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Chapter 1: CP and CPT Violation
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CP Violation in $K \to 3\pi$ Decays

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Abstract

We review the Standard Theory estimates of direct CP violation in charged $K \to 3\pi$ decays, using next-to-leading QCD Wilson coefficients in the $|\Delta S| = 1$ nonleptonic Hamiltonian, and updated determinations of CKM matrix elements and of the *top* quark mass. We also briefly consider the case of neutral $K \to 3\pi$ decays.

1 Introduction

The observation of CP violation in $K \to 3\pi$ decays would greatly enrich our understanding of this, still mysterious, phenomenon.

CP violation in $K_S \to 3\pi$, even at the level where mass-mixing effects only are observed, is very interesting, because any deviation of ϵ_S from ϵ is a direct signal of CPT violation. On the other side, CP-odd charge asymmetries in $K^\pm$ decays would give a univocal sign of the milliweak nature of CP violation. If current Standard Theory estimates are correct, the direct CP violation parameter for $K \to 2\pi$, ϵ'/ϵ, tends to be quite small because of a cancellation of the dominant gluon penguin diagram contribution with the one arising from the electroweak penguin diagrams, made competitive by the large value of the t quark mass [1]. The same two classes of contributions do not cancel in the asymmetry of $K^\pm \to 3\pi$ decays, which then could provide the crucial test of the scheme. Furthermore, while ϵ'/ϵ is

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essentially suppressed by the $\Delta I = 1/2$ rule as being proportional to the small ratio of the $\Delta I = 3/2$ to the $\Delta I = 1/2$ amplitude, in $K \to 3\pi$ there are two independent $\Delta I = 1/2$ amplitudes whose CP-violating interference can avoid this suppression, and in principle could be expected to produce an enhanced effect.

Unfortunately, in general the predicted Dalitz-plot slope asymmetries are in the order of few units per million (subject to a theoretical error which could be perhaps of a factor ten). In particular, the interference between the two $\Delta I = 1/2$ amplitudes turns out to occur only in higher orders in the chiral perturbation theory expansion (χPT), and the corresponding enhancement is therefore somewhat reduced. On purely statistical grounds, DAΦNE experiments can reach the level of a few units in 10^{-4} and therefore are rather far from the crucial region, with presently foreseen luminosities. Concerning neutral kaon decays, DAΦNE experiments can observe a few $K_S \to 3\pi^0$ decays [2], enough to establish the effect, but to a precision unlikely to reveal any CPT anomaly in ϵ_S.

Nonetheless, since DAΦNE should significantly improve presently existing data and/or bounds on $K \to 3\pi$ decays, it is appropriate to discuss in this Handbook the present status-of-the-art of this field.

2 $K^{\pm}$ decays

In this case one can study CP-odd charge asymmetries in the total rates and in the linear slope of the Dalitz plot. Referring to [3] for more details on the kinematics and the relevant definitions, we recall that, due to the smallness of the outgoing pion energies, in general the Dalitz plot distributions for $K \to 3\pi$ can be expanded in powers of the independent kinematical variables as follows (higher powers are not requested by experimental data):

$$|A(K \to 3\pi)|^2 \propto 1 + gY + jX + hY^2 + kX^2. \tag{1}$$

Here, for the process $K(p) \to \pi_1(p_1)\pi_2(p_2)\pi_3(p_3)$, Y and X are the familiar kinematical variables, $Y = (s_3 - s_0)/m_\pi^2$ and $X = (s_1 - s_2)/m_\pi^2$, with $s_i = (p - p_i)^2$, $\sum_i s_i = 3s_0 = m_K^2 + 3m_\pi^2$, and '3' indicates the 'odd charge' pion. The term linear in X in Eq. (1) is CP violating and can appear only in the case of $K_L \to \pi^+\pi^-\pi^0$, so that it is not relevant here. For $K^\pm \to 3\pi$ decays the charge asymmetries are then defined as the relative differences

$$\delta_\Gamma \equiv \frac{\Delta\Gamma}{2\Gamma} = \frac{\Gamma(K^+ \to 3\pi) - \Gamma(K^- \to 3\pi)}{\Gamma(K^+ \to 3\pi) + \Gamma(K^- \to 3\pi)}, \tag{2}$$

and

$$\delta_g \equiv \frac{\Delta g}{2g} = \frac{g(K^+ \to 3\pi) - g(K^- \to 3\pi)}{g(K^+ \to 3\pi) + g(K^- \to 3\pi)}, \tag{3}$$

for both τ (i.e. $\pi^\pm\pi^\pm\pi^\mp$) and τ' (i.e. $\pi^0\pi^0\pi^\pm$) decay modes.

We expand the relevant $K \to 3\pi$ amplitudes up to linear terms. Assuming $\Delta I \leq 3/2$, and including imaginary parts originating from final state strong interaction effects, this expansion can be written as

$$\begin{aligned}A(K^+ \to \pi^+\pi^+\pi^0) &= 2a_c\left(1 + i\alpha_0 + i\frac{\alpha_0'}{2}Y\right) + [b_c\,(1+i\beta_0) + b_2\,(1+i\delta_0)]\,Y \\ A(K^+ \to \pi^0\pi^0\pi^+) &= a_c\,(1 + i\alpha_0 - i\alpha_0' Y) - [b_c\,(1+i\beta_0) - b_2\,(1+i\delta_0)]\,Y, \qquad (4)\end{aligned}$$

where a_c, b_c, b_2 can be chosen as real constants if CP is conserved, and α_0, α_0' β_0 and δ_0 are also real. With these definitions, b_2 is purely $\Delta I = 3/2$, so that the $\Delta I = 1/2$ rule would imply the condition:

$$b_2 = 0 \qquad\qquad (\Delta I = 1/2). \tag{5}$$

Furthermore, lowest order in chiral perturbation theory and the $\Delta I = 1/2$ rule give:

$$a_c + \frac{1}{3}\frac{m_K^2}{m_\pi^2} b_c = 0 \qquad\qquad (\Delta I = 1/2 \textit{ and } O(p^2)\ \chi\mathrm{PT}). \tag{6}$$

In the presence of direct CP violation, the coefficients a_c, b_c and b_2 are complex numbers, and nonvanishing charge asymmetries such as (2) and (3) are generated. For example, specializing to the centre of the Dalitz plot, for the τ-decay slope asymmetry we obtain:

$$(\delta_g)_\tau = \frac{\alpha_0 - \beta_0}{a_c(b_c + b_2)}\,[a_c \Im b_c - b_c \Im a_c] + \frac{\alpha_0 - \delta_0}{a_c(b_c + b_2)}\,[a_c \Im b_2 - b_2 \Im a_c]\,, \tag{7}$$

where the quantities in the denominators are taken in the CP-exact limit and therefore are understood to be real. Notice that α_0', which appears in (4), does not contribute to the charge asymmetry (7).

The r.h.s. of Eq. (7), and the analogous asymmetry for τ' decay, explicitly vanish in the limit in which relations (5) and (6) are obeyed, since in that case there is only one weak decay amplitude whose phase can be transformed away [4, 5].

The effective Hamiltonian at the constituent quark level consists, in fact, of various components, transforming according to the $(8_L, 1_R)$, $(27_L, 1_R)$ and $(8_L, 8_R)$ representations of chiral $SU(3) \times SU(3)$.

If the t quark mass were small, the $(8_L, 8_R)$ component, which arises from the electroweak penguin diagrams, could be safely neglected. In this case, in lowest order χPT a non-vanishing asymmetry would arise solely from the interference of the $(8_L, 1_R)$ and $(27_L, 1_R)$ amplitudes, and, in this limit, would be necessarily suppressed by the small ratio

$$\omega = \frac{a_{3/2}}{a_{1/2}} = 0.045, \tag{8}$$

quite similar to ϵ'/ϵ. With m_t very large, of the order of $170\,GeV$, the effect of the electroweak operators becomes non-negligible. This rise of the $(8_L, 8_R)$ contribution with m_t is in fact responsible for the decrease of the Standard Theory prediction of ϵ'/ϵ (for more quantitative details, see [1]). As for the $K^\pm \to 3\pi$ charge asymmetry, it turns out that the contribution of the $(8_L, 8_R)$ operator increases the leading order prediction of χPT for the Dalitz plot slope asymmetry, and decreases the prediction for the asymmetry of the widths. Another source of corrections to the lowest order χPT prediction is the isospin breaking u-d quark mass difference, which feeds contributions proportional to the (large) coefficient of the $(8_L, 1_R)$ component into the $\Delta I = 3/2$ channel.

Making contact with Ref. [6], we describe now with some detail the lowest order χPT calculation, accounting for the electroweak penguin diagram contribution and for isospin breaking corrections (see also [7, 8]). The first ingredient is the evaluation of strong interaction rescattering phases. One finds in χPT [9, 6] to one loop:

$$\alpha_0 = \frac{q_0}{32\pi F_\pi^2}(2s_0 + m_\pi^2) \simeq 0.13, \tag{9}$$

$$\beta_0 = -\delta_0 = \frac{q_0}{32\pi F_\pi^2}(s_0 - m_\pi^2) \simeq 0.048, \tag{10}$$

where $q_0 = \sqrt{1 - (4m_\pi^2/s_0)}$.

As it has been introduced in [3], the $|\Delta S| = 1$ effective Hamiltonian now consists of the familiar $(8_L, 1_R)$ and $(27_L, 1_R)$ components, represented in leading order χPT as [10]

$$\mathcal{L}_W^{(2)}|_{(8,1)+(27,1)} = c_2 Tr\lambda_6 L_\mu L^\mu + c_3 t^{jl}_{ik}(TrQ^i_j L_\mu)(TrQ^k_l L^\mu), \tag{11}$$

plus the $(8_L, 8_R)$ component [9]:

$$\mathcal{L}_W^{(2)}|_{(8,8)} = 2c_8 F_\pi^2 Tr\lambda_6 U^\dagger Q U, \tag{12}$$

where $Q = diag(2, -1, -1)$, $L_\mu = iU^\dagger \partial_\mu U$, and $U = \exp(i\sqrt{2}\Phi/F_\pi)$ with Φ the pseudoscalar meson $SU(3)$ matrix.

In terms of the coefficients c_2, c_3 and c_8 one has:[2]

$$a_c = \frac{m_K^2}{F_\pi^3 F_K}\left[-\frac{1}{3}c_2 + \frac{4}{3}c_3 - 6\frac{F_\pi^2}{m_K^2}c_8\right], \tag{13}$$

$$b_c = \frac{m_\pi^2}{F_\pi^3 F_K}\left[\left(1 + \frac{9}{4\sqrt{2}}\omega\Omega_{IB}\right)c_2 + \frac{7m_K^2 - 22m_\pi^2}{2(m_K^2 - m_\pi^2)}c_3 + \frac{9}{2}\frac{F_\pi^2}{m_K^2}c_8\right], \tag{14}$$

$$b_2 = \frac{m_\pi^2}{F_\pi^3 F_K}m_K^2\left[-\frac{9}{4\sqrt{2}}\omega\Omega_{IB}c_2 + \frac{15}{2}\frac{3m_K^2 - 2m_\pi^2}{m_K^2 - m_\pi^2}c_3 - \frac{9}{2}\frac{F_\pi^2}{m_K^2}c_8\right], \tag{15}$$

where Ω_{IB} represents the effect of isospin breaking, and numerically we shall use $\Omega_{IB} \simeq 0.25$.

CP violation is related to the imaginary parts of the coefficients c_2, c_3 and c_8. Introducing for convenience the dimensionless quantities:

$$\Im L_2 = \frac{m_K^2}{F_\pi^3 F_K}\Im c_2, \tag{16}$$

[2]Normalizations here are such that $c_2 - (2/3)c_3$, $(10/3)c_3$ and c_8 are equal to the coefficients c_2, c_3 and c_8 used in [6], multiplied by F_K/F_π.

$$\Im L_3 = \frac{m_K^2}{F_\pi^3 F_K}\frac{10}{3}\Im c_3, \tag{17}$$

$$\Im L_8 = \frac{2}{F_\pi F_K}\Im c_8, \tag{18}$$

one can express the asymmetry as a superposition of the $\Im L_i$ with coefficients determined by the experimental (real) amplitudes. In turn, the $\Im L_i$ are derived from a theoretical determination of the matrix elements of the various components of the effective Hamiltonian ($1/N_c$ expansion [8], or Lattice QCD [11, 12, 13, 14]).

Eq. (7) then takes the form

$$\left(\delta_g\right)_\tau = \frac{\alpha_0-\beta_0}{a_c(b_c+b_2)}\sum_{2,3,8}\mathcal{A}_i\Im L_i + \frac{\alpha_0-\delta_0}{a_c(b_c+b_2)}\sum_{2,3,8}\mathcal{B}_i\Im L_i, \tag{19}$$

where numerically:

$$\mathcal{A}_2 = \left(1+\frac{9}{4\sqrt{2}}\omega\Omega_{IB}\right)\frac{m_\pi^2}{m_K^2}a_c + \frac{1}{3}b_c \simeq 6.49\times 10^{-9} \tag{20}$$

$$\mathcal{A}_3 = \frac{3}{10}\frac{7m_K^2-22m_\pi^2}{2(m_K^2-m_\pi^2)}\frac{m_\pi^2}{m_K^2}a_c - \frac{2}{5}b_c \simeq -1.61\times 10^{-7} \tag{21}$$

$$\mathcal{A}_8 = \frac{9}{4}\frac{m_\pi^2}{m_K^2}a_c + 3b_c \simeq 5.69\times 10^{-7} \tag{22}$$

$$\mathcal{B}_2 = -\frac{9}{4\sqrt{2}}\omega\Omega_{IB}\frac{m_\pi^2}{m_K^2}a_c + \frac{1}{3}b_2 \simeq -1.17\times 10^{-8} \tag{23}$$

$$\mathcal{B}_3 = \frac{9}{4}\frac{3m_K^2-2m_\pi^2}{m_K^2-m_\pi^2}\frac{m_\pi^2}{m_K^2}a_c - \frac{2}{5}b_2 \simeq -5.08\times 10^{-7} \tag{24}$$

$$\mathcal{B}_8 = -\frac{9}{4}\frac{m_\pi^2}{m_K^2}a_c + 3b_2 \simeq 4.84\times 10^{-8} \tag{25}$$

The values of $\Im L_i$ needed in (19) are obtained, as in Ref. [6], by combining the short-distance, perturbative QCD coefficients in the expansion of the nonleptonic $\Delta I = 1$ Hamiltonian in terms of four-quark operators, with the nonperturbative B-factors computed in Lattice QCD [14].

As an indication, in Figs. 1 and 2 we represent the results which would be obtained for the τ slope asymmetry (3), by applying the same kind of numerical analysis performed for ϵ'/ϵ in Ref. [1]. This analysis should account for the most recent determinations of the Standard Model parameters (m_t, CKM matrix elements, Λ_{QCD}, *etc.*), including present theoretical uncertainties on the relevant nonperturbative parameters (such as f_B), as well as the experimental statistical and systematical uncertainties.[3] Specifically, in the histogram of Fig. 1, the solid line represents the predicted values of δ_g without any 'cut'

[3] We are grateful to E. Franco and G. Isidori for providing these updated figures.

on the possible values of f_B (see Ref. [1] for details), while the dashed one is obtained by imposing such a 'cut' (namely, by constraining f_B to the range of values predicted by Lattice calculations). We recall that this 'cut' would imply $\cos\delta > 0$ as the most likely possibility, δ being the CP violating CKM phase. As anticipated in Sec. 1, the predicted values of δ_g are of the order of some units per million. Fig. 2 is a combined plot of δ_g *vs.* ϵ'/ϵ, the solid and dashed lines representing the allowed contours with 5% and 68% probability, respectively, limiting to the case of the f_B 'cut' which privileges $\cos\delta > 0$. One can remark from this figure that, for the large values of $m_t = 174 \pm 17\,GeV$ [15], there is little correlation between the values of δ_g and ϵ'/ϵ, whereas a rather strong correlation would have occurred for small values of m_t [6]. Also, Fig. 2 indicates that, while ϵ'/ϵ can vanish, δ_g should be safely different from zero.

Finally, in Tab. 1 we summarize the numerical values for the various charge asymmetries, as obtained from the procedure outlined above, in the case $\cos\delta > 0$. This table

	exp. limits	theory
$(\delta_g)_\tau$	$-(0.70 \pm 0.53) \times 10^{-2}$	$-(2.3 \pm 0.6) \times 10^{-6}$
$(\delta_\Gamma)_\tau$	$(0.04 \pm 0.06) \times 10^{-2}$	$-(6.0 \pm 2.0) \times 10^{-8}$
$(\delta_g)_{\tau'}$	-	$(1.3 \pm 0.4) \times 10^{-6}$
$(\delta_\Gamma)\|_{\tau'}$	$(0.0 \pm 0.3) \times 10^{-2}$	$(2.4 \pm 0.8) \times 10^{-7}$

Table 1: Experimental values and theoretical predictions for the CP violating asymmetries in τ and τ' modes at lowest order in χPT, including $(8,8)$ and Ω_{IB} effects.

shows that the estimated CP violating asymmetries are much larger for the Dalitz plot slopes than for the total decay widths. Qualitatively, similar results and orders of magnitudes for the asymmetry have been obtained in [8], using the $1/N_c$ expansion at leading order.

Eq. (7) is also useful for a simple discussion of possible higher order χPT effects, which have been advocated in [16] as the source of an enhancement of the asymmetry at the level of two orders of magnitude. Indeed, higher order chiral corrections could be important, because they can introduce further octet effective operators, to give independent phases to the amplitudes of the two I=1 states. CP violating interference of these two amplitudes would avoid the suppression factor ω in Eq. (8). Therefore, one could naively hope to gain an enhancement factor $\mathcal{F} \propto 1/\omega$.

Actually, essentially following the discussion of Ref. [17], let us reconsider Eq. (7) limiting ourselves to the first term only (the second one may be neglected as it involves the $\Delta I = 3/2$ Hamiltonian in an essential way), and, after using Eq. (5), rewrite it as

$$(\delta_g)_\tau \cong \frac{\alpha_0 - \beta_0}{a_c}\left[\left(\frac{a_c}{b_c} + \frac{1}{3}\frac{m_K^2}{m_\pi^2}\right)\Im b_c - \left(\Im a_c + \frac{1}{3}\frac{m_K^2}{m_\pi^2}\Im b_c\right)\right]. \tag{26}$$

The first term in the square bracket is anyway suppressed by the combination of experimental amplitudes, which give

$$\left(\frac{a_c}{b_c}+\frac{1}{3}\frac{m_K^2}{m_\pi^2}\right)\simeq 0.04\simeq\omega. \tag{27}$$

We can expand the second term, according to χPT, into leading $O(p^2)$ and next-to-leading $O(p^4)$ terms, as

$$\left(\Im a_c+\frac{1}{3}\frac{m_K^2}{m_\pi^2}\Im b_c\right)=\left(\Im a_c+\frac{1}{3}\frac{m_K^2}{m_\pi^2}\Im b_c\right)^{(2)}+\left(\Im a_c+\frac{1}{3}\frac{m_K^2}{m_\pi^2}\Im b_c\right)^{(4)}+\cdots \tag{28}$$

Therefore, to a good approximation, the relative size of the correction is represented by the factor $\mathcal{F}$:

$$\mathcal{F}\simeq\frac{\left(\Im a_c+\frac{1}{3}\frac{m_K^2}{m_\pi^2}\Im b_c\right)^{(4)}}{\left(\Im a_c+\frac{1}{3}\frac{m_K^2}{m_\pi^2}\Im b_c\right)^{(2)}} \tag{29}$$

Coefficients are such that:

$$\left(\Im a_c+\frac{1}{3}\frac{m_K^2}{m_\pi^2}\Im b_c\right)^{(2)}\simeq\omega\,(\Im a_c)^{(2)}, \tag{30}$$

and therefore one derives:

$$\mathcal{F}\simeq\frac{(\Im a_c)^{(4)}}{\omega\,(\Im a_c)^{(2)}}. \tag{31}$$

More precisely, it can be seen that in the decomposition of the $O(p^4)$ $K\to 3\pi$ amplitudes into loop and counterterm contributions, only the weak counterterms are relevant to (31). Indeed, in principle loops can generate large $O(p^4)$ imaginary parts. However, these contributions to the two $\Delta I=1/2$ amplitudes must have the same weak phase (that of the unique $\Delta I=1/2$ $O(p^2)$ operator in (11)), and therefore cannot build up a large CP violating interference [6, 17, 18].

The rigorous inclusion of chiral loops and local counterterms for CP violating $K\to 3\pi$ decays has not been accomplished yet, due to many unknown constants. Nevertheless, chiral corrections are seen to be quite reasonable for CP conserving amplitudes, introducing in that case effects of the order of 40% or less (see [3]). Consequently, we can make the assumption that the chiral expansion similarly works also for the CP violating amplitudes (although not yet verified experimentally, this assumption seems quite plausible). Thus, the order p^4 enhancement factor in (31) should be expected to be of the order of $\mathcal{F}\sim 10-20$ or so (the upper figure corresponding to the extreme case $(\Im a_c)^{(4)}=(\Im a_c)^{(2)}$), which would enhance the predictions in Tab. 1 by an order of magnitude at most. This is confirmed, e.g., by the findings of Ref. [19] using the chiral σ-model, and of Ref. [8] in the framework of the $1/N_c$ expansion.

3 K^0 decays

The $\pi^+\pi^-\pi^0$ decay of the K_S is in principle not forbidden by CP conservation. It is so, however, at the centre of the Dalitz plot, where symmetry in pion momenta forbids the appearance of odd values of the relative angular momenta. $3\pi^0$ decay of K_S is altogether CP violating.

Taking into account the mass-matrix mixing effect, one writes:

$$\eta_{+-0} = \frac{A(K_S \to \pi^+\pi^-\pi^0;\ I=1)}{A(K_L \to \pi^+\pi^-\pi^0)} = \epsilon + \epsilon'_{+-0}, \tag{32}$$

and similarly for the decay $K_S \to 3\pi^0$, with an analogous definition of η_{000} and ϵ'_{000}.

Theoretical analyses of the direct CP violation parameters, $\epsilon'_{3\pi}$, proceed along much similar lines to those discussed previously for charged kaon decays (see, e.g., Refs. [4, 7, 8]). In lowest order χPT, one finds [4]:

$$\epsilon'_{+-0} = \epsilon'_{000} = -2\epsilon'_{2\pi}. \tag{33}$$

The more recent discussions [7, 8, 20] include the electroweak penguin contributions, as well as higher order chiral corrections. Analogously to the charged kaon case, it seems unlikely that the lowest order prediction (33) can be increased by more than one order of magnitude. In this case, the $K_S \to 3\pi^0$ branching ratio is predicted to be:

$$BR(K_S \to 3\pi^0) = |\epsilon_S|^2 \, \frac{\tau_S}{\tau_L} BR(K_L \to 3\pi^0) = 2.52 \times 10^{-9} \, |\frac{\epsilon_S}{\epsilon}|^2, \tag{34}$$

to be compared with the present experimental limit $BR(K_S \to 3\pi^0) < 0.37 \times 10^{-4}$ [21]

For $K_S \to \pi^+\pi^-\pi^0$, the problem is the presence of the CP conserving amplitude which can obscure the CP violating one. Since CP conserving and CP violating amplitudes do not interfere in the total rate due to their different dependences on the kinematical variables, the effect of (mass-mixing) CP violation in the rate for this channel is of order $|\epsilon|^2$, bringing it down to the level of $\sim 1.2 \times 10^{-9}$.

A possible strategy to search for this effect at DAΦNE should be the measurement of the time interference between K_S and K_L amplitudes, as discussed in [2]. Concerning the expected sensitivity on $\eta_{3\pi}$, it is difficult to assess it without a detailed simulation. As an indication, on statistical grounds the sensitivity should be of the order of $(7-8) \times 10^{-3}$, which is rather far from the expected value $|\eta_{3\pi}| \simeq |\epsilon| \simeq 2.3 \times 10^{-3}$. On the other hand, the most recent experimental findings are [22]:

$$\Re\eta_{+-0} = 0.019 \pm 0.027; \qquad \Im\eta_{+-0} = 0.019 \pm 0.061, \tag{35}$$

from an unconstrained fit to the data, and

$$\Im\eta_{+-0} = -0.015 \pm 0.017(stat.) \pm 0.025(syst.) \tag{36}$$

if the theoretical constraint $\Re\eta_{+-0} = \Re\epsilon$ is used in such a fit. Thus, it might still be worthwhile to make an attempt to obtain an alternative experimental limit at DAΦNE.

In the decay $K_L \to \pi^+\pi^-\pi^0$, the mass-mixing CP violating amplitude is suppressed by an angular momentum barrier and by the $\Delta I = 1/2$ rule, analogously to the CP conserving amplitude of the K_S decay into the same channel. This CP violation should manifest itself in the appearance of the term linear in X, i.e. $j\,X$, in the Dalitz plot (1). This term should arise from the interference between CP-even and CP-odd $K_L \to \pi^+\pi^-\pi^0$ amplitudes. According to the estimate of [23], the order of magnitude of the CP-violating coefficient j should be $j \simeq 1.2 \times 10^{-4}$, while the present determination is $j = (0.11 \pm 0.08) \times 10^{-2}$ [21]. Thus, the statistics at DAΦNE would be somewhat marginal to the predicted value for this parameter. Nonetheless, a significant improvement of the present experimental value might still be possible.

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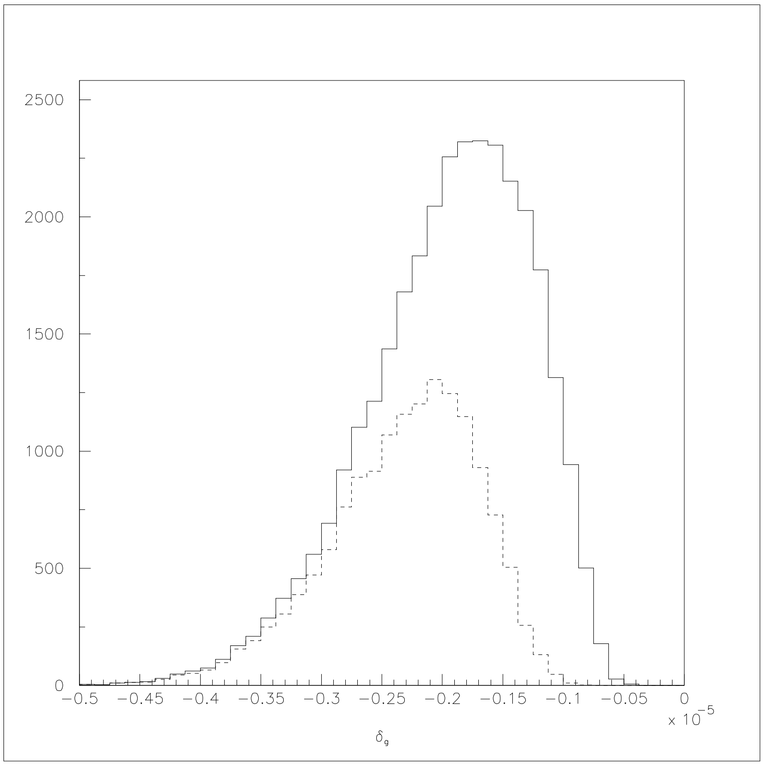

Figure 1: Predicted value of the asymmetry $(\delta_g)_\tau$.

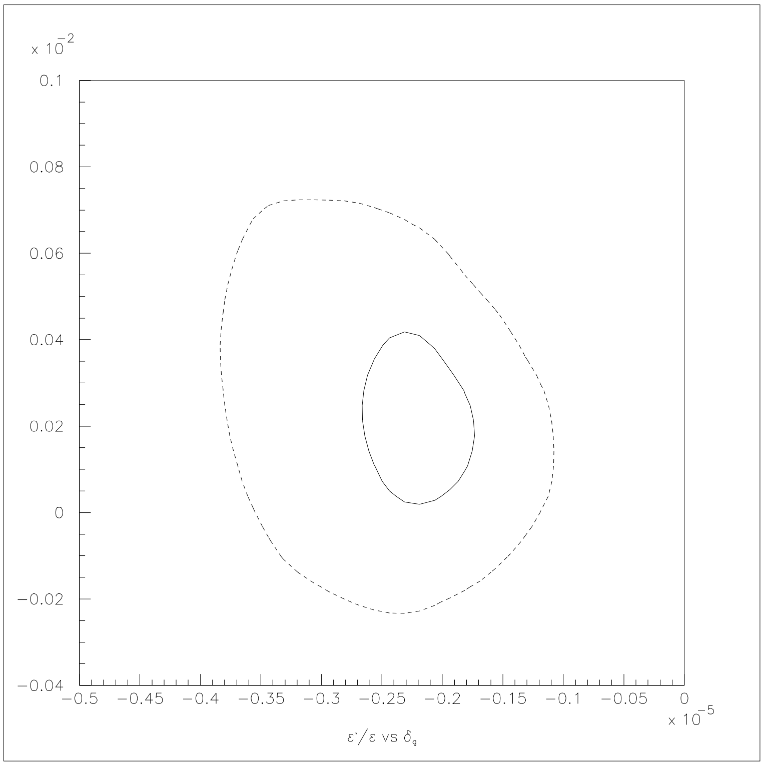

Figure 2: Combined determination of $(\delta_g)_\tau$ (horizontal axis) and ϵ'/ϵ (vertical axis).