Hyperfine Physics

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Abstract

Various aspects of the physics of the hypernuclei are reviewed.

1 Hyperons and hypernuclei

Strangeness has been introduced in particle physics to account for lifetimes in the baryon and meson spectrum much longer (by orders of magnitude) than those expected for strongly interacting systems. Indeed in 1953, Gell-Mann \cite{1} interpreted this experimental finding in terms of selection rules associated with a new (at the time) quantum number, namely the strangeness, $S$, defined as

$$Z = T_z + \frac{S + A}{2} = T_z + \frac{Y}{2}$$

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in terms of the charge number \( Z \), isospin (third component of) \( T_z \) and baryon number \( A \). Alternatively, instead of the strangeness \( S \), one may as well consider the hypercharge \( Y = S + A \) as indicated in formula (1).

Because of the charge and baryon number conservation, (1) entails the equivalence between isospin and strangeness conservation, a requirement indeed respected by the strong and electromagnetic forces, but violated by the weak interactions, which do not conserve neither \( T \) nor \( S \).

As is well-known the introduction of this additional quantum number allows to classify in a larger scheme based on the unitary symmetry SU(3), which encompasses both isospin and strangeness, a new generation of baryons, i.e. the hyperons, endowed with the strangeness degree of freedom \( S \neq 0 \). Thus in this frame the \( \Lambda \) and \( \Sigma \) particles, in addition to a pair of first generation quarks (namely the \( u \) and \( d \)) embody as well a strange quark \( s \), whereas the \( \Xi \) possesses two strange quarks in addition to a first generation one.

The stability of the \( S \neq 0 \) particles, which moreover interact with the protons and neutrons with a force comparable to, although somewhat weaker than the one acting among the latter, permits the existence of strange \( (S \neq 0) \) nuclei, i.e. hypernuclei. On the other hand nuclei with an additional \( K^+ \) meson are not bound since the interaction between a \( K^+ \) and a nucleon is predominantly repulsive.

Our present knowledge on hypernuclei is limited and cannot be compared with the one available on conventional nuclei \( (S = 0) \), yet it has already provided important clues on nuclear structure; most importantly it has much broadened the concept of nuclear structure itself.

Since the \( \Lambda \) particle, with isospin \( I = 0 \), strangeness \( S = -1 \) and mass \( M_\Lambda = 1115.6 \) MeV, is about 80 MeV lighter than the \( \Sigma \) (which also has \( S = -1 \)), the most "stable" among the \( S = -1 \) hypernuclei are those made up of nucleons and a \( \Lambda \) particle. However because

\[
M_\Lambda - (M_N + M_\pi) \approx 40 \text{ MeV} \gg B_\Lambda,
\]

\( M_N \) and \( M_\pi \) being the nucleon and the pion mass respectively and

\[
B_\Lambda = M(\frac{A}{A-1}Z) + M_\Lambda - M(\frac{A}{A-1}Z)
\]

the binding energy of the \( \Lambda \), the hypernucleus will eventually decay by weak interactions.

Yet it lives long enough to be detected and indeed a number of hypernuclei have been observed: they are shown in Fig.1 and the binding energies of the lighter among them are listed in Table 1. In this Table \( B_\Lambda \) is seen to be indeed comparable with the binding energy \( B_N \) of a nucleon in a \( S = 0 \) nucleus, but the striking difference between \( B_\Lambda \) and \( B_N \) lies in the fact that the former grows with the mass number \( A \) whereas, as it is well-known, \( B_N \) saturates around a value of about -8 MeV per particle. This outcome reflects the freedom of the \( \Lambda \) to occupy in the host nucleus orbits which are forbidden to a nucleon since it does not have to obey to the Pauli principle. In turn the low energy spectrum of a hypernucleus exhibits new states not present in the normal \( (S = 0) \) nuclei.
<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$T$</th>
<th>$M_T$</th>
<th>$B_\Lambda (\text{MeV})$</th>
<th>$I^\pi$</th>
</tr>
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<tr>
<td>$^3\Lambda H$</td>
<td>0</td>
<td>0</td>
<td>0.13±0.05</td>
<td>$1/2^+$</td>
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<tr>
<td>$^4\Lambda H$</td>
<td>1/2</td>
<td>1/2</td>
<td>2.04±0.04</td>
<td>$0^+$</td>
</tr>
<tr>
<td>$^4\Lambda He$</td>
<td>1/2</td>
<td>-1/2</td>
<td>2.39±0.03</td>
<td>$0^+$</td>
</tr>
<tr>
<td>$^6\Lambda He$</td>
<td>0</td>
<td>0</td>
<td>3.12±0.02</td>
<td>$1/2$</td>
</tr>
<tr>
<td>$^6\Lambda He$</td>
<td></td>
<td></td>
<td>4.18±0.10</td>
<td>(1)</td>
</tr>
<tr>
<td>$^7\Lambda Li$</td>
<td></td>
<td></td>
<td>4.50± ?</td>
<td></td>
</tr>
<tr>
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<td>1</td>
<td>4.4 ±0.7</td>
<td></td>
</tr>
<tr>
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<td>0</td>
<td>5.58±0.03</td>
<td>(1/2)</td>
</tr>
<tr>
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<td>-1</td>
<td>5.16±0.08</td>
<td>$1/2$</td>
</tr>
<tr>
<td>$^8\Lambda He$</td>
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<td></td>
<td>7.16±0.7</td>
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<td>1/2</td>
<td>6.80±0.03</td>
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</tr>
<tr>
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<td>1</td>
<td>8.50±0.12</td>
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</tr>
<tr>
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<td>1/2</td>
<td>9.11±0.22</td>
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<tr>
<td>$^{10}\Lambda B$</td>
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<td>-1/2</td>
<td>8.89±0.12</td>
<td></td>
</tr>
<tr>
<td>$^{11}\Lambda B$</td>
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<td>10.24±0.05</td>
<td>$5/2$</td>
</tr>
<tr>
<td>$^{12}\Lambda B$</td>
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<td>1/2</td>
<td>11.37±0.06</td>
<td>1</td>
</tr>
<tr>
<td>$^{12}\Lambda C$</td>
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<td></td>
<td>10.76±0.19</td>
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</tr>
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<tr>
<td>$^{14}\Lambda C$</td>
<td>1/2</td>
<td>1/2</td>
<td>12.17±0.33</td>
<td></td>
</tr>
<tr>
<td>$^{14}\Lambda N$</td>
<td>1/2</td>
<td>1/2</td>
<td>12.17± ?</td>
<td></td>
</tr>
<tr>
<td>$^{15}\Lambda N$</td>
<td></td>
<td></td>
<td>13.59±0.15</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: The binding energies of some hypernuclei (taken from ref. [3], p.55 and ref.[4], p.4028). In the second and third columns are indicated the isospin and its third component. In the fourth column the binding energy and in last column the ground state spin and parity, when available, are quoted.
Figure 1: Chart of observed Λ hypernuclei as of 1988 (from ref.[2]).

In this connection Feshbach [2] observes that this might not be exactly true since the u and d quarks in the nucleon and in the Λ have to satisfy the Pauli principle. Actually, to assess the impact on the spectra of the hypernuclei of the Pauli principle, which must be obeyed by the quarks, will represent a challenging theme of investigation for the future hypernuclear physics.

Leaving aside this observation a rough estimate of the potential well binding the Λ in a hypernucleus might be obtained by considering the kinetic energy of a Λ particle in the lowest orbit it can occupy in a "shell model" nucleus, namely the $1s_{1/2}$. One gets [3]

$$E_{\text{kin}} \approx \frac{\pi^2 R^2}{2M \Lambda R^2} \approx (118 \text{ MeV}) \Lambda^{-2/3}$$

$R$ being the nuclear radius.

For example in $^{13}C$ one has $E_{\text{kin}} \approx 20$ MeV. Hence, since in this nucleus $B_{\Lambda} \approx -10$ MeV (see Table 1), from the relationship

$$B_{\Lambda} = E_{\text{kin}} - V_{\Lambda}$$

it follows that $V_{\Lambda} \approx 30$ MeV to be compared with about 55 MeV [5], which is the depth of the potential well felt by a nucleon in carbon. Of course the above should only be considered as a rough estimate for the depth of the well acting on the Λ particle in a nucleus. More realistic evaluations will be later considered.

Concerning the spin and parity assignments in Table II they were obtained from angular correlations measurements as well as from the determination of branching ratios for different decays modes (note that to the Λ an intrinsic parity +1 is assigned).
2  Production of Hypernuclei

Since the strong interactions conserve strangeness \((\Delta S = 0)\) one must necessarily make use of particles (essentially mesons) endowed with strangeness to produce \(S \neq 0\) hyperons via strong interactions. The following are the physical processes commonly considered:

i) reactions where a \(s\) quark is exchanged, namely

\[
K(-1) + N \rightarrow Y(-1) + \pi. \tag{6}
\]

Here the \(s\) quark is transferred from a kaon \(K\) to a hyperon \(Y\), both having strangeness \(S = -1\) (in the above \(N\) and \(\pi\) represent a nucleon and pion, respectively).

In this reaction the momentum transfer can be quite small. Actually if a \(K^-\) with a momentum \(p_k\) hits a neutron \(n\) at rest, then in the reaction

\[
K^- + n \rightarrow \Lambda + \pi^- \tag{7}
\]

a "magic" momentum exists such that the \(\Lambda\) also stays at rest while the \(\pi^-\) moves in the forward direction.

The equation fixing such a momentum is easily found to be

\[
M_n + \sqrt{M_k^2 + p_k^2} = M_\Lambda + \sqrt{M_{\pi}^2 + p_{\pi}^2} \tag{8}
\]

which yields \(p_k = 531\) MeV/c.

If, instead of a \(\Lambda\), a \(\Sigma\) particle is considered one gets \(p_k = 284\) MeV/c. In Fig.2 the momentum transferred from the kaon to the pion (in the forward direction) is displayed as a function of the incident kaon momentum. Note that because of the smallness of the momentum transfer also the polarization \(P_\Lambda\) of the \(\Lambda\) is almost negligible. On the other hand for \(p_k = 1.2\) and 1.6 GeV/c it is found that \(P_\Lambda \approx -1\) and \(P_\Lambda \approx +1\) respectively.

ii) Reactions where a \(s\bar{s}\) pair is created.
In this instance typical processes are \((\pi^+, K^+), (\gamma, K^+)\) and \((p, K^+)\). Here a \(s\)-quark is transferred to a nucleon yielding an hyperon, whereas the antiquark \(\bar{s}\) becomes a constituent of the final \(K^+\).

At variance with the case i) now the momentum transfers are large (because the final particles, namely the \(Y\) and the \(K^+\), are heavy): accordingly the \(P_A\) will be large as well.

iii) Reactions in which the two processes above described are combined into a single one.

Examples of this case are

\[
K^- + N \rightarrow K^+ + \Xi \tag{9}
\]

and

\[
N + N \rightarrow K^+ + K^+ + \Xi \tag{10}
\]

where the so-called "cascade particles" \(\Xi^-\) and \(\Xi^0\) are produced.

The reactions we have referred to above are indeed exploited to obtain hyperons inside nuclei. A number of \((K^-, \pi^-)\) experiments have been performed at CERN and at Brookhaven (BNL). With the direct \((K^-, \pi^-)\) mechanism a neutron can be replaced with a \(\Lambda\) inside the nucleus so gently that the wave function of the nuclear system remains essentially unchanged (hence the name of "substitutional reaction" for the process). If the pion is emitted in the forward direction then the angular momentum change occurring in the reaction is \(\Delta l = 0\), otherwise \(\Delta l = 1\) and \(\Delta l = 2\) transitions become possible as well.

As an illustration in Fig.3 the spectra of the \(^{12}_A\Lambda C\) and \(^{16}_A\Lambda O\) hypernuclei, obtained via the \((K^-, \pi^-)\) process, are shown. In both cases it is clearly apparent that when the \(\Lambda\) particle and the neutron hole, which together identify the excited state of the hypernuclear system, are in the same single particle orbit (and coupled to a state of zero angular momentum and positive parity, i.e. \(J^\pi = 0^+\)), then the corresponding peak in the cross-section is quite pronounced, whereas for the other configurations a sizable reduction of the cross-section is seen to occur (the associated peaks are much less evident). This finding reflects the marked preference of the \((K^-, \pi^-)\) reaction for the \(\Delta l = 0\) transitions rather than for those having \(\Delta l = 1, 2\ldots\). In turn this explains why the \((K^-, \pi^-)\) process is not appropriate for exciting low-lying \(\Lambda\) hypernuclear states in heavy nuclei: indeed here the neutron has in general a high angular momentum thus entailing large \(\Delta l\) reactions when the \(\Lambda\) sits in low lying orbits.

For exciting high spin hypernuclear states actually the process

\[
\pi^+ + n \rightarrow \Lambda + K^+ \tag{11}
\]

is preferable to (7) (actually (7) and (11) are in fact complementary).

This reaction indeed prefers to create the \(\Lambda\) in a high angular momentum state at large excitation energy (quasi-free scattering). It is, accordingly, well-suited for unfolding the shell structure of nuclei heavier than those previously considered. As an example in Fig.4 the spectrum of the hypernucleus \(^{51}_8\Lambda V\) thus obtained is shown. It is indeed impressive to see in the figure how the \(\Lambda\) is probing all the nuclear shells down to the inner one, namely

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Figure 3: Production of hypernuclei $^{12}_C$ and $^{16}_O$ by the $(K^-,\pi^-)$ reaction (from ref.[2]).
the 1s\(_{1/2}\) (remember that in \(^{51}\)V the neutron shells are closed and the 1f\(_{7/2}\) single particle level is fully occupied).

In connection with the reaction (11) it should be observed that the \(\Lambda\), being produced in a high angular momentum state and above its emission threshold (quasi-free region), may either escape from the hypernucleus (with a width \(\Gamma_{esc}\)) or be captured inside the hypernucleus (with a width \(\Gamma_{spread}\)) where it spreads its energy with the other constituents. In the latter instance a compound nucleus is formed eventually decaying by the emission of several nucleons in addition to a number of \(\gamma\)-rays: at the end of the process a hypernucleus is left in a variety of quantum states.

In the range of excitation energies from 30 to 120 MeV in \(^{12}\)C the following value

\[
\frac{\Gamma_{spread}}{\Gamma_{esc} + \Gamma_{spread}} \approx 0.15 - 0.20, \tag{12}
\]

has been experimentally found [6]

Now the above energy range is dominated by the quasi-free scattering, whose cross-section \(\sigma(QF)\) is larger by about two orders of magnitude than the cross section \(\sigma_d(HY)\) for the formation of an hypernucleus directly in a bound state, via, e.g., the process (7). Thus, even by taking the lower value for the ratio (12), we obtain for the cross section \(\sigma_c(HY)\), corresponding to the formation of an hypernucleus through the compound nucleus mechanism via (11), the estimate

\[
\sigma_c(HY) = 0.15\sigma(QF) \tag{13}
\]
still an order of magnitude larger than $\sigma_d(HY)$.

Due to its importance it is appropriate in conclusion to this Section to shortly return to consider the polarization of the $\Lambda$-hypernuclei. It is largely due to the polarization of the hyperons produced in the elementary processes like $(K^-,\pi^-), (\pi^+, K^+), (\gamma, K^+)$ and $(e,e'K^+)$ which occurs, as we have already seen, if the momenta involved are sufficiently large and reflects the quite significant spin dependence of the $\Lambda$-N interaction (for example, a notable feature of the latter is that rather than yielding the strongest attraction in the triplet state $^3S$, like the N-N force, appears to be preferring the singlet state $^1S$). However it is also contributed to by the distortion of the pionic waves either in the initial or in the final state, which orients the angular momentum transferred in the reaction.

Estimates of the polarization of the $\Lambda$ hypernuclei, obtained through the $(e,e'K^+)$ process have been performed since this type of experiments are expected to be performed at CEBAF. On the other hand the $(\pi^+, K^+)$ mechanism for obtaining polarized hypernuclei has been explored by Ejiri [6]. This author considers closed shell nuclei having the lowest spin–orbit partner filled. As we already know the considered reaction proceeds primarily through the compound nucleus formation and the most populated final hypernuclear states, reached at the end of the statistical decaying process, are those with a $(j_n = l + 1/2)^{-1}$ neutron hole and a $(j_A = l - 1/2)$ $\Lambda$ particle coupled to $J = 2l$. In other words stretched states with maximum spin are preferentially excited by the $(\pi^+, K^+)$ reaction. Large polarizations of the hypernuclei (of the order of $30 \approx 50\%$) are found. The corresponding polarizations of the spin of the $\Lambda$ turn out to be of the order of $20 \approx 40\%$.

Another interesting case is provided by the elementary $(K^-,\pi^-)$ reaction: indeed the latter yields, for momenta larger than 1 GeV/c, a $\Lambda$ almost 100% polarized [7]. When applied to populate natural parity states of the hypernucleus $^{12}_{\Lambda}C$ this reaction leads to polarizations very small, large and positive and large and negative for momenta of the incident $K^-$ of 0.7, 1.1 and 1.5 GeV/c, respectively. This shows how sensitive the polarization of an hypernucleus can be to the kinematics of the reaction selected for its own production.

Because the produced hypernuclei are polarized, it becomes possible to measure their magnetic moments. This is an important measurement as it will test the hypothesis that the hypernucleus consists of a host nucleus plus a $\Lambda$. One would learn if the hyperon state consists of just a $\Lambda$ or if there is a $\Sigma$ component [14].

3 The structure of hypernuclei

In the past Sections we have already pointed out that the dynamic of an hypernucleus is essentially ruled by the following facts:

i) the $\Lambda$ is not Pauli-blocked,

ii) the $\Lambda$ experiences an interaction with the nucleons appreciably weaker than the N-N force.

Point ii) is to a substantial extent related to the zero isospin of the $\Lambda$, which prevents the latter from exchanging an isovector meson, like the pion or the rho, with a nucleon and
bears relevant implications.

Indeed the relative weakness of the Λ-N interaction entails, as it was already observed, that the shell structure is not disrupted by the insertion of the Λ into a nucleus and that the mean field binding the Λ is not as deep as the one binding the nucleon.

Moreover, and importantly, the lack of a strong tensor component, largely carried by the pion and the rho in the Λ-N interaction, might render the spin-orbit central potential in the hypernuclei very weak. And in fact the spectrum of $^{12}_Λ C$, shown in Fig.3, is consistent with an energy difference of less than 0.3 MeV between the spin-orbit partners $1p_{1/2}$ and $1p_{3/2}$. Thus the conjecture that the strong one-body spin-orbit potential, which is the central element for obtaining the proper organization of the shells in normal nuclei, is actually due to the two-body tensor force, is significantly supported by the experimental findings in hypernuclei.

An alternative explanation for the puny spin-orbit mean field in hypernuclei may come from the recognition that in the constituent quark model the u and d quarks in the Λ are coupled to zero angular momentum: hence the spin of the Λ appears to be carried by the s quark alone. One could accordingly conjecture that the latter is rather inert as far as the interaction with the other quarks is concerned. This explanation needs further elaboration in view of the recent experiments on the spin structure of the nucleon.

Although two examples have already been provided concerning the survival of the shell structure (Figs. 3 and 4) in a hypernucleus yet the spectrum of the Yttrium (Z=40) displays the pattern of the single particle levels so strikingly that it is still worth to be shown (Fig.5). Especially because it is the heaviest among the hypernuclei till now explored where the shells are so neatly seen. It is obtained via the reaction

$$ \pi^+ + ^{89}Y \rightarrow ^{89}_\Lambda Y + K^+ $$  \hspace{1cm} (14)

at an incident pion momentum of 1.05 GeV/c and for a scattering angle $\theta = 10^0$.

Again the hypernuclear states show up as prominent peaks appearing in the spectrum. They belong to the configuration having a neutron hole in the $1g_{9/2}$ single particle level and the Λ in all the orbits starting from the $1g_{9/2}$ down to the $1s_{1/2}$. We thus see that the Λ indeed sits in the inner s-level when the hypernucleus is in its ground state.

As previously discussed this occurrence accounts for the lack of saturation experimentally found in the Λ binding energy. The latter is displayed in Fig.6, which complements the data quoted in Table 1, as a function of $A^{-2/3}$. The power of $A$ should in fact correspond to the next to the leading term in the expansion of $B_Λ$ (here defined as a positive quantity) in terms of the mass number $A$. Physically it mainly represents a kinetic energy correction (see(4)) to the bulk value of the Λ binding energy. One can thus write

$$ B_Λ = B^{∞, m}_Λ - (90 \text{ MeV})/A^{2/3} + ... $$  \hspace{1cm} (15)

where the coefficient just provides a reasonable fit and the infinite nuclear matter value $B^{∞, m}_Λ = 27$ MeV is obtained in a relativistic mean field theory. Its difference from the corresponding quantity for a nucleon in ordinary nuclear matter, namely $B_N = 16$ MeV, is clearly due to the kinetic energy.
Figure 5: The excitation spectrum for the $^89Y(\pi^+, K^+)_{\Lambda}^89Y$ reaction at 1.05 GeV/c and $\theta_K = 10^\circ$. Solid line: a theoretical calculation. Dotted line: the $[1j_{3/2}]_{\Lambda}$ contributions (from ref.[15]).

In Fig.6 it is indeed observed that for the available data, which end at the hypernucleus $^89Y$, the heavier the hypernucleus, the larger the binding energy. Also shown in the Figure is the behaviour with $\Lambda$ of the energies of the excited states of the hypernuclei corresponding to configurations with a $\Lambda$ sitting in the various shell model orbits, as previously discussed: thus a family of curves, one for each single particle level, shows up, all of them converging to the nuclear matter value $B^{\infty,m}_{\Lambda}$. They are calculated starting from a Woods–Saxon potential well, namely

$$V(r) = \frac{V_0}{1 + \exp \frac{r-R}{a}}$$  \hspace{1cm} (16)$$

with

$$V_0 = -31 \text{ MeV}, \quad R = 1.1 \Lambda^{1/3} \text{ fm}, \quad a = 0.6 \text{ fm}$$  \hspace{1cm} (17)$$

and no spin-orbit potential.

Although the curves appear to nicely fit the available experimental data, a closer scrutiny reveals that some amount of non-locality is actually required in the binding potential. Clearly the Woods-Saxon well above referred to is only meant to phenomenologically represent the mean field acting on a $\Lambda$ in a hypernucleus. It is of significance that while the radius $R$ and the surface thickness of (17) are close to the values characterizing the shell model potential of the $S = 0$ nuclei, its depth is about one third less. In this connection it should also be mentioned that the binding energy of the $^5\Lambda He$ is presently not understood.

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Figure 6: The observed binding energies of $\Lambda$ single-particle states. The curves correspond to a calculation with an adjusted density-dependent non-local interaction (from ref.[15]).

This might suggest, in line with the previous considerations on the spin-orbit term, that the $s$ quark inside a $\Lambda$ interacts only very weakly with the $u$ and $d$ quarks of the other nucleons. In fact such a conjecture would also account for the value of about 27 MeV above quoted for $B_{\Lambda}^{\infty,n,m}$, which is in fact roughly two thirds of the potential energy per nucleon (about 40 MeV) in infinite nuclear matter (we have seen that a $\Lambda$ implanted in nuclear matter would have essentially no kinetic energy).

Another feature of relevance in the hypernuclear structure relates to the nature of the single particle states of the $\Lambda$. These appear to experience little, if any at all, fragmentation, even the deeply bound ones. However one must recall that the energy resolution involved in this experiment is several MeV so that no conclusions can be drawn at present time.

In accord with the above is the analysis of the dynamical behaviour of the hypernuclei, in the energy regime above the $\Lambda$ emission threshold, carried out in terms of a $\Lambda$-nucleus optical potential. The real part of the latter has in fact been found to be positive and about 30 MeV strong, whereas the strength of the negative imaginary part does not exceed a couple of MeV, much less that the corresponding quantity experienced by a nucleon.

This remarkable stability of the single particle quantum states of the $\Lambda$ in the mean field generated by the nuclear medium is indeed remarkable and worth to be explored in the appropriate theoretical framework. In fact the above discussed Wood–Saxon phenomenological potential should actually be obtained through a Hartree calculation starting from some realistic $\Lambda$-N interaction.

In this connection if is worth noticing that a $\Lambda$, implanted into a nucleus, not only
feels the Hartree field generated by its own interaction with all the other nucleons, but in turn affects the Hartree-Fock field felt individually by each nucleon. In this perspective the Hartree-Fock problem acquires a new dimension in hypernuclei. Formally this can be stated by saying that the two one-body hamiltonian

$$h^N_{\alpha} = t_{\alpha \alpha} + \sum_{\alpha'} < \alpha \alpha' | V_{NN} | \alpha \alpha' >_{\alpha \alpha} + < \alpha_{\Lambda} \alpha | V_{AN} | \alpha_{\Lambda} \alpha >$$

(18)

and

$$h^\Lambda_{\alpha \alpha} = t^\Lambda_{\alpha \alpha} + \sum_{\alpha'} < \alpha_{\Lambda} \alpha | V_{AN} | \alpha_{\Lambda} \alpha >$$

(19)

should be self-consistently dealt with simultaneously.

Some estimates of the impact of the $\Lambda$ on the HF mean field acting on a nucleon predict a change in the single particle energies as large as $3 \div 5$ MeV for the deeply bound nucleons and of about one MeV for valence nucleons. No experimental evidence for this effect is presently available. This is unfortunate because it has been argued [8] that a partial deconfinement of the quarks inside the $\Lambda$ could be reflected by a wide spacing, say 5 MeV, among the most bound single particle levels inside a heavy hypernucleus like $^{208}_{\Lambda}$Pb.

Concerning the two-body force to be utilized as an input the HF framework it should be observed that in hypernuclear physics a new self-consistency requirement, to be added to the usual one, is met at a deeper level in the sense that the two nucleon–nucleon ($V_{NN}$) and $\Lambda$–nucleon ($V_{AN}$) interactions are clearly interrelated.

One of the few approaches dovetailed to deal with this difficulty has been carried out by Nagels et al.[9] starting from the so-called Nijmegen OBE (one boson exchange) $NN$ potential, specifically designed to account for the $NN$ scattering data. These authors then solve the associated Bethe–Goldstone equation for the $\Lambda$–N scattering in the medium thus obtaining a G–matrix, i.e. a density–dependent $\Lambda$–N interaction.

For convenience the latter, commonly referred to as the YNG force, has all its components expressed via the combination of three gaussians of different ranges. The YNG potential turns out to have the following structure

$$V_{AN}(r, k_F) = V_{AN}^C(r) + V_{AN}^{LS}(r) \vec{L} \cdot \vec{S} + V_{AN}^{ALS}(r) \vec{L} \cdot \frac{\vec{\sigma}_1 - \vec{\sigma}_2}{2} + V_{AN}^T(r) S_{12},$$

(20)

i.e. it is made up of a central, spin–orbit, anti spin–orbit and tensor term. In (20) $r = |\vec{r}_N - \vec{r}_\Lambda|$, $\vec{L}$ is the relative orbital angular momentum, $\vec{S} = \vec{\sigma}_1 + \vec{\sigma}_2$ the total spin, $S_{12}$ the tensor operator and $k_F$ the Fermi momentum.

A notable feature of the YNG force, in addition to its rather weak tensor component, is its strong short range repulsion. The latter is analogous to the repulsive core of the $NN$ interaction, but has radically different consequences. In fact when the diagonal matrix elements of the force are taken with, e.g., the $\Lambda$–N pair sitting in the configuration $(1f_{7/2})^2$, then a "repulsive" $J = 0$ matrix element is found to occur (see Fig.7).

This "anti-pairing" AN force is in stunning contrast with the "attractive" pairing $NN$ potential, which has, as it is well-known, momentous consequences on the nuclear dynamics. It reflects a different balance between the short range repulsion and the intermediate
range attraction occurring in the ΛN channel: indeed a Λ can approach more closely a nucleon than another nucleon can do.

At a more fundamental level attempts have been made to derive the baryon-baryon potential from first principles. One model calculates the effect of the exchange of bosons (OBE). In another the quark-quark force obtained from a one gluon exchange is used in a cluster model. In addition the influence of three-body forces has not been evaluated. A detailed discussion of these attempts would take us too far afield. Suffice to say the fundamental basis of the hyperon-nucleon interaction is still not known.

The YNG potential, as well as others more phenomenological versions of the ΛN interaction, has been extensively applied to analyze and predict a number of hypernuclear spectra. Since in a hypernucleus three kinds of particles, the proton, the neutron and the Λ are coexisting, states characterized by new symmetries should appear. To display the latter more transparently it is of much convenience to classify the states according to the $SU(3)$ symmetry.

An often considered example in this connection is the hypernucleus $^{9}_\Lambda Be$ viewed as $^{8}Be$, whose simple shell model configuration ($s^4p^4$) belongs to the irreducible representation $(\lambda, \mu) = (4,0)$ of $SU(3)$ with in addition a Λ particle. One thus gets three sets
(bands) of states, which with a compact notation can be labelled as \((s^5p^4)(\lambda, \mu) = (4, 0), (s^4p^5)(\lambda, \mu) = (5, 0)\) and \((s^4p^6)(\lambda, \mu) = (3, 1)\). Now the first and third set are nothing new with respect to \(^8\text{Be}\) and \(^9\text{Be}\) respectively. But the second set \((\lambda, \mu) = (5, 0)\) corresponds to a new symmetry not found in ordinary nuclei.

Clearly the interest in this type of investigations is partly to search for states displaying new aspects of symmetry and partly to assess the amount by which \(SU(3)\) is broken in hypernuclei. Are the breakings of \(SU(3)\) at the many body level (spectra of hypernuclei) and at the baryonic level (the octet) related?

It should also be mentioned that along the same lines above outlined a \(\Sigma N\) potential has been derived. Since the experimental information, not to say the very existence, of the \(\Sigma\) hypernuclei is still somewhat controversial, they will not be discussed here. Suffices it to say that the mean field acting on a \(\Sigma\) appears to be quite shallow, with a substantial spin–orbit component (a \(\Sigma\) and a nucleon can exchange a pion or a rho) and the associated states are characterized by a large width (30 or 40 MeV), mainly associated with the strong decay channel

\[
\Sigma + N \rightarrow \Lambda + N. \tag{21}
\]

Various mechanisms have been explored that could reduce such a width, like Pauli blocking, dispersion effects, isospin selection rules etc., but the matter appears to be far from being settled.

In concluding this Section a brief reference should be made to hypernuclei with two \(\Lambda\) particles. In principle they can be obtained from a \(\Xi\)-hypernucleus formed through the process (9). In fact the \(\Xi\)-hyperon can interact with a proton inside the nucleus according to

\[
\Xi + p \rightarrow \Lambda + \Lambda + 28.5 \text{ MeV}. \tag{22}
\]

and a double-\(\Lambda\) hypernucleus can thus be formed with a certain probability. In practice the data are very scarce.

The importance of double-\(\Lambda\) hypernuclei relates to a large extent to the existence and stability of the \(H\)-particle predicted long ago by [10] as a six–quark bound state exceptionally stable and with a mass about 80 MeV below the \(\Lambda\Lambda\) decay threshold. The \(H\) particle is made up of two \(u, d\) and \(s\) quarks coupled to a flavor \(SU(3)\) singlet and, when expressed in terms of two baryons configurations, reads

\[
|H| = \sqrt{1/8}|\Lambda\Lambda| + \sqrt{4/8}|N\Xi| - \sqrt{3/8}|\Sigma\Sigma|. \tag{23}
\]

The \(H\) particle is still intensively searched for, but has not yet been convincingly found.

4 Weak decay of hypernuclei

The decay of \(\Lambda\)-hypernuclei \((S \neq 0)\) to normal nuclei \((S = 0)\) occurs through the weak interaction which can change the strangeness. In contrast to a \(\Sigma\)-hypernucleus a strong decay channel \((\Delta S = 0)\) is open.
Figure 8: Weak-decay diagrams for mesonic decays (left-hand side), and for non-mesonic decays (right-hand side) of a \( \Lambda \) (from ref.[6]).

The study of the weak decay of a \( \Lambda \)-hypernucleus aims to the understanding of the stability of the system on the one side and of how the many-body nuclear system affects the products of the \( \Lambda \) decay on the other (clearly the two issues are related). Moreover the decay process also reflects the influence of the medium on the \( \Lambda \) itself and on the weak interactions it experiences.

Basically two are the mechanisms for the decay of an hypernucleus:

i) the mesonic decay (for a review see ref. [11]), namely

\[
\Lambda \to N + \pi
\]  

which is just the free decay of the \( \Lambda \) but now occurring in the medium;

ii) the non-mesonic (NM) decay, commonly viewed as being driven by the process

\[
\Lambda + N \to N + N
\]  

which is only possible in a nucleus.

They are displayed in Fig.8.

For the mesonic decay the following two branches are open in free space

\[
\Lambda \to p + \pi^- \ (64\%) \quad \text{and} \quad \Lambda \to n + \pi^0 \ (36\%).
\]  

The above branching ratios are in approximate accord with the isospin rule \( \Delta I = 1/2 \) as it can be easily checked with a Clebsch–Gordan analysis. It is disturbing that such a rule, discovered at the empirical level, has never been convincingly understood on a sound theoretical ground.

Now, if the process (24), whose \( Q \)-value is about 40 MeV (see (2)), occurs with the \( \Lambda \) at rest, then most of the energy is carried away by the pion and it is easily verified that the outgoing proton remains with only about 5 MeV of energy. Accordingly its momentum turns out to be

\[
p_N = \sqrt{2m_N E_N} \approx 100 \text{ MeV}/c
\]
Figure 9: The calculated mesonic decay rates of the free $\Lambda$ decay rate (from ref.[6]).

which is much less than the Fermi momentum $k_F$ (in the above $m_N$ is the nucleon mass). Thus the mesonic decay in nuclei is substantially hindered by the Pauli principle. However some high momentum components in the proton wave function are supplied by the interaction both with the outgoing pion and with the medium: thus the restriction on the mesonic $\Lambda$ decay of a hypernucleus is less severe than implied by (27), even in heavy nuclei, although the suppression remains impressive. In this respect the situation is well illustrated in Fig.9.

The NM decay, on the other hand, is not affected by the Pauli blocking. Indeed, assuming the available energy to be equally splitted between the two nucleons, one gets

$$E_N \approx (m_\Lambda - m_N)/2 \approx 89 \text{ MeV},$$

hence

$$p_N = \sqrt{2m_N E_N} \approx 400 \text{ MeV}/c >> k_F.$$  \hfill (29)

Owing to the large energy and momentum transfer characterizing the NM decay channel it is clear that the short range component of the $\Lambda N$ interaction is going to play an important role in the process.

Indeed Alberico et al. [12] showed that in nuclear matter the stronger the $\Lambda N$ short-range repulsion is, the larger the reduction of the non-mesonic decay is going to be. This result was achieved by parametrizing the short range interaction with the Landau–Migdal parameter $g'$ and allowing for sizable variations of the latter.

Central questions to be asked concerning the NM decay of hypernuclei are:
Figure 10: Ratio $Q^-$ of non-mesonic to $\pi^-$ mesonic decay rates as a function of $A$.

i) how does the NM decay compare with the free $\Lambda$ life-time?

ii) which of the two mechanisms

$$\Lambda + p \rightarrow p + n$$  \hspace{1cm} (30)

and

$$\Lambda + n \rightarrow n + n$$  \hspace{1cm} (31)

is more relevant for the NM decay?

iii) how the NM decay is realized in a many-body framework?

Fig.10 provides an answer to the first question: indeed it is clearly seen that for $A \geq 10$ the NM decay mode not only is the dominant one, but also brings the total width for the decay of a hypernucleus $\Gamma$ back to the free value $\Gamma_{\text{free}}$ (if not to values larger than $\Gamma_{\text{free}}$).

Concerning the second question, which of course is not disjoint from the third one, a simple evaluation of Clebsch–Gordan coefficients leads to the geometrical estimate

$$\frac{\Gamma_{T_f=1}^{(A_n\rightarrow nn)}}{\Gamma_{T_f=1}^{(A_p\rightarrow np)}} = 2$$  \hspace{1cm} (32)

(of course (32) ignores a $T_f = 0$ contribution in the denominator).
Figure 11: The intersection in the \((\omega, q)\) plane of the parabola (solid line) representing the energy–momentum relation for the \(\pi\) emitted in the decay of a \(\Lambda\) at rest with the excitation spectrum of nuclear matter. Dashed region: particle–hole excitation. Dashed line: the collective pionic branch. Also shown (dotted line) the free pion branch (from ref.[12]).

The experimental indication, on the other hand, is more consistent with a value for the ratio (32) close to one (although the data are rather scarce).

A variety of calculations have been performed to improve upon the geometrical estimate. These approaches range from models accounting for all possible mesons exchanges between \(\Lambda N\) and \(NN\) propagation lines with one strong and one weak vertex to models accounting for the quarks degrees of freedom. Not surprisingly the results thus obtained show a marked model dependence.

This brings us to address the third question, which is more easily answered in a nuclear matter context. In fact in this framework is transparently seen that the occurrence of the elementary processes (30) and (31) might be prevented. Indeed it is easily realized that if the pion emitted from the \(\Lambda\), an allowed process now since it is taking place in a weak vertex, is close to the mass–shell ( in the considered process this correspond to a pion carrying relatively large energy and little momentum) then it cannot be absorbed by a single nucleon because, as shown in Fig.11, this would induce a particle–hole excitation in a region forbidden in nuclear matter. It can do so only when it is highly virtual (Fig.11).

Therefore a close to the mass–shell pion can only be absorbed by a pair of nucleons, leading to two particle–two hole excitations. More specifically the absorption can occur in the pionic branch, a collective nuclear state embodying a coherent superposition of pionic and \(\Delta\)–hole elementary excitations, which lies at an energy lower than the physical pion.
Its decay mode in the kinematical region of concern for the decay of a hypernucleus is by two nucleons emission. It thus appears that the $\Lambda$ decay, particularly in heavy nuclei, is occurring by a substantial fraction not via the (25), but rather via the process

$$\Lambda + p + n \rightarrow p + n + n.$$  \hspace{1cm} (33)

The above outlined situation is somewhat reminiscent of the absorption of a real photon which, even when of relatively high energy, carries nevertheless little momentum and accordingly can only be absorbed on a correlated pair of nucleons (hence the quasi-deuteron model) in order to conserve both energy and momentum. In ref.(11) it is shown that these excitations provide an important contribution to the width for the NM decay.

The identification of the process, because is associated with the emission of three nucleons, poses however a serious experimental challenge.

## 5 Conclusions

Strangeness has opened a new dimension in nuclear physics: owing to the presence of the strange quark the $\Lambda$N interaction displays unique features which are reflected in many of the facets of the hypernuclear structure. Through the understanding of these features one can obtain information on the $\Lambda$N interaction.

Examples of these are the remarkable stability of the $\Lambda$ Hartree field (even the deep lying orbits are well defined), the absence of spin-orbit splitting in the single particle levels, the occurrence in the spectra of states unfolding new symmetries. It would also be fascinating to explore the manifestations of the antipairing nature of the $\Lambda$N force in the response functions of hypernuclei, although their transient existence makes it difficult to achieve this goal.

Note moreover that the $\Lambda$ can act as a probe of the host nucleus. One can expect that macroscopic parameters such as the nuclear radius and the moment of inertia will change because of the presence of the $\Lambda$. These effects are small and thus will require the high resolution experiments mentioned earlier to be detected.

Beyond these phenomena, which are novel with respect to the traditional nuclear ones, hypernuclear physics is moving in the direction of unifying nuclear and particle physics. It offers indeed the opportunity, e.g., of investigating the single hadron properties or, via the $\Lambda$ decay, the weak interactions in the nuclear medium.

Moreover the non-mesonic decay of a hypernucleus allows to test the $\Lambda$N interaction at short distances, which, as we have seen, has crucial consequences on the dynamics of the hypernuclei and which is sensitive to sub-hadronic degrees of freedom.

Finally, in a broader perspective, one can conceive a large set of many-body baryon systems enlarging the concept of flavour to include the atomic nuclei. From this viewpoint the $\Lambda$ hypernucleus should just be conceived as a first step of a ladder. The second step is represented by the $S = -2$ systems (double strange). These encompass not only the $\Lambda\Lambda$ hypernuclei, but the $\Xi^-$ as well, and in particular the $H$ hypernuclei.
Farther up in the ladder lie the fascinating \textit{strangelets}, the stable droplets of strange matter conjectured by Witten [13], whose existence may be revealed by central collisions of relativistic heavy ions.

A broad spectrum of themes is indeed open to the investigation in the field of flavour nuclei. Unfortunately adverse circumstances (especially the death of KAON) have severely restricted the number of laboratories where this physics is actively pursued: left open are KEK and Brookhaven (BNL).

Dafne, thanks to the effort of the physicists engaged in the FINUDA experiment [16], has thus the opportunity of promoting important advances in a field where a number of interesting questions wait to be answered.

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