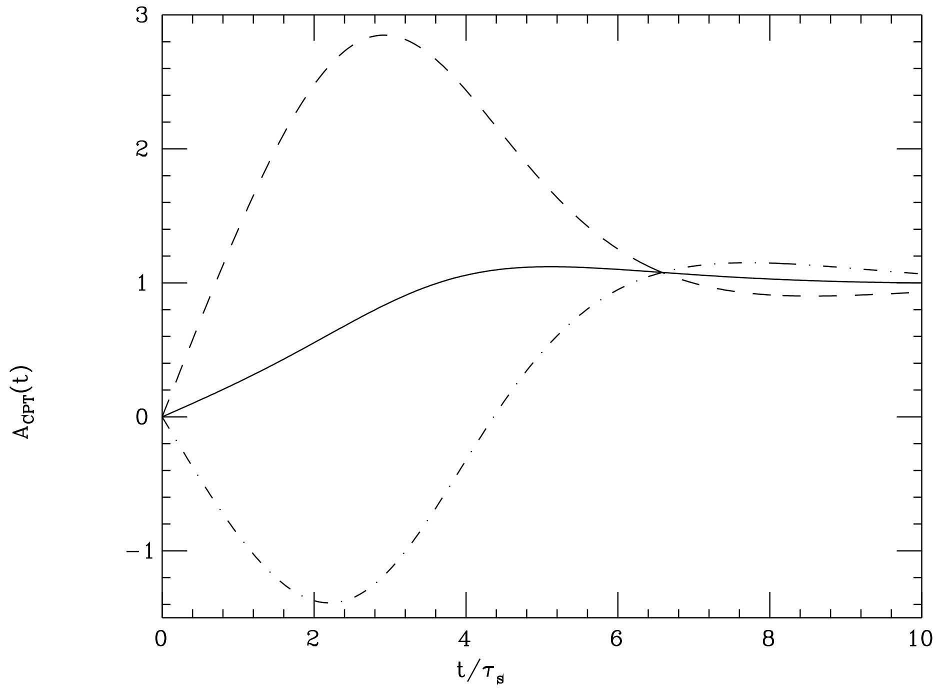

Figure 3: The asymmetry $A_{CPT}(t)$ as a function of the time difference for $\Delta_R = 1/4$. The full, dashed and dot-dashed lines correspond to $\Delta_I = 0$, $\Delta_I = 5\Delta_R$ and $\Delta_I = -5\Delta_R$ respectively.

implies that the phase of ϵ_M is equal to $\phi_{SW} \equiv \arctan(2\Delta m/\Delta\Gamma) = (43.64 \pm 0.15)^\circ$ and the phase of $[\Delta - \Re(B_0)/A_0]$ is $\left[\phi_{SW} \pm \frac{\pi}{2}\right]$; therefore, one has [11, 22]:

$$\frac{1}{3}(2\eta^{+-} + \eta^{00}) = |\epsilon_M| e^{i\phi_{SW}} \pm i|\Delta - \frac{\Re B_0}{A_0}| e^{i\phi_{SW}}. \tag{37}$$

The present experimental data on η^{+-} and η^{00} [24] give:

$$|\Delta - \frac{\Re B_0}{A_0}| = (1.7 \pm 2.8) \times 10^{-5} \qquad \text{and} \qquad |\epsilon_M| = (2.266 \pm 0.017) \times 10^{-3}. \tag{38}$$

As can be seen, the CPT constraint is at present very well satisfied and, assuming CPT conservation in decay amplitudes, the limit in $K^0\bar{K}^0$ mass difference is

$$\frac{|M_{11} - M_{22}|}{m_K} \leq 10^{-18}, \tag{39}$$

close to the natural scale factor m_K/M_{Planck}.

In addition, from the measured value of K_L charge asymmetry one gets:

$$-\Re\Delta + \Re\left(\frac{b}{a}\right) + \Re\left(\frac{d^*}{a}\right) = (-0.06 \pm 0.6) \times 10^{-4}. \tag{40}$$

The future measurement of δ_S at DAΦNE would lead also to the determination of $\Re(b/a)$, while the CPLEAR experiment will give direct measurement of $\Re\epsilon_S$ and of $\Im(c^*/a)$. Therefore all the parameters that appear in the observables introduced above can be disentangled,

Equations	Parameters	σ
$\Re\Delta = \Re\epsilon_S - \Re\epsilon_M$	$\Re\Delta$	4×10^{-4}
$\frac{\Re B_0}{A_0} = \Re\epsilon_S - \Re\epsilon_M - \Re w$	$\frac{\Re B_0}{A_0}$	4×10^{-4}
$\Re(\frac{b}{a}) = \frac{1}{4}(\delta_L + \delta_S) - \Re\epsilon_M$	$\Re(\frac{b}{a})$	2×10^{-4}
$\Re(\frac{d^*}{a}) = \Re\epsilon_S - \Re\epsilon_M + \frac{1}{4}(\delta_L - \delta_S)$	$\Re(\frac{d^*}{a})$	5×10^{-4}
$\frac{1}{4}(\delta_L + \delta_S) - \frac{1}{4}A_T = 0$	–	6×10^{-4}
$\frac{1}{4}(\delta_L - \delta_S) + \Delta_R = 0$	–	6×10^{-4}
$\Im(\frac{c^*}{a}) + \Im w - \Delta_I = 0$	–	$\sim 10^{-2}$

Table 2: Table 2: Statistical errors on parameters and consistency relations, using present experimental data [24] (for η^{+-}, η^{00} and δ_L) together with DAΦNE (for δ_S, A_T, Δ_R and Δ_I) and CPLEAR (for $\Re\epsilon_S$ and $\Im(c^*/a)$) future results. The σ of the last equation in the table is only a guess.

and some consistency relations must be satisfied. The preliminary data of the CPLEAR collaboration [25] have large errors and still do not give significant bounds. We report in Table 2 the relations between the observables and the theoretical parameters with the corresponding statistical errors from present and future experiments, together with the consistency equations and the corresponding sensitivity. To simplify the notations of the table we define

$$w = \Delta - \frac{\Re B_0}{A_0} = \epsilon_M - \frac{1}{3}(2\eta^{+-} + \eta^{00}). \tag{41}$$

5 $K_S \to 3\pi$

The $K_S \to 3\pi^0$ decay is a pure CP-violating transition, while the $K_S \to \pi^+\pi^-\pi^0$ decay receives both CP-conserving and CP-violating contributions.

The CP-conserving $K \to 3\pi$ decay amplitudes are well described by Chiral Perturbation Theory (ChPT). They have been calculated, including the next-to-leading-order corrections, in Ref. [27] and turn out to be in good agreement with the experimental data. The CP-conserving $K_S \to \pi^+\pi^-\pi^0$ decay amplitude is odd under $\pi^+ - \pi^-$ momenta exchange and thus, neglecting final states with high angular momenta, it is induced by a $\Delta I = 3/2$ transition. The ChPT calculation of Ref. [27] leads to the prediction:

$$\mathrm{Br}(K_S \to \pi^+\pi^-\pi^0)_{CP+} = (2.4 \pm 0.7) \times 10^{-7}, \tag{42}$$

consistent with the recent data:

$$\mathrm{Br}(K_S \to \pi^+\pi^-\pi^0)_{CP+} = (3.9^{+5.4}_{-1.8}\,{}^{+0.9}_{-0.7}) \times 10^{-7}, \quad \text{E621[28]} \tag{43}$$

$$\mathrm{Br}(K_S \to \pi^+\pi^-\pi^0)_{CP+} = (7.8^{+5.7}_{-4.1}\,{}^{+7.3}_{-4.9}) \times 10^{-7}. \quad \text{CPLEAR[25]} \tag{44}$$

As in $K \to 2\pi$ decays, for the CP-violating amplitudes it is convenient to define the ratios:

$$\eta^{+-0} = \frac{(A_S^{+-0})_{CP-}}{A_L^{+-0}} = \epsilon_S + \epsilon'_{+-0}, \tag{45}$$

$$\eta^{000} = \frac{A_S^{000}}{A_L^{000}} = \epsilon_S + \epsilon'_{000}. \tag{46}$$

The direct CP-violating parameters ϵ'_{+-0} and ϵ'_{000} have been evaluated at lowest order in ChPT [29] and turn out to be of the same order as ϵ'. As shown in [30], higher-order terms can substantially enhance ϵ'_{+-0} and ϵ'_{000}, which are nevertheless negligible[8] compared to ϵ. The predicted branching ratios are:

$$\mathrm{Br}\,(K_S \to (\pi^+\pi^-\pi^0)_{CP-}) \simeq 1.06 \times 10^{-9}, \tag{47}$$

$$\mathrm{Br}(K_S \to 3\pi^0) \simeq 1.94 \times 10^{-9}, \tag{48}$$

much smaller then the present upper limits [24, 25, 31].

Due to the smallness of the branching ratios it is very hard to detect $K_S \to 3\pi$ decays, especially the CP-violating ones. Tagging the K_S as in the case of the semileptonic decays (eqs. (25) and (26)) and inserting the numerical values, one gets for $3\pi^0$ final state:

$$N_S(3\pi^0) = N_0\mathrm{Br}(K_L \to 3\pi^0)[3.8 \times 10^{-4}|\eta^{000}|^2 + 8.9 \times 10^{-10} - 1.8 \times 10^{-10}\Im(\eta^{000})]. \tag{49}$$

The total number of events is very small ($\simeq$ 6 per year) and the ratio of right events (those with a $K_S \to 3\pi^0$ decay) to wrong ones (those with a $K_L \to 3\pi^0$ decay) is only 2.2.

In the case of the CP-conserving $K_S \to \pi^+\pi^-\pi^0$ decay, the expected number of events is about 440 with a negligible background.

A more promising way to detect the CP-violating $K_S \to 3\pi$ decays is to study the interference terms of $I(a, b; t)$, in eq. (8), choosing $|a\rangle = |3\pi\rangle$ and $|b\rangle = |l^\pm\pi^\mp\nu\rangle$, as suggested in Refs. [32, 33]. For the $K_S \to 3\pi^0$ it is useful to define the asymmetry:

$$A^{000}(t) = \frac{\int [I(3\pi^0, l^+\pi^-\nu; t) - I(3\pi^0, l^-\pi^+\nu; t)]\, d\phi_{3\pi}\, d\phi_{l\pi\nu}}{\int [I(3\pi^0, l^+\pi^-\nu; t) + I(3\pi^0, l^-\pi^+\nu; t)]\, d\phi_{3\pi}\, d\phi_{l\pi\nu}} \tag{50}$$

which, integrating over the 3π and $\pi l\nu$ Dalitz plots, becomes:

$$A^{000}(t) = \frac{2\Re\epsilon e^{+\frac{\Delta\Gamma}{2}t} - 2\Re\left(\eta^{000}e^{+i\Delta mt}\right)}{e^{+\frac{\Delta\Gamma}{2}t} + |\eta^{000}|^2 e^{-\frac{\Delta\Gamma}{2}t}}. \tag{51}$$

For positive and large values of the time difference t, eq. (51) reads: $A^{000} \simeq 2\Re\epsilon$; on the other hand, for negative value of t, one gets an interesting interference effect between ϵ and η^{000}, as shown in Fig. 4. The asymmetry for $t < 0$ is quite large, but the total number of events is small, about 2×10^3 per year.

[8] In $K \to 3\pi$ we will neglect possible CPT-violating effects, thus in the following we will assume $\epsilon_S = \epsilon_L = \epsilon$.

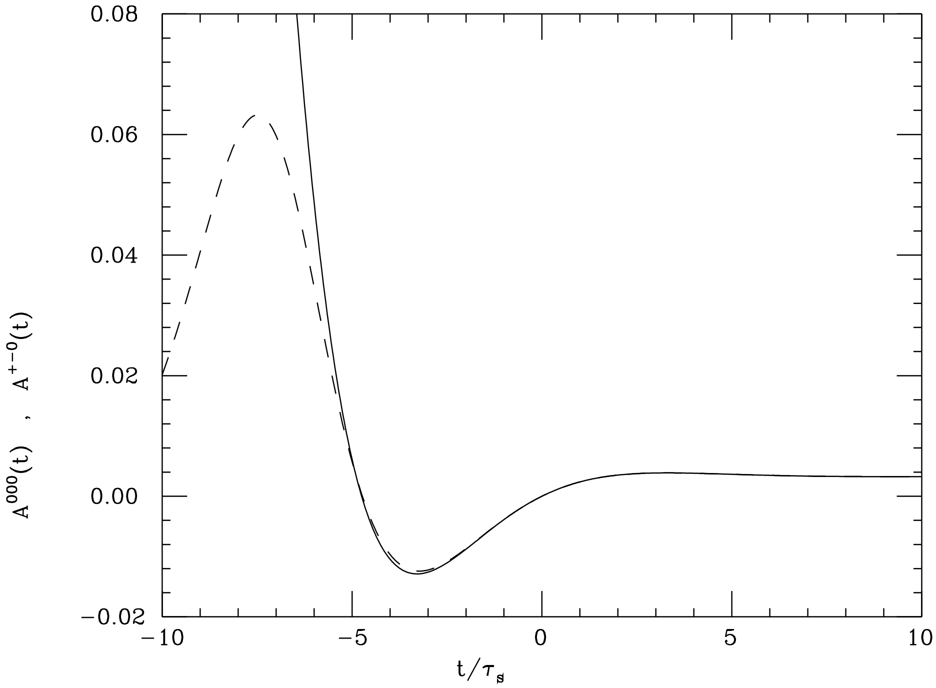

Figure 4: The asymmetries $A^{000}(t)$ (full line) and $A^{+-0}(t)$ (dashed line). We have fixed $\eta^{000} = \eta^{+-0} = |\epsilon| e^{i\phi_{SW}}$.

In the case of the $\pi^+\pi^-\pi^0$ final state, the CP-violating and CP-conserving amplitudes have opposite symmetry under $\pi^+ - \pi^-$ momentum exchange. Therefore it is possible to select the CP-violating and the CP-conserving part of the interference term in eq.(8) with an even or an odd integration over the 3π Dalitz plot. Analogously to the $3\pi^0$ case, for the CP-violating part we define the asymmetry:

$$\begin{aligned} A^{+-0}(t) &= \frac{\int [I(\pi^+\pi^-\pi^0, l^+\pi^-\nu; t) - I(\pi^+\pi^-\pi^0, l^-\pi^+\nu; t)]\, d\phi_{3\pi}\, d\phi_{l\pi\nu}}{\int [I(\pi^+\pi^-\pi^0, l^+\pi^-\nu; t) + I(\pi^+\pi^-\pi^0, l^-\pi^+\nu; t)]\, d\phi_{3\pi}\, d\phi_{l\pi\nu}}, \\ &= \frac{2(\Re\epsilon) e^{+\frac{\Delta\Gamma}{2}t} - 2\Re\left(\eta^{+-0} e^{+i\Delta m t}\right)}{e^{+\frac{\Delta\Gamma}{2}t} + \frac{\Gamma_S^{+-0}}{\Gamma_L^{+-0}} e^{-\frac{\Delta\Gamma}{2}t}}, \end{aligned} \tag{52}$$

while the CP-conserving part can be singled out by the ratio:

$$\begin{aligned} R^{\pm}(t) &= \frac{\int_+ I(\pi^+\pi^-\pi^0, l^\pm\pi^\mp\nu; t) d\phi_{3\pi}\, d\phi_{l\pi\nu} - \int_- I(\pi^+\pi^-\pi^0, l^\pm\pi^\mp\nu; t) d\phi_{3\pi}\, d\phi_{l\pi\nu}}{\int [I(\pi^+\pi^-\pi^0, l^\pm\pi^\mp\nu; t)]\, d\phi_{3\pi}\, d\phi_{l\pi\nu}}, \\ &= \pm 2 \frac{\int_+ \Re\left(A_L^{+-0} A_S^{+-0*}\right) d\phi_{3\pi}}{\Gamma_L^{+-0} e^{+\frac{\Delta\Gamma}{2}t} + \Gamma_S^{+-0} e^{-\frac{\Delta\Gamma}{2}t}} \left[\cos(\Delta m t) + \tilde{\delta} \sin(\Delta m t)\right], \end{aligned} \tag{53}$$

where $\int_\pm d\phi_{3\pi}$ indicates the integration in the region $E_{\pi^\pm} > E_{\pi^\mp}$.

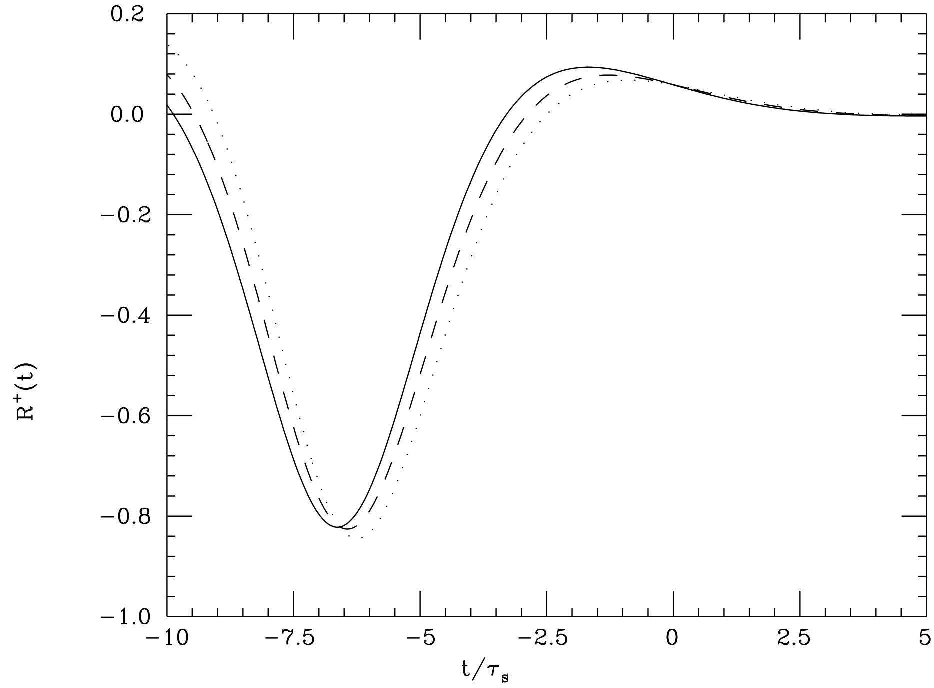

Figure 5: The ratio $R^+(t)$ defined in eq.(53). The full, dashed and dotted lines correspond to $\tilde{\delta} = 0$, $\tilde{\delta} = 0.2$ and $\tilde{\delta} = 0.4$ respectively.

The behaviour of $A^{+-0}(t)$ is completely analogous to the one of $A^{000}(t)$ (Fig. 4). As discussed in refs. [32, 33] the study of $R^{\pm}(t)$ will certainly lead to a determination of the $(A_S^{+-0})_{CP+}$ amplitude, performing an interesting test of ChPT in the $\Delta I = 3/2$ transitions, and perhaps could also lead to a direct measurement of the $\pi^+\pi^-\pi^0$ rescattering functions. The phase $\tilde{\delta}$ of eq. (53) can be written as:

$$\tilde{\delta} \simeq \alpha_0 - \delta_0, \tag{54}$$

where α_0 and δ_0 are the first terms in the expansion of the 3π rescattering functions of the $I = 2$ and of the symmetric $I = 1$ final states, respectively. The ChPT prediction is $\tilde{\delta} = (10 \pm 1)^\circ$ [33] and the first measurement [28] gives $\tilde{\delta} = (59 \pm 48)^\circ$. With a different integration over the Dalitz plot, also the rescattering function of the non-symmetric $I = 1$ final state could be selected [32, 33]. Figure 5 shows the behaviour of $R^{\pm}$ for different values of $\tilde{\delta}$.

6 Interference in $K \to \pi\pi\gamma$

In this section we will discuss the possibility to detect CP violation through the study of the time difference distribution $I(a, b; t)$, defined in eq. (8), choosing $|a\rangle = |\pi^+\pi^-\gamma\rangle$ and $|b\rangle = |\pi^\pm l^\mp \nu\rangle$ [32]. To this purpose we recall some useful decomposition of the $K \to \pi^+\pi^-\gamma$ decay amplitude, referring to [9] for a more detailed discussion of these decays.

The amplitude for $K(p_K) \to \pi^+(p_+)\pi^-(p_-)\gamma(q,\varepsilon)$ decays can be generally decomposed as the sum of two terms: the inner bremsstrahlung (A_{IB}) and the direct emission (A_{DE}) [9]. The first term, which has a pole at zero photon energy, is completely predicted by QED in terms of the $K \to \pi^+\pi^-$ amplitude [34]:

$$A_{IB}(K_{S,L} \to \pi^+\pi^-\gamma) = e\left(\frac{\varepsilon \cdot p_-}{q \cdot p_-} - \frac{\varepsilon \cdot p_+}{q \cdot p_+}\right) A(K_{S,L} \to \pi^+\pi^-). \tag{55}$$

The second term, which is obtained by subtracting A_{IB} from the total amplitude, depends on the structure of the $K \to \pi^+\pi^-\gamma$ effective vertex and provides a test for mesonic interaction models.

The $K \to \pi^+\pi^-\gamma$ amplitude is usually decomposed also in a different way, separating the electric and the magnetic terms. Defining the dimensionless amplitudes E and M as in [35], we can write:

$$A(K_{S,L} \to \pi^+\pi^-\gamma) = \varepsilon_\mu(q)\left[E(z_i)(p_+ \cdot q\, p_-^\mu - p_- \cdot q\, p_+^\mu) + M(z_i)\epsilon^{\mu\nu\rho\sigma} p_{+\nu} p_{-\rho} q_\sigma\right]/m_K^3, \tag{56}$$

where

$$z_\pm = \frac{p_\pm \cdot q}{m_K^2}, \qquad \text{and} \qquad z_3 = \frac{p_K \cdot q}{m_K^2} = z_+ + z_-. \tag{57}$$

As can be seen from eq. (55), the inner bremsstrahlung amplitude can contribute only to the $E(z_i)$ term, while the direct emission amplitude can contribute to both the $E(z_i)$ and the $M(z_i)$ terms. Summing over photon helicities there is no interference between electric and magnetic terms:

$$\mathrm{d}\Gamma = \frac{m_K}{(4\pi)^3}(|(E(z_i)|^2 + |M(z_i)|^2)\left[z_+z_-(1 - 2z_3 - 2r_m^2) - r_m^2(z_+^2 + z_-^2)\right]\mathrm{d}z_+\mathrm{d}z_- \tag{58}$$

($r_m = m_\pi/m_K$). Thus the two contributions A_{IB} and $A_{DE}^{electric}$ can interfere in the $E(z_i)$ amplitude, contrary to the case of the amplitude $M(z_i)$ where only a direct emission contribution appears:

$$|A(K_{S,L} \to \pi^+\pi^-\gamma)|^2 = |A_{IB}|^2 + 2\Re\{A_{IB}^* A_{DE}^{electric}\} + |A_{DE}^{electric}|^2 + |A_{DE}^{magnetic}|^2. \tag{59}$$

Finally the magnetic and the electric direct emission amplitudes can be decomposed in a multipole expansion (see Refs. [9, 36]). Since higher multipoles are suppressed by angular momentum barrier, in the following we will consider only the lowest multipole component (the dipole one). In this approximation the electric amplitude is CP-conserving in the K_S decay and CP-violating in the K_L one, while the magnetic amplitude is CP-conserving in the K_L decay and CP-violating in the K_S one. For this reason, since A_{IB} is enhanced by the pole at zero photon energy, the K_S decay is completely dominated by the electric transition, while electric and magnetic contributions are of the same order in the K_L decay.

Similar to $K \to 2\pi$ and $K \to 3\pi$ cases it is convenient to introduce the CP-violating parameter:

$$\eta_{+-\gamma} = \frac{A(K_L \to \pi^+\pi^-\gamma)_{IB+E1}}{A(K_S \to \pi^+\pi^-\gamma)_{IB+E1}}, \tag{60}$$

where the subscript E_1 indicates that only the lowest multipole component of the electric direct emission amplitude has been considered. Using eq. (55) we can write:

$$\eta_{+-\gamma} = \eta_{+-} + \epsilon'_{\pi\pi\gamma} \frac{A(K_S \to \pi^+\pi^-\gamma)_{E1}}{A(K_S \to \pi^+\pi^-\gamma)_{IB+E1}} \simeq \eta_{+-} + \epsilon'_{\pi\pi\gamma} \frac{A(K_S \to \pi^+\pi^-\gamma)_{E1}}{A(K_S \to \pi^+\pi^-\gamma)_{IB}}, \qquad (61)$$

where η^{+-} is the usual $K \to 2\pi$ CP-violating parameter. The term proportional to $\epsilon'_{\pi\pi\gamma}$ in eq. (61) is a direct CP-violating contribution, not related to the $K \to \pi\pi$ amplitude and consequently not suppressed by the 1/22 factor of the $\Delta I = 1/2$ rule. However, although $\epsilon'_{\pi\pi\gamma}$ could be much larger than $\epsilon'_{\pi\pi}$, the second term in eq. (61) is suppressed by the factor $A_{E1}/A_{IB} \ll 1$.

The $\eta_{+-\gamma}$ parameter has already been measured at Fermilab obtaining for the IB contribution [37]

$$|\eta_{+-\gamma(IB)}| = \left|\frac{A(K_L \to \pi^+\pi^-\gamma)_{IB}}{A(K_S \to \pi^+\pi^-\gamma)_{IB}}\right| = (2.414 \pm 0.065 \pm 0.062) \times 10^{-3}, \qquad (62)$$

$$\phi_{+-\gamma(IB)} = \arg(\eta_{+-\gamma(IB)}) = (45.47 \pm 3.61 \pm 2.40)^\circ. \qquad (63)$$

DAΦNE should improve these limits by studying the time evolution of the decay.

Referring to [32] for an extensive analysis, here we show how to take advantage of the ϕ-factories possibilities to measure $\eta_{+-\gamma}$. Choosing as final states $f_1 = \pi^\pm l^\mp \nu$, $f_2 = \pi^+\pi^-\gamma$ and following the notation of Section 2, the time difference distribution, integrated over final phase space, is given by[9]:

$$I(\pi^\pm l^\mp \nu, \pi^+\pi^-\gamma; t<0) = \frac{\Gamma(K_L \to \pi^\pm l^\mp \nu)\Gamma(K_S \to \pi^+\pi^-\gamma)}{2\Gamma}\Big\{R_L e^{-\Gamma_L|t|} + e^{-\Gamma_S|t|}$$
$$\pm \frac{2\, e^{-\Gamma|t|}}{\Gamma(K_S \to \pi^+\pi^-\gamma)}\Big[\Re\langle E\rangle_{int} \cos(\Delta m|t|) + \Im\langle E\rangle_{int} \sin(\Delta m|t|)\Big]\Big\}, \qquad (64)$$

where $R_L = \Gamma(K_L \to \pi^+\pi^-\gamma)/\Gamma(K_S \to \pi^+\pi^-\gamma)$ and

$$\langle E\rangle_{int} \equiv \frac{m_K}{(4\pi)^3}\int\int dz_1 dz_2 (E^S_{IB} + E^S_1)^*(E^L_{IB} + E^L_1) \times$$
$$\times[z_+ z_-(1 - 2z_3 - 2r_m^2) - r_m^2(z_+^2 + z_-^2)]. \qquad (65)$$

Neglecting the phase space dependence of $\eta_{+-\gamma}$ one should have $\langle E\rangle_{int} = \eta_{+-\gamma}\Gamma(K_S \to \pi^+\pi^-\gamma)$, and therefore the interference term of eq. (64) measures the CP-violating $K_L \to \pi^+\pi^-\gamma$ amplitude. The expression for $t > 0$ is obtained by interchanging $\Gamma_S \leftrightarrow \Gamma_L$ and changing the sign of the $\sin(\Delta m|t|)$ term. By fitting the experimental data with the theoretical expression of eq. (64), one should be able to measure the interference term and then improve the measurement of $\eta_{+-\gamma}$. Very useful to this purpose will be the difference among the fluxes defined in eq. (64) with positive and negative lepton charges, as discussed for $K_S \to 3\pi$ decays.

To conclude, we remark that not only the semileptonic tagging but also the $K \to 2\pi$ one can be used to measure $\eta_{+-\gamma}$ [32].

[9] In the following we neglect possible violations of CPT and of the $\Delta S = \Delta Q$ rule.

7 C-even background

Some years ago it was pointed out that the radiative ϕ decay $(\phi \to \gamma f_0(980) \to \gamma(K^0\bar{K}^0)_{C=+})$ could have a non-negligible branching ratio and therefore could spoil the power of a ϕ factory in measuring $\frac{\epsilon'}{\epsilon}$ and to single out the suppressed K_S decays [12]. More recent determinations of the resonant contribution consider also the $a_0(980)$, $f_0(980)$ interference effect and give a much lower value: the ratio

$$r = \frac{\text{Br}(\phi \to \gamma\,(K^0\bar{K}^0)_{C+})}{\text{Br}(\phi \to (K^0\bar{K}^0)_{C-})}, \tag{66}$$

is estimated to be in the range (3×10^{-7})–(5×10^{-9}) [13].

These predictions are strongly model-dependent and larger values could perhaps be obtained; however we can trust that r is certainly smaller than 10^{-4}. The non-resonant contribution to $\phi \to \gamma\,K^0\bar{K}^0$ has been evaluated in the current algebra framework [38] to be of the order of 10^{-9}, comparable to the lower predictions of [13]. As we will show, the effects of the C-even background on the DAΦNE measurements are negligible also for unrealistically large values of r.

The C-even $K^0\bar{K}^0$ state can be written as:

$$\begin{aligned} &\frac{1}{\sqrt{2}}[K^{0(q)}\bar{K}^{0(-q)} + \bar{K}^{0(q)}K^{0(-q)}] \\ &\simeq \frac{1}{\sqrt{2}}[K_S^{(q)}K_S^{(-q)} - K_L^{(q)}K_L^{(-q)} - 2\Delta(K_S^{(q)}K_L^{(-q)} + K_L^{(q)}K_S^{(-q)})], \end{aligned} \tag{67}$$

where terms of order ϵ^2 have been neglected but the effect of possible CPT violation has been included.

The K_SK_S component has a CP-conserving decay into the $|2\pi, 2\pi\rangle$ final states and could be a dangerous background in the $\frac{\epsilon'}{\epsilon}$ measurement. However, the time difference distribution of these events,

$$\overline{F(t)} = \frac{\Gamma_S^{+-}\Gamma_S^{00}}{2\Gamma_S} e^{-\Gamma_S|t|}, \tag{68}$$

is symmetric and cannot simulate the effect of ϵ'. Furthermore, it vanishes very rapidly for large values of $|t|$, and therefore it does not affect the determination of $\Re(\frac{\epsilon'}{\epsilon})$. In effect, as shown in [39], by means of a suitable cut the background contribution can be eliminated in the event sample used to determine $\Re(\frac{\epsilon'}{\epsilon})$, also for very large values of r.

The background contribution overlaps the signal just in the interference zone, $d \simeq d_S$, worsening the resolution on $\Im\left(\frac{\epsilon'}{\epsilon}\right)$. However the signal $(K_LK_S \to \pi^0\pi^0, \pi^+\pi^-)$ and the background $(K_SK_S \to \pi^0\pi^0, \pi^+\pi^-)$ have different spatial behaviour, which is of help in disentangling the signal contribution from the background. The C-even background has been added to the signal in the fitting procedure of [20] and the accuracy achievable on $\Im\left(\frac{\epsilon'}{\epsilon}\right)$ has been estimated again. The result is that for a realistic vertex resolution the worsening is around 5%, even if r would be as large as 10^{-4}.

The $K_L K_L$ component can affect the determination of the suppressed K_S branching ratios and the corresponding CP-violating asymmetries, as we will discuss in the following.

If the K_S semileptonic decays are tagged as in eq. (25), the number of events generated by a single C-even $K^0\bar{K}^0$ pair is:

$$\overline{N(l^\pm)} = \text{Br}(K_L \to l^\pm\pi^\mp\nu)\text{Br}(K_L \to x_L)S_1(1 - e^{-10\Gamma_L/\Gamma_S}), \tag{69}$$

then eq. (27) is modified in:

$$N_S(l^\pm)_{exp} = 0.22 \times N_0 \left[\text{Br}(K_S \to l^\pm\pi^\mp\nu) + r \cdot \frac{10\Gamma_L}{\Gamma_S}\text{Br}(K_L \to l^\pm\pi^\mp\nu)\right], \tag{70}$$

and the measured charge asymmetry becomes:

$$(\delta_S)_{exp} = \frac{N_S(l^+)_{exp} - N_S(l^-)_{exp}}{N_S(l^+)_{exp} + N_S(l^-)_{exp}} = \frac{\delta_S + 10r\delta_L}{1 + 10r}. \tag{71}$$

As can be seen the correction is absolutely negligible and cannot simulate CPT violation, in fact $(\delta_S)_{exp} - \delta_L = (\delta_S - \delta_L)/(1 + 10r)$.

The number of equal-sign dilepton events generated by $(K^0\bar{K}^0)_{C=+}$ is:

$$\overline{L^{++}} = \frac{1}{2}\text{Br}(K_L \to l^+\pi^-\nu)\text{Br}(\mathrm{K_L} \to \mathrm{l}^+\pi^-\nu)\mathrm{S_L} \cdot \mathrm{S_L}, \tag{72}$$

and that of opposite sign is:

$$\overline{L^{+-}} = \overline{L^{-+}} = \frac{1}{2}\text{Br}(K_L \to l^+\pi^-\nu)\text{Br}(\mathrm{K_L} \to \mathrm{l}^-\pi^+\bar{\nu})\mathrm{S_L} \cdot \mathrm{S_L}. \tag{73}$$

Therefore the experimentally measured T- and CPT-violating asymmetries are:

$$(A_T)_{exp} = \frac{\delta_S + \delta_L + 87r(2\delta_L)}{1 + 87r} \tag{74}$$

and

$$(A_{CPT})_{exp} = \frac{A_{CPT}}{1 + 87r}. \tag{75}$$

Also in this case the effect of the C-even background is negligible and the CPT prediction $A_T = 2\delta_L$ is still valid.

Finally we discuss the effect of C-even background in $K_S \to 3\pi$ decays, where the largest influence is expected.

The inclusion of the background contribution in eq. (49) gives:

$$\begin{aligned}(N_S(3\pi^0))_{exp} &= N_0\text{Br}(K_L \to 3\pi^0) \times \\ &\times \left[3.8 \times 10^{-4}|\eta^{000}|^2 + 8.9 \times 10^{-10} - 1.8 \times 10^{-10}\Im(\eta^{000}) + 3.8 \times 10^{-3}r\right]. \end{aligned} \tag{76}$$

Therefore, for $r \geq 10^{-7}$ the background is comparable to the signal, enforcing the conclusion that the direct tag of the K_S is not useful to determine $\text{Br}(K_S \to (3\pi)_{CP-})$. On the

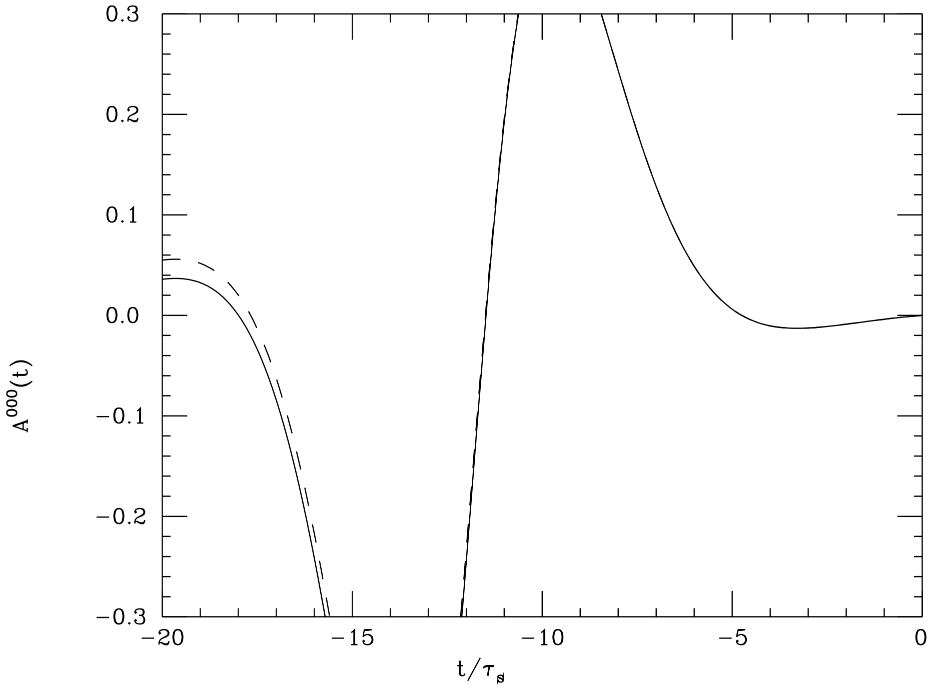

Figure 6: The effect of the C-even background on $A^{000}(t)$. The full line corresponds to $r=0$ and the dashed one to $r=3\times 10^{-7}$.

contrary, if the $K_S \to (\pi^+\pi^-\pi^0)_{CP+}$ branching ratio is $\sim 10^{-7}$, in agreement with ChPT predictions and preliminary CPLEAR results, the background contribution to this decay can be neglected and the corresponding branching ratio can be measured.

The modifications induced in interference measurements require a more detailed study. The analogue of the time difference distribution defined in eq. (8) for an initial $K_L\,K_L$ state is

$$\overline{I(a,b;t)} = \frac{1}{2\Gamma_L}|A_L(a)A_L(b)|^2 e^{-\Gamma_L|t|}(1-e^{-2D/d_L+2\Gamma_L|t|}), \tag{77}$$

where the last factor accounts for having both K_L decays inside the detector. Thus the C-even background, which is symmetric in $\pi^+\pi^-$ momenta, modifies eqs. (51, 52) in the following way:

$$(A^{000}(t))_{exp} = \frac{2\Re\epsilon\left[e^{+\frac{\Delta\Gamma}{2}t} + r\frac{\Gamma}{\Gamma_L}e^{+\frac{\Delta\Gamma}{2}|t|}(1-e^{-2D/d_L+2\Gamma_L|t|})\right] - 2\Re\left(\eta^{000}e^{+i\Delta m t}\right)}{e^{+\frac{\Delta\Gamma}{2}t} + |\eta^{000}|^2 e^{-\frac{\Delta\Gamma}{2}t} + r\frac{\Gamma}{\Gamma_L}e^{+\frac{\Delta\Gamma}{2}|t|}(1-e^{-2D/d_L+2\Gamma_L|t|})}, \tag{78}$$

and

$$(A^{+-0}(t))_{exp} = \frac{2\Re\epsilon\left[e^{+\frac{\Delta\Gamma}{2}t} + r\frac{\Gamma}{\Gamma_L}e^{+\frac{\Delta\Gamma}{2}|t|}(1-e^{-2D/d_L+2\Gamma_L|t|})\right] - 2\Re\left(\eta^{+-0}e^{+i\Delta m t}\right)}{e^{+\frac{\Delta\Gamma}{2}t} + \frac{\Gamma_S^{+-0}}{\Gamma_L^{+-0}}e^{-\frac{\Delta\Gamma}{2}t} + r\frac{\Gamma}{\Gamma_L}e^{+\frac{\Delta\Gamma}{2}|t|}(1-e^{-2D/d_L+2\Gamma_L|t|})}. \tag{79}$$

The contribution of the C-even background becomes important (especially in the numerator) for large and negative values of the time difference, where the number of events is absolutely negligible, but, as can be seen from Fig. 6, does not affect the results for $t \geq -10\tau_S$.

In the CP-conserving ratio $R^{\pm}(t)$ the C-even background contributes in the denominator only and can be safely neglected.

8 Quantum mechanics violations

Research in the theory of quantum gravity [14] led to a proposal for a modification of quantum mechanics time evolution, which might transform a pure initial state into an incoherent mixture [16]. This effect becomes particularly interesting in the kaon system, where quantum oscillations can be accurately measured. Moreover the ϕ-factory, where the initial $\bar{K}^0 K^0$ is an antisymmetric coherent state, is a very suitable facility to test this idea. We will not discuss the theoretical models of quantum mechanics violations, which are discussed elsewhere in this handbook [40], but, following the analysis of Ref. [18], we shall analyze some examples of observable effects at DAΦNE.

To describe the time evolution to incoherent states, one has to introduce the formalism of the density matrix. In Ref. [16] it has been proposed to modify the quantum mechanics time evolution equation in the following way:

$$i\frac{\mathrm{d}}{\mathrm{d}t}\rho = H\rho - \rho H^{\dagger} + \widetilde{\mathcal{H}}\rho, \tag{80}$$

where ρ is the 2×2 kaon density matrix, $H = M - i\Gamma/2$ is the usual non-Hermitian kaon Hamiltonian (see eq. (18)) and $\widetilde{\mathcal{H}}\rho$ is the quantum mechanics violating term. For $\widetilde{\mathcal{H}} = 0$ the eigenmatrices of eq. (80) are the usual matrices:

$$\begin{aligned} \rho_{LL} &= |K_L\rangle\langle K_L|, \qquad & \rho_{SS} &= |K_S\rangle\langle K_S|, \\ \rho_{SL} &= |K_S\rangle\langle K_L|, \qquad & \rho_{LS} &= |K_L\rangle\langle K_S|. \end{aligned} \tag{81}$$

Under reasonable assumptions (probability conservation, not decreasing entropy and strangeness conservation) $\widetilde{\mathcal{H}}$ can be expressed in terms of the three real parameters α, β and γ of Ref. [16]. With this parametrization the new eigenmatrices become:

$$\begin{aligned} \tilde{\rho}_{LL} &= \rho_{LL} + \left[\frac{\gamma}{\Delta\Gamma} + 4\beta\frac{\Delta m}{\Delta\Gamma}\Im\left(\frac{\epsilon_L}{\Delta\lambda^*}\right) - \frac{\beta^2}{|\Delta\lambda|^2}\right]|K_1\rangle\langle K_1| \\ &\quad + \frac{\beta}{\Delta\lambda}|K_1\rangle\langle K_2| + \frac{\beta}{\Delta\lambda^*}|K_2\rangle\langle K_1| \\ \tilde{\rho}_{SS} &= \rho_{SS} - \left[\frac{\gamma}{\Delta\Gamma} + 4\beta\frac{\Delta m}{\Delta\Gamma}\Im\left(\frac{\epsilon_S}{\Delta\lambda^*}\right) - \frac{\beta^2}{|\Delta\lambda|^2}\right]|K_2\rangle\langle K_2| \\ &\quad - \frac{\beta}{\Delta\lambda^*}|K_1\rangle\langle K_2| + \frac{\beta}{\Delta\lambda}|K_2\rangle\langle K_1| \\ \tilde{\rho}_{SL} &= \rho_{SL} - \frac{\beta}{\Delta\lambda^*}|K_1\rangle\langle K_1| + \frac{\beta}{\Delta\lambda}|K_2\rangle\langle K_2| - i\frac{\alpha}{2\Delta m}|K_2\rangle\langle K_1| \\ \tilde{\rho}_{LS} &= \rho_{LS} - \frac{\beta}{\Delta\lambda}|K_1\rangle\langle K_1| + \frac{\beta}{\Delta\lambda^*}|K_2\rangle\langle K_2| - i\frac{\alpha}{2\Delta m}|K_1\rangle\langle K_2|, \end{aligned} \tag{82}$$

where $|K_{1,2}\rangle$ are the usual CP eigenstates and

$$\Delta\lambda = \Delta m + i\frac{\Delta\Gamma}{2} = |\Delta\lambda| e^{i(\pi/2-\phi_{SW})}. \tag{83}$$

The analysis of fixed target experiments has led the authors of Ref. [18] to put stringent bounds on the quantum mechanics violating parameters β and γ:

$$\begin{aligned} \beta &= (0.32 \pm 0.29) \times 10^{-18}\mathrm{GeV}, \\ \gamma &= (-0.2 \pm 2.2) \times 10^{-21}\mathrm{GeV}. \end{aligned} \tag{84}$$

Quite similar results were already obtained in [17]. In [41] also a bound for α ($\alpha \leq 4.8 \times 10^{-16}$ GeV) is derived. It is interesting to note that, using the values in eq. (84), the limits on β/m_K and γ/m_K turn out to be of the order of m_K/M_{Planck}, which could be the natural suppression factor for these parameters.

These limits have been obtained assuming that there is no CPT violation in the decay amplitudes. In the more general case the effects of β, γ and those of the conventional CPT-violating terms are mixed together.

This situation could be improved at DAΦNE. In effect, quantum mechanics predicts a vanishing amplitude for the transition to the final state $|f(t_1), f(t_2)\rangle$, with $t_1 = t_2$, independently from possible CPT violations. Therefore, as pointed out in [18], any measurement of equal time $f_1 = f_2 = f$ events can give a bound for pure quantum mechanics violations. It should be stressed however that the finite experimental resolution will partially wash out these effects (see Fig. 2). Moreover, also the C-even background gives rise to equal time events. Thus only the time distribution, which is different for C-even background and quantum mechanics violations, could help in disentangling them.

In Ref. [18] the consequences of quantum mechanics violation to several DAΦNE observables have been analyzed. In particular the explicit formula of the $|\pi^+\pi^-, \pi^0\pi^0\rangle$ time difference distribution has been derived, discussing the quantum mechanics violating effects in the measurement of $\frac{\epsilon'}{\epsilon}$. For $t \gg \tau_S$ the time asymmetry of eq. (11) becomes:

$$A(t) \stackrel{t\gg\tau_S}{\longrightarrow} 3\Re(\tfrac{\epsilon'}{\epsilon})\left[1 - \frac{\gamma}{\sqrt{2}|\Delta\lambda||\epsilon|^2} - 2\frac{\beta}{|\Delta\lambda||\epsilon|}\right] - 3\Im(\tfrac{\epsilon'}{\epsilon})\left[2\frac{\beta}{|\Delta\lambda||\epsilon|}\right]. \tag{85}$$

In Fig. 7 we have plotted $A(t)$ in the usual quantum mechanics case and in the quantum mechanics violating case (following the analysis of [18]), for $\Re(\frac{\epsilon'}{\epsilon}) = 5 \times 10^{-4}$ and $\Im(\frac{\epsilon'}{\epsilon}) = -4 \times 10^{-3}$, 0 and $+4 \times 10^{-3}$ (close to the predicted DAΦNE sensitivity). The quantum mechanics violating parameters have been chosen to be: $\beta = 0.71 \times 10^{-18}$GeV and $\gamma = 2.2 \times 10^{-21}$GeV, in order to maximize their effects. As one can see, for very small values of the time difference t the effects of quantum mechanics violations are striking, but probably beyond any realistic experimental resolution. The determination of $\Im(\frac{\epsilon'}{\epsilon})$ should not be affected, but an effect could be present in the asymptotic value of the asymmetry where, contrary to the usual quantum mechanics case, also $\Im(\frac{\epsilon'}{\epsilon})$ contributes.

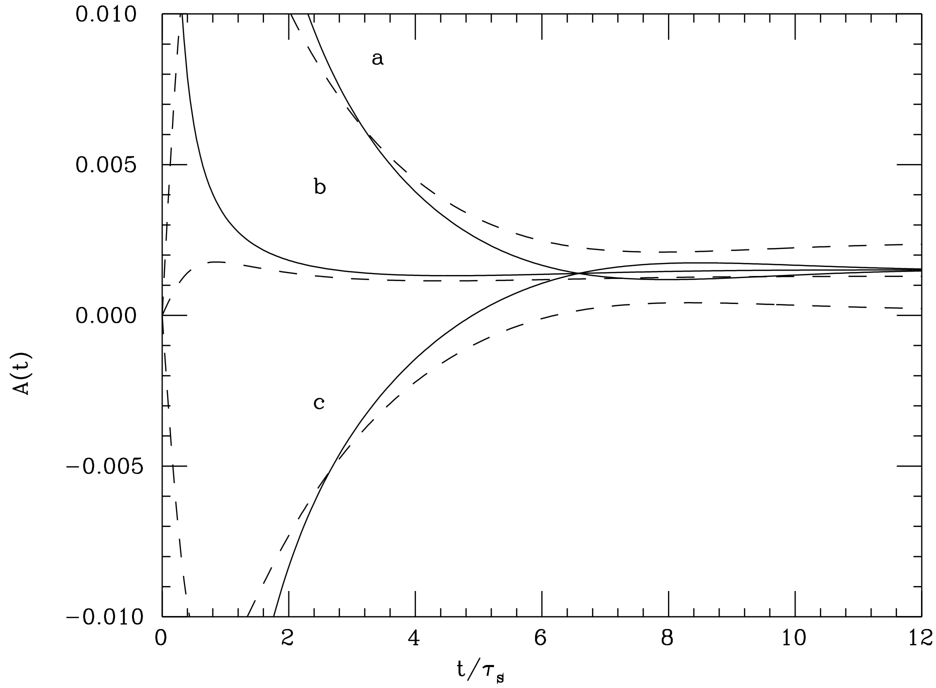

Figure 7: The time asymmetry $A(t)$ for $\Re(\frac{\epsilon'}{\epsilon}) = 5 \times 10^{-4}$. The full lines correspond to quantum mechanics predictions of eqs. (11) and (12), with $\Im(\frac{\epsilon'}{\epsilon}) = -4 \times 10^{-3}$ (a), 0 (b) and 4×10^{-3} (c). The dashed lines correspond to the upper bounds of quantum mechanics violating parameters $\beta = 0.71 \times 10^{-18}$GeV and $\gamma = 2.2 \times 10^{-21}$GeV, for the previous values of $\Im(\frac{\epsilon'}{\epsilon})$.

Using the expressions of Ref. [18] and neglecting a possible violation of the $\Delta S = \Delta Q$ rule, we have calculated also the quantum mechanics violating effects for eq. (25). The measured charge asymmetry for K_S semileptonic decays would become:

$$(\delta_S)_{exp} \equiv \frac{(N_S(l^+))_{exp} - (N_S(l^-))_{exp}}{(N_S(l^+))_{exp} + (N_S(l^-))_{exp}} = \frac{\delta_S - 2\Re\left(\frac{\beta}{\Delta\lambda}\right) + 4\sqrt{2}\frac{\beta}{|\Delta\lambda|} + 20\frac{\gamma}{\Delta\Gamma}\delta_L}{1 + 20\frac{\gamma}{\Delta\Gamma}}, \qquad (86)$$

where $\delta_{S,L}$ are the usual asymmetries, as defined in eq. (23). The effect of the γ term simulates a C-even background and interestingly, with the limit of eq. (84), this correction turns out to be of the order of the one estimated in the previous section (for $r \sim 10^{-7}$). The β term simulates a CPT violation, however, using the bound in eq. (84), this effect turns out to be smaller than the DAΦNE sensitivity.

Concluding, we can say that quantum mechanics violating effects can be neglected in integrated asymmetries at DAΦNE. Nevertheless, if α, β and γ were suppressed only linearly by m_K/M_{Planck}, some effects in time-dependent distributions could be observable. An estimate of DAΦNE sensitivity on these effects would require an accurate simulation of the experimental apparatus, which is beyond the purpose of this work.

9 Conclusions

From the previous analysis it is clear that a ϕ factory is very suitable for an accurate study of the origin of CP violation in K_S and K_L decays and to test the Standard Model predictions.

The real part of the ratio $\frac{\epsilon'}{\epsilon}$, which is a clear signal of direct CP violation, can be measured with high precision, about 10^{-4}. A non-vanishing value of the imaginary part of $\frac{\epsilon'}{\epsilon}$, which would imply CPT violation, can be detected up to some units in 10^{-3}. Even if the fixed target experiments will reach a similar sensitivity, the KLOE apparatus has a completely different systematics and such experimental result will be very important.

The presence of a pure K_S beam will allow, for the first time, the direct detection of CP violation in K_S decays. Moreover many interesting tests of T and CPT symmetries (in addition to $\Im(\frac{\epsilon'}{\epsilon})$) can be performed.

The combined analysis of DAΦNE, CPLEAR, E731 and NA31 experiments will allow to disentangle the CPT-violating contributions in decay amplitudes from those in mass matrix and also the $\Delta S = -\Delta Q$ transitions can be singled out. All the real parts of the parameters could be bounded up to $\simeq 5 \times 10^{-4}$ while a lower sensitivity ($\sim 10^{-3}$–10^{-2}) is expected for the imaginary parts.

More doubtful is the situation for $K_S \to 3\pi$ decays. We think that the CP-conserving decays will certainly be measured and some information on the rescattering phases could be obtained. For the CP-violating ones we observe, without doing a complete statistical analysis, that the shape of the interference effect of Fig. 4 is very characteristic and could easily be detected over a flat background. The analysis of the possibility to measure $\eta_{+-\gamma}$ leads to similar conclusions.

Also the recent suggestions on possible quantum mechanics violations might be tested at DAΦNE.

Appendix

The KLOE detector measures the positions $\mathbf{x}_1$ and $\mathbf{x}_2$ of the two decay vertices. To compare the theoretical expressions of time dependent asymmetries with the experimental distributions, the decay times t_1 and t_2 introduced in the text[10] must be transformed to the corresponding decay distances. Usually this was simply done by using the classical equations of motion, i.e. through the identity

$$t_{1,2} = d_{1,2}/\bar{v}, \qquad (A.1)$$

where $\bar{v}$ is the mean velocity of the two kaons and $d_{1,2} = |\mathbf{x}_{1,2}|$. This leads to the replacement $\Gamma_{S,L}\tau_{1,2} \to d_{1,2}/d_{S,L}$, as in [20].

Recently, the possibility that the different classical velocities of K_S and K_L could modify Eq. (A.1) has been suggested in Ref. [42]. However, such conclusion seems to follow from

[10]In all the previous formulae $t_{1,2}$ denote the proper times. In this appendix, $\tau_{1,2}$ indicate the proper times and $t_{1,2}$ the laboratory times.

an incomplete account of the particle quantum properties, which ultimately overlooks the distinction between the possible values of a quantum observable and the corresponding classical value. Indeed, we present here the proof of Eq. (A.1) based on the description of the $K_L - K_S$ state, in terms of localized wave-packets, of Ref. [43].[11]

The plane wave state $|\mathbf{q}\rangle$ of eq. (1), is replaced with a wave packet peaked at $\mathbf{p} = \mathbf{q}$ and similarly for the other state, $|-\mathbf{q}\rangle$:

$$|\mathbf{q}\rangle \to |\psi\rangle\ ; \langle\mathbf{p}|\psi\rangle = \psi(\mathbf{p}) \approx C\exp[-a(\mathbf{p}-\mathbf{q})^2]$$
$$|-\mathbf{q}\rangle \to |\phi\rangle\ ; \langle\mathbf{p}|\phi\rangle = \phi(\mathbf{p}) \approx C\exp[-a(\mathbf{p}+\mathbf{q})^2] \qquad (A.2)$$

We take, for simplicity, gaussian wave packets, but any other, well-localized, function would give similar results. At time t, one has:

$$\begin{aligned}\psi(\mathbf{x},t) &= C\int d^3p \exp\left[-a(\mathbf{p}-\mathbf{q})^2 + i\mathbf{p}\cdot\mathbf{x} - iE(\mathbf{p})t\right] \\ &\approx C\int d^3p \exp\Big[-a(\mathbf{p}-\mathbf{q})^2 + i\mathbf{p}\cdot\mathbf{x} - itE(\mathbf{q}) - it\frac{\partial E(\mathbf{p})}{\partial p_i}\Big|_{\mathbf{q}}(p-q)_i - \\ &\quad -\frac{it}{2}\frac{\partial^2 E(\mathbf{p})}{\partial p_i \partial p_j}\Big|_{\mathbf{q}}(p-q)_i(p-q)_j - \ldots\Big]\end{aligned}$$

where ellipses represent further terms in the expansion, which can be neglected for a sufficiently narrow packet.

We have to introduce a complex $E(\mathbf{p})$ to describe an unstable particle. The relativistically invariant propagator for a weakly unstable particle reads:

$$D(p^2) = \frac{1}{p^2 - (mJ - J\frac{i}{2}J\Gamma)^2}$$

from which, at the pole, we derive:

$$E(\mathbf{p}) = \sqrt{\mathbf{p}^2 + (mJ - J\frac{i}{2}\Gamma)^2} \approx \sqrt{\mathbf{p}^2 + m^2} - \frac{i}{2}\frac{m\Gamma}{\sqrt{\mathbf{p}^2+m^2}} \approx m + \frac{\mathbf{p}^2}{2m} - \frac{i}{2}\Gamma\Big(1 - \frac{\mathbf{p}^2}{2m^2}\Big)$$

The gaussian integral is easily performed by completing the square, with the result:

$$\psi(\mathbf{x},t) \propto \exp[-itE(\mathbf{q}) + i\mathbf{q}\cdot\mathbf{x}]g(\mathbf{x}-\mathbf{v}_0 t),$$

where

$$\mathbf{v}_0 = \frac{\partial E(\mathbf{p})}{\partial p_i}\Big|_{\mathbf{q}},$$

g is a gaussian wave-packet with a time-dependent size:

$$g(\mathbf{x}-\mathbf{v}_0 t) \approx \exp\Big[-\frac{(\mathbf{x}-\mathbf{v}_0 t)^2}{4A}\Big]$$

[11] We are very grateful to L. Maiani for clarifying discussions, and for providing us with the appendix of his work [43].

$$A = a - \frac{\Gamma t}{4m^2} + i\,\frac{t}{2m}$$

and m is the particle mass. With respect to the stable particle case, there is a contribution to the real part of A from the particle width which, in our case, turns out to be very small.

We consider now the two particle state at time $t = 0$, replacing the plane wave state given in eq. (1) of the text with:

$$|t=0\rangle = \frac{1}{\sqrt{2}}[|\psi_S\rangle|\phi_L\rangle - |\psi_L\rangle|\phi_S\rangle], \tag{A.3}$$

where the subscripts indicate which particle (K_L or K_S) is in which wave packet. At later times, the two packets evolve independently, according to the rules given above.

The amplitude[12] for a $\pi^+\pi^-$ pair to appear at time t_1 and location $\mathbf{x}_1$, and a $\pi^0\pi^0$ pair at time t_2 and location $\mathbf{x}_2$, is given by:

$$A(\mathbf{x}_1, t_1; \mathbf{x}_2, t_2) = \frac{1}{\sqrt{2}}[A_2 - A_1]$$

$$\begin{aligned} A_1 &= \psi_L(\mathbf{x}_1, t_1)\langle\pi^+\pi^-|H_W|K_L\rangle\phi_S(\mathbf{x}_2, t_2)\langle\pi^0\pi^0|H_W|K_S\rangle \\ &\propto \eta^{+-}\psi_L(\mathbf{x}_1, t_1)\phi_S(\mathbf{x}_2, t_2) \\ A_2 &= \psi_S(\mathbf{x}_1, t_1)\langle\pi^+\pi^-|H_W|K_S\rangle\phi_L(\mathbf{x}_2, t_2)\langle\pi^0\pi^0|H_W|K_L\rangle \\ &\propto \eta^{00}\psi_S(\mathbf{x}_1, t_1)\phi_L(\mathbf{x}_2, t_2) \end{aligned}$$

(we are assuming $\mathbf{x}_1$ and $\mathbf{x}_2$ to be on opposite sides of the interaction region, so that the amplitude that, say, the particle in ψ gives rise to an event at $\mathbf{x}_2$ is negligible). According to the previous discussion:

$$\psi_L(\mathbf{x}_1, t_1) = \exp[-it_1 E_L(\mathbf{q}) + i\mathbf{q}\cdot\mathbf{x}_1]g(\mathbf{x}_1 - \mathbf{v}_L t_1) \tag{A.4}$$

$$E_L \approx m_L + \frac{\mathbf{q}^2}{2m_L} - \frac{i}{2}\Gamma_L\Big(1 - \frac{\mathbf{q}^2}{2m_L^2}\Big); \quad \mathbf{v}_L = \frac{\mathbf{q}}{m_L}$$

and similarly for the other cases. Numerically, in the decay $\Phi \to K_L K_S$, with Φ approximately at rest:

$$q = 109 \text{ MeV}, \qquad \bar{v} = \frac{q}{m_K} = \frac{v_L + v_S}{2} = 0.22,$$

$$\frac{m_L - m_S}{m_K} = \frac{\Delta m}{m_K} = -\frac{v_L - v_S}{\bar{v}} = -\frac{\Delta v}{v} = 7.08\cdot 10^{-15},$$

$$\frac{\bar{v}}{\Gamma_S} = d_S = 0.59 \text{ cm}, \qquad \frac{\bar{v}}{\Gamma_L} = d_L = 3.4 \text{ m},$$

$$E_L^* - E_S = \Big[\Delta m + \frac{i}{2}(\Gamma_L + \Gamma_S)\Big]\Big(1 - \frac{\bar{v}^2}{2}\Big) = \Delta E + \frac{i}{2}(\tilde{\Gamma}_L + \tilde{\Gamma}_S).$$

[12] We choose the final state $|a,b\rangle = |\pi^+\pi^-, \pi^0\pi^0\rangle$, however the results can be easily generalized to any $|a,b\rangle$ final state.

The total number of events (for one initial $K_S\,K_L$ pair) with the $\pi^+\pi^-$ pair in $\mathbf{x}_1$ and the $\pi^0\pi^0$ pair in $\mathbf{x}_2$ is found by integrating the probability over t_1 and t_2:

$$
\begin{aligned}
F(\mathbf{x}_1,\mathbf{x}_2) &= \frac{1}{2}\int dt_1 \int dt_2 |A_2 - A_1|^2 \\
&\propto \int dt_1 \int dt_2 \Big\{ |\eta^{+-}|^2 |\psi_L(\mathbf{x}_1,t_1)|^2 |\phi_S(\mathbf{x}_2,t_2)|^2 + |\eta^{00}|^2 |\psi_S(\mathbf{x}_1,t_1)|^2 |\phi_L(\mathbf{x}_2,t_2) \\
&\qquad -2\Re[(\eta^{+-})^*\eta^{00}\psi_L(\mathbf{x}_1,t_1)^*\psi_S(\mathbf{x}_1,t_1)\phi_S(\mathbf{x}_2,t_2)^*\phi_L(\mathbf{x}_2,t_2)]\Big\} \\
&= \int dt_1 \int dt_2 \Big\{ |\eta^{+-}|^2 \exp(-\tilde{\Gamma}_S t_2 - \tilde{\Gamma}_L t_1) |g(\mathbf{x}_1 - \frac{\mathbf{q}}{m_L}t_1)|^2 |g(\mathbf{x}_2 + \frac{\mathbf{q}}{m_S}t_2)|^2 \\
&\qquad + |\eta^{00}|^2 \exp(-\tilde{\Gamma}_S t_1 - \tilde{\Gamma}_L t_2) |g(\mathbf{x}_1 - \frac{\mathbf{q}}{m_S}t_1)|^2 |g(\mathbf{x}_2 + \frac{\mathbf{q}}{m_L}t_2)|^2 \\
&\qquad -2e^{-\frac{1}{2}(\tilde{\Gamma}_S+\tilde{\Gamma}_L)(t_1+t_2)}\Re\{(\eta^{+-})^*\eta^{00}e^{i\Delta E(t_1-t_2)}[g(\mathbf{x}_1 - \frac{\mathbf{q}}{m_L}t_1)^* g(\mathbf{x}_1 - \frac{\mathbf{q}}{m_S}t_1)] \\
&\qquad \times [g(\mathbf{x}_2 + \frac{\mathbf{q}}{m_S}t_2)^* g(\mathbf{x}_2 + \frac{\mathbf{q}}{m_L}t_2)]\}\Big\}. \qquad (A.5)
\end{aligned}
$$

In the direct terms, we may replace each factor $|g|^2$ by a $\delta-$function, thereby obtaining, after integration, the result of eq. (A.1). In the interference term, we can do the same, *provided* that the difference in the peak position of g and g^*, due to the different velocities of K_L and K_S, is negligible with respect to the width of the wave-packet:

$$(\Delta v)t_{1,2} < \sqrt{a + \frac{t_{1,2}^2}{4am^2}}$$

The l.h.s of the inequality is less than a Fermi for $t_{1,2}\,\bar{v} = d_S$, which is where the interference term starts being suppressed by K_S decay. Neglecting the velocity difference in the interference is thus justified in all the region of interest, for any reasonable value of the size of the wave-packet. Therefore the total number of events with the $\pi^+\pi^-(\pi_0\pi_0)$ pair at distance $d_1(d_2)$ from the interaction vertex is given by:

$$
\begin{aligned}
F(d_1,d_2) &= \Gamma_S^{+-}\Gamma_L^{00}\Big\{|\eta^{+-}|^2 e^{-\tilde{\Gamma}_S t_2 - \tilde{\Gamma}_L t_1} + |\eta^{00}|^2 e^{-\tilde{\Gamma}_S t_1 - \tilde{\Gamma}_L t_2} \\
&\qquad -2e^{-\frac{1}{2}(\tilde{\Gamma}_S+\tilde{\Gamma}_L)(t_1+t_2)} Re[(\eta^{+-})^*\eta^{00}\, e^{i\Delta E(t_1-t_2)}]\Big\}, \qquad (A.6)
\end{aligned}
$$

with $t_{1,2} = d_{1,2}/\bar{v}$.

Considerations leading to analogous results, for the case of single Kaon and B oscillations, have been recently presented in Ref. [44].

A few comments.

i) In the case of stable particles, e.g. solar neutrino oscillations, we may reach sufficiently large distances, where the overlap factor:

$$g(\mathbf{x}_1 - \frac{\mathbf{q}}{m_L}t_1)^* g(\mathbf{x}_1 - \frac{\mathbf{q}}{m_S}t_1)$$

vanishes. In the far-distant region, coherent oscillations would disappear to gives rise to an incoherent superposition of a K_L and K_S beam.

ii) the velocity dependent terms in the exponents have reconstructed the correct Lorentz factors. We have obtained exponents of the form:

$$\Delta E\ t = \Delta m t\Big(1 - \frac{v^2}{2}\Big) \approx \Delta m t\sqrt{1-v^2} = \Delta m \tau$$

$$\tilde{\Gamma} t = \Gamma t\Big(1 - \frac{v^2}{2}\Big) \approx \Gamma t\sqrt{1-v^2} = \Gamma \tau$$

where τ is the particle proper time, as one could have expected on general ground.

iii) One may have preferred to assume exactly vanishing total momentum, rather then independently distributed momenta for the two particles, as done in eq. (A.3). This, however, is *incorrect.* A zero- momentum state is translation invariant. If this were the case, the intensity $F(\mathbf{x}_1, \mathbf{x}_2)$ would depend only upon the difference, $\mathbf{x}_1 - \mathbf{x}_2$, which is obviously wrong. Electrons and positrons annihilate in the interaction region and not elsewhere, and the interaction region is small with respect to the general dimensions of the experiment. As a consequence, $F(\mathbf{x}_1, \mathbf{x}_2)$ must depend from the individual distances of the two points from the interaction region, as in eq. (A.5).

Clearly, the initial wave packet may be not in the factorized form of eq. (A.3), generally will be a superposition of all the possible factorized forms (one possibility is to factorize the c.m. and the relative motion). However an explicit calculation shows that the result of eq. (A.6) holds for any form of the initial state, provided it satisfies the physical constraints: $< \mathbf{p}_1 > = - < \mathbf{p}_2 > = \mathbf{q}$ and it has reasonable widths. Should the result depend on more particular assumptions, the outcome experiment would be entirely unpredictable.

References

[1] G. Vignola, in "Workshop on Physics and Detectors for DAΦNE" (Frascati, 1991), edited by G. Pancheri p. 11; M. Piccolo, *ibid.*, p. 707.

[2] T. Ohshima, *ibid.*, p. 29.

[3] P. Franzini, *ibid.*, p. 733; KLOE Collaboration (A. Aloisio et al.), LNF-92/019 (IR) and LNF-93/002 (IR).

[4] NA31 Collaboration, Phys. Lett. B206 (1988) 169; B237 (1990) 303; B317 (1992) 233.

[5] E731 Collaboration, Phys. Rev. Lett. 64 (1990) 1491; 70 (1993) 1199 and 1203.

[6] A.J. Buras, M. Jamin and M.E. Lautenbacher, Nucl. Phys. B408 (1993) 209; M. Ciuchini, E. Franco, G. Martinelli and L. Reina, Phys. Lett. B 301 (1993) 263; Nucl. Phys. B 415 (1994) 403.

[7] M. Ciuchini, E. Franco, G. Martinelli and L. Reina, in this Handbook, and references therein.

[8] L. Maiani and N. Paver, in "The DAΦNE Physics Handbook", edited by L. Maiani, G. Pancheri and N. Paver, p. 191 and references therein; L. Maiani and N. Paver, in this Handbook.

[9] G. D'Ambrosio, G. Ecker, G. Isidori and H. Neufeld, in this Handbook, and references therein.

[10] Y. Fukushima et al., KEK preprint 89-159.

[11] C. Buchanan, R. Cousin, C. Dib, R.D. Peccei and J. Quackenbush, Phys. Rev. D45 (1992) 4088;
M. Hayakawa and A.I. Sanda, Phys. Rev. D48 (1993) 1150.

[12] N.N. Achasov and V.N. Ivanchenko, Nucl. Phys. B315 (1989) 465; S. Nussinov and I.N. Truong, Phys. Rev. Lett. 63 (1989) 2003.

[13] F.E. Close, in "Workshop on Physics and Detectors for DAΦNE" (Frascati, 1991), edited by G. Pancheri, p. 309, and F.E. Close, in "The DAΦNE Physics Handbook", edited by L. Maiani, G. Pancheri and N. Paver, p. 465.

[14] S.W. Hawking, Phys. Rev. D14 (1975) 2460.

[15] D.N.Page, Gen. Rel. Grav. 14 (1982).

[16] J. Ellis, J.S. Hagelin, D.V. Nanopoulos, M. Srednicki, Nucl. Phys. B241 (1984) 381.

[17] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B293 (1992) 142 and preprint CERN-TH 6755/92

[18] P. Huet and M.E. Peskin, Nucl. Phys. B434 (1995) 3; P. Huet, SLAC preprint 6491, May/1994, to appear in the proceedings of the "First International Conference on Phenomenology of Unification from Present to Future", Rome, 1994.

[19] I. Dunietz, J. Hauser and J.L. Rosner, Phys. Rev. D35 (1987) 2166; J. Bernabeu, F.J. Botella and J. Roland, Phys. Lett. B211 (1988); F.J. Botella in "Workshop on Physics and Detectors for DAΦNE" (Frascati, 1991), edited by G. Pancheri, p.325.

[20] V. Patera and A. Pugliese in "The DAΦNE Physics Handbook", edited by L. Maiani, G. Pancheri and N. Paver, p. 87

[21] C.O. Dib and B.Guberina, Phys. Lett. B255 (1991) 113; M. Luke, Phys. Lett. B256 (1991) 265.

[22] L. Maiani in "The DAΦNE Physics Handbook", edited by L. Maiani, G. Pancheri and N. Paver, p. 21.

[23] R. Peccei, in "Results and perspectives in particle physics", Proc. of Rencontres de Physique de la Vallée d'Aoste, La Thuile, March 1993, edited by M. Greco.

[24] Particle Data Group, Review of Particle Properties, Phys. Rev. D50 (1994) 1173.

[25] CPLEAR Collaboration (R. Adler et al.), Phys. Lett. B286 (1992) 180; talk given by A. Ealet at Rencontres de Physique de la Vallée d'Aoste, La Thuile, March 1994, to appear in the proceedings; talk given by T. Ruf at the 27th Int. Conf. on High Energy Physics, Glasgow, July 1994, to appear in the proceedings.

[26] P.K. Kabir, Phys. Rev. D2 (1970) 540; see also N.W. Tanner and R.H. Dalitz, Ann. Phys. 171 (1986) 463.

[27] J. Kambor, J. Missimer and D. Wyler, Phys. Lett. B261 (1991) 496.

[28] E621 Collaboration (G.B. Thomson et al.), Phys. Lett. B337 (1994) 411.

[29] L.F. Li and L. Wolfenstein, Phys. Rev. D21 (1980) 178.

[30] J.F. Donoghue, B.R. Holstein and G. Valencia, Phys. Rev. D36 (1987) 798;
H.Y. Cheng, Phys. Rev. D43 (1991) 1579 and Phys. Rev. D44 (1991) 919 (addendum).

[31] E621 Collaboration (Y. Zou et al.), Phys. Lett. B329 (1994) 519.

[32] G. D'Ambrosio and N. Paver, Phys. Rev. D49 (1994) 4560.

[33] G. D'Ambrosio, G. Isidori, A. Pugliese and N. Paver, Phys. Rev. D50 (1994) 5767 and D51 (1995) 3975 (erratum).

[34] F.E. Low, Phys. Rev. **110** (1958) 974.

[35] G. Ecker, H. Neufeld and A. Pich, Nucl. Phys. **B 314** (1994) 321.

[36] G. D'Ambrosio, M. Miragliuolo and F. Sannino, Z. Phys. **C 59** (1993) 451; G. D'Ambrosio and G. Isidori Z. Phys. C65 (1995) 649.

[37] E773 Collaboration, talks given by G.D. Gollin and W.P. Hogan at the 27th Int. Conf. on High Energy Physics, Glasgow, July 1994, to appear in the proceedings.

[38] N.Paver and Riazuddin, Phys. Lett. B246 (1990) 240.

[39] D. Cocolicchio, G.L. Fogli, M. Lusignoli and A. Pugliese, Phys. Lett. B238 (1990) 417 and B249 (1990) 552 (erratum).

[40] N.E. Mavromatos, in this Handbook.

[41] J.L. Lopez, CTP-TAMU-38/93.

[42] Y.N. Srivastava,"On Two-time formulae for CP-Violation decays", in II EURO-DAΦNE Collaboration Meeting, LNF-94/003; Y.N. Srivastava, E. Sassaroli and A.Widom, "Spatial Correlations In Two Neutral Kaon Decays", to appear in Zeit. Phys. C.

[43] L. Maiani, "Matter-antimatter asymmetry with DAΦNE", lectures given at the 1994 Erice School, to appear in the proceedings.

[44] H. Lipkin, WIS-95/1/Jan-PH, TAUP 2225-95, Jan. 1995.