INFN - Laboratori Nazionali di Frascati, May 1995
From: **The Second DaΦne Physics Handbook**[1]
Chapter 11: Light Quark Spectroscopy
Eds. L. Maiani, G. Pancheri, N. Paver

Electron-Positron Annihilation below 1.6 GeV

A. Donnachie
Department of Physics and Astronomy, University of Manchester,
Manchester, M13 9PL, UK
and
A. B. Clegg
School of Physics and Materials, University of Lancaster,
Lancaster, LA1 4YB, UK

Abstract

The latest analyses of low-energy e^+e^- annihilation data, and the complementary channels in τ-decay, are summarised. The energy range accessible to DAΦNE promises to be particularly rich as, although there are strong contributions from the first radial and orbital excitations of the ρ and ω, the reality is much more complex. Mixing with non-$q\bar{q}$ states provides the most natural interpretation of the current data, and several such states are predicted to lie in the DAΦNE energy range. A systematic study is required to identify unambiguously departures from nonets.

1 Introduction

There have been many analyses of e^+e^- annihilation data, at relatively low energies, with the objective of establishing the existence and decay modes of light-quark vector mesons which couple strongly to e^+e^-. These have been summarised and updated in [1], from which the details in this report have been taken. The analyses of the $I = 1$ data provide definite evidence for at least two ρ'-mesons with one having a mass of ~ 1.46 GeV and one having a mass of ~ 1.73 GeV. Making the initial assumption that these are each single mesons they are labelled as ρ'_1 and ρ'_2 respectively. The corresponding analyses of the data on production of $I = 0$ states provide evidence for an ω' partner of ρ'_2 at ~ 1.61 GeV,

[1]Supported by the INFN, by the EC under the HCM contract number CHRX-CT920026 and by the authors home institutions

labelled as ω_2', and for a ϕ_1' state at ~ 1.7 GeV, together with weaker indication of an ω' partner of ρ_1' at ~ 1.44 GeV, labelled as ω_1'.

The existence of the ρ_1' is evident in the production of $\omega\pi^0$ [2] [5], $\pi^+\pi^-$ [3] [4], $\pi^+\pi^-\pi^+\pi^-$ [5] [6] and $\pi^+\pi^-\pi^0\pi^0$ [5] [7] in e^+e^- annihilation, complemented by $\omega\pi^\pm$ [8], and $\pi^\pm\pi^0\pi^0\pi^0$ [9] mass spectra measured in τ-lepton decay (using the relation between τ-decays and production of $I = 1$ states in e^+e^- annihilation described in [10] [11]). For the $\omega\pi$ and $\pi\pi$ channels the production of a ρ_1'-resonance is shown by the distinctive interference between the resonant amplitude and the nearly real amplitude due to the tail of the ρ^0-meson, while for the 4π channels a contribution of a ρ_1'-meson is only indicated by a peak in the mass spectrum produced.

The only definite signal for a ρ_2' resonance is an interference with the tail of the ρ^0-meson in $e^+e^- \to \pi^+\pi^-$. These results are complemented by a $\pi\pi$ branching ratio for the ρ_2' deduced from measurements of $\pi\pi$ elastic scattering [12]. The analysis of 4π mass spectra, from e^+e^- annihilation and from τ-lepton decay, indicates a possible contribution from ρ_2', but it is imprecise and small. All results consistently indicate that production of ρ_2' in electron-positron annihilation is appreciably weaker than is the production of ρ_1'.

The properties of the $I = 0$ states produced in electron-positron annihilation are not so well determined, as the $I = 0$ states are produced with smaller cross sections and therefore with worse statistical accuracy. Analyses of the $e^+e^- \to \rho\pi$ and $e^+e^- \to \omega\pi\pi$ reactions [13], [14] yield two distinct solutions: one with two resonant states, with masses consistent with their being the ω_1' and ω_2' partners of ρ_1' and ρ_2' respectively, and the other with only one resonant state, with a mass consistent with being the ω_2' partner of ρ_2'. Other I=0 states, whose production is observed in electron-positron annihilation, are $K\overline{K^*} + \overline{K}K^*$ [15], K^+K^- [16] [17] and $K^0\overline{K^0}$ [18]. The cross sections for these channels are consistent with there being major contributions from three different decay modes of a single ϕ'-meson.

Other $J^P = 1^-$ mesons have been reported, although not in e^+e^- annihilation.

(i) The first is a $\pi^+\pi^-$ state at a mass of 1.302 ± 0.028 GeV with a width of 0.14 ± 0.05 GeV, seen in the reaction $K^-p \to \Lambda\pi^+\pi^-$ [19]: this is labelled as ρ_x. The mass of this state shows that it cannot be identified with ρ_1' while the small upper limit for production of this state in electron-positron annihilation implies that it cannot readily be considered as a $q\overline{q}$ radial excitation. The weak production in electron-positron annihilation could be taken as an indication that the ρ_x is a $q\overline{q}$ orbital excitation, but it is at too low a mass for such an interpretation to be viable.

(ii) The second is the C(1480) state which is claimed to be seen decaying into $\phi\pi^0$ in the reaction $\pi^-p \to \phi\pi^0 n$ [20]. On the other hand there is no observable signal in the reaction $\pi^+p \to \phi\pi^+p$ [21]. Its reported mass and width are consistent with those of the ρ_1'-meson. However it has been shown [22] that specific relations between partial widths, obtained from experimental results, demonstrate that C(1480), if it exists, and ρ_1' must be two different states.

(iii) Many years ago evidence was presented [23] for two $J^{PC} = 1^{--}$ states with masses 1097^{+16}_{-19} MeV and 1266 ± 5 MeV, and with widths 31^{+24}_{-20} MeV and 110 ± 35 MeV respectively. The reaction studied was $\gamma p \to e^+e^-p$ and the evidence for these two states was obtained from the interference between the Bethe-Heitler amplitude and the real part of the hadronic photproduction amplitude. The coincidence of the mass and width of the ρ_x with the upper of these two states is intriguing.

Systematic comparisons of the partial widths found for decays of the ρ' and ω' states show considerable departure from expectation based on the assumption that they are members of pure quark-antiquark nonets. A more complicated interpretation must be considered, for example by including considerable mixing of additional states which do not have a quark-antiquark structure. The possibility of this mixing being with four-quark states or with hybrid states has been presented [24] [25], and demonstrates the general principles of such a solution.

2 $I = 1$ resonances in e^+e^- annihilation

The most precise determination of the mass and width of the $\rho_1^{'}$-meson comes from production of the $\omega\pi$ state in e^+e^- annihilation (data from [2] and, more recently, from [5]) and from the related production in the decay $\tau \to \omega\pi\nu$ (data from [8]). The data can be fitted with only two amplitudes: one from the tail of the ρ-meson and another from the $\rho_1^{'}$-meson, with no observable contribution from the $\rho_2^{'}$-meson being required. The signal for the $\rho_1^{'}$ is a very strong interference between its resonant amplitude and the more slowly varying and nearly real amplitude from the tail of the ρ. The data require $+-$ phases for the ρ and $\rho_1^{'}$ amplitudes. Typical fits (see Fig.1 for an example) give $M = 1.463 \pm 0.025$ GeV, $\Gamma = 0.311 \pm 0.062$ GeV for the ρ_1', and an intensity corresponding to a peak cross section of 12.5 ± 2.0 nb. Note that the quoted errors include contributions from different assumptions about the nature of the ρ tail and are not merely statistical.

Extending this fit to include a third resonant amplitude, to allow for a contribution from ρ_x [19] decaying into the $\omega\pi$ state, gives no definite signal, and puts a limit on the peak cross section for production of ρ_x of 0.6 nb. This in turn implies an upper limit on $\Gamma_{ee}\Gamma_{\omega\pi}/\Gamma_{tot}$ for ρ_x of 9.8 eV.

Structure in the measured mass spectra of the reaction $e^+e^- \to \pi^+\pi^-$ provided the first clear evidence for believing there are two $\rho^{'}$-mesons in the region of 1.4-1.7 GeV [26] [27]. Fits have been made to these cross sections, assuming amplitudes from ρ^0, $\rho_1^{'}$ and $\rho_2^{'}$. The data can accomodate considerable variation in the magnitude of the tail of the ρ^0-meson and, therefore, allow considerable variation in the parameters found for the $\rho^{'}$-mesons. However all the fits found, with reasonable values of χ^2, require two $\rho^{'}$-mesons, with the three amplitudes having phases $+-+$ (in accord with expectation [28]). The masses found for the two $\rho^{'}$-mesons in these fits are, respectively, around 1.4 and 1.75 GeV. To reach more definite conclusions it is necessary to hold the mass and width of the $\rho_1^{'}$-meson at the values found from its decay to $\omega\pi$. The resulting fits are equally good, and provide the best estimates of the mass and width of $\rho_2^{'}$, giving $M = 1.73 \pm 0.03$ GeV and $\Gamma = 0.40 \pm 0.10$ GeV. Best estimates of the peak cross sections are 4.0 ± 1.0nb for $\rho_1^{'}$ and 2.3 ± 0.5nb for $\rho_2^{'}$. Examples of the fits are shown in Fig.2.

Including a further amplitude for the ρ_x, leads to an upper limit for $\sigma(\rho_x \to \pi^+\pi^-)$

of 0.35 nb. This cross section limit yields an upper limit on $\Gamma_{ee}\Gamma_{\pi\pi}/\Gamma_{tot}$ for ρ_x of 5.7 eV, and assuming a lower limit of 5% on the $\pi\pi$ branching ratio of the ρ_x [19] the limit of $\Gamma(\rho_x \to e^+e^-) < 0.11$ keV is obtained. This limit is very much less than one would expect if the ρ_x were a radial excitation of the ρ. Of course the electromagnetic width can be increased by decreasing the $\pi\pi$ width, but this then implies an increasingly strong signal in other channels in e^+e^- annihilation which, as we shall see, is not evident in the data.

Useful complementary information is available from measurements of elastic $\pi\pi$ scattering amplitudes obtained from the study [12] of the reaction

$$\pi^-p \to \pi^-\pi^+n.$$

Structure is observed in the p-wave amplitude which, if due to a single resonance, indicates that it has $M = 1.59\pm0.02$ GeV, $\Gamma = 0.18\pm0.05$ GeV and $\Gamma_{\pi\pi}/\Gamma = 0.25\pm0.05$. Donnachie and Mirzaie [26] showed that this structure could also be fitted with the two ρ'-mesons, using values of masses and widths from e^+e^- annihilation. Using the current values of masses and widths of the ρ_1' and ρ_2' the elasticities are < 0.08 for ρ_1' and 0.27 ± 0.02 for ρ_2'. Combining these with the peak cross sections for production of $\pi^+\pi^-$ in e^+e^- annihilation, one deduces total peak cross sections, summed over all decay modes, of $> 50 \pm 12.5$ nb for ρ_1' and of 8.5 ± 1.8 nb for ρ_2'.

In discussing the measurements of decays of ρ'-mesons into 4π states one needs to note the classification developed by Pais [29] in terms of the permutation symmetries of the isotopic spin wave functions. This classifies $n\pi$ states in terms of partitions of the integer n into three, or fewer, integers. In particular, for 4π states with $I = 1$ there are two states corresponding, respectively, to the partitions [31] and [211]. Wave functions corresponding to different partitions are orthogonal so results must be analysed in terms of the intensities of these states. Writing I_{31} and I_{211} for the intensities, one has for the neutral states from electron-positron annihilation

$$\pi^+\pi^-\pi^+\pi^- = \frac{4}{5}I_{31}$$

and

$$\pi^+\pi^-\pi^0\pi^0 = \frac{1}{5}I_{31} + I_{211},$$

Paradoxically, the 4π channels (other than the $\omega\pi$ channel) are the most difficult to interpret despite the dominant peak in the $\pi^+\pi^-\pi^+\pi^-$ mass spectrum, observed in e^+e^- annihilation and in diffractive photoproduction, being the original evidence for the ρ_1'. The problem lies both in our lack of detailed knowledge and understanding of the non-resonant backgrounds under the $\pi^+\pi^-\pi^+\pi^-$ and $\pi^+\pi^-\pi^0\pi^0$ peaks, and in the presence of a number of broad thresholds (e.g. $\rho\pi\pi$, $a_1\pi$) which can considerably distort the peaks. Neither of these problems affects the other channels we have considered. The relevant threshold is either far away, as in $\pi\pi$, or is sharply defined, as in $\omega\pi$. Additionally the resonance signals in these latter channels are seen as the interference between a relatively small resonance amplitude and a relatively large (and almost real) background amplitude, giving a very characteristic and unmistakeable "anomalous dispersion" shape. In these circumstances, the data can be used to reduce the background uncertainties and the resonance parameters are insensitive to the parametrisation of the background.

Thus to make progress with the 4π channels one has to inject some theoretical prejudice. The choice made is to assume that the background in $e^+e^- \to 4\pi$ in the [31] state is due to $\rho \to a_1\pi$ and that the background in the [211] state in $e^+e^- \to 4\pi$ is due to $\rho \to h_1\pi$. These assumptions are based on the calculation of [31] who found relatively large cross sections for these processes assuming vector dominance and the known properties of the $a_1 \to \rho\pi$ and $h_1 \to \rho\pi$ decays. The assumption is also reasonably consistent with observation [30]. The background amplitude is allowed to interfere with the ρ' amplitudes, using the masses and widths found in the two previous subsections.

Figure 3 shows the measured values of $M^2\sigma(e^+e^- \to [31])$, where M denotes the mass of the e^+e^- centre-of-mass energy. Two sets of e^+e^- annihilation data have been available for some time, which have had the problem that the overlap of their energy ranges is small so that a check on possible differences in normalisation is weak. However the newer results for $\tau^\pm \to \pi^\pm\pi^0\pi^0\pi^0\nu$, which are also shown on Fig.3, overlap both previous data sets, and the agreement seen between these results of quite different measurements of $\sigma(e^+e^- \to [31])$ indicate that there are no major discrepancies among the data sets.

The factor M^2 is a kinematic factor which is useful to incorporate as without it there is an apparent shift of the peak. Dynamic energy dependence, such as arising from threshold factors, is retained. The data have been fitted with a combination of three amplitudes. The first is the background from $\rho \to a_1\pi$, calculated from the formalism of [31] with a mass and width for the a_1 of 1260 MeV and 400 MeV respectively. The second and third amplitudes are those from the ρ_1' and the ρ_2'. As the data lack detailed structure, unlike the $\pi\pi$ and $\omega\pi$ data, it is necessary to fix the masses and widths of the two ρ' states at their standard values and only vary their intensities and relative phases. A wide range of parameters is permitted by the data. To limit the spread of results it is necessary to impose the constraints found above, namely that the peak cross section for ρ_1', summed over all decay modes, is $> 50 \pm 12.5$ nb and that for ρ_2' is 8.5 ± 1.8 nb. As the $\omega\pi$ and $\pi\pi$ channels contribute 16.5 ± 2.2 nb and 2.3 ± 0.5 nb to the peak cross sections for ρ_1' and ρ_2' respectively, these constraints are quite tight. The data favour $+ - +$ relative phases. The peak cross sections are found to lie in the ranges $22nb < \sigma(\rho_1' \to [31]) < 46nb$ and $0 < \sigma(\rho_2' \to [31]) < 7.3nb$.

An additional possibility is to include a contribution from the ρ_x, which simply gives the result that the upper limit on the peak cross section for $e^+e^- \to \rho_x \to \pi^+\pi^-\pi^+\pi^-$ is 0.24 nb.

Cross sections for production of 4π systems in the [211] state are obtained by subtracting $\frac{1}{4}\sigma(e^+e^- \to \pi^+\pi^-\pi^+\pi^-)$ from $\sigma(e^+e^- \to \pi^+\pi^-\pi^0\pi^0)$, and by subtracting $\frac{3}{4}\sigma(e^+e^- \to \pi^+\pi^-\pi^+\pi^-)$ from the annihilation cross section deduced from the τ decay $\tau \to \pi^\pm\pi^+\pi^-\pi^0\nu$. The $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ results and the τ-decay results are in good agreement for production of the [211] state. As these cross sections include a contribution from $\sigma(e^+e^- \to \omega\pi)$ this can be subtracted using the fit to the $\omega\pi$ data to leave the residual cross sections for production of the rest of the [211] state. These cross sections are presented in Fig 4. They show a fuzzy threshold at about 1.1 GeV which can be identified as the ρ-tail decaying to

$h_1\pi^0$. A quantitative calculation of this effect, developed from the related calculation of the ρ-meson tail decaying to $a_1\pi$ by Kramer and Walsh [31], agrees well with the measured cross section.

Precisely the same procedures are followed as for the analysis of 4π in the [31] state, the only change being to calculate the background from $\rho \to h_1\pi$. As before the masses and widths of the $\rho_1^{'}$ and $\rho_2^{'}$ need to be held at their standard values and only their intensities varied. The data require $+-+$ phases, although the magnitudes of the $\rho_1^{'}$ and $\rho_2^{'}$ contributions are not well determined. The upper limits on peak cross sections are 6.3 nb for $\rho_1^{'}$ and 3.5 nb for $\rho_2^{'}$, and the data appear to be dominated by $\rho \to h_1\pi$. An example of a fit is shown as the full line in Fig 4.

Once again, this fit can be extended to include a possible contribution from $\rho_x \to \pi^+\pi^-\pi^0\pi^0$, but no evidence can be found. The absence of a ρ_x signal leads to the limit on a peak cross section of 0.72 nb. An alternative approach is to replace the combination of the ρ-tail and $\rho_1^{'}$ contributions by one from the ρ_x while retaining the $\rho_2^{'}$ contribution. These fits are found to require $++$ phases. Leaving the mass and width of the lower state free leads to values for the mass and width of around 1.2 and 0.25 GeV respectively. Constraining this mass and width to be within one standard deviation of those found for the ρ_x gives reasonable fits, Such a fit, giving for ρ_x a mass of 1.25 GeV, a width of 0.20 GeV, a peak cross section of 10 nb, and for $\rho_2^{'}$ a peak cross section of 4 nb, is plotted as the dashed line in Fig 4. This fit, with its indication of a nonzero cross section for $e^+e^- \to \rho_x$, is to be contrasted with all the other searches for production of ρ_x in e^+e^- annihilation reported here, which find cross sections consistent with zero. It must be emphasised that, if this indication of production of ρ_x is valid, then it must be restricted to production of the [211] state, as if

$$R = \frac{\sigma(\pi^+\pi^-\pi^+\pi^-)}{\sigma(\pi^+\pi^-\pi^+\pi^-) + \sigma(\pi^+\pi^-\pi^0\pi^0)} > 0.03$$

the cross section for $\rho_x \to \pi^+\pi^-\pi^+\pi^-$ is above the limit reported above.

The data for the reaction $e^+e^- \to \eta\pi^+\pi^-$ are shown in Fig.5. At first sight the interpretation of the data looks straightforward, the process being dominated by $\rho_1^{'}$ with a possible small additional contribution from $\rho_2^{'}$. A fit to the data with two Breit-Wigners [30] gives a mass and width of 1470 ± 20 MeV and 230 ± 30 MeV for the $\rho_1^{'}$, and of 1740 ± 20 MeV and 150 ± 30 MeV for the $\rho_2^{'}$. Apart from the width of the $\rho_2^{'}$, these are very close to the standard values. Fixing the masses and widths at the standard values and using all the available data gives an acceptable fit with peak cross sections of 2.0 nb for the $\rho_1^{'}$ and 0.4 nb for the $\rho_2^{'}$. This is shown as the solid line in Fig.5. However an adequate fit can also be found with the $\rho_1^{'}$ by itself, the peak cross section for the $\rho_1^{'}$ then being 3.2 nb. This fit is shown as the dashed line in Fig.5. Obviously there is a continuum of solutions between these two limits, with an ever-increasing fraction of $\rho_1^{'}$ and an ever-decreasing fraction of $\rho_2^{'}$.

These fits ignore any possible contribution from the tail of the ρ, which assumption would appear to be unrealistic given that there is a well defined $\rho \to \eta\gamma$ decay. The known width of this decay can be used to estimate the contribution to $e^+e^- \to \eta\pi^+\pi^-$ from the tail of the ρ and it is non-negligible. Incorporating this permits another broad range of fits in which the peak cross section for the $\rho_1^{'}$ lies in the range 1.0 to 3.0 nb. The peak cross section for $\rho_2^{'}$ is always small. That such a range of fits should exist is not surprising, but what is puzzling in this case is that the data demand ++ phases for the ρ, $\rho_1^{'}$ unlike all other data sets for which +− phases are the natural outcome.

Thus a consistent picture emerges from the e^+e^- annihilation data for the $\rho_1^{'}$ and the $\rho_2^{'}$, particularly the former. There is unambiguous evidence for the $\rho_1^{'}$ in all five channels considered. The evidence for the $\rho_2^{'}$ is less extensive, but its existence is undoubted. The key channel here is $\pi\pi$, which not only provides unambiguous proof of the $\rho_2^{'}$ but also specifies its mass and width rather precisely. There is no evidence whatsoever for the $\rho_2^{'}$ in the $\omega\pi$ channel; there may be a contribution to $\eta\pi\pi$ but it is small; there is no convincing evidence of the $\rho_2^{'}$ in 4π in the [31] state although it cannot be excluded; and a similar situation holds for 4π in the [211] state. The main ρ' parameters are as follows:

parameter	$\rho_1^{'}$	$\rho_2^{'}$
Mass(GeV)	1.463 ± 0.025	1.73 ± 0.03
Width(MeV)	311 ± 62	400 ± 100
$\Gamma_{e^+e^-}$(keV)	$1.6 - 3.4$	0.69 ± 0.15
$\Gamma_{\omega\pi}$(MeV)	$52 - 78$	~ 0
$\Gamma_{\pi\pi}$(MeV)	$17 - 25$	~ 100
$\Gamma_{\eta\pi\pi}$(MeV)	$4 - 19$	< 30
$\Gamma_{[31]}$(MeV)	190	?
$\Gamma_{[211]}$(MeV)	$0 - 39$	?
$\Gamma_{4\pi}$(MeV)	$190 - 230$	~ 300

The partial widths quoted for $\rho_1^{'}$ and for $\rho_2^{'}$ are known to about $\pm 25\%$ accuracy, a major contribution to this arising from the uncertainty with which the total widths are measured.

3 $\omega^{'}$ resonances in e^+e^- annihilation

The main evidence for the $\omega^{'}$ resonances comes from production of $\rho\pi$ and $\omega\pi\pi$ states, making the assumption, which can be fully justified, that states involving strange particles result primarily from the decay of $\phi^{'}$ resonances.

Two groups of fits have been made to $e^+e^- \to \rho\pi$ mass spectra, with contributions from the tails of the ω-meson and of the ϕ-meson and from one or two $\omega^{'}$ states. The fits with one $\omega^{'}$ require ++ phases, and the $\omega^{'}$ peak has a mass of 1.635 ± 0.035 GeV, a width of 0.35 ± 0.08 GeV and a peak cross section of 2.25 ± 0.45 nb. The fits with two $\omega^{'}$ states are not significantly better, but they are interesting because of their relation to the results of the fits to $I = 1$ mass spectra with two $\rho^{'}$ states. These fits with two $\omega^{'}$ states require $+-+$ phases. For $\omega_1^{'}$ the mass is 1.44 ± 0.07 GeV, the width is 0.24 ± 0.07 GeV and the peak cross section is 4.4 ± 0.4 nb, while for ω_2' the mass is 1.60 ± 0.03 GeV, the width is 0.14 ± 0.05 GeV and the peak cross section is 5.3 ± 1.0 nb.

Data (with preliminary DM2 results [5]) and typical fits are shown in Fig.6. The steep fall in the experimental $\rho\pi$ mass spectrum at an energy of about 1.65 GeV, signals interference between a resonance and the nearly real amplitude of the $\omega - \phi$ tail. Just as a similar effect in the $\omega\pi$ channel provides a rather precise value of the ρ_1' parameters, this fall determines the single ω' amplitude in the one-ω' fits, and equivalently the ω_2' amplitude in the two-ω' fits. The two-ω' fits are particularly sensitive to variations in the background and it is easy to overestimate the values of the peak cross sections in these fits.

Two fits to early $\omega\pi\pi$ data [35], which relate to the fits found to the $\rho\pi$ mass spectra, are presented in Fig.7. The simplest fit to the $\omega\pi\pi$ data uses a simple Breit-Wigner formula. This fit gives a mass of 1.625 ± 0.021 GeV, a width of 0.401 ± 0.063 GeV and a peak cross section of 3.1 ± 0.4 nb. This mass and width are completely compatible with those found in the one-ω' fit to the $\rho\pi$ data.

The other fit requires an appreciable empirical background. Such a fit, with a mass of the ω' of 1.607 ± 0.010 GeV and a width of 0.086 ± 0.020 GeV, has $\chi^2 \approx 9.7$ for 9 d.o.f., and a peak cross section of 1.8 ± 0.2 nbarn. This fit has the merit of yielding resonance parameters which agree well with the parameters of ω_2' found in the two-ω' fits to the $\rho\pi$ data.

The presence of such a background is perhaps not surprising. Just as there are significant contributions to 4π states in e^+e^- annihilation from $\rho \to a_1\pi$ and $\rho \to h_1\pi$ so too is there a contribution to 5π states from $\omega \to b_1\pi$. This can be readily estimated in the Kramer and Walsh formalism [31]. The results are compatible with the empirical background, lending some support to the validity of the two-ω' fit.

It should be emphasised that these two fits to the $\omega\pi\pi$ mass spectra are two separate, and discrete, classes of fit, and are not just two points on a continuum of fits.

Combining the results from the $\rho\pi$ and $\omega\pi\pi$ fits, which are satisfactorily consistent, a reasonable estimate of the parameters of the ω' candidates can be made. For the one-ω' fit, the estimates of these parameters are:

	ω_1'
Mass(GeV)	1.68 ± 0.018
Width(MeV)	381 ± 49
Γ_{ee}(keV)	0.37
$\Gamma_{\rho\pi}$(MeV)	160
$\Gamma_{\omega\pi\pi}$(MeV)	220

The two candidate resonances from the two ω' fit are referred to as ω_1' and ω_2' respectively. Estimates of the ω' parameters in this case are:

	ω_1'	ω_2'
Mass(GeV)	1.44 ± 0.07	1.61 ± 0.01
Width(MeV)	240 ± 70	113 ± 20
Γ_{ee}(keV)	0.15	0.14
$\Gamma_{\rho\pi}$(MeV)	~ 240	~ 84
$\Gamma_{\omega\pi\pi}$(MeV)	~ 0	~ 29

All the partial widths are determined to about ±25% accuracy.

4 ϕ' resonances in e^+e^- annihilation

As the ϕ' mass is above the range accessible to DAΦNE only a brief summary of results is presented. The production of a $K\overline{K}$ decay interfering with a ϕ-tail amplitude is a positive demonstration of a resonance, and shows it has a relatively high mass. Comparing this mass with those indicated from the fits to the mass spectrum for $K\overline{K}^* + \overline{K}K^*$ shows that $+-$ phases are required, which is what would be expected if the ϕ' is a radial excitation of the ϕ [28].

Estimates of the parameters of this resonance are then:

Mass(GeV)	1.70 ± 0.02
Width(MeV)	300 ± 60
Γ_{ee}(keV)	~ 0.7
$\Gamma_{K\overline{K}}$(MeV)	~ 8
$\Gamma_{K\overline{K}^*+\overline{K}K^*}$(MeV)	~ 292

These partial widths are determined to about ±25% accuracy.

5 Discussion

Masses predicted [36] for the first radial, $^3S_1$, and orbital, $^3D_1$, excitations of ρ, ω and ϕ are

State	Mass(GeV)
ρ_S'	1.45
ρ_D'	1.66
ω_S'	1.46
ω_D'	1.66
ϕ_S'	1.69
ϕ_D'	1.88

These predictions are in reasonable agreement with the masses obtained for the two ρ'-mesons, the single ϕ'-meson and the two ω'-meson candidates found in the two ω' fit, thereby encouraging the interpretation of these states as being the $q\overline{q}$ radial and orbital excitations of the ρ, ω and ϕ. However the details of their decay modes indicate that this simple interpretation cannot be the whole story.

The first problem lies with the electromagnetic couplings. Using the results of the two-ω' fit, the measured values of Γ_{ee}, in keV, are

	$\rho_i^{'}$	$\omega_i^{'}$	$\phi_i^{'}$
$i = 1$	1.6-3.4	0.15	0.7
$i = 2$	0.69	0.14	?

The values of $\Gamma_{e^+e^-}$ for the $\rho_2^{'}$ and $\omega_2^{'}$ are too large relative to those for $\rho_1^{'}$ and $\omega_1^{'}$, particularly for the $\omega^{'}$ states. In the non-relativistic limit the leptonic widths for orbitally excited $q\overline{q}$ states are expected to be zero, and even when relativistic corrections are applied they are still considerably suppressed. For example Godfrey and Isgur [36] quote a ratio for $\Gamma_{e^+e^-}(1^3D_1)/\Gamma_{e^+e^-}(2^3S_1)$ of $\sim 1/4$. From the Table we see that the experimental value of this ratio is $\sim$ 0.2 - 0.5 for the $\rho^{'}$ states and is $\sim$ 1.0 for the $\omega^{'}$ states in the two-$\omega^{'}$ fit. In the one-$\omega^{'}$ fit, the $\omega_1^{'}$ must decouple almost completely from e^+e^-. Thus by taking the upper limit of $\Gamma_{e^+e^-}$ for the $\rho_1^{'}$ the $\rho^{'}$ results can be brought into accord with expectation, but the $\omega^{'}$ results are not at all compatible and cannot be made so.

The problem is emphasised by looking at the ratios $\rho_i^{'} : \omega_i^{'} : \phi_i^{'}$ of the e^+e^- partial widths, for which we would expect

$$\rho_i^{'} \rightarrow e^+e^- : \omega_i^{'} \rightarrow e^+e^- : \phi_i^{'} \rightarrow e^+e^- = 9 : 1 : 2$$

The table again shows considerable discrepancy with these expected ratios. The ratio $\Gamma(\phi_1^{'} \rightarrow e^+e^-)/\Gamma(\rho_1^{'} \rightarrow e^+e^-)$ is consistent with prediction at the upper limit of the range for $\rho_1^{'}$, but the ratio $\Gamma(\omega_1^{'} \rightarrow e^+e^-)/\Gamma(\rho_1^{'} \rightarrow e^+e^-)$ is only consistent with prediction at the other extreme of the $\rho_1^{'}$ range. Further, the ratio $\Gamma(\phi_1^{'} \rightarrow e^+e^-)/\Gamma(\omega_1^{'} \rightarrow e^+e^-)$ differs from prediction by a factor of about two, as does the one ratio which is experimentally measured for i = 2.

Additional problems are evident in the hadronic decays. For example, the expected values of the ratios of the partial widths for vector-pseudoscalar decays are:

$$\rho_i^{'} \rightarrow \omega\pi : \omega_i^{'} \rightarrow \rho\pi : \phi_i^{'} \rightarrow K\overline{K^*} + \overline{K}K^* = 1 : 3 : 2$$

To test this we again use the results of the two-$\omega^{'}$ fit. The vector-pseudoscalar partial widths, in MeV, are then

	$\rho_i^{'}$	$\omega_i^{'}$	$\phi_i^{'}$
$i = 1$	52-78	240	292
$i = 2$	0	84	?

The ratio of the $\rho_1^{'}$ and $\omega_1^{'}$ decays is in reasonable agreement with expectation, but the $\phi_1^{'}$ width is too large. The absence of any visible decay of $\rho_2^{'}$ to $\omega\pi$ contrasts markedly with the unambiguous decay of $\omega_2^{'}$ to $\rho\pi$.

The results for the hadronic decay widths of the ρ'_1, ω'_1 and the ρ'_2, ω'_2 can be compared with predictions from quark model calculations, identifying them with the $^3S_1$ radial excitations, and with the $^3D_1$ orbital excitations of the ρ and ω respectively. Estimates of partial widths for hadronic states have been made using a model of π-meson emission from quarks [36], the $^3P_0$ model [37], and a flux tube model [38].

Partial widths calculated for decays of ρ'_S, in MeV, are:

Decay to	Ref [36]	Ref [37]	Ref [38]
$\pi\pi$	125	0	12-61
$\omega\pi$	125	53	78
$\eta\pi\pi$	14	41	17
$4\pi, [31]$		16	1.4
$4\pi, [211]$		99	18-78

The considerable variation in the predicted widths for the $\pi^+\pi^-$ decay mode is due to its sensitivity to a node in the radial wave function.

The partial widths, in MeV, calculated for decays of ρ'_D are:

Decay to	Ref [36]	Ref [38]
$\pi\pi$	100	55
$\omega\pi$	25	28
$4\pi, [31]$		88
$4\pi, [211]$		115

For the decays of ρ'_S the notable prediction is that the decay to 4π in the [31] state is expected to be very small, contrary to observation. The decays predicted for ρ'_D are in reasonable accord with the experimental results.

For the ω'_S, the predictions of partial widths in MeV are:

parameter	Ref [36]	Ref [37]	Ref [38]
$\Gamma_{\rho\pi}$	300	158	257
$\Gamma_{\overline{K}K^*+\overline{K}K^*}$		72	19
$\Gamma_{b_1\pi}$		23	2
$\Gamma_{\overline{K}K}$			12-45

while those of ω'_D are

parameter	Ref [36]	Ref [38]
$\Gamma_{\rho\pi}$	100	72
$\Gamma_{K\overline{K}}$	25	45
$\Gamma_{b_1\pi}$		440

These tables include prediction of partial widths for production of pairs of strange particles, which have not considered explicitly here. Most of the observed production of $K\overline{K}$ and $K\overline{K}^*$, $\overline{K}K^*$ is due to a ϕ' state and does not result from ω' decay, a conclusion which is in reasonable accord with the above predictions. The predictions of appreciable $\rho\pi$ widths are in plausible accord with the general pattern of the experimental results. At first sight, the large partial width predicted for $\omega_D^{'} \to b_1\pi$ disagrees with the results from the two-$\omega^{'}$ fit, particularly with the small width found for $\omega_2^{'}$, but is in accord with outcome of the one-$\omega^{'}$ fit, thereby apparently favouring the latter. However as the $\omega_D^{'}$ does not couple to the electromagnetic current in the naive quark model, and corrections to this do no more than generate a small electromagnetic coupling, the e^+e^- widths found experimentally for the $\omega_1^{'}$ and ω'_2 imply that there must be considerable mixing between them.

This is the only indication of ρ_x being produced in e^+e^- annihilation with a cross section which is not consistent with zero. As the ρ_x is believed to have a $\pi\pi$ branching ratio of only $5-10\%$ [19] this signal in the [211] state can be used to derive a lower limit on the electromagnetic width of the ρ_x from the peak cross section of $\sim$ 10 nb. This gives $\Gamma_{e^+e^-} \geq 240$ eV which is in conflict with the upper limit of 110 eV derived from the $\pi\pi$ analysis. For this reason, and because $\rho \to h_1\pi$ provides a satisfactory alternative explanation for the same signal, it is unlikely that the interpretation of the 4π data in the [211] state in terms of the ρ_x and $\rho_2^{'}$ is correct. It is certainly not demanded by the data.

An obvious interpretation to put on these various inconsistencies and discrepancies is that there has been considerable mixing of states, for whatever reason, resulting in all the e^+e^- decay probability being redistributed in a "non-standard" way. Assuming that there is no mixing between the $\omega^{'}$ states and the $\phi^{'}$ states, the partial widths for the e^+e^- decays can be summed separately for the $\rho^{'}$-states, the $\omega^{'}$-states and the $\phi^{'}$-states and are (in keV)

$\rho^{'}$	$\omega^{'}$	$\phi^{'}$
4.1-2.3	0.29/0.37	0.7

The two values for the two $\omega^{'}$ states are for the two–$\omega^{'}$ and one–$\omega^{'}$ fits respectively. These results are in excellent agreement with the expected ratio of $9:1:2$, although it should be noted that any contribution from a $\phi_2^{'}$ has been neglected.

Invoking S-D mixing in the $q\overline{q}$ sector does not help resolve these discrepancies, as the only source of mixing is via the tensor force of the $q\overline{q}$ potential, which is small and gives only $\sim 4\%$ for S-D mixing for vectors. Thus the mixing has to involve systems other than $q\overline{q}$ i.e. glueballs, hybrids or four-quark states. further, if one is going to invoke mixing, then it would appear that it has to occur in both the isoscalar and isovector sectors. This excludes the sole use of glueballs, although they have been invoked to explain anomalies between diffractive photoproduction and e^+e^- annihilation in isoscalar channels [39]. Specific models have been presented [24] for both hybrid states and four-quark states, although they cannot be taken to be definitive.

Mixing of the hybrid states with those of the $q\overline{q}$ system permits a good qualitative description of all the observed features of the hadronic decays of the $\rho_1^{'}$, $\rho_2^{'}$, $\omega_1^{'}$ and $\omega_2^{'}$. The model is economical and the mixing is straightforward. However it is not possible to incorporate the ρ_x in the hybrid mixing scheme. On the other hand the four-quark model can incorporate the ρ_x at the price of a much more complex mixing scheme, with particular difficulties in the isoscalar sector. The four-quark model without the ρ_x does not have such problems and can provide as good a description of the e^+e^- annihilation data as the hybrid model, albeit with a richer structure.

The four-quark model with the ρ_x does have an intriguing consequence as the corresponding isoscalar state is predicted to be at $\sim$ 1.1 GeV. Many years ago evidence was presented [23] for two $J^{PC} = 1^{--}$ states with masses $\sim$ 1.1 GeV and $\sim$ 1.27 GeV, and recently it has been shown [25] that they are compatible with present evidence and can be incorporated in an extended mixing scheme of $q\bar{q}$ and $qq\bar{q}\bar{q}$ states.

The situation of the ρ_x is particularly interesting. The e^+e^- data show no direct evidence for it, and without the prompt from the LASS experiment [19] one would not have considered including it in any analysis of e^+e^- annihilation data. In contrast to the $\rho_1^{'}$ and the $\rho_2^{'}$ which are clearly demanded by this data, the ρ_x is not explicitly required. However if it is included, then there is a possible weak signal in the $\pi\pi$ channel. However as this is correlated with the shape of the non-resonant background only an upper limit on the cross section can be quoted. There is also a possible strong signal, at the expense of the ρ-tail and $\rho_1^{'}$ contributions, in production of 4π in the [211] state, although it apparently implies a lower limit for the e^+e^- width of the ρ_x in excess of the upper limit derived from the analysis of the $\pi\pi$ channel, and a straightforward explanation of the data can be found in terms of $\rho \to h_1\pi$. On the other hand, the analysis of the $\pi^+\pi^-\pi^+\pi^-$ channel in the LASS data [40] puts an upper limit on this decay of the ρ_x: $\sigma(\rho_x \to \pi^+\pi^-\pi^+\pi^-/\sigma(\rho_x \to \pi^+\pi^-) < 3$, implying that the decay must be primarily in $\pi^+\pi^-\pi^0\pi^0$. (There is no evidence for significant production in the $\eta\pi^+\pi^-$ channel [40].)

6 Conclusions

The fits presented here represent the maximum information on vector mesons which can be obtained from present e^+e^- annihilation data and τ-decay. The fits are purely phenomenological, based on the assumption of vector dominance. In determining the basic parameters of the resonances found, it is essential to use all available data, as generally a single data set will permit a wide variation. Nonetheless considerable quantitative uncertainties remain, but the qualitative picture is clear. The data require there to be two $\rho^{'}$ states, $\rho_1^{'}(1.46)$and $\rho_2^{'}(1.73)$; they support the existence of two corresponding $\omega^{'}$ states, $\omega_1^{'}(1.44)$ and $\omega_2^{'}(1.61)$, although an equally satisfactory fit can be found with $\omega_2^{'}(1.63)$ only; and they unambiguously identify the $\phi_1^{'}(1.70)$. The vector state $\rho_x(1.30)$ found by LASS couples relatively weakly to e^+e^- and is not therefore a candidate for an excitation of the ρ.

The masses found for these states are such that it seems natural to identify the $\rho_1^{'}$ and $\omega_1^{'}$ with the first radial excitations, $\rho_S^{'}$ and $\omega_S^{'}$, of the ρ and ω; to identify the $\rho_2^{'}$ and $\omega_2^{'}$ with the first orbital excitations, $\rho_D^{'}$ and $\omega_D^{'}$; and to identify the $\phi_1^{'}$ with the first radial excitation, $\phi_S^{'}$, of the ϕ. However the details of their decays provide clear evidence that the reality is more complicated,and that mixing with other, non-$q\overline{q}$ states must occur. It is not possible to specify precisely what these non-$q\overline{q}$ states are, as model calculations of mixing with hybrid or four-quark states show that both can provide a suitable scheme. If the ρ_x is included as part of the scheme, then only mixing with four-quark states is permissible.

Further progress in our understanding can only be made by a considerable improvement in the data, particularly in channels with neutrals. The advent of DAΦNE should do much to clarify the lower mass range, which promises to be particularly rich. It has frequently been said that one of the best ways to identify exotic non–$q\bar{q}$ states is by establishing departures from nonets. It certainly appears that systematic study of vector mesons is a very promising approach for progress on such a programme.

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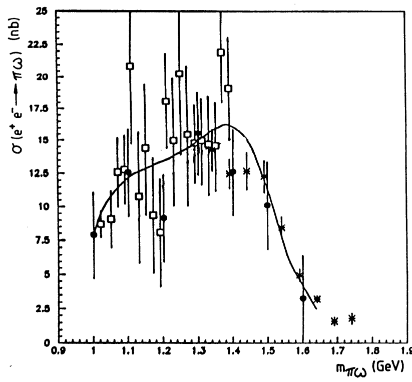

FIG. 1

Cross sections for $\epsilon^+\epsilon^- \to \omega\pi$, together with a typical fit. The data are from Dolinsky et al [2] (dashes), Bisello et al [5] (closed squares) and deduced from the $\tau \to \omega\pi\nu$ measurements of Albrecht et al [8]. The full curve shows a typical fit and the dashed curve shows the ρ_1' contribution on its own.

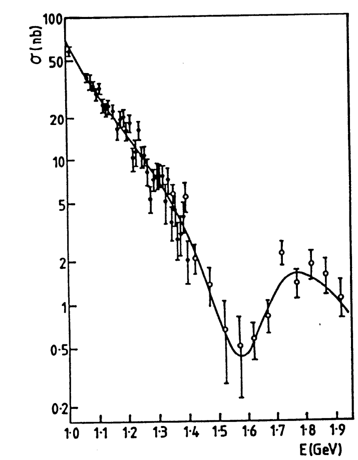

FIG. 2

Cross sections for $\epsilon^+e^- \to \pi^+\pi^-$ above the ρ peak from Bisello et al [3] (open circles) and Barkov et al [4] (closed circles), with a typical fit.

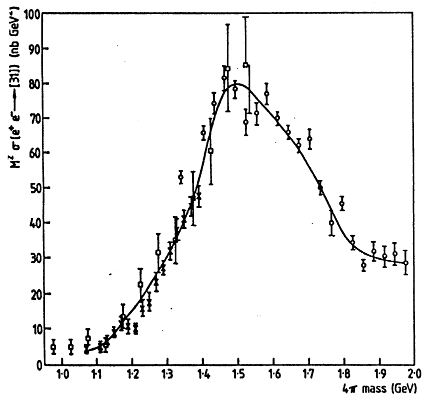

FIG. 3

Measured values of $M^2\sigma(e^+e^- \to 4\pi$ in [31] state) from Kurdadze et al [6] (squares), Bisello et al [5] (circles) and from the τ-lepton decay data of Procario et al [9] (crosses), with a typical fit.

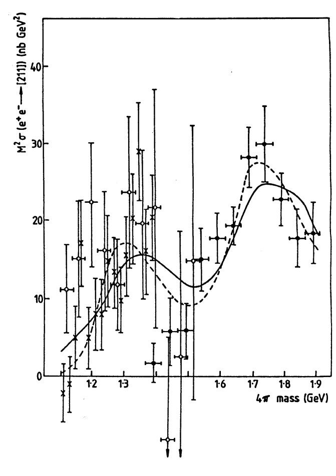

FIG. 4

Measured values of $M^2\sigma(e^+e^- \to 4\pi$ in [211] state), from which the contribution of $\omega\pi$ has been subtracted, from Kurkadze et al [7] (crosses), Bisello et al [5] (filled circles) and from the τ-lepton decay data of [8] (open circles), with a typical fit.

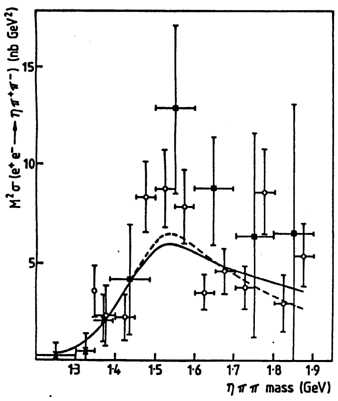

FIG. 5

Measured values of $M^2\sigma(e^+e^- \rightarrow \eta\pi^+\pi^-)$ from Druzhin et al [32] (squares), Delcourt et al [33] (crosses) and Antonelli et al [34] (circles). The solid line shows the result of a fit assuming two ρ' amplitudes, and the dashed line the result of a fit assuming a ρ_1' amplitude only.

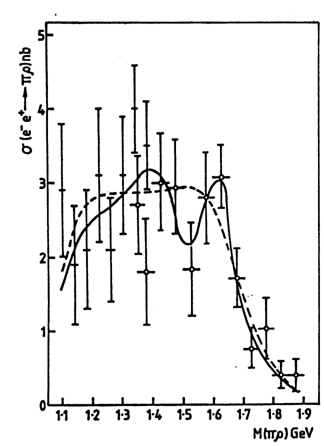

FIG. 6

Cross section for the reaction $e^+e^- \rightarrow \rho\pi$ from Bisello et al [5] and Dolinsky et al [14]. The solid line is a two-resonance fit, the broken line a one-resonance fit.

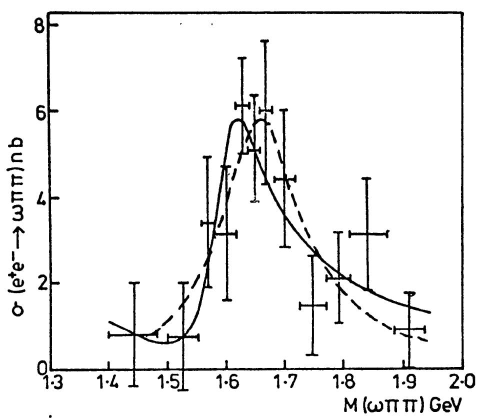

FIG. 7
Cross section for the reaction $e^+e^- \to \omega\pi\pi$ [35]. The solid line is a two resonance fit, the broken line a one-resonance fit.