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Scalar Mesons and $\phi \to \pi\pi\gamma$ at DAΦNE

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Abstract

The possible detection of the reaction $\phi \to f_0(975)\gamma \to \pi^+\pi^-\gamma$ at Daϕne is discussed together with the initial and final state background radiation. The size of the effect is shown to depend crucially on the relative sign of the $e^+e^- \to \rho \to \pi^+\pi^-\gamma$ and $e^+e^- \to \phi \to f_0\gamma \to \pi^+\pi^-\gamma$ amplitudes. The $\phi \to \pi^0\pi^0\gamma$ channel is also discussed briefly.

Projected high-luminosity, low-energy e^+e^- machines, such as the Daϕne Φ-Factory, will allow for the detection and measurement of rare decay modes of the well-known, low-lying vector mesons. The $\phi \to \pi^+\pi^-\gamma$ decay, whose branching ratio is known to be smaller than $7 \cdot 10^{-3}$ [1], will probably be studied in the near future due to the (relatively) clean signature of a charged pion pair with a rather energetic photon. The possibility of studing the less clean $\phi \to \pi^0\pi^0\gamma$ decay is open too. In both cases, not only the properties of the ϕ-meson will be explored but also those of the final pion-pair and, in particular, their resonant states such as the controversial f_0-scalar meson at 975 MeV [2]. Recent discussions on the controversial nature of scalar mesons can be seen in ref. [3].

The main purpose of this note is to discuss the possible detection of such an f_0-signal under the considerably large background affecting the charged decay channel. Much of the considerations discussed here have been reported in previous publications [4]. The

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main two sources for the latter are expected to be initial-state bremsstrahlung and ρ-formation followed by its (off-shell) decay into $\pi^+\pi^-\gamma$. In the first case, the pion pair is in a negative charge-conjugation state, while the opposite is true for the second as well as for the pion pair in any genuine $\phi \to \pi^+\pi^-\gamma$ radiative decay. Interference effects will be important between these two latter (C=+) amplitudes, but those with the first one (C=–) will disappear when integrating over pion angles disregarding their charges [5]. Background for the neutral decay channel $\phi \to \pi^0\pi^0\gamma$ is expected to be much smaller and will be briefly discussed at the end of this note.

The most obvious and important background for $\phi \to \pi^+\pi^-\gamma$ comes from initial-state hard photon radiation. One obtains [6]

$$d\sigma/dE|_{in} = 2\sigma_0(s(1-x))H(x,s,\theta_{min})/\sqrt{s} \tag{1}$$

where σ_0 stands for the non-radiative $e^+e^- \to \rho \to \pi^+\pi^-$ cross-section

$$\sigma_0(s) = \frac{\pi}{3}\frac{\alpha^2}{s}|F_\rho(s)|^2(1-\xi)^{3/2}, \tag{2}$$

x is related to the photon energy E through $x \equiv 2E/\sqrt{s}$, $\xi \equiv 4m_\pi^2/s$, and

$$H(x,s,\theta_{min}) = \frac{\alpha}{\pi}\frac{2(1-x)+x^2}{x}\left[ln\left(\frac{1+\beta cos\theta_{min}}{1-\beta cos\theta_{min}}\right) - \frac{1}{\gamma^2}\frac{cos\theta_{min}}{1-\beta^2cos^2\theta_{min}}\right]. \tag{3}$$

In eq. (3), θ_{min} is the minimal angle (with respect to the beam direction) allowed for the photon to be detected. The radiator $H(x,s,\theta_{min})$ is obtained from the angular distribution $d\sigma/dcos\theta_\gamma \simeq sin^2\theta_\gamma/(1-\beta^2cos^2\theta_\gamma)^2$ and reduces to the well known integrated radiator

$$H(x,s) = \frac{\alpha}{\pi}\left[\frac{2(1-x)+x^2}{x}\left(ln\frac{s}{m_e^2} - 1\right)\right], \tag{4}$$

for $\theta_{min} = 0$. Finally $F_\rho(s)$ in eq. (2) stands for the ρ-dominated pion form-factor [7]

$$F_\rho(s) = \frac{M_\rho^2}{M_\rho^2 - s - i\sqrt{s}\Gamma_\rho}. \tag{5}$$

The resulting differential cross-section for initial-state radiation is shown in Fig.1 for $\theta_{min} = 0$ (solid line) and for $\theta_{min} = 10^o$ (dotdashed line).

The second source of background comes from off-shell ρ-formation followed by radiative decay into a C=+ pion pair. The corresponding, gauge- invariant amplitude is given by

$$A(\rho \to \pi^+\pi^-\gamma) = 2\sqrt{2}eg\left[\frac{\epsilon p_+}{qp_+}\ \epsilon^*\left(p_- \ - \frac{1}{2}q^*\right)\ + \frac{\epsilon p_-}{qp_-}\ \epsilon^*\left(p_+ \ - \frac{1}{2}q^*\right)\ + \epsilon\epsilon^*\right] \tag{6}$$

where g=4.2 comes from the total ρ width of 149 MeV, p_+, p_- and q are the pion and photon four-momenta, and ϵ, ϵ^* are the polarizations of the photon and the ρ. The correctness of this amplitude can easily be tested by comparing with the measured decay rate of on-shell

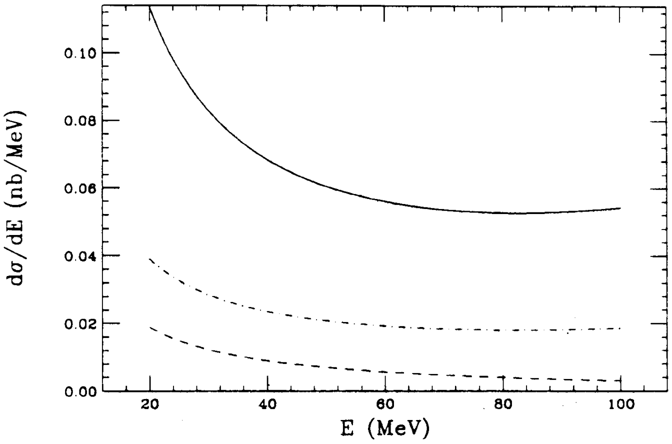

Figure 1: Differential cross-sections as a function of the photon energy E for initial state, hard photon radiation, $e^+e^- \to \rho^0\gamma \to \pi^+\pi^-\gamma$ (solid line for $\theta_{min} = 0$ and dotdashed line for $\theta_{min} = 10^o$) and for off-shell ρ-formation and decay $e^+e^- \to \rho \to \pi^+\pi^-\gamma$ (dashed line).

ρ mesons into a pion pair and a photon of energy E larger than 50 MeV. One immediately obtains a partial width of 1.62 MeV in good agreement with the observed value of 1.48±0.24 MeV ([1, 8], see also [9]). The effect of the above amplitude around the ϕ-peak requires the introduction of the ρ dominated form factor $F_\rho(s)$. One easily obtains the following differential cross-section

$$d\sigma/dE|_\rho = 2\sigma_0(s)\ F(x,s)/\sqrt{s} \tag{7}$$

with

$$\begin{aligned} F(x,s) = \frac{\alpha}{\pi}\frac{2}{(1-\xi)^{3/2}}\Big[\Big(x-(1-\xi)\frac{1-x}{x}\Big)\sqrt{1-\frac{\xi}{1-x}} \\ +(1-\xi)(1-x-\xi/2)\frac{1}{x}\ ln\Big|\frac{1+\sqrt{1-\frac{\xi}{1-x}}}{1-\sqrt{1-\frac{\xi}{1-x}}}\Big|\Big] \end{aligned} \tag{8}$$

where small corrections coming from higher order terms in x have been neglected. For s on the ϕ-peak, the values of the above expression are shown (dashed line) in Fig.1. Tipically, they are one order of magnitude below the previously considered background coming from the initial-state radiation with $\theta_{min} = 0$.

We now turn to the ϕ signal as produced through the $\phi \to f_0\gamma \to \pi^+\pi^-\gamma$ decay chain.

Amplitudes and couplings for each step are defined according to

$$A(\phi \to f_0\gamma) = eG_s[(\epsilon^*\epsilon)\ (q^*q)\ - (\epsilon^*q)\ (\epsilon q^*)] \equiv eG_s\{a\},$$
$$A(f_0 \to \pi^+\pi^-) = g_s. \tag{9}$$

where now ϵ^* and q^* stand for the ϕ polarization and four-momentum. These amplitudes lead to the following decay rates

$$\Gamma(\phi \to f_0\gamma) = \frac{1}{12\pi}G_s^2 e^2 \mid \vec{q}_\gamma \mid^3$$
$$\Gamma(f_0 \to \pi^+\pi^-) = \frac{2}{3}\Gamma(f_0 \to \pi\pi) = \frac{1}{8\pi}g_s^2 \frac{\mid \vec{p}_+ \mid}{m_f^2} \tag{10}$$

¿From these expressions and the experimental values quoted in [1] one obtains the modulus of the product of the relevant coupling constants in terms of the unknown branching ratio BR($\phi \to f_0\gamma$)

$$\mid G_s g_s \mid = (144 \pm 15)(BR(\phi \to f_0\gamma))^{1/2} \tag{11}$$

The amplitude for the ϕ decay into $f_0\gamma \to \pi^+\pi^-\gamma$ then follows

$$A(\phi \to \pi^+\pi^-\gamma) = eg_sG_sP_f(s')\{a\} \tag{12}$$

with

$$P_f(s') = \frac{1}{m_f^2 - s' - im_f\Gamma_f} \tag{13}$$

$\{a\}$ as in eq.(9), $s' = s(1-x)$ and $s = q*^2 = M_\phi^2$ on the ϕ-peak.

As previously stated, the off-shell ρ-amplitude in an e^+e^- experiment interferes with the just derived (near on-shell) one for the ϕ. The total amplitude (with C= +) may be written as

$$A^+(\gamma^* \to \pi^+\pi^-\gamma) = \frac{e}{f_\rho}F_\rho(s)A(\rho \to \pi^+\pi^-\gamma) + \frac{e}{f_\phi}F_\phi(s)A(\phi \to \pi^+\pi^-\gamma) \tag{14}$$

where $f_\phi = -3f_\rho/\sqrt{2} = -3g$, $F_{\rho,\phi}(s)$ are form factors and $A(\rho, \phi \to \pi^+\pi^-\gamma)$ are given in eqs.(6) and (12).

Integrating over the pion energies the C-even amplitude (14) leads to the differential cross-section

$$d\sigma/dE|_{\rho+\phi} = d\sigma/dE|_\rho + d\sigma/dE|_\phi + d\sigma/dE|_{int} \tag{15}$$

where the ρ-term has been given in eq.(7), and the ϕ- and interference-terms (the ϕ-signal) are given by

$$d\sigma/dE|_\phi = \frac{2\alpha^3}{3s}\left(\frac{g_sG_s}{3g}\right)^2 |F_\phi(s)P_f(s(1-x))|^2\ E^3\sqrt{1-\frac{\xi}{1-x}} \tag{16}$$

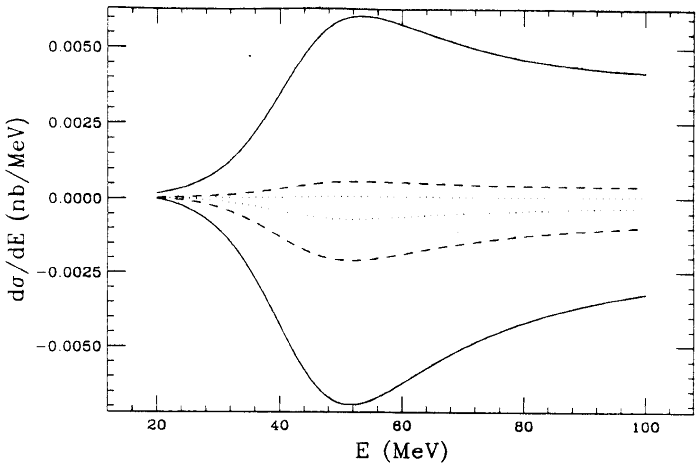

Figure 2: Contributions to the differential cross-section for $e^+e^- \to \pi^+\pi^-\gamma$ on the ϕ-peak as a function of the photon energy E coming from the ϕ-terms in eq.(16) (upper curves) and the interference term in eq.(17) (lower curves). $G_s g_s$ is taken from the positive root in eq.(10) for BR($\phi \to f_0\gamma$) = 10^{-4}, 10^{-5} and 10^{-6} (solid, dashed and dotted lines, respectively). $\theta_{min} = 10^o$.

$$d\sigma/dE|_{int} = \frac{-4\alpha^3 g_s G_s}{9sg}\ Re\left(F_\rho^*(s)F_\phi(s)P_f(s')\right)\ E\left(\sqrt{1-\frac{\xi}{1-x}}+\frac{\xi}{2}\ln|\frac{1-\sqrt{1-\frac{\xi}{1-x}}}{1+\sqrt{1-\frac{\xi}{1-x}}}|\right) \tag{17}$$

The corresponding expressions before integration over the angle θ_γ and pion energies, can be found in refs. [4, 10]. Some detailed studies of the angular distributions are given in ref. [11]. Eqs.(16) and (17) contain the unknown product g_sG_s, i.e., relevant information on the f_0-meson and, more specifically, on the $BR(\phi \to f_0\gamma)$ as indicated in eq.(11). Theoretical estimates of this branching ratio range from 10^{-3} to 10^{-6}. Recent analyses by Brown and Close [2] and by Close, Isgur and Kumano [12] indicate $BR(\phi \to f_0\gamma) = 1.36 \cdot 10^{-4}$ (if a calculation along the lines of [13] is performed), 10^{-4} (if the f_0 is a $q\bar{q}q\bar{q}$ system) or 10^{-5} (if the f_0 were pure $s\bar{s}$). Since the last possibility is not entirely reasonable for an f_0-meson decaying mainly in two pions, smaller values for the branching ratio should also be considered. For all these reasons we present our results in Figs.2, 3 and 4 for three values of $BR(\phi \to f_0\gamma)$ in eq.(11), namely, 10^{-4} (solid lines), 10^{-5} (dashed lines) and 10^{-6} (dotted lines). Fig.2 shows the photonic spectra coming from the terms quoted in eq.(16) (positive curves) and eq.(17) (negative curves).

This corresponds to choosing the positive sign for g_sG_s in eq.(11). In this case the

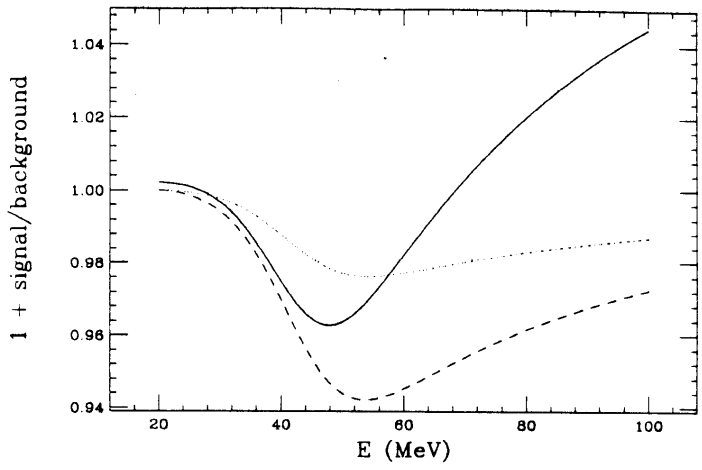

Figure 3: Ratio between the $\phi \to f_0\gamma \to \pi^+\pi^-\gamma$ signal and the background as a function of the photon energy E. Solid, dashed and dotted curves correspond to $BR(\phi \to f_0\gamma) = 10^{-4}$, 10^{-5} and 10^{-6} in eq.(10) with the positive root (destructive interference). $\theta_{min} = 10^o$

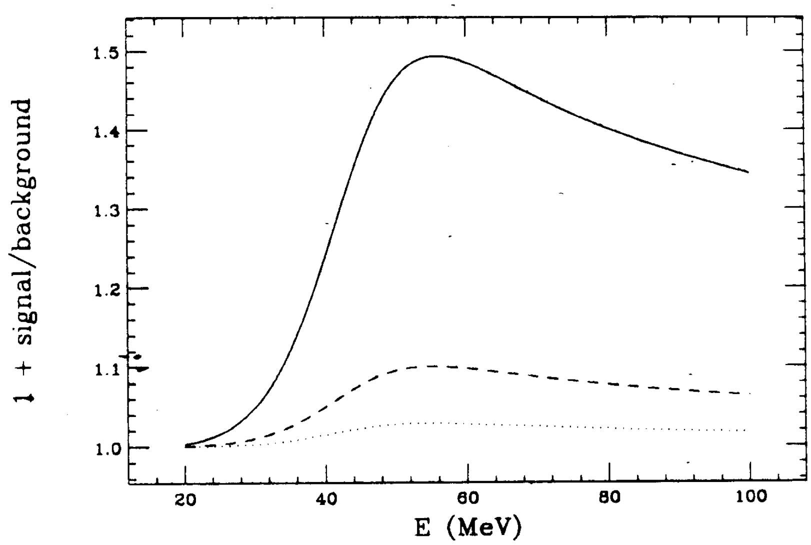

Figure 4: Same as fig. 3 but with the choice of the positive root (constructive interference).

effects of the ϕ-decay tend to cancel when interfering with the off-shell $\rho \to \pi^+\pi^-\gamma$- decay. Fig.3, where the values for $1 + d\sigma(\text{signal})/d\sigma(\text{background})$ have been plotted, shows the remaining effects of the order of a few percent. Choosing the negative sign for $g_s G_s$ reverses the sign of the interference term shown in the lower half of Fig.2, thus enhancing the effect. This is clearly seen in Fig.4 where now the ratio between the signal and the background can reach a 50%. In all these curves we have assumed $\theta_{min} = 10^o$, which seems reasonable for a ϕ-factory [11]. If we had integrated over the whole radial angle, the signal over background ratio would have reduced by a factor of 2.5.

In a recent paper [10], Lucio and Napsuciale have reconsidered the $\rho \to \pi^+\pi^-\gamma$ decay under a specific model for the $\phi \to f_0\gamma$ transition, namely a model consisting of a loop of charged kaons in the $\phi \to f_0\gamma$ transition. The analysis of ref. [4] for the absolute magnitude of the background is confirmed, as well as for the ratio of f_0-signal to background in the case of constructive interference. The details of the model [10], however, modify our findings in the more delicate case of destructive interference, showing the importance of the loop effects.

Let's now briefly discuss the $\rho \to \pi^0\pi^0\gamma$ decay channel. The f_0-signal is just one half of the preceding one, but the background is much smaller. Indeed, none of the sources of background for the charged case is now present. The only source (which could be neglected in the charged case) comes from the effects of charged kaon loops radiating a photon and converting into a neutral pion pair. This process has been analysed in detail in ref. [14], where the result $d\Gamma(\phi \to \pi^0\pi^0\gamma)/dE = 3 \cdot 10^{-7}$ was obtained for photon energies $E = 44$ MeV (see Fig. 14 in [14]). The amplitude for this E, which is the energy of a photon recoiling against an on-mass-shell f_0 in $\phi \to f_0\gamma$ decays, is purely real. Conversely the amplitude for the $\phi \to f_0\gamma \to \pi^0\pi^0\gamma$ signal is expected to be mainly imaginary around the f_0 peak with $d\Gamma(\phi \to \pi^0\pi^0\gamma)/dE = 15 \cdot 10^{-7},\ 15 \cdot 10^{-8},\ 15 \cdot 10^{-9}$ for $\text{BR}(\phi \to f_0\gamma) = 10^{-4}$, 10^{-5} and 10^{-6}.

In summary, the possibilities of detecting and analysing the decay chain $\phi \to f_0\gamma \to \pi^+\pi^-\gamma$ depend rather crucially on the relative sign between this amplitude and the one describing off-shell $\rho \to \pi^+\pi^-\gamma$ decay. The latter is only a minor part of the whole background dominated by non-interfering radiation from the e^+e^- initial state. In spite of this, a rather clean effect is predicted if the ϕ- and ρ-amplitudes are on-phase, while the effect tends to disappear in the opposite case, which is a priori equally probable. On the other hand the type of background and the ratio of signal to background are completely different for the $\phi \to f_0\gamma \to \pi^0\pi^0\gamma$ case.

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