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Small Angle Radiative Bhabha Scattering in the No-recoil Approximation

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Abstract

A simple analytical formula is derived for the total rate of small angle radiative Bhabha scattering, using the no-recoil approximation. This expression illustrates how radiative processes soften the forward angle singularity so that the cross-section for all electrons which have radiated an energy larger that a given amount ΔE, is only logarithmically divergent as $m_e \to 0$.

In this note we present a simple expression which can be of use to approximate the differential and integrated cross-section for the process

$$e^-(p_1) + e^+(p_2) \to e^-(p_3) + e^+(p_4) + \gamma(k) \tag{1}$$

in the forward region. This process, the so-called radiative Bhabha scattering, is used at electron-positron colliders as a luminosity monitor [1] and the relevant cross-section is well established in the literature [2, 3, 4, 5, 6]. Our aim in this note is to derive an expression which is a good approximation to the exact results, and from which one can easily estimate the expected total rates.

As discussed in the previous section in this handbook, process (1) constitutes an important background to $e^+e^- \to (\gamma\gamma) \to \pi\pi\ e^+e^-$or $e^+e^- \to \pi^o/\eta\ e^+e^-$[7]. In order to disentangle this process from the hadronic background due to the annihilation channel, it is being considered to provide the planned detectors with tagging facilities in the forward region.

In the forward region, only small values of the momentum transfer are considered and the no-recoil approximation can be applied. In this approximation, and to lowest order in

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α, the cross-section for process (1), can be written as

$$\frac{d\sigma}{dt} = \frac{d\sigma_0}{dt} d^3\bar{n}_k \tag{2}$$

where the elastic electron-positron cross-section is given by

$$\frac{d\sigma_0}{dt}(e^+e^- \to e^+e^-) = \frac{4\pi\alpha^2}{s^2}\left[\frac{s^2+u^2}{2t^2} + \frac{u^2+t^2}{2s^2} + \frac{u^2}{st}\right] \tag{3}$$

and $d^3\bar{n}_k$ is the probability that photons with momentum k are emitted. The relativistic invariants for the elastic scattering process, are defined as

$$s = (p_1+p_2)^2, \quad t = (p_2-p_4)^2, \quad u = (p_2-p_3)^2$$

and the probability $d^3\bar{n}_k$ is given by[8, 9]

$$d^3\bar{n}_k = \frac{d^3\vec{k}}{2k}\left[-j^\mu(k) j^\dagger_\mu(k)\right] \tag{4}$$

with

$$j^\mu(k) = \frac{ie}{(2\pi)^{3/2}} \sum_{i=1,4} \epsilon_i \frac{p_{i\mu}}{p_i \cdot k} \tag{5}$$

for radiation emitted by particles of 4-momentum p_i. The factor $\epsilon_i = \pm 1$ according to the charge of the emitting particle or antiparticle : the (-) sign corresponds to photon emission from an electron (positron) in the final(initial) state, and the (+) sign to emission from an electron (positron) in the initial (final) state. Upon integration over the photon directions, one obtains the energy spectrum as

$$d\bar{n}(k) = \int_\Omega d^3\bar{n}_k = \beta(E,\theta)\frac{dk}{k} \tag{6}$$

with E the beam energy and θ the electron's scattering angle. This photon spectrum correctly describes collinear and almost collinear photons, but only in the soft photon approximation. To include hard collinear emission, one can make the substitution

$$d\bar{n}(k) \longrightarrow d\bar{n}_k^{hard} = \beta(E,\theta)\frac{dk}{k}\frac{E^2+(E-k)^2}{2} \tag{7}$$

The function $\beta(E,\theta)$ is obtained performing the angular integration on the photon direction

$$\beta(E,\theta) = -\frac{\alpha}{(2\pi)^2} \sum_{i,j=1,4} \epsilon_i\epsilon_j (p_i \cdot p_j) \int \frac{d\Omega_n}{(p_i \cdot n)(p_j \cdot n)} \tag{8}$$

with the null 4-vector, $n^2 = 1 - \hat{n}^2 = 0$. Since $\beta(E,\theta)$ is a relativistic invariant[8], it can be conveniently expressed in terms of the Mandelstam variables, i.e.

$$\beta(s,t,u) = \frac{2\alpha}{\pi}\left(I_{12} + I_{13} - I_{14} - 2\right) \tag{9}$$

with

$$I_{ij} = 2(p_i \cdot p_j) \int_0^1 \frac{dy}{m_e^2 + 2y(1-y)[(p_i \cdot p_j) - m_e^2]}$$

In the large energy, fixed t limit, the terms with s and u dependence cancel out[10], since

$$I_{12} = I(s) \longrightarrow 2\log\frac{s}{m_e^2}, \quad I_{14} = I(u) \longrightarrow 2\log\frac{-u}{m_e^2} \approx 2\log\frac{s}{m_e^2}, \; s, u \gg m_e^2 \tag{10}$$

and the radiative spectrum β takes the simple form

$$\beta(t) = \lim_{large\ s, small\ t} \beta(s,t,u) = \frac{2\alpha}{\pi}\left[(2m_e^2 - t)\int_0^1 \frac{dy}{m_e^2 - ty(1-y)} - 2\right] \tag{11}$$

i.e.

$$\beta(t) = \frac{4\alpha}{\pi}\left[\frac{2m_e^2 - t}{\sqrt{-t}\sqrt{4m_e^2 - t}} log \frac{\sqrt{4m_e^2 - t} + \sqrt{-t}}{\sqrt{4m_e^2 - t} - \sqrt{-t}} - 1\right] \tag{12}$$

The function $\beta(t)$ is plotted in fig.1 together with two curves which approximate its behaviour for small and large values of $-t/m_e^2$. One notices that, as t goes to zero, $\beta(t)$ also goes to zero, i.e.

$$\lim_{|t|<<m_e^2} \beta(t) = \frac{2\alpha}{\pi}\left[-\frac{2}{3}\frac{t}{m_e^2} - \frac{1}{10}\left(\frac{t}{m_e^2}\right)^2\right] \tag{13}$$

In order to obtain the total rate, one integrates eq.(2) over the momentum transfer t, between the two limits obtained from the exact kinematics of process (1) for small angle scattering. In the forward region the maximum value for the momentum transfer $-t$ is approximated as

$$-t_1 = E^2\theta_{max}^2$$

where θ_{max} is the maximum angular opening of the electron detector in the forward region. For the experimental set up at DAΦNE, the largest scattering angle allowed by the small angle tagging system, SAT, is expected to be $\approx 7\ mrad$.

For the smallest momentum transfer (see the Appendix) one obtains

$$-t_0 = \frac{m_e^6 x^2}{s^2(1-x)^2} \quad with \quad x = \frac{k}{E}$$

The total cross-section in the forward region, with a radiative energy loss of at least ΔE, is now given by

$$\sigma_{rad} = \int_{\Delta E}^{E} dk \int_{t_0}^{t_1} \frac{d\sigma}{dkdt} dt \tag{14}$$

with

$$\frac{d\sigma}{dkdt} \approx 4\pi\alpha^2 \frac{dt}{t^2}\beta(t)\frac{dk}{k}\frac{E^2 + (E-k)^2}{2E^2} \tag{15}$$

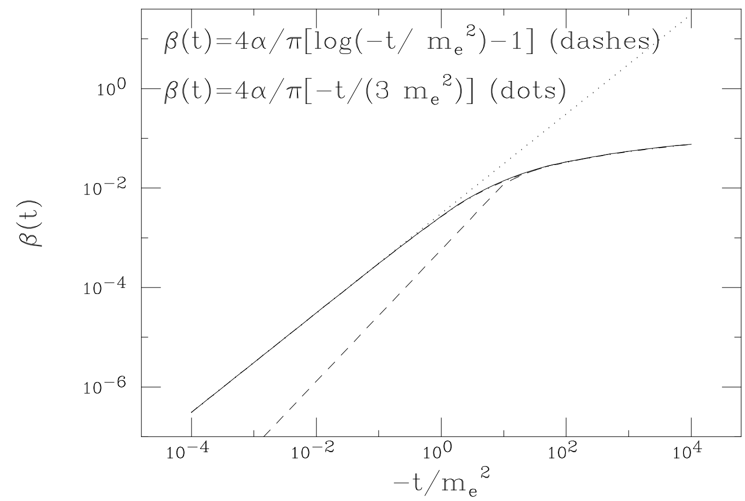

Figure 1: The radiative spectrum $\beta(t)$ is plotted as a full line for the exact expression from eq.(12), and for the two approximations respectively valid at small (dots) and large -t/m_e^2 (dashes).

While the exact integration of eqs.(14) and (15) contains non leading t/m_e^2 terms, the main contribution to the integral comes from the small t region, where the differential cross-section for process (1) takes the form

$$\frac{d\sigma}{dkdt} = \frac{4\pi\alpha^2}{t^2}\frac{4}{3}\frac{\alpha}{\pi}\left(\frac{-t}{m_e^2}\right)\frac{dk}{k}\frac{E^2+(E-k)^2}{2E^2} \qquad |t| \ll m_e^2 \tag{16}$$

We have used the small t, large s limit for the elastic electron-positron cross-section $\frac{d\sigma_0}{dt}$ and have neglected the second (higher order term) in the expression for $\beta(t)$. This expression shows how the radiative spectrum regularizes the t^{-2} singularity. We notice that this phenomenon is a coherence effect, obtained from an exact cancellation between all the terms of eq.(9), including the constant ones. Because of this cancellation, the singular, small t behaviour is softened and the cross-section exhibits only a logarithmic singularity.

A particularly simple expression, for the total radiative Bhabha cross section, can be obtained in the soft photon approximation, i.e.

$$\sigma_{RB}^{soft}(E) = \int_{\Delta E}^{E} dk \int_{t_0}^{t_1} dt \frac{d\sigma}{dtdk} = \frac{16}{3}\alpha r_0^2 \int_{\Delta E}^{E} \frac{dk}{k} \int_{t_0}^{t_1} \frac{dt}{t} \tag{17}$$

with $r_0 = \alpha/m_e = 2.8\ fm$. Using the kinematic limits, with $t_1 = -m_e^2$ and t_0 as described, the integrated cross-section takes the form

$$\sigma_{RB}^{soft} \approx \frac{32\alpha}{3} r_0^2 \int_{x_0}^{1} \frac{dx}{x} log\left(\frac{s}{m_e^2}\frac{1-x}{x}\right) \tag{18}$$

with $x_0 = \frac{\Delta E}{E}$. Retaining only the leading logarithmic contribution, one then gets

$$\sigma_{RB}^{LLO} \approx \frac{64\alpha}{3} r_0^2 log(\frac{E}{\Delta E}) log(2\gamma) = \frac{16}{3}\pi r_0^2 \beta_e log(\frac{E}{\Delta E}) \tag{19}$$

with

$$\beta_e = \frac{4\alpha}{\pi} log(2\gamma)$$

and $\gamma = E/m_e$. The LLO expression just obtained coincides with the one from ref. [2, 3], in the same limit. For radiative Bhabha scattering, from all four charged particles, these authors obtain the differential expression

$$d\sigma = 8\alpha r_0^2 \frac{dx}{x}\left[\frac{4}{3}(1-x) + x^2\right](\mathbf{L} - \frac{1}{2}), \quad \mathbf{L} = log\left(\frac{s}{m_e^2}\frac{1-x}{x}\right)$$

whose integrated rate coincides with eq.(18) in the small x limit. The total rate of electrons hitting a small angle detector along one of the beam directions, is obtained (i) by halving the above cross-sections, since only photons from either electrons or positrons are detected in single tagging [11] and (ii) by multiplying with the expected luminosity.

We find that the expected rate of electrons which have lost an energy larger that 70 MeV is of the order of 30 MHz, corresponding to 0.03 electrons per nanosecond, an acceptable rate, which should not interfere with the tagging of electrons from the radiative $\gamma\gamma$ process

$$e^+e^- \to e^+e^- \ hadrons$$

of interest at DAΦNE.

A more precise and exact calculation for both the integrated and the differential cross sections can be found in the next section of this Handbook.

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Appendix : Evaluation of t_0

Let us consider the detected photon to be emitted from an electron moving along the positive z- direction, either incoming or outgoing, with momentum $p_{1\mu}$ or $p_{3\mu}$ respectively. The momentum transfer between the two positrons is then

$$t = (p_2 - p_4)^2 = 2m_e^2 - 2EE_4 + 2|\vec{p}|\ |\vec{p}_4| cos\theta_{24}$$

The energy E_4 of the outgoing positron can be expressed in terms of the total c.m. energy squared, $s = 4E^2$ and the invariant mass of the electron-photon system $s_{e\gamma} = (p_3 + k)^2$ as

$$E_4 = \frac{s - s_{e\gamma} + m_e^2}{2\sqrt{s}}$$

Approximating

$$pp_4 \approx EE_4 \left(1 - \frac{m_e^2}{2E^2} - \frac{m_e^2}{2E_4^2}\right)$$

and using

$$s_{e\gamma} \approx m_e^2(1 + \frac{k}{E_3}) \qquad for \quad \cos\theta_{e\gamma} \approx 0,$$

in the limit $\theta_{24} = 0$ one obtains

$$-t_0 \approx m_e^2 \frac{(E - E_4)^2}{EE_4} = m_e^6 \frac{k^2}{4sEE_4E_3^2} \qquad with \quad E_3 = 2E - E_4 - k \approx E - k$$

and thus

$$-t_0 = \frac{m_e^6 x^2}{s^2(1-x)^2} \qquad x = \frac{k}{E}$$

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