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Chapter 1: CP and CPT Violation
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CP and CPT Violation in Neutral Kaon Decays

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Introduction

The study of CP violation in neutral K decays has been, from the outset, the primary goal of a high-luminosity $\Phi-$factory, like DAΦNE. At the beginning, attention was focussed almost exclusively on the measurement of the direct CP-violation parameter, ϵ'/ϵ. Due to the very interesting work of Buchanan et al. [1], it has been realized that, in addition, a $\Phi-$factory offers the unique possibility to make a clean test of the CPT symmetry, at the level of precision of few parts 10^{-4} of CP$-$violating amplitudes, independently from possible conspiracies still allowed in presently measured quantities.

The aim of the present article is to provide a self-consistent introduction to the phenomenology of CP and CPT violation in neutral Kaon decays and to give a first illustration of the impact of DAΦNE on the issue of the CPT symmetry.

More details are provided in the subsequent papers [2, 3]. The calculation of ϵ'/ϵ in the Standard Theory is also reviewed here (but discussed in [3] in more detail). CP violation in charged Kaon decays is deferred to a later part of this Chapter [4].

Of course, there exist in the literature many excellent reviews of the subject, starting from the classical and influential article of Lee and Wu [5]. In the more recent literature, CPT symmetry in Kaon decays has been analyzed by Barmin et al. [6] and, with reference to the experiments at a $\Phi-$factory, in Refs. [7, 1].

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1 Hamiltonian matrix, eigenvalues and mixing

The hamiltonian of the neutral Kaon system, in the particle rest frame, is a complex, 2x2 matrix:

$$\mathbf{H} = \begin{pmatrix} h & l \\ m & n \end{pmatrix} = M - \frac{i}{2}\Gamma \tag{1.1}$$

We are in the basis:

$$up = |K^0>; \quad down = |\bar{K}^0>;$$

h, l, m, n are complex numbers (so that $\mathbf{H}$ depends upon 8 real parameters), and M and Γ are hermitian matrices.

The sign of the antihermitian part of $\mathbf{H}$ is determined by the requirement that it must lead to exponential damping of the wave-function, for $t \to +\infty$. With the sign given in Eq. (1.1), the time-dependent factor of the wave-function is: $\exp(-iEt) = \exp(-iMt - \frac{1}{2}\Gamma t)$, which is correct, provided that Γ has non$-$negative eigenvalues.

We shall also write:

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}; \quad \Gamma = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}$$

Symmetries.

CP:

$$|K^0> \to |\bar{K}^0>; \quad |\bar{K}^0> \to |K^0> \tag{1.2}$$

T :

$$makes \quad the \quad complex \quad conjugate \tag{1.3}$$

It follows (τ_i = Pauli matrices):
CP:

$$M \to \tau_1 M \tau_1; \quad \Gamma \to \tau_1 \Gamma \tau_1 \tag{1.4}$$

T:

$$M \to M^*; \quad \Gamma \to \Gamma^* \tag{1.5}$$

If we develop both M and Γ in the basis of the Pauli matrices plus the identity,

$$X = c_0\mathbf{1} + \sum_i c_i\tau_i; \quad (\mathrm{c}_0,..,\mathrm{c}_3 \text{ real})$$

we are led to the following Table, which gives the sign taken by each component under CP, T and CPT transformations.

Tab.1

	c_0	c_1	c_2	c_3
$CP:$	$+$	$+$	$-$	$-$
$T:$	$+$	$+$	$-$	$+$
$CPT:$	$+$	$+$	$+$	$-$

We recover the familiar result that CPT implies equal diagonal elements. Also, CP conservation implies CPT conservation (unlike in 2π decay amplitudes, see below).

There is some freedom to redefine the phases of the states $|K^0>$ and $|\bar{K}^0>$. Strangeness is conserved by strong and e.m. interactions, so we can make the change:

$$\mid n, S > \rightarrow e^{-i\alpha S} \mid n, S >$$

i.e.:

$$\mid K^0 > \rightarrow e^{-i\alpha} \mid K^0 >; \quad \mid \bar{K}^0 > \rightarrow e^{i\alpha} \mid \bar{K}^0 > \tag{1.6}$$

In the new basis, the CP transformation is not given by (1.2) anymore, so that the matrix **H** needs not satisfy the rules given above, even if CP is conserved. All this amounts to say that CP is conserved *if and only if* there exists a change of phase of the form (1.6) such that, in the new basis, M and Γ have vanishing components along τ_2 and τ_3. This happens when M_{12} and Γ_{12} have the same phase. We conclude that the phase-invariant condition for CP conservation is:

$$arg(\frac{\Gamma_{12}}{M_{12}}) = 0 \quad \text{(CP symmetry)} \tag{1.7}$$

Diagonal elements are not affected by the phase change, so that the condition for CPT symmetry is always:

$$M_{11} = M_{22}; \quad \Gamma_{11} = \Gamma_{22} \quad \text{(CPT symmetry)} \tag{1.8}$$

Eigenvalue equation:

$$det \begin{pmatrix} h - \lambda & l \\ m & n - \lambda \end{pmatrix} = 0$$

leads to:

$$\lambda_{\pm} = \frac{1}{2}\Big[h + n \pm \sqrt{(h-n)^2 + 4lm}\Big] \tag{1.9}$$

note: $(\lambda_+ - h)(\lambda_- - h) = -lm$.
Eigenvectors:

$$v = \begin{pmatrix} p \\ q \end{pmatrix}$$

$$(\frac{q}{p})_{\pm} = \frac{\lambda_{\pm} - h}{l} = \frac{n - h \pm \sqrt{(h-n)^2 + 4lm}}{2l} \tag{1.10}$$

The identification of K_L and K_S states is obtained by going to the symmetric limit of exact CP (and CPT). In this limit, see (1.7) and (1.8):

$$\lambda_{\pm} = h \pm \sqrt{lm}$$

$$\frac{l}{m} = \frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*}} = \frac{M_{12}}{M_{12}^{*}} = \text{phase factor} =_{DEF} e^{-2i\alpha}$$

so that:

$$v_{+} = p_{+}\begin{pmatrix} 1 \\ e^{i\alpha} \end{pmatrix} = p_{+}e^{i\frac{\alpha}{2}}\begin{pmatrix} e^{-i\frac{\alpha}{2}} \\ e^{i\frac{\alpha}{2}} \end{pmatrix} =$$

$$= CP - even \quad \text{(after phase redefinition)}$$

$$v_{-} = -q_{-}\begin{pmatrix} e^{-i\alpha} \\ -1 \end{pmatrix} = -q_{-}e^{-i\frac{\alpha}{2}}\begin{pmatrix} e^{-i\frac{\alpha}{2}} \\ -e^{-i\frac{\alpha}{2}} \end{pmatrix} =$$

$$= CP - odd \quad \text{(after phase redefinition)}$$

Therefore, by continuity, we identify:

$$\mid K_S >= v_{+}; \quad \mid K_L >= v_{-} \tag{1.11}$$

Conventionally, eigenvectors are written in terms of two complex numbers, $\epsilon_{L,S}$, defined by:

$$\mid K_S >= v_{+} = N_S(\mid K_1 > + \epsilon_S \mid K_2 >); \quad N_S^{-2} = 1 + \mid \epsilon_S \mid^2 \tag{1.12}$$

$$\mid K_L >= v_{-} = N_S(\mid K_2 > + \epsilon_L \mid K_1 >); \quad N_L^{-2} = 1 + \mid \epsilon_L \mid^2 \tag{1.13}$$

with:

$$\mid K_{1,2} >= \frac{1}{\sqrt{2}}(\mid K_0 > \pm \mid \bar{K}^0 >) \tag{1.14}$$

The Hamiltonian matrix is determined by 8 real parameters, which we can substitute with the 2 complex eigenvalues and the 2 complex mixing parameters, $\epsilon_{L,S}$. The relevant formulae are:

$$(m_S + m_L) - \frac{i}{2}(\Gamma_S + \Gamma_L) = h + n \tag{1.15}$$

$$(m_L - m_S) + \frac{i}{2}(\Gamma_S - \Gamma_L) = -\sqrt{(h-n)^2 + 4lm} \tag{1.16}$$

$$\frac{1-\epsilon_S}{1+\epsilon_S} = \frac{n - h + \sqrt{(h-n)^2 + 4lm}}{2l} \tag{1.17}$$

$$\frac{1+\epsilon_L}{1-\epsilon_L} = \frac{n-h+\sqrt{(h-n)^2+4lm}}{2m} \tag{1.18}$$

In the CPT symmetric limit, $h = n$ and there is only one mixing parameter, $\epsilon = \epsilon_S = \epsilon_L$, determined by:

$$\frac{1+\epsilon}{1-\epsilon} = \sqrt{\frac{l}{m}} = \sqrt{\frac{M_{12}-\frac{i}{2}\Gamma_{12}}{M_{12}^*-\frac{i}{2}\Gamma_{12}^*}} \qquad \text{(CPT exact)}$$

To keep contact with the CPT-invariant limit, the ϵ's are conveniently written as:

$$\epsilon_S = e_M + \Delta \tag{1.19}$$

$$\epsilon_L = e_M - \Delta \tag{1.20}$$

2 $K \to 2\pi$ decay amplitudes, I

The strong interaction, $S-$wave, phase shifts for $\pi - \pi$ scattering are defined as:

$$< 2\pi, I; out \mid 2\pi, I; in >= e^{2i\delta_I} \tag{2.1}$$

where I denotes the total isospin ($I = 0, 2$) and a c.o.m. energy equal to the Kaon mass is understood. In fact, since a 2π state cannot transform into 3π or 4π states by energy conservation, Eq. (2.1) can be rewritten as:

$$\mid 2\pi, I; in >= e^{2i\delta_I} \mid 2\pi, I; out > \tag{2.2}$$

2π decay amplitudes of K^0 and $\bar{K}^0$ are defined according to:

$$A(K^0 \to 2\pi, I) =< 2\pi, I; out \mid H_W \mid K^0 >=_{DEF} \sqrt{\frac{3}{2}}(A_I + B_I)\, e^{i\delta_I} \tag{2.3}$$

$$A(\bar{K}^0 \to 2\pi, I) =< 2\pi, I; out \mid H_W \mid \bar{K}^0 >=_{DEF} \sqrt{\frac{3}{2}}(A_I^* - B_I^*)\, e^{i\delta_I} \tag{2.4}$$

(The factor $\sqrt{3/2}$ is inserted to simplify later formulae). Symmetry relations are as follows.

$$CP: \qquad < 2\pi, I; out \mid H_W \mid K^0 >=< 2\pi, I; out \mid CP^{-1}(H_W)CP \mid \bar{K}^0 > \tag{2.5}$$

$$\begin{aligned} T: \qquad < 2\pi, I; out \mid H_W \mid K^0 >^* &=< 2\pi, I; in \mid T^{-1}(H_W)T \mid K^0 >= \\ &= e^{-2i\delta_I} < 2\pi, I; out \mid T^{-1}(H_W)T \mid K^0 > \end{aligned} \tag{2.6}$$

Writing, for fixed I:

$$A = A^{(1)} + iA^{(2)}$$

$$B = B^{(1)} + iB^{(2)}$$

the symmetry properties of the various components of the amplitude are those given in the Table.

Tab.2

	$A^{(1)}$	$A^{(2)}$	$B^{(1)}$	$B^{(2)}$
$CP:$	$+$	$-$	$-$	$+$
$T:$	$+$	$-$	$+$	$-$
$CPT:$	$+$	$+$	$-$	$-$

In this case, we may have CP conserved and CPT violated or viceversa. B is exclusively a signal of CPT violation.

3 The Wu-Yang phase convention

It is very important (and helpful!) to keep track of the phase arbitrariness embodied by Eq. (1.6). From the formulae above, one has:

$$\mathbf{X} \to e^{-iS\alpha}\mathbf{X} \tag{3.1}$$

with:

$$S = 0; \quad -2; \quad +2; \quad (\text{for } \mathbf{X}= \text{h or n; l; m}) \tag{3.2}$$

$$S = 1; \qquad (\text{for } \mathbf{X}= \text{A}_I \text{ or } \text{B}_I) \tag{3.3}$$

We can use the phase arbitrariness to make one of the quantities in (3.2) and (3.3) to be real. The Wu-Yang convention [8] requires:

$$A_0 = \textit{real and positive} \tag{3.4}$$

The Wu-Yang convention is phenomenologically very useful, as we shall see presently. It corresponds to shift as much T-violation as possible away from the dominant, $\Delta I = 1/2$, non-leptonic amplitude into the suppressed, $\Delta I = 3/2$ one.

In the usual parametrization of the KM matrix precisely the opposite occurs, namely T-violation appears predominantly in the $\Delta I = 1/2$ amplitude, due to t-quark exchange in penguin diagrams, while the $\Delta I = 3/2$ amplitude is predominantly real (except for electroweak penguin effects, which may become important for large values of the t-quark mass, see Sect. 5). Of course, amplitudes computed in the latter (KM) convention can be transformed back to the Wu-Yang phase convention by the transformation (3.1) with:

$$\alpha = -\frac{ImA_0}{A_0} \tag{3.5}$$

We adopt the Wu-Yang convention in the following.

4 $K \to 2\pi$ decay amplitudes, II

From Eqs. (2.3) and (2.4) and the isospin Clebsch-Gordan coefficients, one finds the decay amplitudes in the $\pi^+\pi^-$ and $\pi^0\pi^0$ channels:[2]

$$A(K^0 \to \pi^+\pi^-) = (A_0 + B_0)e^{i\delta_0} + \frac{1}{\sqrt{2}}(A_2 + B_2)e^{i\delta_2} \tag{4.1}$$

$$A(K^0 \to \pi^0\pi^0) = (A_0 + B_0)e^{i\delta_0} - \sqrt{2}(A_2 + B_2)e^{i\delta_2} \tag{4.2}$$

For $\bar{K}^0$ amplitudes, $A \to A^*$, $B \to -B^*$. Also:

$$A(K \to \pi^+\pi^0) = \frac{3}{2}(A_2 + B_2)e^{i\delta_2} \tag{4.3}$$

From these formulae and from the experimental K_S and K^+ decay rates we can derive the values of the CP and CPT-conserving amplitudes:

$$A_0 = 2.7 \cdot 10^{-7} GeV \tag{4.4}$$

$$\omega = \frac{A_2}{A_0} = 0.045 \tag{4.5}$$

as well as:

$$cos(\delta_2 - \delta_0) = 0.52; \quad \mid \delta_2 - \delta_0 \mid = 59^0 \tag{4.6}$$

The value of the phase is in reasonable agreement with the one found from pion production in π-Nucleon scattering and in K_{e4} decay:

$$\delta_0 - \delta_2 = (41.4 \pm 8.1)^0 \quad \text{(Devlin and Dickey [9])}$$

$$\delta_0 - \delta_2 = (46.3^{+2.7}_{-4.0})^0 \quad \text{(Shenk [10])} \tag{4.7}$$

After these preliminaries, we proceed to derive the formulae for the 2π decays of K_L. One defines:

$$\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}; \quad \eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \tag{4.8}$$

and finds, from Eqs. (1.13) and (4.1-2) (Wu-Yang convention is used throughout, terms of second order in CP/CPT violation and first order multiplied by ω^2 are neglected):

$$\eta_{+-} = \epsilon_L + \frac{A(K_2 \to \pi^+\pi^-)}{A(K_1 \to \pi^+\pi^-)} = \epsilon + \epsilon' \tag{4.9}$$

[2]The decay rates into $\pi^+\pi^-$ or $\pi^+\pi^0$ are given by: Rate= $\frac{p_{decay}}{8\pi M^2} \mid A \mid^2$. For $\pi^0\pi^0$ there is an additional factor of 1/2, for the identical particles.

$$\eta_{00} = \epsilon_L + \frac{A(K_2 \to \pi^0\pi^0)}{A(K_1 \to \pi^0\pi^0)} = \epsilon - 2\epsilon' \tag{4.10}$$

with:

$$\epsilon = \epsilon_M - (\Delta - \frac{Re\, B_0}{A_0}) \tag{4.11}$$

$$\epsilon' = ie^{i(\delta_2-\delta_0)}\frac{\omega}{\sqrt{2}}\Big[\frac{Im\, A_2}{A_2} - i(\frac{Re\, B_2}{A_2} - \frac{Re\, B_0}{A_0})\Big] \tag{4.12}$$

Eqs. (4.9) and (4.10) are formally identical to those of the exact CPT limit, but with a different relation between ϵ and the mass mixing parameters, and with an additional contribution of the B's to ϵ'.

In the limit of vanishing Δ and B, one obtains in (4.12) the usual expression for ϵ', in the notation appropriate to the Wu-Yang convention. As noted before, given the amplitudes computed with a different phase convention, e.g. with the usual KM phases, we obtain the amplitudes in the Wu-Yang frame by making, for any $\Delta S = -1$ amplitude $\mathbf{X}$, the replacement:

$$Im\,\mathbf{X} \to Im\,\mathbf{X} - Re\,\mathbf{X}\,\frac{Im\, A_0}{A_0} \tag{4.13}$$

In this way, we obtain from Eq. (4.12) the phase-convention independent expression:

$$\epsilon' = ie^{i(\delta_2-\delta_0)}\frac{\omega}{\sqrt{2}}\Big[\frac{Im\, A_2}{A_2} - \frac{Im\, A_0}{A_0} - i(\frac{Re\, B_2}{A_2} - \frac{Re\, B_0}{A_0})\Big] \tag{4.14}$$

The structure of Eq. (4.14) can be read very simply, with reference to the definition of ϵ' given in Eq. (4.9). The factor $\exp i(\delta_2 - \delta_0)$ arises from the final state interaction, the dominant final states being $I = 2$ and $I = 0$ for the numerator and denominator of the ratio in Eq. (4.9), respectively. The further factor of i arises because ϵ', being CP violating, must violate time-reversal in a CPT conserving theory (i.e. be imaginary, apart from the final state interaction phases). CPT violation, indeed, appears as a further, T-conserving, imaginary contribution to the square bracket in Eq. (4.14).

5 Standard Theory prediction of ϵ'/ϵ

The parameter ϵ' is uniquely related to CP (and CPT) violation in the transition amplitude, see Eq. (4.14). In the usual terminology, $\epsilon' \neq 0$ characterizes *milli-weak* theories, i.e. theories in which the weak interaction itself has a small but detectable CP violating component. The mass mixing parameter, ϵ, arises as a 2^{nd} order weak effect, in which the CP-odd and CP-even parts combine to give the $\Delta S = \pm 2$ quantities M_{12} and Γ_{12}.

Another, logically independent, possibility is that the observed CP-violation is the 1^{st} order manifestation of a new interaction with $\Delta S = \pm 2$. In the mass matrix, the new interaction competes with the 2^{nd} order weak contribution. Therefore, a very weak interaction is required, of strength $\simeq 10^{-3} \cdot (G_F M_P^2)^2 \simeq 10^{-13}$, to give rise to the observed CP violation. This is called the "superweak" theory [11], and it has the obvious prediction that no CP-violation is visible in channels available to 1^{st} order weak transitions, i.e. it predicts $\epsilon' = 0$.

Milliweak theories are in danger to contradict the very tight experimental limits to the electric dipole of the neutron, a T- and P-violating, $\Delta S = 0$, effect. Unless a special cancellation occurs, we expect any hadron to have an e.d.m. of the order of:

$$\mu_E \simeq e \cdot r_P (G_F M_P^2) \mid \epsilon \mid \simeq 10^{-21} e \cdot cm \tag{5.1}$$

$r_P \simeq 10^{-13} cm$ is the proton radius, which gives the general dimension, the Fermi constant G_F is associated to P-violation while ϵ, in a milliweak theory, characterizes the generic strength of T-violation. The present experimental upper bound to the neutron e.d.m. is [12, 13]:

$$(\mu_E)_{neutron} \leq 1.2 \cdot 10^{-25} e \cdot cm \tag{5.2}$$

much too small to be compatible with (5.1), which therefore calls for a very special cancellation.

The Standard, six flavour, Theory [14] is a milliweak theory in which such a special cancellation *does* occur [15]. CP-violation (rather, T-violation) arises because different components of the weak charged current have non-vanishing relative phases. However, the one-loop correction to the electromagnetic current of a given quark, e.g. the d-quark, is given by a sum of terms in which each complex entry, corresponding to, say, $d \to c$ (V_{cd}), is multiplied by the amplitude for the inverse process, $c \to d$ (V_{cd}^*). Thus, the correction is real and the e.d.m vanishes to one loop, which brings the estimate (5.1) already down to $\simeq 10^{-24} \cdot e \cdot cm$.

A further suppression is due to the fact that one can *rotate away* the CP-violating phase when any two quark of the same charge are degenerate in mass [15]. Thus, any CP-violating effect in the Standard theory must involve light quark mass differences, which brings in powers of m_{quark}/M_W.

Finally, as shown in Ref. [16], the quark e.d.m. vanishes also at two-loops, which brings in another factor of 10^{-3}.

In conclusion, current estimates are that the e.d.m. of the neutron in the Standard Theory is essentially unobservable [16]:

$$(\mu_E)_{neutron} \simeq 10^{-31} e \cdot cm \quad \text{(Standard Theory)} \tag{5.3}$$

The above discussion underlines the importance of a positive measurement of ϵ'/ϵ.

The first calculation of ϵ'/ϵ in the Standard Theory is due to Gilman and Wise [17]. We summarize here the most recent analyses[18, 19]. The calculation of ϵ'/ϵ goes through several steps.

i) Determination of the effective weak, non-leptonic Hamiltonian, H_{eff}. The coefficients of the effective Hamiltonian depend upon the chosen value of the subtraction point. Provided we choose the subtraction point large enough, the result is dominated by short-distance effects which, in QCD, are controlled by perturbation theory. Different terms in H_{eff} can be

classified according to their transformation properties under chiral SU(3)×SU(3)[20]. The dominant term transforms as $(8_L, 1_R)$, corresponding to the familiar octet-enhancement, while $\Delta I = 3/2$ transitions are produced by a $(27_L, 1_R)$ component. The $(8_L, 1_R)$ component has a complex coefficient which arises, in the usual KM phase convention, because of t-quark exchange in the so-called penguin diagrams. Electroweak penguin diagrams give rise instead to $(8_L, 8_R)$ components, also with a complex coefficient.
ii) The $(8_L, 1_R)$ term gives rise to a non-vanishing value of $Im\, A_0$, thus giving a first contribution to Eq. (4.14). Contributions to $Im\, A_2$ arise from two different sources. The first is due to isospin breaking: the octet component contributes to $Im\, A_2$ a term proportional to the quark mass difference, $m_d - m_u$. Although this difference is small:

$$\frac{(m_d - m_u)}{m_s} \simeq \frac{3\, MeV}{150\, MeV} = 0.02$$

it is partly compensated by the fact that such a term appears in Eq. (4.14) divided by $Re\, A_2$ and is therefore enhanced by a factor of ω^{-1} with respect to the previous one, see Eq. (4.5). A second contribution to $Im\, A_2$ arises from the $(8_L, 8_R)$ component. The small Wilson coefficient with which it appears in H_{eff} is partly compensated by the factor ω^{-1} and also by the fact that chiral symmetry does not require the matrix element of the $(8_L, 8_R)$ to vanish for vanishing external momenta, as is the case for both the $(8_L, 1_R)$ and $(27_L, 1_R)$. The raising with the t-quark mass of the Wilson coefficient of the $(8_L, 8_R)$ term is responsible for the decrease of ϵ'/ϵ.
iii) Matrix elements of the effective Hamiltonian are parametrized in terms of the so-called B-factors, scale factors which measure the deviation of the true matrix element from the one computed in the vacuum insertion approximation. At present, systematic calculations of the B-factors have been carried on with lattice QCD, QCD sum rules and the expansion in the inverse of the number of colours, $1/N_c$.

Predictions of ϵ'/ϵ vs. the top-quark mass are discussed later in this report [3]. For illustration, we show in Fig. 1 the theoretical prediction, for B-factors computed in lattice QCD.

The value of ϵ'/ϵ is generally predicted in the 10^{-3} range. A very small value results for $m_t \simeq 200\, GeV$, due to the electroweak penguin effects. The top-quark mass recently discovered by CDF [21] is:

$$M_{top} = 174 \pm 17 \quad \text{GeV}$$

For this value, the best prediction of lattice QCD calculation is [3]:

$$[\frac{\epsilon'}{\epsilon}]_{lattice} = (3.1 \pm 2.5)\, 10^{-4} \tag{5.4}$$

6 Semileptonic amplitudes

We focus on K_{e3} decays of K^0 and $\bar{K}^0$. On general grounds, there are 4 independent matrix elements, related to the (complex) form factors of the transitions:

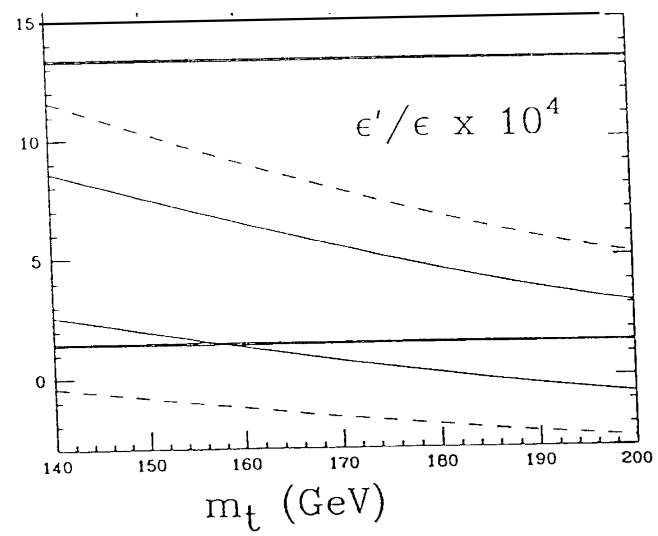

Figure 1: ϵ'/ϵ as function of m_t, obtained by applying the f_B-cut [3]. The zones delimited by the solid and dashed curves represent the allowed regions containing respectively 68% and 95% of the generated events. The region between the two double lines is the experimental result coming from $E731$, see text.

$\Delta S = \Delta Q$:

$$\begin{pmatrix} K^0 \to e^+\nu_e\pi^- \\ \bar{K}^0 \to e^-\bar{\nu}_e\pi^+ \end{pmatrix} \tag{6.1}$$

$\Delta S = -\Delta Q$:

$$\begin{pmatrix} K^0 \to e^-\bar{\nu}_e\pi^+ \\ \bar{K}^0 \to e^+\nu_e\pi^- \end{pmatrix} \tag{6.2}$$

Time-reversal relates each form factor to its complex conjugate, CP relates K^0 to $\bar{K}^0$ form factors. This suggests to parametrize the amplitudes according to:

$$< e^+\nu_e\pi^- \mid H_W \mid K^0 >= a + b \tag{6.3}$$

$$< e^-\bar{\nu}_e\pi^+ \mid H_W \mid \bar{K}^0 >= a^* - b^* \tag{6.4}$$

$$< e^-\bar{\nu}_e\pi^+ \mid H_W \mid K^0 >= c + d \tag{6.5}$$

$$< e^+\nu_e\pi^- \mid H_W \mid \bar{K}^0 >= c^* - d^* \tag{6.6}$$

a and b (c and d) obey the same symmetry properties as the non-leptonic amplitudes A_I and B_I (see Tab. 2), i.e.: b and d are CPT violating, imaginary parts are all T-violating; c

and d describe possible violations of the $\Delta S = \Delta Q$ rule. We consider $Re\,a$ of order unity, and keep first order terms in all the other quantities.

Of course, one should introduce analogous amplitudes for muonic decays, but we will leave this understood, in the following, to avoid a too heavy notation.

The following notations are also used [2]:

$$y = -\frac{b}{a} \tag{6.7}$$

$$x = \frac{c^* - d^*}{a+b}; \quad \bar{x}^* = \frac{c^* + d^*}{a-b} \tag{6.8}$$

with:

$\Delta S = \Delta Q$ exact:

$$x = \bar{x} = 0 \tag{6.9}$$

CPT exact:

$$x = \bar{x}^*, \quad y = 0 \tag{6.10}$$

T exact:

$$x, \quad \bar{x}, \quad y = real \tag{6.11}$$

CP exact:

$$x = \bar{x}, \quad y = imaginary \tag{6.12}$$

For convenience, we shall also define:

$$\Delta' = \Delta - \frac{Re\,d}{Re\,a} - i\frac{Im\,c}{Re\,a} \tag{6.13}$$

$$\Gamma^e_{L/S} = \Gamma(K_{L/S} \to e^+ + ..) + \Gamma(K_{L/S} \to e^- + ..) \tag{6.14}$$

with Δ defined in Eqs. (1.19) and (1.20) and:

$$\Gamma^{sl}_{L/S} = \Gamma^e_{L/S} + \Gamma^\mu_{L/S}$$

The following relations are immediate:

$$A_L = \frac{\Gamma(K_L \to e^+\nu_e\pi^-) - \Gamma(K_L \to e^-\bar{\nu}_e\pi^+)}{sum} = 2(Re\,\epsilon_L + \frac{Re\,b}{Re\,a} + \frac{Re\,d}{Re\,a}) =$$

$$= 2(Re\,\epsilon_M - Re\,\Delta' + \frac{Re\,b}{Re\,a}) \tag{6.15}$$

$$A_S = 2(Re\,\epsilon_M + Re\,\Delta' + \frac{Re\,b}{Re\,a}) \tag{6.16}$$

$$\Delta_e = \frac{\Gamma^e_S - \Gamma^e_L}{sum} = 2\frac{Re\, c}{Re\, a} \tag{6.17}$$

There are in all four semileptonic rates, which can be expressed in terms of the three combinations given above plus the average rate, which determines $(Re\, a)^2$. In addition, to study the correlated decays of the K_L-K_S pair produced at a $\Phi-$factory, it is convenient to introduce the complex quantities:

$$\eta_{l+} = \frac{< e^+\nu_e\pi^- \mid H_W \mid K_L >}{< e^+\nu_e\pi^- \mid H_W \mid K_S >} = 1 - 2\frac{Re\, c}{Re\, a} - 2\Delta' - 2i\frac{Im\, d}{Re\, a} \tag{6.18}$$

$$\eta_{l-} = \frac{< e^-\bar{\nu}_e\pi^+ \mid H_W \mid K_L >}{< e^-\bar{\nu}_e\pi^+ \mid H_W \mid K_S >} = -(1 - 2\frac{Re\, c}{Re\, a} + 2\Delta' - 2i\frac{Im\, d}{Re\, a}) \tag{6.19}$$

In the Standard Theory, CPT and CP are conserved in semileptonic processes and the $\Delta S = \Delta Q$ rule is obeyed to a very good precision [22], with ($g_8 \simeq 5$ is the relative strength of the octet non-leptonic amplitude):

$$\mid x \mid \simeq g_8 \frac{G_F}{\sqrt{2}} f_\pi^2 \simeq 7 \cdot 10^{-7} \tag{6.20}$$

In the current x current picture there is, in fact, little space for the violation of these symmetries, given our very good knowledge of the currents themselves.

Violation of CP or of the $\Delta S = \Delta Q$ rule could arise from contact interactions of quark and leptons (e.g. in composite models) and one should keep an open mind on the possible presence of anomalies in the semileptonic amplitudes. However, $\Delta S = -\Delta Q$ transitions require hadronic operators transforming as 10 + 27 of flavour SU(3), see e.g. the second paper of Ref. [22], that can be induced only by effective quark and lepton operators of dimension higher than four. A typical example is:

$$\Delta H_{eff} = \frac{4\pi}{\Lambda^6}\Big[\frac{\partial}{\partial x^\nu}(\bar{e}_L\gamma_\mu\nu_{eL})\Big](\bar{u}_L\gamma_\nu d_L)(\bar{s}_L\gamma_\mu d_L) \tag{6.21}$$

with Λ the compositeness scale, which leads to the (rather generous) estimate:

$$\mid x \mid \simeq \frac{4\pi}{G_F\Lambda^2}\Big(\frac{1GeV}{\Lambda}\Big)^4 \simeq 10^{-10}\Big(\frac{1TeV}{\Lambda}\Big)^6 \tag{6.22}$$

The result (6.22) justifies the neglect of $\Delta S = -\Delta Q$ amplitudes, still keeping open the possibility of CPT violation.

For the sake of brevity, the case in which semileptonic amplitudes are assumed to conserve both CPT and the $\Delta S = \Delta Q$ rule will be called *Scheme I*, in the following. *Scheme II* will be the case in which CPT is relaxed, still keeping exact the $\Delta S = \Delta Q$ rule. We shall also comment on *Scheme III*, where Eqs. (6.3) to (6.6) are considered in full generality.

7 Comparison of notations

The recent analyses [6, 1] of neutral Kaon mixing and decays without assumption of CPT do not follow a unified set of notations. To facilitate the comparison, we give below the translation table from ours to their notations.

Tab.3

our notation *Scheme III*	**Barmin et al. Ref. [6]** *Scheme I*	**Buchanan et al. Ref. [1]** *Scheme II*
$\epsilon_{S,L} = \epsilon_M \pm \Delta$	$\epsilon_{S,L} = \epsilon \pm \Delta$	$\epsilon_{S,L} = \epsilon_K \pm \delta_K$
$\eta_{+-} = \epsilon + \epsilon'$	$\eta_{+-} = \epsilon_0 + \epsilon'$	$\eta_{+-} = \epsilon + \epsilon'$
ϵ'	ϵ'	ϵ'
$\frac{Re\, B_0}{A_0}$	a	$\frac{Re\, B_0}{A_0}$
a, b; Eqs.(6.3-4)	$b = 0$	a, b
c, d; Eqs.(6.5-6)	$c = d = 0$	$c = d = 0$

8 Unitarity constraints

We know little about the real part of the Hamiltonian, the mass matrix M, which is sensitive to virtual particle, high-energy effects (this is, for instance, the case in the Standard Theory, where T violation in M is determined by top-quark exchange). On the other hand, unitarity relates the matrix Γ, the imaginary part of $\mathbf{H}$, to the real decays of the neutral Kaons, about which we have considerably more information:

$$\Gamma_{ab} = \sum_n 2\pi\ \delta(M_K - E_n) < a \mid H_W \mid n >< n \mid H_W \mid b > \tag{8.1}$$

In particular, we know that the $2\pi, I = 0$ final state is by far the most prominent one in K^0 and $\bar{K}^0$ decays, and this simple fact gives interesting restrictions on the parameters $\epsilon_{L,S}$.

We start from Eqs. (1.16) to (1.18), which are easily solved to obtain $h - n$, l and m in terms of the physical parameters. Separating real and imaginary parts, one finds six relations:

$$2Re\, M_{12} = -(m_L - m_S) \tag{8.2}$$

$$2Im\, M_{12} = -(\Gamma_S - \Gamma_L)\, [Re(\frac{\epsilon_S + \epsilon_L}{2}) + tan\phi_{SW}\ Im\,(\frac{\epsilon_S + \epsilon_L}{2})] \tag{8.3}$$

$$2Re\,\Gamma_{12} = (\Gamma_S - \Gamma_L) \tag{8.4}$$

$$Im\,\Gamma_{12} = -(\Gamma_S - \Gamma_L)\, [tan\phi_{SW}\ Re(\frac{\epsilon_S + \epsilon_L}{2}) - Im\,(\frac{\epsilon_S + \epsilon_L}{2})] \tag{8.5}$$

$$M_{11} - M_{22} = -(\Gamma_S - \Gamma_L)\,[tan\phi_{SW}\; Re(\frac{\epsilon_S - \epsilon_L}{2}) - Im\,(\frac{\epsilon_S - \epsilon_L}{2})] \quad (8.6)$$

$$\frac{1}{2}(\Gamma_{11} - \Gamma_{22}) = (\Gamma_S - \Gamma_L)\,[Re(\frac{\epsilon_S - \epsilon_L}{2}) + tan\phi_{SW}\; Im\,(\frac{\epsilon_S - \epsilon_L}{2})] \quad (8.7)$$

We have introduced the "superweak phase", ϕ_{SW} , defined by:

$$tan\phi_{SW} =_{DEF} \frac{2(m_L - m_S)}{\Gamma_S - \Gamma_L} = 0.9565 \pm 0.0051$$

$$\phi_{SW} = (43.73 \pm 0.15)^0 \quad (8.8)$$

and will denote by v and w the complex numbers:

$$v = \frac{1}{(1 + tan^2\phi_{SW})^{\frac{1}{2}}}(1 + i\; tan\phi_{SW})$$

$$w = \frac{1}{(1 + tan^2\phi_{SW})^{\frac{1}{2}}}(-tan\phi_{SW} + i) \quad (8.9)$$

Eqs. (8.3) and (8.5) specify the components of $(\epsilon_S + \epsilon_L)$ along v and w, regarded as mutually orthogonal vectors in the complex plane. Eqs. (8.6) and (8.7) do the same for the CPT violating parameter $(\epsilon_S - \epsilon_L)$.

In the first case, we use the fact that the dominant 2π, $I = 0$ amplitude is exactly real, in the Wu-Yang convention. Correspondingly, $Im\,\Gamma_{12}$ receives contribution from 2π with $I = 2$, 3π and semileptonic decay modes. In general, the scale of these contributions is suppressed, with respect to the r.h.s. of Eq.(8.5), by a factor of $\Gamma(K^+)/\Gamma_S$ or Γ_L/Γ_S. Thus, to be competitive with the r.h.s., CP violation on the l.h.s. of Eq. (8.5) should be of order unity, which is not the case (rather, as we have indicated in Sect. 5, ϵ' effects in 2π decays are much more suppressed).

More in detail, one may classify the contribution of the most prominent intermediate states as follows (first order terms only are retained).

2π:

$$(Im\,\Gamma_{12})_{(2\pi)} = \frac{3}{2}Im\,[(A_0^2 - B_0^2)^* + (A_2^2 - B_2^2)^*] \times (phase\ space) =$$

$$= -\frac{4}{3}B(K^+ \to \pi^+\pi^0)\frac{\Gamma_{K^+}}{\Gamma_S}\frac{Im\,A_2}{Re\,A_2}\Gamma_S \simeq -2.04 \cdot 10^{-3}(\frac{Im\,A_2}{Re\,A_2})\Gamma_S$$

3π:

We approximate 3π decay amplitudes with their value at the center of the Dalitz plot and consider only the $\Delta I = 1/2$ contribution to the CP$-$conserving transition. Defining CP-violating parameters according to [4]:

$$\epsilon'_{+-0} = \frac{A(K_1 \to \pi^+\pi^-\pi^0; I = 1)}{A(K_2 \to \pi^+\pi^-\pi^0; I = 1)};$$

$$\epsilon'_{000} = \frac{A(K_1 \to 3\pi^0; I=1)}{A(K_2 \to 3\pi^0; I=1)}$$

one finds:

$$(Im\,\Gamma_{12})_{(3\pi)} = \Gamma(K_2 \to \pi^+\pi^-\pi^0) Im\,\epsilon'_{+-0} + \Gamma(K_2 \to 3\pi^0) Im\,\epsilon'_{000} =$$

$$\simeq 2.14 \cdot 10^{-3}(Im\,\epsilon'_{+-0} + 1.74 Im\,\epsilon'_{000})\Gamma_S$$

semileptonic:

$$(Im\,\Gamma_{12})_{sl} = -Im\,(x^* + \bar{x})\Gamma_L^{sl} \simeq -2.26 \cdot 10^{-3} Im\,(\frac{x^* + \bar{x}}{2})\Gamma_S$$

(we have averaged the electron and muon contributions). The semileptonic contribution is suppressed, since it requires $\Delta Q = -\Delta S$.

Neglecting completely the l.h.s of (8.5), we conclude that $(\epsilon_S + \epsilon_L)$ is orthogonal to w:

$$Arg(\epsilon_S + \epsilon_L) \simeq \phi_{SW} \tag{8.10}$$

The component along v is determined by the short-distance sensitive quantity, $Im\,M_{12}$:

$$\frac{\epsilon_S + \epsilon_L}{2} = \epsilon_M \simeq (\frac{-Im\,M_{12}}{m_L - m_S})\frac{tan\phi_{SW}(1 + i\,tan\phi_{SW})}{(1 + tan^2\phi_{SW})}) = \frac{-iIm\,M_{12}}{m_L - m_S - \frac{i}{2}(\Gamma_L - \Gamma_S)} \tag{8.11}$$

In addition, since:

$$Arg[ie^{i(\delta_2 - \delta_0)}] \simeq 45^0 \simeq \phi_{SW} \tag{8.12}$$

it follows from Eq. (4.12) that ϵ' is approximately parallel to ϵ_M, except for CPT-violating effects.

To make a similar analysis for the CPT-violating quantity, $\Delta = (\epsilon_S - \epsilon_L)/2$, we first extract the 2π contribution to the r.h.s. of Eq. (8.7). Explicitly (ρ_{+-}, ρ_{00} are the 2π phase-space factors; $\rho_{+-} \simeq 2\rho_{00}$ for exact isospin symmetry):

$$(\Gamma_{11} - \Gamma_{22})_{(2\pi)} = \rho_{+-}(\mid A(K^0 \to \pi^+\pi^-) \mid^2 - \mid A(\bar{K}^0 \to \pi^+\pi^-) \mid^2) +$$

$$+\rho_{00}(\mid A(K^0 \to \pi^0\pi^0) \mid^2 - \mid A(\bar{K}^0 \to \pi^0\pi^0) \mid^2) =$$

$$= \frac{3}{2}\rho_{+-}\left(4A_0^2\frac{Re\,B_0}{Re\,A_0} + 4A_2^2\frac{Re\,B_2}{Re\,A_2}\right) \simeq$$

$$\simeq 2\Gamma(K_S \to 2\pi)[\frac{Re\,B_0}{A_0} + \omega^2\frac{Re\,B_2}{A_2}] \tag{8.13}$$

With this result, Eq. (8.7) becomes:

$$\frac{1}{2}(\Gamma_{11}-\Gamma_{22})_{res}+\delta\Gamma(\frac{Re B_0}{A_0}) = (\Gamma_S-\Gamma_L)\Big[Re(\Delta-\frac{Re\, B_0}{A_0})+tan\phi_{SW} Im(\Delta-\frac{Re\, B_0}{A_0})\Big] \quad (8.14)$$

The suffix *res* indicates the sum over intermediate states different from 2π, $I=0$, and:

$$\delta\Gamma = \Gamma(K_S \to 2\pi) - \Gamma_S + \Gamma_L \simeq \Gamma_L - \Gamma_S^{sl} \simeq \Gamma_L - \Gamma_L^{sl} \simeq 0.59\cdot 10^{-3}\Gamma_S$$

(assuming the semileptonic rates af K_S and K_L to be approximately equal).

The term proportional to $\delta\Gamma$ can be safely neglected.[3] Proceeding as before, we write the most important contributions to the l.h.s. of Eq. (8.14) as follows.
2π, $I=2$:

$$\frac{1}{2}(\Gamma_{11}-\Gamma_{22})_{(2\pi,I=2)} = B(K_S\to 2\pi)\omega^2\frac{Re\, B_2}{A_2}\Gamma_S \simeq 2.02\cdot 10^{-3}\frac{Re\, B_2}{A_2}\Gamma_S$$

3π:

$$\frac{1}{2}(\Gamma_{11}-\Gamma_{22})_{(3\pi)} = \Gamma(K_2\to\pi^+\pi^-\pi^0)Re\,\epsilon'_{+-0}+\Gamma(K_2\to 3\pi^0)Re\,\epsilon'_{000} =$$

$$\simeq 2.14\cdot 10^{-3}(Re\,\epsilon'_{+-0}+1.74 Re\,\epsilon'_{000})\Gamma_S$$

semileptonic:

$$\frac{1}{2}(\Gamma_{11}-\Gamma_{22})_{sl} = 2\frac{Re\, b}{Re\, a}\Gamma_L^{sl} \simeq 2.26\cdot 10^{-3}\frac{Re\, b}{Re\, a}\Gamma_S$$

(electron and muon contributions averaged).

It is difficult to say anything more precise about the first term in the l.h.s. of (8.14), except that it should be very small, for the same reasons which justified the neglect of $Im\,\Gamma_{12}$. DAΦNE can improve substantially on the present limits to the above CPT-violating quantities and therefore lead to improved bounds to the unitarity sum.

If we take the l.h.s. to vanish, Eq. (8.14) leads to the elegant result that the complex number $(\Delta - Re\, B_0/A_0)$ is parallel to w, i.e. it is orthogonal to ϵ:

$$Arg(\Delta-\frac{Re\, B_0}{A_0}) = \phi_{SW}\pm 90^0$$

so that:

$$\Delta-\frac{Re\, B_0}{A_0} \simeq -\frac{\frac{Re\, B_0}{A_0}(m_L-m_S)+\frac{1}{2}(M_{11}-M_{22})}{m_L-m_S-\frac{i}{2}(\Gamma_L-\Gamma_S)} \quad (8.15)$$

or, equivalently:

$$\Delta \simeq -\frac{1}{2}\frac{(M_{11}-M_{22})-i\frac{Re\, B_0}{A_0}(\Gamma_S-\Gamma_L)}{m_L-m_S-\frac{i}{2}(\Gamma_L-\Gamma_S)} \simeq -\frac{1}{2}\frac{M_{11}-M_{22}-\frac{i}{2}(\Gamma_{11}-\Gamma_{22})_{(2\pi)}}{m_L-m_S-\frac{i}{2}(\Gamma_L-\Gamma_S)}$$

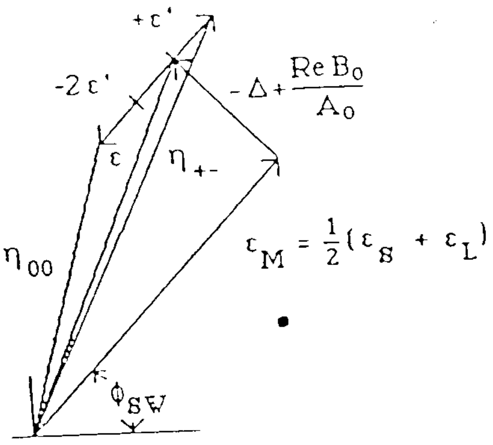

Figure 2: Schematic representation in the complex plane, of the relations between ϵ_M, ϵ, η_{+-} and η_{00}. ϵ' is drawn approximately parallel to ϵ, as appropriate in the exact CPT limit. Relative sizes are not in scale.

The situation is illustrated in Fig. 2 (without paying attention to the relative proportions).

The phases of ϵ_M and Δ are sometime discussed, in the literature, in connection with the Bell-Steinberger (BS) relation [23]:

$$[-i(m_S - m_L) + \frac{1}{2}(\Gamma_S + \Gamma_L)] < K_S \mid K_L >= \sum_f A^*(K_S \to f)A(K_L \to f) \qquad (8.16)$$

The BS relation can be derived directly from the conservation of probability.

By substituting Eqs. (1.12) and (1.13) in the r.h.s. of (8.16), it is immediate to see that the real and imaginary parts of the BS relation coincide with Eqs. (8.7) and (8.5), respectively.

It could have not been differently. The unitarity condition Eq.(8.1) is all we can say about probability conservation. The BS equation involves a total of 4 real quantities: $Re\,\Gamma_{12}$, $Im\,\Gamma_{12}$, Γ_{11} and Γ_{22}. $Re\,\Gamma_{12}$ and the average $\Gamma_{11} + \Gamma_{22}$ are related to the CP and CPT conserving total widths, Γ_S and Γ_L; the CP-violating (CPT-conserving) $Im\,\Gamma_{12}$ determines the phase of ϵ_M, while the CPT and CP violating difference, $\Gamma_{11} - \Gamma_{22}$, fixes the phase of the combination $\Delta - Re\,B_0/A_0$.

There can be no other general restrictions.

[3] Even if we want to leave open the possibility that $Re\,(\Delta - Re\,B_0/A_0)$ is considerably smaller than its individual components [1], we consider it very unlikely a cancellation by three orders of magnitude.

9 Comparison with present data

We can analyse the data at different levels, according to whether CPT symmetry and the $\Delta S = \Delta Q$ rule are kept exact or released in the semileptonic transitions. Exact CPT and the $\Delta S = \Delta Q$ rule are assumed by Barmin et al. [6], who adopt what we have called *Scheme I*, while Buchanan et al. [1] adopt *Scheme II*. We illustrate in detail, in this Section, the results in the *Scheme I*, and will comment, in the next Section, on the impact of DAΦNE on the complex of Kaon parameters in *Schemes II* and *III*.

Data [13]:

$$\begin{pmatrix} \mid \eta_{+-} \mid = (2.269 \pm 0.023)10^{-3} \\ \frac{|\eta_{00}|}{|\eta_{+-}|} = 0.9955 \pm 0.0023 \end{pmatrix} \tag{9.1}$$

$$\begin{pmatrix} Arg(\eta_{+-}) = \phi_{+-} = (44.3 \pm 0.8)^0 \\ \phi_{+-} - \phi_{00} = (1.0 \pm 1.0)^0 \end{pmatrix} \tag{9.2}$$

$$\begin{pmatrix} A_L = (3.27 \pm 0.12)10^{-3} \quad [13] \\ Re\Delta = (0.7 \pm 5.3_{stat} \pm 4.5_{syst}) \quad [24] \end{pmatrix} \tag{9.3}$$

$$Re(\frac{\epsilon'}{\epsilon}) = \begin{pmatrix} (23 \pm 7)10^{-4} \text{ NA31[25]} \\ (6 \pm 7)10^{-4} \text{ E731[26]} \end{pmatrix} \tag{9.4}$$

Analysis:

The smallness of ϵ'/ϵ implies that ϵ is very close to η_{+-} and η_{00}:

$$\mid \epsilon \mid = \mid \frac{2\eta_{+-} + \eta_{00}}{3} \mid = (2.266 \pm 0.023)10^{-3} \tag{9.5}$$

$$Arg(\epsilon) = \phi_{+-} + \frac{1}{3}(\phi_{00} - \phi_{+-}) = (44.0 \pm 1.0)^0 \tag{9.6}$$

With the most recent analysis of the data there is no indication of a possible CPT-violating difference between ϵ and ϵ_M. With ϕ_{SW} the superweak phase, Eq. (8.8), we have:

$$Arg(\epsilon) - \phi_{SW} = Arg(\epsilon) - Arg(\epsilon_M) = (0.3 \pm 1.0)^0$$

Since $\Delta - Re\, B_0/A_0$ is at right angle with respect to ϵ_M, see Fig. 2, the above result translates into:

$$\mid -\Delta + \frac{Re\, B_0}{A_0} \mid \simeq \mid \epsilon \mid [Arg(\epsilon) - Arg(\epsilon_M)] = (0.1 \pm 0.3)10^{-4} \tag{9.7}$$

In *Scheme I*, the K$_L$ lepton asymmetry, Eq. (9.3), already allows a separate determination of Δ and $Re\, B_0/A_0$. From (9.3) we get:

$$A_L = 2Re(\epsilon_M - \varDelta) = 2cos\phi_{SW} \mid \epsilon \mid -2Re\,\Delta$$

whence:

$$Re\,\Delta = (0.0 \pm 0.6)10^{-4}$$

Since, as we saw:

$$-\Delta + \frac{Re\,B_0}{A_0} = w\,(0.1 \pm 0.3)10^{-4} =$$

$$= (0.1 \pm 0.3)10^{-4}(-0.6912 + i0.7226)$$

we find at once:

$$\frac{Re\,B_0}{A_0} = (-0.1 \pm 0.6)10^{-4} \tag{9.8}$$

and:

$$\Delta = [(0.0 \pm 0.6) - i(0.1 \pm 0.2)]10^{-4} \tag{9.9}$$

Note that this result is still more precise than the direct measurement of CP-LEAR [24] quoted in (9.3).

Using Eq. (8.6), we find a limit to the CPT-violating mass difference,[4] M_{11}-M_{22}:

$$\frac{M_{11} - M_{22}}{m_K} = 1.48 \cdot 10^{-14}(\frac{M_{11} - M_{22}}{\Gamma_S - \Gamma_L}) = (0.0 \pm 0.9)10^{-18} \tag{9.10}$$

A last possible CPT test is given by the phase of ϵ'/ϵ. As seen from Eq. (4.12), the phase of ϵ'/ϵ is made of two components:

$$Arg\frac{\epsilon'}{\epsilon} = \phi' + \phi_{CPT/CP}$$

$$\phi' = (\frac{\pi}{2} + \delta_2 - \delta_0 - Arg\epsilon) = (0 \pm 4)^0$$

$$\phi_{CPT/CP} = -(\frac{Re\,B_2}{A_2} - \frac{Re\,B_0}{A_0})(\frac{Im\,A_2}{A_2})^{-1} \tag{9.11}$$

A precise measurement of the real and imaginary parts of ϵ'/ϵ allows, in principle, a determination of the CPT violating phase, $\phi_{CPT/CP}$. The smallness of $Re(\epsilon'/\epsilon)$, Eq. (9.4), implies $Im\,A_2/A_2 \simeq 10^{-4}$, so that we can obtain anyway an interesting bound to the CPT violating part of ϵ'/ϵ. In formulae, from Eqs. (4.9), (4.10) and (4.12), one finds:

[4] For comparison, the results quoted by Carosi et al., Ref. [27], are: $Arg\,(\epsilon) = (47.0 \pm 2.0)^0$; $(M_{11} - M_{22})/m_K < 5 \cdot 10^{-18}$ (95% c.l.).

$$Im(\epsilon'/\epsilon) = Im(\frac{\eta_{+-} - \eta_{00}}{2\eta_{+-} + \eta_{00}}) \simeq \frac{1}{3}(\phi_{+-} - \phi_{00}) =$$

$$= \frac{-\omega}{\sqrt{2} \mid \epsilon \mid}[cos\phi'(\frac{ReB_2}{A_2} - \frac{ReB_0}{A_0}) - sin\phi'(\frac{ImA_2}{A_2})] \simeq \frac{-\omega}{\sqrt{2} \mid \epsilon \mid} cos\phi'(\frac{ReB_2}{A_2} - \frac{ReB_0}{A_0}) \quad (9.12)$$

so that:

$$(\frac{ReB_2}{A_2} - \frac{ReB_0}{A_0}) = -2.4 \cdot 10^{-2}(\phi_{+-} - \phi_{00}) = (-4 \pm 8)10^{-4} \quad (9.13)$$

Note that the error of the strong interaction phase drops out in first order, because $\phi' \simeq 0$.

It will be difficult to make this into a much more precise test. DAΦNE can produce, anyway, a substantial improvement on the determination of $Im(\epsilon'/\epsilon)$ and of the strong interaction phase.

10 The impact of DAΦNE on CPT violating parameters

In *Schemes II* and *III*, the presence of the new parameters b, c and d, in the semileptonic sector considerably complicates the situation, with respect to what found in the previous Section. Eq. (9.9) still holds for $Im\Delta$, i.e.:

$$Im\Delta = (-0.1 \pm 0.2)10^{-4}$$

but the present data give a bound only to the combination $\mid \Delta - ReB_0/A_0 \mid$, which raises the possibility that the smallness of the r.h.s. of (9.7) may be due to (fortuitous?) cancellations of larger effects.

As pointed out in Ref. [1], the observation of correlated $K_L - K_S$ decays at DAΦNE will permit to disentangle the individual CPT violating parameters, in *Scheme II*. In the last instance, this is made possible by the fact that $\Phi-$decay provides a beam of tagged K_S. With reference to the integrated luminosity of the present Report ($2.5 \cdot 10^{32}$ cm^{-2} sec^{-1} for an effective year of 10^7 sec) one estimates a yearly production of $1.7 \cdot 10^9$ K_S, which corresponds to a statistical error on the asymmetry A_S, Eq. (6.16):

$$\Delta A_S \simeq 7 \cdot 10^{-4} \quad (10.2)$$

As seen from Eqs. (6.12) to (6.16), the observation of the three possible semileptonic asymmetries allows a separate determination of the three parameters: $Re\,c/Re\,a$, $Reb/Re\,a$ and $Re\Delta'$. The same conclusion is reached starting from the measurement of $\mid \eta_{l\pm} \mid$ and of the CP-conserving asymmetry, Δ^{sl}.

In *Scheme II*, $Re\,\Delta' = Re\,\Delta$, while $Im\,\Delta$ is still given by Eq. (9.9), and we succeed in disentangling Δ from $Re\,B_0/Re\,A_0$, as anticipated. The observation of correlated semileptonic and hadronic decays allows, in addition, to determine the phase of $\eta_{l^\pm}$, which, in *Scheme II*, gives a further check of the imaginary part of Δ.

In *Scheme III*, the phases of $\eta_{l^\pm}$ determine $Im\,c$ and $Im\,d$, but the determination of $Re\,\Delta'$ is no more sufficient to fix Δ, if we allow a non vanishing value of $Re\,d$.

A conspiracy between CPT-violating parameters in $\Delta S = \Delta Q$ and $\Delta S = -\Delta Q$ amplitudes may seem unlikely. It remains that, in this situation, a full separation of Δ from the other parameters requires a further experimental input. One possibility is given by experiments where a K^0 (or $\bar{K}^0$) can be tagged on the basis of the strangeness, like e.g. in CP-LEAR, rather than the semileptonic decay. In this case, we have access to the further experimental quantity:

$$R(t) = \frac{|< e^- + .. \mid H_W \mid K^0(t) >|^2}{|< e^+ + .. \mid H_W \mid K^0(t) >|^2}$$

which is sensitive to $Re\,d$, for small times. The imaginary parts of a and b remain undetermined, at this level.

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