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Theoretical Predictions for Pion Polarizabilities

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Abstract

The polarizabilities of the pion have been predicted in several different theoretical frameworks. The status of these is reviewed.

1 Introduction

The concept of polarizability first appeared in the realm of particle physics twenty years ago [TE73, TE74, FR75, BT76] as a quantity which characterizes an elementary particle, like its charge radius, magnetic moment, etc. In *classical* physics the polarizability of a medium (or a composite system in general) is a well known concept related to the response of the system to the presence of an external electromagnetic field, representing a measure of how easy it is to polarize it. The translation of this quantity into *quantum* physics involves Compton scattering on the corresponding target. For an electrically charged system, scattering at threshold is determined by the charge of the system. This is the Thompson limit. The polarizabilities give the corrections to Thompson scattering —corrections to the next order in the energy of the photons. For neutral targets, the corrections parameterized by the polarizabilities are the leading answer.

Since the introduction of this new quantity, considerable work has been performed both theoretically and experimentally, largely on the nucleon and pion polarizabilities. In this article we focus attention on the theoretical predictions for the pion polarizabilities.

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All the numerical results about polarizabilities given in this paper are expressed in units of $10^{-43}\text{cm}^3 = 10^{-4}\text{fm}^3$, which are not quoted.

2 Definition of pion polarizabilities

Let us consider Compton scattering on a pion

$$\gamma(q_1)\,\pi(p_1) \longrightarrow \gamma(q_2)\,\pi(p_2) \tag{1}$$

where q_i are the 4–momenta of the photons and p_i those for the pions. The amplitude for this process can then be expanded in powers of the energies of the photons near threshold:

$$A(\gamma\pi \to \gamma\pi) \;=\; \left[-\frac{\alpha}{m_\pi}\delta_{\pi\pi^\pm} + \alpha_\pi\,\omega_1\omega_2\right]\hat{\epsilon}_1\cdot\hat{\epsilon}_2^* + \beta_\pi\,\omega_1\omega_2\,(\hat{\epsilon}_1\times\hat{q}_1)\cdot(\hat{\epsilon}_2^*\times\hat{q}_2) + \ldots \tag{2}$$

where $q_i = \omega_i\,(1,\hat{q}_i)$ for $i = 1,2$ and $\hat{\epsilon}_i$ is the polarization vector of the photon with momentum q_i. $\delta_{\pi\pi^\pm} = 1$ for charged pions and is zero for neutral ones. Note that some authors define α_π and β_π with a factor 4π, so care must be taken in comparing results and predictions.

In Eq. (2) α_π and β_π are the electric and magnetic polarizabilities, respectively, and α is the fine structure constant. Let us note:

1.- At zeroth order in the energies of the photons only the *point-like* structure of the target survives and it is accordingly zero for a neutral pion.

2.- There is no linear term in the energy of photons. This is because we are dealing with a spinless target. In the case of the nucleon, for example, there is a non–vanishing linear term.

3.- The polarizabilities are not specified by symmetry arguments alone.

As can be seen from their definition, the polarizabilities carry information about the electromagnetic structure of the target in the Compton process.

3 General remarks on pion polarizabilities

Before detailing various theoretical predictions, there are results we know about the pion polarizabilities on quite general grounds:

1/ Chiral dynamics demands that in the exact chiral limit the relation

$$\alpha_\pi + \beta_\pi = 0 \tag{3}$$

must be obeyed [DH89].

2/ A dispersion relation for the forward scattering amplitude gives [TE74, HO90]

$$\alpha_\pi + \beta_\pi = \frac{1}{2\pi^2}\int_0^\infty d\omega \ \frac{\sigma_{tot}(\omega)}{\omega^2} \quad , \tag{4}$$

where σ_{tot} is the total photoproduction cross section on pions; this, of course, implies

$$\alpha_\pi + \beta_\pi \geq 0 \quad . \tag{5}$$

3/ Crossing symmetry relates the polarizabilities to the helicity amplitudes of the process $\gamma\gamma \to \pi\pi$ at the crossed–channel threshold. If we call [2] M_{++} the helicity 0 amplitude and M_{+-} the helicity 2 amplitude [KS86, BG94] for $\gamma\gamma \to \pi\pi$,

$$\begin{aligned}(\alpha_\pi \pm \beta_\pi)^C &= \frac{\alpha}{m_\pi}\left[M^C_{+\mp} - M_{BORN}\right]\Big|_{s=0,t=m_\pi^2} \\ (\alpha_\pi \pm \beta_\pi)^N &= \frac{\alpha}{m_\pi} M^N_{+\mp}\,|_{s=0,t=m_\pi^2} \quad ,\end{aligned} \tag{6}$$

where the superscript C or N denotes *charged* or *neutral* pions, respectively. In the charged case, the Born amplitude must be subtracted first to obtain the corresponding combination of electric and magnetic polarizabilities. The relations, Eq. (6), allow us to see that the combination $(\alpha_\pi - \beta_\pi)$ is pure S–wave, while $(\alpha_\pi + \beta_\pi)$ is pure D–wave in the $\gamma\gamma \to \pi\pi$ channel.

4/ Current Algebra and PCAC demand that the polarizabilities of the charged pion be directly related to the axial (h_A) and vector (h_V) structure–dependent form factors of the radiative pion decay $\pi^+ \to e^+\nu_e\gamma$ as [TE73, DH89]

$$\alpha_{\pi^\pm} = \frac{\alpha}{8\pi^2 m_\pi F_\pi^2}\frac{h_A}{h_V} \tag{7}$$

where $F_\pi \sim 93\,\mathrm{MeV}$ is the decay constant of pion. At $\mathcal{O}(p^6)$ in Chiral Perturbation Theory (χPT) this relation no longer holds and there can be additional contributions to either side of Eq. (7) [3].

[2] Note that these $M_{+\pm}$ differ from the helicity amplitudes called $\mathcal{M}_{+\pm}$ of [PE92] by kinematic factors.
[3] We thank J. Gasser for pointing this out to us.

5/ A sum rule for the electric polarizability was proposed by Petrun'kin [PE64] (see also [EH73, BH88]) using a classical approach. This sum rule says that α_π can be split into two parts

$$\alpha_\pi = \alpha_\pi^{cl} + \alpha_\pi^{intr} \quad . \tag{8}$$

The term α_π^{cl} is related to the electromagnetic pion size (it is proportional to the charge radius squared)

$$\alpha_\pi^{cl} = \frac{\alpha}{3m_\pi}\langle r_\pi^2\rangle \quad , \tag{9}$$

while α_π^{intr} is the intrinsic polarizability associated with possible excited states of the pion accessed by electric dipole transitions and vacuum polarization effects:

$$\alpha_\pi^{intr} = 2\alpha \sum_{n\neq 0} \frac{|\langle n|\mathcal{D}|0\rangle|^2}{E_n - E_\circ} \quad , \tag{10}$$

where $\mathcal{D}$ is the electric dipole operator. This description has been criticized by Terent'ev [TE74] (on grounds that assumptions involved in taking the non–relativistic limit are dubious), but Holstein [HO90] gives an interpretation of α_π^{intr} relating it to the spectral functions of vector and axial–vector mesons, $\rho^V(s)$ and $\rho^A(s)$, respectively, by comparing with the known current algebra result [DM67]. This gives,

$$\sum_{n\neq 0} \frac{|\langle n|\mathcal{D}|0\rangle|^2}{E_n - E_\circ} = \frac{1}{4m_\pi F_\pi^2} \int ds \; \frac{\rho^A(s) - \rho^V(s)}{s^2} \tag{11}$$

Bernard et al. [BH88] have computed the intrinsic electric polarizability, Eq. (10), and the analogous intrinsic magnetic polarizability in a valence quark model (MIT bag). However, they obtain the opposite sign for the magnetic polarizability compared to that given by phenomenology. This result the authors claim means that the pion cannot be regarded as a single bound $q\overline{q}$ valence pair, but rather the pion must be treated as a collective degree of freedom to obtain consistent results.

4 Experimental situation

It is evident that experimentally Compton scattering on pions is not easy. Fortunately, there are processes, when properly analyzed, that allow information about pion polarizabilities to be extracted. In this section we comment briefly on the status of these experimental determinations and refer the reader to the reference [BB92] for a more complete discussion.

The experimental situation is very different for charged and neutral pions:

Charged pion polarizabilities

All the experimental results (except when otherwise stated) are analysed assuming the constraint on the sum of polarizabilities, Eq. (3). The experimental sources of information and results are:

1/ *Radiative pion nucleon scattering (Primakoff effect)* $[\,\pi^- Z \to \pi^- Z\gamma\,]$
The SERPUKOV group gives [AN83]

$$\alpha_{\pi^\pm} = 6.8 \pm 1.4(stat) \pm 1.2(syst) \qquad . \tag{12}$$

When they relax the condition Eq. (3), they find [AN85]

$$(\alpha_\pi + \beta_\pi)^C = 1.4 \pm 3.1(stat) \pm 2.5(syst) \qquad , \tag{13}$$

which is consistent with Eq. (3).

2/ *Pion photoproduction in photon–nucleon scattering* $[\,\gamma p \to \gamma\pi^+ n\,]$
The LEBEDEV group obtains [AI86]

$$\alpha_{\pi^\pm} = 20 \pm 12(stat) \qquad . \tag{14}$$

3/ *Photon–photon into two pions* $[\,\gamma\gamma \to \pi^+\pi^-\,]$ in χPT.
The data of the MARK II group [BO90] have been fitted in [BB92] using $\mathcal{O}(p^4)$ χPT. This gives

$$\alpha_{\pi^\pm} = 2.2 \pm 1.6(stat + syst) \qquad . \tag{15}$$

As can be seen the results are far from consistent with each other. Clearly more experimental effort is needed.

Neutral pion polarizabilities in χPT

There are no real experimental measurements of neutral pion polarizabilities. However, Babusci et al. [BB92] have analyzed data on $\gamma\gamma \to \pi^\circ\pi^\circ$ taken by the Crystal Ball Collaboration [MA90] using the theoretical calculation in Chiral Perturbation Theory to be discussed in Sect. 5.1 (with the constraint on the sum of polarizabilities Eq. (3)) and find,

$$|\alpha_{\pi^\circ}| = 0.69 \pm 0.07(stat) \pm 0.04(syst) \qquad . \tag{16}$$

Other analyses involve parameterizations using dispersion relations [KS86, KS92] and then fitting the data. We will comment on these in Sect. 5.4.

5 Theoretical predictions

In spite of the fact that the concept of the polarizability of an elementary particle was introduced long ago [PE64, TE73, TE74], only after the first measurement of charged pion polarizabilities was this issue taken up by theorists. In the last two years there has been a burst of theoretical predictions on charged and neutral pion polarizabilities. These we collect here. We detail the theoretical frameworks employed, their respective results and give a brief analysis of these.

5.1 Chiral Perturbation Theory (χPT)

The study of the cross–section for the process $\gamma\gamma \to \pi\pi$ in χPT gave the first predictions. The leading contribution is $\mathcal{O}(p^4)$ in the chiral expansion and was computed by Bijnens and Cornet [BC88] using the $SU(3)_L \otimes SU(3)_R$ chiral Lagrangian [GL85]. At this order, even with $m_\pi \neq 0$, they found that

$$(\alpha_\pi + \beta_\pi)^{C,N}|_{\chi PT[\mathcal{O}(p^4)]} = 0 \tag{17}$$

and the results

$$\begin{aligned} \alpha_{\pi^\pm} &= \frac{4\alpha}{m_\pi F_\pi^2}\,(\ L_9^r + L_{10}^r\ ) = 2.68 \pm 0.42 \\ \alpha_{\pi^\circ} &= -\frac{\alpha}{96\pi^2 m_\pi F_\pi^2} = -0.50 \qquad , \end{aligned} \tag{18}$$

where the error in $\alpha_{\pi^\pm}$ comes from the phenomenological determination of L_9^r and L_{10}^r. It is worth emphasizing that there is no contribution from the 1–loop graphs to $\alpha_{\pi^\pm}$ and that only the pion loop contributes to α_{π°.

Recently the next–to–leading order corrections to $\gamma\gamma \to \pi^\circ\pi^\circ$ in $SU(2)_L \otimes SU(2)_R\, \chi PT$ have been calculated by Bellucci et al. [BG94]. The authors have computed the $\mathcal{O}(p^6)$ contribution that involves a full two–loop calculation [4]. In order to handle the divergences, the $\mathcal{L}^6$ contact terms of the effective Lagrangian must be included. The strengths of these terms are given by coupling constants, the values of which are extracted by saturating with vector mesons (1^{--}), C–odd axial vector mesons (1^{+-}), scalars (0^{++}) and tensors (2^{++}).

At this order Eq. (3) is no longer satisfied. Now

$$(\alpha + \beta)^{C,N}_{\chi PT[\mathcal{O}(p^6)]} \neq 0 \qquad . \tag{19}$$

The complete results are shown in Table I. The errors in the total predictions come from the uncertainties in the phenomenological determination of the coupling constants in the

[4] See S. Bellucci, J. Gasser and M.E. Sainio in this Handbook.

chiral Lagrangian.

Table I *Predictions of χPT for pion polarizabilities*

Polarizability	$\mathcal{O}(p^4)$ [BC88]	$\mathcal{O}(p^6)$ [BG94] *tree*	$\mathcal{O}(p^6)$ [BG94] *two − loop*	*Total*
α_{π°	-0.50	0.21	-0.07	-0.35 ± 0.10
β_{π°	0.50	0.79	0.24	1.50 ± 0.20
$(\alpha_\pi + \beta_\pi)^N$	0	1.00	0.17	1.15 ± 0.30
$(\alpha_\pi - \beta_\pi)^N$	-1.00	-0.58	-0.31	-1.90 ± 0.20
$\alpha_{\pi^\pm}$	2.68	$-$	$-$	2.68 ± 0.42

5.2 Lowest order χPT with explicit resonance contributions

Another less systematic way to incorporate corrections to the leading order result comes from explicitly computing the resonance contributions. This is the approach taken by Ko [KO90, KO93] and by Babusci et al. [BB93]. In order to take into account the momentum dependence of the resonance propagators chiral power counting establishes that only axial vector mesons (1^{++}) start to contribute to $\mathcal{O}(p^4)$, while vectors (1^{--}), C–odd axial vectors (1^{+-}), scalars (0^{++}) and tensors (2^{++}) start at $\mathcal{O}(p^6)$. Such calculations mean that part of the higher order corrections are automatically included in these resonance terms. They serve as a guide, but not a substitute, for higher order corrections, the $\mathcal{O}(p^6)$ having been computed in [BG94].

As the leading order is $\mathcal{O}(p^4)$ and in order to avoid double counting the contribution of axial vectors $a_1(1^{++})$ is not included. This is because these resonances only contribute to $\alpha_{\pi^\pm}$, but the combination $L_9^r + L_{10}^r$ in Eq. (18) comes from pure a_1 annihilation in the s–channel for $\gamma\pi^+ \to \gamma\pi^+$ [DH93] [5]. We now enumerate the contribution of these resonances

[5] This is so when the Weinberg relation of masses $m_{a_1} = \sqrt{2} m_\rho$ is used.

in turn:

Vector mesons $(1^{--})\ [\rho,\omega]$

These resonances only affect the value of β_π. The contribution of their direct channel exchange in $\gamma\pi\to\gamma\pi$ has been calculated in [KO90, BB93] and found to be:

$$\begin{aligned}(\alpha_\pi+\beta_\pi)\,|_\rho^C &= 0.07\\ (\alpha_\pi+\beta_\pi)\,|_{\rho,\omega}^N &= 0.83 \quad .\end{aligned} \tag{20}$$

Since the couplings of the light vector mesons are well–known , the calculation of their contributions should be reliable.

C–odd axial vector mesons $(1^{+-})\ [b_1,h_1]$

Ko has worked out [KO93] their contribution to $\gamma\pi^\circ\to\gamma\pi^\circ$, and hence to the neutral pion polarizabilities. These resonances do not modify β_{π° and Ko finds [6]

$$\alpha_{\pi^\circ}|_{b_1,h_1} = 0.21 \tag{21}$$

to be added to

$$\alpha_{\pi^\circ}|_{\chi PT[\mathcal{O}(p^4)]} = -0.50 \quad . \tag{22}$$

We note that since this resonance sector is not so well known, the result in Eq. (21), which is obtained assuming no mixing and exact nonet symmetry, could change when more realistic approximations are made. [7]

Tensor mesons $(2^{++})\ [a_2,f_2]$

After [AB92], Babusci et al. [BB93] have considered the contribution of tensor resonances to $\gamma\pi^\circ\to\gamma\pi^\circ$. However a problem arises here. Since these exchanges contribute in the t–channel, an ambiguity results in the phenomenological determination of the signs of the couplings. The contribution given by tensor exchange is thus

$$|\alpha_{\pi^\circ}|_T = 0.04 \qquad , \qquad |\beta_{\pi^\circ}|_T = 0.07 \quad . \tag{23}$$

Since we only know the magnitudes, both these contributions must be included as an uncertainty in the chiral prediction.

In Table II we collect the different resonance structure contributions. It is worth emphasizing that the final results in Table II are not directly comparable with those in Table

[6] A numerical error has been corrected here (we thank S. Bellucci for pointing this out).

[7] By the way this warning must be extended also to the $\mathcal{O}(p^6)$ calculation in χPT [BG94] that model the C–odd axial vector mesons in the same way.

I because of differences in the order of chiral powers included (though individual contributions may be comparable).

Table II *Lowest order χPT with explicit resonance contributions to pion polarizabilities*

Polarizability	$\chi PT\|_{\mathcal{O}(p^4)}$	1^{--}	1^{+-}	2^{++}	*Total*
α_{π°	-0.50	0	0.21	± 0.04	-0.29 ± 0.04
β_{π°	0.50	0.83	0	± 0.07	1.33 ± 0.07
$\alpha_{\pi^\pm}$	2.68 ± 0.42	0	$-^*$	$-^*$	2.68 ± 0.42
$\beta_{\pi^\pm}$	-2.68 ± 0.42	0.07	0	$-^*$	-2.61 ± 0.42

$(*)$*This contribution has not been calculated*

5.3 Generalized Chiral Perturbation Theory ($G\chi PT$)

The cross–section of $\gamma\pi^\circ \to \gamma\pi^\circ$ has recently been studied in the framework of $G\chi PT$ [SS93] by Knecht et al. [KM94]. This has allowed predictions to $\mathcal{O}(p^5)$ for the neutral pion polarizabilities to be calculated. The authors have also worked out an $\mathcal{O}(p^4)$ result for the charged pion polarizability. To these orders, they still find

$$(\alpha_\pi + \beta_\pi)^{C,N} = 0 \qquad . \tag{24}$$

The neutral pion polarizability depends on 5 unknown parameters : $\alpha_{\pi\pi}, \beta_{\pi\pi}$ (related to $\mathcal{O}(p^3)$ tree level $\pi\pi$ scattering), $\alpha_{\pi K}, \beta_{\pi K}$ (related to $\mathcal{O}(p^3)$ tree level πK scattering) and a combination of the coupling constants of the $\mathcal{L}^5_{G\chi PT}$ contact term they call c. Disregarding the kaon contribution (i.e. setting $\alpha_{\pi K} = \beta_{\pi K} = 0$), taking values for $\alpha_{\pi\pi}, \beta_{\pi\pi}$ from a fit to $\pi\pi$ scattering data [SS93] and constructing a low energy sum rule for c evaluated by saturating with the vector resonances, ρ, ω and ϕ, they obtain

$$\alpha_{\pi^\pm}|_{\mathcal{O}(p^4)} = 3.47 \qquad , \qquad \alpha_{\pi^\circ}|_{\mathcal{O}(p^5)} = +0.44 \qquad . \tag{25}$$

Let us note that the prediction to $\mathcal{O}(p^5)$ of $G\chi PT$ for α_{π° has a different sign than the prediction to $\mathcal{O}(p^6)$ in χPT (Table I).

5.4 Dispersive descriptions

The S–wave isospin amplitudes for the processes $\gamma\gamma \to \pi\pi$ were worked out by Morgan and Pennington [MP91] using twice–subtracted dispersion relations

$$f_I(s) = \Omega_I(s)\left\{ p_I(s)\Omega_I^{-1}(s) + c_I + s d_I - \frac{(s-s_\circ)^2}{\pi}\int_{4m_\pi^2}^{\infty} dx \frac{p_I(x)\Im m[\Omega_I^{-1}(x)]}{(x-s_\circ)^2(x-s-i\epsilon)}\right\} \quad (26)$$

for $I = 0, 2$, where $\Omega_I(s)$ is the Omnès function, $p_I(s)$ gives the structure of the left hand cut, and c_I, d_I are two subtraction constants which depend on the subtraction point $s_\circ$. Analogous relations hold for higher waves, their known threshold behaviour fixing their subtraction constants [MP88].

Once the $f_I(s)$ are determined it is possible to deduce the combination $(\alpha_\pi - \beta_\pi)$ (since it is pure S–wave, Eq. (6)). Thus

$$\begin{aligned}(\alpha_\pi - \beta_\pi)^N &= \frac{4\alpha}{m_\pi}\lim_{s\to 0}\frac{f^N(s)}{s} \\ (\alpha_\pi - \beta_\pi)^C &= \frac{4\alpha}{m_\pi}\lim_{s\to 0}\frac{[f^C(s) - f_{BORN}]}{s}\end{aligned} \quad (27)$$

with

$$f^N(s) = \frac{2}{3}\left[\, f_0(s) - f_2(s)\,\right] \qquad , \qquad f^C(s) = \frac{1}{3}\left[\, 2f_0(s) + f_2(s)\,\right] \quad . \quad (28)$$

The Omnès functions, $\Omega_I(s)$ in Eq. (26), can be determined from experimental phase shifts. The nearby structure of the left hand cut $p_I(s)$ and the subtraction constants can be worked out from QED at low energy and chiral dynamics: $PCAC$ zeros [PE92], matching with χPT [DH93] or $G\chi PT$ [KM94], respectively.

The analysis of Morgan and Pennington [MP91, PE92] inputs the appearance of a near threshold zero in the S–wave $\gamma\gamma \to \pi^\circ\pi^\circ$ amplitude from chiral dynamics at $s = s_N$ to fix subtraction constants in Eq. (26). The remaining input information is taken from experiment. The S–wave amplitudes for $\gamma\gamma \to \pi\pi$ are thereby determined and so predictions can be made for the neutral pion polarizability $(\alpha_\pi - \beta_\pi)^N$. Not surprisingly, given Eq. (27), this combination is found to be directly proportional to how far the chiral zero is from $s = 0$. (Note that at $\mathcal{O}(p^4)$ in χPT , this zero is at $s_N = m_\pi^2$, while at $\mathcal{O}(p^6)$ is at [8] $s_N \simeq 1.1\, m_\pi^2$).

Alternatively, others [DH93, KM94], rather than inputting experimental information, have attempted to match these dispersion relations with detailed predictions of χPT (to a given order) for a range of energies about some point. Donoghue and Holstein [DH93] use $s_\circ = 0$, while Knecht et al. [KM94] try a range from $s_\circ = 0$ to $s_\circ = 4m_\pi^2$. Knecht et

[8] This can be seen in Fig. 8 of [BG94].

al. find that the resulting neutral pion polarizability depends strongly on the subtraction point chosen for this matching [9]. Indeed, even the sign of the combination of neutral pion polarizabilities, Eq. (27), changes as $s_\circ$ is varied. This implies that the χPT predictions for $0 < s < 4m_\pi^2$ must have a significantly different dependence on energy than given by the other inputs to Eq. (26). This problem requires further work to isolate the source of the discrepancy. We note that the dependence is far softer for the charged pion polarizabilities.

In these approaches, predictions are not just made for the polarizabilities but for the amplitudes for $\gamma\gamma \to \pi\pi$ scattering at low energies too. Pennington [PE92] illustrates that an equally good description of low energy data is obtained whether the sub–threshold zero is at $s_N = \frac{1}{2}m_\pi^2$ or $2m_\pi^2$, not just m_π^2, given by lowest order χPT. However, the combination $(\alpha_\pi - \beta_\pi)^N$ varies by a factor 4 in this range of s_N. Similarly, the $\mathcal{O}(p^6)$ χPT prediction of [BG94] and the $\mathcal{O}(p^5)$ GχPT prediction of [KM94] describe low energy $\gamma\gamma \to \pi^\circ\pi^\circ$ data equally well, yet can have $(\alpha_\pi - \beta_\pi)^N$ with opposite signs.

In a related way Kaloshin et al. [KS86, KS92, KS94] directly parameterize the dispersion relations of Eq. (26) in terms of polarizabilities as the subtraction constants. Then by fitting the charged and neutral channel data [BO90, BE92, MA90] they find the results for $(\alpha_\pi - \beta_\pi)^{C,N}$ summarized in Table III. Note the large uncertainty in the neutral combination reflects the insensitivity of the low energy data to the exact position of the chiral zero already mentioned. In [KP94], Kaloshin et al. have attempted a similar analysis for the D–wave combination $(\alpha_\pi + \beta_\pi)^{C,N}$ and claim to have determined these with remarkable precision —again their results are tabulated in Table III. In view of the detailed amplitude analysis of the same data by [MP90] which shows the D–wave cross–section may be uncertain by as much as 50 % even in the $f_2(1270)$ region, the errors quoted by Kaloshin et al. [KP94] appear unbelievably small [PE94] [10].

[9] In doing this Knecht et al. assume the combination $(\alpha_\pi + \beta_\pi)^N$ is saturated by vector and C–odd axial vector mesons.

[10] *Note added in proof*: The above remarks refer to [KP94]. Kaloshin et al. now have a new analysis of $\gamma\gamma \to \pi\pi$ data [KP95], from which we quote: $(\alpha_\pi + \beta_\pi)^C = 0.41 \pm 0.08(stat.) \pm 0.01(syst.)$ (CELLO) , $(\alpha_\pi + \beta_\pi)^C = 0.28 \pm 0.09(stat.) \pm 0.05(syst.)$ (MARK-II) and $(\alpha_\pi + \beta_\pi)^N = 1.43 \pm 0.10(stat.) \pm 0.20(syst.)$ (Crystal Ball).

Table III *Results of fits to the experimental data on* $\gamma\gamma \rightarrow \pi\pi$ *by Kaloshin et al.* [KS94, KP94] *to determine the pion polarizabilities*

Polarizability	*Data fitted*		
	CELLO	*MARK − II*	*Crystal Ball*
$(\alpha_\pi + \beta_\pi)^C$	$0.30 \pm 0.04^*$	$0.22 \pm 0.06^*$	–
$(\alpha_\pi - \beta_\pi)^C$	–	5.3 ± 1.0	–
$(\alpha_\pi + \beta_\pi)^N$	–	–	$1.00 \pm 0.05^*$
$(\alpha_\pi - \beta_\pi)^N$	–	–	-0.6 ± 1.8

$*$ *See text for discussion of these errors*

5.5 Quark Confinement Model (QCM)

The QCM developed by Efimov et al. [IM92, EI93] has also been employed to predict values for the pion polarizabilities. The basic characteristic of the model is a confinement ansatz that allows loop calculations to be made finite, avoiding the need for a regularization procedure.

Efimov et al. [EI93] consider the contribution to pion polarizabilities coming from pure loop terms and vector, axial vector and scalar resonances. Their results are given in Table IV.

Table IV *Predictions of QCM for the pion polarizabilities by Ivanov et al.* [IM92]

Polarizability	*Loops*	0^{++}	1^{--}	1^{++}	*Total*
α_{π°	-3.01	3.76	0	0	0.75
β_{π°	2.95	-3.76	0.51	0	-0.30
$\alpha_{\pi^\pm}$	-0.14	3.76	0	0.02	3.64
$\beta_{\pi^\pm}$	0.30	-3.76	0.05	0	-3.41

As can be seen the most important contribution in this model is given by the scalar resonances. Moreover, in the case of neutral pion polarizabilities, there is a strong cancellation between loops and the scalar contribution. The contribution of vector mesons is quite similar to that found in χPT, but not for axial vector mesons. As mentioned earlier the main contribution to the leading order in χPT to $\alpha_{\pi^\pm}$ comes from these axial–vector resonances, but in the QCM scheme their contribution is negligible.

Let us comment on these predictions of the Quark Confinement Model :

1/ As stated before, the charged electric pion polarizability is related to the quotient of axial and vector structure–dependent form factors of the radiative pion decay Eq. (7). In the QCM $\alpha_{\pi^\pm}$ and h_A/h_V are independent quantities. How this model legitimately avoids this constraint of Current Algebra and PCAC is unclear.

2/ There is a large model dependence in the scalar sector of QCM. Moreover, the experimental parameters used in their model have changed significantly [PD94]. The authors themselves consider that the contribution of scalars has an error of at least 30%. It could easily be bigger. However, the scalar contributions cancel in $(\alpha+\beta)$, giving

$$(\alpha_\pi+\beta_\pi)^C|_{QCM} = 0.22 \qquad , \qquad (\alpha_\pi+\beta_\pi)^N|_{QCM} = 0.44 \quad . \tag{29}$$

3/ Finally, there remains the question of double counting between loops and meson resonances. This is an unresolved issue for models that consider contributions at both the quark and meson level simultaneously.

5.6 QCD sum rules

Recently a calculation of the electric polarizability for the charged pion has been performed using QCD sum rules [LN94]. The authors take the current algebra sum rule of Das, Mathur and Okubo [DM67],

$$\frac{\alpha}{8\pi^2 m_\pi F_\pi^2}\,\frac{h_A}{h_V} = \frac{\alpha}{3m_\pi}\langle r_\pi^2\rangle + \frac{\alpha}{2m_\pi F_\pi^2}\int_{4m_\pi^2}^{\infty} ds\,\frac{1}{s^2}\,[\,\rho^A(s) - \rho^V(s)\,] \qquad , \tag{30}$$

where the spectral functions are defined as in Eq. (11). They input the experimental [AM86] pion charge radius $\langle r_\pi^2\rangle = (0.439 \pm 0.008)\, fm^2$ and then compute the integral of the vector and axial spectral functions using QCD sum rule methods. Identifying the left hand side of Eq. (30) with Eq. (7), they obtain

$$\alpha_{\pi^\pm}\,|_{QCDsr} = 5.6 \pm 0.5 \qquad , \tag{31}$$

the integral in Eq. (30) largely cancelling the charge radius term. Fortunately, Lavelle et al. find that their calculation depends little on the poorly known quark condensate and so they deduce a rather precise value for $\alpha_{\pi^\pm}$. This value, Eq. (31), is in good agreement with Eq. (12), but not with the prediction of one loop χPT, Eq. (18).

6 Pion polarizabilities at DAΦNE

The DAΦNE electron–positron collider at $\sqrt{s} \sim 1.02\,\text{GeV}$ provides the opportunity to study $\gamma\gamma \to \pi\pi$ processes at low energy. This is due to the possibility of double tagging of $e^+e^- \to e^+e^-\pi\pi$ in the KLOE multi–particle detector [AL93] that will eliminate much of the background.

Assuming full acceptance of the detector and with a machine luminosity of $L \simeq 5 \times 10^{32}$ cm^{-2}s^{-1}, the expected rates are

- $N[\gamma\gamma \to \pi^\circ\pi^\circ] \simeq 10^4$ events/year
- $N[\gamma\gamma \to \pi^+\pi^-] \simeq 1.8 \times 10^6$ events/year

Such event rates will allow the low energy cross–sections to be measured with much higher statistics than previously.

However, the pion polarizabilities are still likely to be poorly known. The maximum $\pi\pi$ mass that can be realistically attained is ~ 0.6 GeV. This will allow the S–wave cross–sections to be accurately measured, but not the difference of the D–wave from its Born component without precise azimuthal correlations. Even then, we have seen that knowing the low energy $\gamma\gamma \to \pi\pi$ cross–section accurately still allows large uncertainties in the polarizabilities [PE94]. Only measurements of Compton scattering will resolve these.

In principle, there are two ways of studying Compton scattering on a pion. Both involve $\gamma\pi$ production, initiated by either a photon or a pion beam. The idea is that $\gamma N \to \gamma\pi N$ proceeds by one pion exchange at small momentum transfers allowing Chew–Low extrapolation to extract the Compton cross–section, while $\pi Z \to \gamma\pi Z$ will occur by one photon exchange if the scattering is on a heavy target, Z, and the photon is almost real. Such a Primakoff production experiment, $E781$, is planned at Fermilab for 1996 with an initial pion momentum of 600 GeV/c, when the virtual photon can have a 4-momentum squared of $-2\times10^{-8}\,\mathrm{GeV}^2$ very close to zero [MO92, FE94]. The radiative corrections have already been computed [AG94] and are estimated to affect polarizability determinations at the level of $\sim\ (3\ -\ 10)\%$ (for nuclei C to Pb). This experiment will also provide data on the radiative width $a_1(1260)\ \to\ \pi\gamma$ [MO94] that, as we discussed in Sect. 5.2, is expected to be directly related with the polarizabilities. Therefore, the consistence of this relationship will be checked. Finally, the process $\gamma N \to \gamma\pi N$ may prove possible at the GRAAL facility at the Grenoble electron storage ring (ESRF) using a back–scattered laser [DA94, RE94]. These experiments are for the future; they are much needed if we are to learn more of the pion's polarizabilities.

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