

# Study of the OZI Rule Violation at DAΦNE

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## 1 Introduction

The OZI veto means a suppression of  $\phi$  decays into final states without strange quarks because the  $\phi$  meson is largely composed of strange quarks ( $s\bar{s}$ ).

The OZI rule is not a science in a literal sense but a more important part of folklore invoked to explain the striking effectiveness of naive (constituent) quark model. Only in some cases we have a qualitative grasp of its origin.

That is why the problem of the study of the OZI rule violation at DAΦNE is a central one.

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## 2 To search four-quark ( $qs\bar{q}\bar{s}$ ) states in the $\phi \rightarrow \gamma\pi\pi$ and $\phi \rightarrow \gamma\pi\eta$ decays

The existence of exotic states would be the most interesting violation of the OZI rule.

As is known, see e.g. the reviews [1, 2, 3, 4, 5, 6], the  $a_0(980)$  and  $f_0(975)$  mesons possess peculiar properties from the point of view of the naive ( $q\bar{q}$ ) model. In the meantime, all challenging properties of these mesons can be understood in the framework of the four-quark ( $q^2\bar{q}^2$ ) model [2, 4, 6].

Along with the  $q^2\bar{q}^2$  nature of the  $a_0(980)$  and  $f_0(975)$  mesons [7] the possibility of they are the  $K\bar{K}$  molecules is discussed [8, 9]. Furthermore, probably the  $a_0(980)$  and  $f_0(975)$  mesons are participators of confinement [10]. Radiative decays of  $\phi$  meson give a possibility to get new unique findings about the nature of the  $a_0(980)$  and  $f_0(975)$  scalar mesons [11].

### 2.1 The OZI suppression of the decays $\phi \rightarrow \gamma a_0(980) \rightarrow \gamma\eta\pi$ and $\phi \rightarrow \gamma f_0(975) \rightarrow \gamma\pi\pi$ in the case of two-quark ( $q\bar{q}$ ) nature of the $a_0(980)$ and $f_0(975)$ mesons

i) Indeed, the  $a_0(980)$  meson is a isovector state and has a symbolic structure

$$a_0(980) = (u\bar{u} - d\bar{d})/\sqrt{2} \quad (1)$$

in case of its two-quark nature. If so the decay  $\phi \rightarrow \rho a_0(980) \rightarrow \gamma a_0(980) \rightarrow \gamma\pi\eta$  is suppressed by the OZI rule. A very rough estimate of

$$BR(\phi \rightarrow \gamma a_0(980) \rightarrow \gamma\eta\pi) \sim \frac{\alpha}{f_\rho^2/4\pi} (\delta_{OZI})^2 \left(\frac{\omega_0}{q_K}\right)^3 \sim 3 \times 10^{-7}, \quad (2)$$

where

$$\begin{aligned} q_K &= \left( \sqrt{m_\phi^2 - 4m_{K^+}^2} + \sqrt{m_\phi^2 - 4m_{K^0}^2} \right) / 4 \simeq 120 \text{ MeV}, \\ \omega_0 &= m_\phi [1 - (m_{a_0}/m_\phi)^2] / 2 \simeq 40 \text{ MeV}, \quad \delta_{OZI} \simeq 1/20. \end{aligned} \quad (3)$$

The parameters of the OZI rule violation  $\delta_{OZI} \simeq 1/20$  is taken from a comparison of the  $\phi \rightarrow 3\pi$  with the  $\omega \rightarrow 3\pi$  decay. It should be pointed out that for the low photon energy the gauge invariance demands the factor  $\omega^3 = (\text{Photon energy})^3$  suppressing this decay despite of the fact that the final states ( $\gamma$  and  $a_0(980)$ ) are in the S wave.

Let us recall that the OZI suppressed decay  $\phi \rightarrow \gamma\pi^0$  has  $BR(\phi \rightarrow \gamma\pi^0) = 1.31 \times 10^{-3}$  [12]. It is instructive that if the  $\pi$  meson mass was equal to 980 MeV then according to the  $\omega^3$  low  $BR(\phi \rightarrow \gamma\pi^0(980))$  would be  $6 \times 10^{-7}$  in close agreement with a estimate akin to Eq. (2).

ii) Things with the  $f_0(975)$  meson are somewhat different since in parallel with the structure

$$f_0(975) = (u\bar{u} + d\bar{d})/\sqrt{2} \quad (4)$$

in the naive ( $q\bar{q}$ ) quark model at times the structure

$$f_0(975) = s\bar{s} \quad (5)$$

was discussed despite of the fact that in the naive quark model it is impossible to understand almost exact degeneracy of the  $a_0$  and  $f_0$  mesons masses in this case. Traditionally, one was hopeful of overcoming this difficulty with taking into account the transitions between the quark and gluon degrees of freedom ( $q\bar{q} \leftrightarrow gg$ ). But at present there are no good grounds for such a hope. The point is the experiment showed that the  $f_0(975)$  meson is coupled with gluons only weakly [13]<sup>2</sup>:

$$BR(J/\psi \rightarrow \gamma gg \rightarrow \gamma f_0(975) \rightarrow \gamma \pi \pi) < 1.4 \times 10^{-5} \quad (CL = 90\%). \quad (6)$$

Please compare with [12]

$$BR(J/\psi \rightarrow \gamma gg \rightarrow \gamma \eta'(958)) = 4,3 \times 10^{-3}. \quad (7)$$

It is clear that in the case of Eq.(2) there is a very rough akin to Eq.(2) estimate of the intensity decay  $\phi \rightarrow (\phi + \omega)f_0(975) \rightarrow \gamma f_0(975)$ :

$$BR(\phi \rightarrow \gamma f_0(975)) \sim \frac{\alpha}{\pi} (\delta_{OZI})^2 \left(\frac{\omega_0}{q_K}\right)^3 \simeq 2 \times 10^{-7}. \quad (8)$$

## 2.2 The OZI breaking in the decays $\phi \rightarrow \gamma a_0(980) \rightarrow \gamma \eta \pi$ and $\phi \rightarrow \gamma f_0(975) \rightarrow \gamma \pi \pi$ in the case of four-quark nature of the $a_0(980)$ and $f_0(975)$ mesons

A completely different type of situation occurs in the case of the four-quark nature of the  $a_0(980)$  and  $f_0(975)$  mesons with the symbolic structure

$$a_0(980) = s\bar{s}(u\bar{u} - d\bar{d})/\sqrt{2}, \quad f_0(975) = s\bar{s}(u\bar{u} + d\bar{d})/\sqrt{2}. \quad (9)$$

In this case the OZI suppression is absent and one can hope that

$$BR(\phi \rightarrow \gamma a_0(980) \rightarrow \gamma \eta \pi) \sim BR(\phi \rightarrow \gamma f_0(975) \rightarrow \gamma \pi \pi) \sim 10^{-4} \quad (10)$$

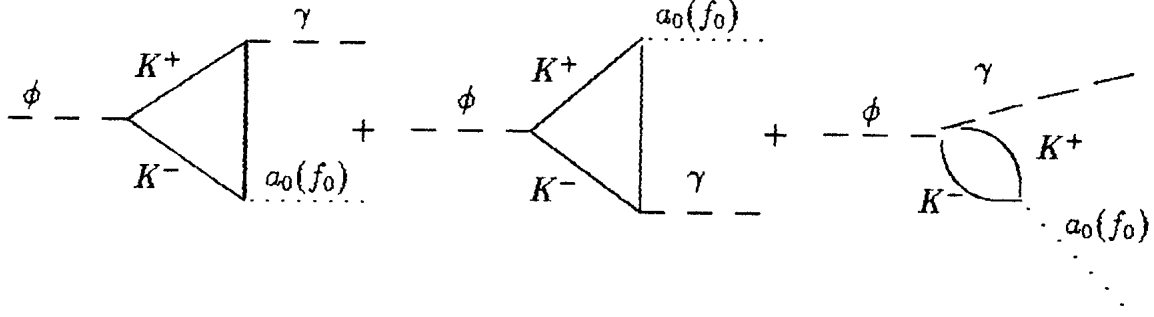
which do not contradict to the present experiment (Neutral Detector, Novosibirsk) [12]:

$$\begin{aligned} BR(\phi \rightarrow \gamma \pi^0 \eta) &< 2.5 \times 10^{-3} \quad (CL = 90\%), \\ BR(\phi \rightarrow \gamma \pi^0 \pi^0) &< 10^{-3} \quad (CL = 90\%). \end{aligned} \quad (11)$$

In 1987 for the decays  $\phi \rightarrow \gamma a_0(980) \rightarrow \gamma \eta \pi$  and  $\phi \rightarrow \gamma f_0(975) \rightarrow \gamma \pi \pi$  we suggested [11] the  $K\bar{K}$  loop model, see Fig.1,  $\phi \rightarrow K^+ K^- \rightarrow \gamma a_0(f_0)$ , motivated by a possible four-quark ( $q^2 \bar{q}^2$ ) nature of  $a_0(980)$  and  $f_0(975)$  mesons.

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<sup>2</sup>Needless to say Eq.(6) rules out a possibility to interpret the  $f_0(975)$  meson as a glueball.



### 2.2.1 Model of $\phi \rightarrow \gamma a_0$ and $\phi \rightarrow \gamma f_0$ decays

Let us introduce the amplitudes

$$M(\phi \rightarrow \gamma R; m) = g_R(m)(\vec{e}(\phi)\vec{e}(\gamma)) \quad R = a_0, f_0, \quad (12)$$

$\vec{e}(\phi)$  and  $\vec{e}(\gamma)$  are the  $\phi$ -meson and  $\gamma$ -quantum polarization vectors in the  $\phi$ -meson rest frame. According to the gauge invariance condition

$$g_R(m) \rightarrow \omega \times const, \quad (13)$$

if  $m \rightarrow m_\phi$ ,  $\omega = (1/2)m_\phi(1 - m^2/m_\phi^2) \rightarrow 0$ . The width of a decay

$$\Gamma(\phi \rightarrow \gamma R; m) = \frac{1}{3} \frac{|g_R(m)|^2}{4\pi} \frac{1}{2m_\phi} \left(1 - \frac{m^2}{m_\phi^2}\right), \quad (14)$$

where  $m$  is the invariant mass of the  $ab$  state into which the  $R$  meson decays,  $m = m_{ab}$ ,  $a$  and  $b$  are the pseudoscalar mesons.

The calculation of the Fig.1 diagrams gives [11] for  $m < 2m_{K^+}$

$$g_R(m) = \frac{e}{2(2\pi)^2} g_{\phi K^+ K^-} g_{RK^+ K^-} \left\{ 1 + \frac{m_\phi^2}{m_\phi^2 - m^2} [2|\rho(m)| \arctan(\frac{1}{|\rho(m)|}) - \right. \\ \left. - \rho(m_\phi)(\lambda(m_\phi) - i\pi) - \frac{4m_{K^+}^2}{m_\phi^2} (\frac{1}{4}(\pi + i\lambda(m_\phi))^2 - (\arctan(\frac{1}{|\rho(m)|}))^2) \right\}. \quad (15)$$

For  $2m_{K^+} \leq m$

$$g_R(m) = \frac{e}{2(2\pi)^2} g_{\phi K^+ K^-} g_{RK^+ K^-} \left\{ 1 + \frac{m_\phi^2}{m_\phi^2 - m^2} [\rho(m)(\lambda(m) - i\pi) - \right. \\ \left. - \rho(m_\phi)(\lambda(m_\phi) - i\pi) - \frac{m_{K^+}^2}{m_\phi^2} ((\lambda(m) - i\pi)^2 - (\lambda(m_\phi) - i\pi)^2)] \right\}. \quad (16)$$

In Eqs.(13) and (14)  $e^2/4\pi = \alpha = 1/137$ , for  $m > 2m_{K^+}$

$$\rho(m) = \sqrt{1 - 4m_{K^+}^2/m^2} = \rho_{K^+ K^-}(m) \quad \lambda = \ln \frac{1 + \rho(m)}{1 - \rho(m)}. \quad (17)$$

The  $g_{\phi K^+ K^-}$  and  $g_{RK^+ K^-}$  are related to the widths by

$$\begin{aligned}\Gamma(\phi \rightarrow K^+ K^-) &= \frac{1}{3} \frac{g_{\phi K^+ K^-}^2}{16\pi} m_\phi \rho^3(m_\phi), \\ \Gamma(R \rightarrow K^+ K^-, m) &= \frac{1}{16\pi} g_{RK^+ K^-}^2 \frac{1}{m} \rho(m).\end{aligned}\quad (18)$$

Using these amplitudes one can find the widths

$$\Gamma(\phi \rightarrow \gamma R \rightarrow \gamma ab) = \frac{2}{\pi} \int_{m_a+m_b}^{m_\phi} m dm \frac{m \Gamma(R \rightarrow ab; m) \Gamma(\phi \rightarrow \gamma R; m)}{|D_R(m)|^2} \quad (19)$$

for  $ab = \pi\pi, \pi^0\eta$  and for  $ab = K^+ K^-, K^0 \bar{K}^0$

$$\begin{aligned}\Gamma(\phi \rightarrow \gamma(a_0 + f_0) \rightarrow \gamma K^+ K^-) &= \\ \int_{m_a+m_b}^{m_\phi} m \Gamma(f_0 \rightarrow K^+ K^-; m) \Gamma(\phi \rightarrow \gamma f_0; m) &\left| \frac{1}{D_{f_0}(m)} + \frac{g_{a_0 K^+ K^-}^2}{g_{f_0 K^+ K^-}^2} \frac{1}{D_{a_0}(m)} \right|^2 \frac{dm^2}{\pi}, \\ \Gamma(\phi \rightarrow \gamma(a_0 + f_0) \rightarrow \gamma K^0 \bar{K}^0) &= \\ \int_{m_a+m_b}^{m_\phi} m \Gamma(f_0 \rightarrow K^0 \bar{K}^0; m) \Gamma(\phi \rightarrow \gamma f_0; m) &\left| \frac{1}{D_{f_0}(m)} - \frac{g_{a_0 K^+ K^-}^2}{g_{f_0 K^+ K^-}^2} \frac{1}{D_{a_0}(m)} \right|^2 \frac{dm^2}{\pi},\end{aligned}\quad (20)$$

where we took into account the isotopic symmetry

$$g_{f_0 K^+ K^-} = g_{f_0 K^0 \bar{K}^0}, \quad g_{a_0 K^+ K^-} = -g_{a_0 K^0 \bar{K}^0}. \quad (21)$$

As is clear from Eq.(20) that in our model the constructive and destructive interferences take place in the  $\phi \rightarrow \gamma(a_0 + f_0) \rightarrow \gamma K^+ K^-$  and  $\phi \rightarrow \gamma(a_0 + f_0) \rightarrow \gamma K^0 \bar{K}^0$  decays respectively.

The width of the decay of the scalar meson into the  $ab$  state is

$$\begin{aligned}\Gamma(R \rightarrow ab; m) &= \frac{g_{Rab}^2}{16\pi} \frac{1}{m} \rho_{ab}(m), \\ \rho_{ab}(m) &= \sqrt{(1 - m_+^2/m^2)(1 - m_-^2/m^2)}, \quad m_\pm = m_a \pm m_b.\end{aligned}\quad (22)$$

The final particle identity in the  $\pi^0\pi^0$  case is taken into account in the determination of  $g_{f_0\pi^0\pi^0}$ .

We use the  $R$  resonance propagator

$$D_R(m) = m_R^2 - m^2 + Re \Pi_R(m_R) - \Pi_R(m); \quad \Pi_R(m) = \sum_{ab} \Pi_R^{ab}(m), \quad (23)$$

where the term  $Re \Pi_R(m_R) - \Pi_R(m)$  takes into account the finite -width corrections [2, 14]. Let  $m_a < m_b$ , then for  $m_+^2 < m^2$

$$\begin{aligned}\Pi_R^{ab}(m) &= \frac{g_{Rab}^2}{16\pi} \left[ L + \frac{1}{\pi} \rho(m) \ln \frac{\sqrt{m^2 - m_-^2} - \sqrt{m^2 - m_+^2}}{\sqrt{m^2 - m_-^2} + \sqrt{m^2 - m_+^2}} \right] + im \Gamma(R \rightarrow ab; m), \\ L &= \frac{m_+ m_-}{\pi m^2} \ln(m_b/m_a).\end{aligned}\quad (24)$$

For  $m_-^2 < m^2 < m_+^2$

$$\Pi_R^{ab}(m) = \frac{g_{Rab}^2}{16\pi} [L - |\rho_{ab}(m)| - \frac{2}{\pi} |\rho_{ab}(m)| \arctan \frac{\sqrt{m_+^2 - m^2}}{\sqrt{m^2 - m_-^2}}]. \quad (25)$$

For  $m^2 < m_-^2$

$$\Pi_R^{ab}(m) = \frac{g_{Rab}^2}{16\pi} [L - \rho_{ab}(m) \frac{1}{\pi} \ln \frac{(m_+^2 - m^2)^{1/2} - (m_-^2 - m^2)^{1/2}}{(m_+^2 - m^2)^{1/2} + (m_-^2 - m^2)^{1/2}}] \quad (26)$$

We considered [11] the finite-width corrections due to the  $\pi^0\eta$ ,  $K\bar{K}$ ,  $\pi^0\eta'$  channels for the  $a_0$  meson and the  $\pi\pi$ ,  $K\bar{K}$ ,  $\eta\eta$ ,  $\eta\eta'$ ,  $\eta'\eta'$  channels for the  $f_0$  meson. But only the  $K\bar{K}$  channels are of prime importance.

### 2.2.2 Numbers and other details in $q^2\bar{q}^2$ and $q\bar{q}$ models

The  $q^2\bar{q}^2$  model [7] gave an idea of the OZI superallowed couplings and decays, the widths of which have values of the order of 1 GeV if superallowed decay channels are not suppressed by the phase space volume. We [2, 14] established that all present data on the  $f_0(975)$  meson can be described by the OZI superallowed coupling constant

$$\begin{aligned} g_{f_0 K^+ K^-}^2 / 4\pi &= 2.3 \text{ GeV} \\ \Gamma(f_0 \rightarrow K\bar{K}; m = 1.1) &\simeq 0.45 \text{ GeV}. \end{aligned} \quad (27)$$

The  $q^2\bar{q}^2$  MIT model [7] predicts (see Eq.(9)) the suppression of  $g_{f_0\pi\pi}$  only but no value itself. We found [2, 14] that

$$R = g_{f_0 K^+ K^-}^2 / g_{f_0 \pi^+ \pi^-}^2 = 4 \div 10, \quad (28)$$

that corresponds the visible width of the  $f_0(975)$  meson

$$\Gamma_{f_0(975)} = 50 \div 25 \text{ MeV} \quad (29)$$

Besides that we showed [2, 6, 14, 15] that all present data on the  $a_0(980)$  meson can be described in the  $q^2\bar{q}^2$  model.

$$\begin{aligned} g_{a_0 K^+ K^-}^2 / 4\pi &= g_{f_0 K^+ K^-}^2 / 4\pi = 2.3 \text{ GeV}^2 \\ g_{a_0 \pi\eta} &= 0.85 g_{a_0 K^+ K^-}, \end{aligned} \quad (30)$$

despite of the fact that in this case  $\Gamma(a_0 \rightarrow \pi\eta; 0.980) = 275 \text{ MeV}$ . The point is that in the four-quark interpretation the narrow structure with the 50 MeV width, observed in the  $\pi^0\eta$  channel, arises against a background of the broad resonance due to the  $K\bar{K}$  channels opening whose thresholds are situated near the resonance mass [2, 6, 14, 15].

Using Eqs.(14)–(19),(22)–(27),(30) and  $R = 8$  in Eq.(28) we got [11]

$$\begin{aligned} BR(\phi \rightarrow \gamma a_0(980) \rightarrow \gamma \pi \eta) &= 2 \times 10^{-4} \\ BR(\phi \rightarrow \gamma f_0(975) \rightarrow \gamma \pi \pi) &= 2.5 \times 10^{-4} \end{aligned} \quad (31)$$

our results does not depend considerably on the range of  $R$ , see Eq.(28). For example, if  $R = 5$  then  $BR(\phi \rightarrow \gamma f_0(975) \rightarrow \gamma \pi \pi)$  increases by 20%.

It is notable that

$$\begin{aligned} BR(\phi \rightarrow \gamma a_0; 0.980) &= \Gamma(\phi \rightarrow \gamma a_0; 0.980)/\Gamma_\phi = 5.4 \times 10^{-4} \\ BR(\phi \rightarrow \gamma f_0; 0.975) &= \Gamma(\phi \rightarrow \gamma f_0; 0.975)/\Gamma_\phi = 5.4 \times 10^{-4}, \end{aligned} \quad (32)$$

$\Gamma(\phi \rightarrow \gamma R; m)$  is specified in Eq.(14).

It follows from the comparison of Eq.(32) with Eq.(31) that the narrow resonance approximation (Eq.(32)) in our case gives the values considerably more than values physically measured (Eq.(31)), that is the narrow resonance approximation is absolutely wrong in our case. It should be noted in this connection that using additionally Eq.(20) we got in the  $q^2\bar{q}^2$  model[11]

$$\begin{aligned} BR(\phi \rightarrow \gamma(a_0 + f_0) \rightarrow K^+ K^-) &= 2.6 \times 10^{-6}, \\ BR(\phi \rightarrow \gamma(a_0 + f_0) \rightarrow K^0 \bar{K}^0) &= 1.3 \times 10^{-8}. \end{aligned} \quad (33)$$

In the  $q\bar{q}$  model we are dealing with the OZI allowed couplings and decays, the widths of which have values of the order of 100 MeV if the OZI allowed decay channels are not suppressed by the phase space volume. In this case

$$g_{a_0 K^+ K^-}^2/4\pi = g_{f_0 K^+ K^-}^2/4\pi \simeq 0.2 \text{ GeV}^2, \quad (34)$$

and the  $K\bar{K}$  loop contributions (see Fig.1) are about an order of magnitude smaller than in Eqs.(31) and (33) [11]. It may appear that there is the considerable contradiction with the estimates in Eq.(2) and (8). But this is not the case. The matter is that from the point of view of dispersion relations the OZI veto is realized at the cost of a high cancellation of the  $K\bar{K}$ ,  $\bar{K}^* K + K^* \bar{K}$ ,  $K^* \bar{K}^*$  and so on intermediate-state contributions to the real part of the  $\phi \rightarrow \gamma a_0(f_0)$  decay amplitude. While each of the contributions by itself as a rule is much more than the sum.

In the four-quark case, generally, there is no need for such a cancellation. But, even though this would take place in the real part of the  $\phi \rightarrow \gamma a_0(f_0)$  decay amplitude only its imaginary one (in which the  $K^+ K^-$  intermediate state dominates [11]) would give the great violation of the OZI rule

$$BR(\phi \rightarrow \gamma a_0(980) \rightarrow \gamma \pi \eta) = BR(\phi \rightarrow \gamma f_0(975) \rightarrow \gamma \pi \pi) \simeq 5 \times 10^{-5} \quad (35)$$

Please compare with the estimations in Eqs.(2) and (8).

### 3 Conclusion

We treated [11] the  $a_0(980)$  and  $f_0(975)$  scalar mesons as compact systems in the same fashion that usual hadrons. It presumes that the hadron form-factor cut off has the range of order  $m_\rho \simeq 0,77 \text{ GeV}$ . In this case the our calculation [11] of the imaginary part of the amplitude is very solid for the low energy photon is essential in our consideration ( $\omega \leq 0,1 \text{ GeV}$ )[11]. Furthermore in the  $q^2\bar{q}^2$  case there is a good chance of the reasonable real part in our model because the dispersion integral for the amplitude is saturated with the low-energy contribution ( $E \leq 1,1 \text{ GeV}$ )[11].

But concurrent with the above there is a treatment of the  $a_0$  and  $f_0$  mesons as extended  $K\bar{K}$  molecular systems with the form-factor cut off having the range much less than  $m_\rho$  [8, 9]. In such a scenario

$$(g_{f_0 K^+ K^-}^2/4\pi) = (g_{a_0 K^+ K^-}^2/4\pi) = 0.6 \text{ GeV}^2, \quad (36)$$

that is intermediate between Eq.(34) and Eqs.(27),(30). The calculation [16] in the narrow resonance approximation gave

$$BR(\phi \rightarrow \gamma a_0; 980) \simeq BR(\phi \rightarrow \gamma f_0; 975) = 4 \times 10^{-5}. \quad (37)$$

But this approximation is wrong in the case under consideration, too.

Let us show it.

First factor is connected with the low energy photon. It is easily shown that the right slope of the resonance suppressed at least by  $(\omega/\omega_0)^3$ ,  $\omega_0 = m_\phi(1 - m_R^2/m_\phi^2)/2$ . For simplicity's sake use the elastic Breit-Wigner, then for the width of decay into  $\pi\eta$  or  $\pi\pi$   $\Gamma_0 = 50 \text{ MeV}$  we have

$$\frac{1}{\pi} \int_{m_R^2}^{m_\phi^2} \left(\frac{\omega}{\omega_0}\right)^3 \frac{\Gamma_0 m_R dm^2}{(m_R^2 - m^2)^2 + m_R^2 \Gamma_0^2} \simeq 0.11 \quad (38)$$

instead of 0.5 in the narrow resonance approximation. The inclusion of the inelasticity only lowers the quantity in Eq.(38) and makes the visible width  $\simeq 30 \text{ MeV}$ . The integral over the left slope of the resonance measurably depends on the way of taking into account the finite-width corrections. Let us use the generally accepted Breit-Wigner. If  $m > 2m_{K^+}, 2m_{K^0}$

$$BW(m) = \frac{\sqrt{\Gamma_0}}{m_R^2 - m^2 - i(\Gamma_0 + \Gamma_{KK})m_R} \\ \Gamma_{KK} = \frac{g_{RK^+ K^-}^2}{16\pi} (\sqrt{1 - 4m_{K^+}^2/m^2} + \sqrt{1 - 4m_{K^0}^2/m^2}) \frac{1}{m_R}. \quad (39)$$

When  $2m_{K^+}, 2m_{K^0} > m$

$$BW(m) = \frac{\sqrt{\Gamma_0}}{m_R^2 - m^2 + \frac{g_{RK^+ K^-}^2}{16\pi} (\sqrt{4m_{K^+}^2/m^2 - 1} + \sqrt{4m_{K^0}^2/m^2 - 1}) - i\Gamma_0 m_R}. \quad (40)$$



For  $\Gamma_0 = 50$  and Eq.(36)

$$\frac{1}{\pi} \int_{(m_a+m_b)^2}^{m_R^2} \frac{\Gamma_0 m_R dm^2}{[m_R^2 - m^2 + \frac{g_{R K^+ K^-}^2}{16\pi} (\sqrt{4m_{K^+}^2/m^2 - 1} + \sqrt{4m_{K^0}^2/m^2 - 1})^2 + \Gamma_0^2 m_R^2]} \simeq 0.15 \quad (41)$$

instead of 0.5 in the narrow resonance approximation. ( $m_a + m_b = 2m_\pi$  or  $m_\pi + \eta$ .)

The intensities of the decays  $\phi \rightarrow \gamma K \bar{K}$  are negligible [11, 17] due to the low energy photon. So, it follows from Eqs.(38) and (41) that the prediction in the  $K \bar{K}$  molecule case [16](see Eq.(37)) must be reduced by a factor 4. Certainly, it is the matter of the authors to find out the physically measured values of the decay intensities. I only call attention to this problem. I dare also to recommend our formulae, Eqs.(22)-(26), to take into account the finite-widths of the resonances. The point is that the generally accepted Breit-Wigner, Eqs.(39) and (40), is inadequate in the case of the strong coupling with the  $K \bar{K}$  channels. Among other things, the position of the peak in  $|BW(m)|^2$  does not coincide with the resonance mass  $m_R$ .

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