an unbroken Yang-Mills gauge field theory featuring asymptotic freedom and confinement

in non-perturbative regime (low $Q^2$) many approaches: lattice, Regge theory, $\chi$ PT, large $N_c$, HQET

in perturbative regime (high $Q^2$) QCD is a precision toolkit for exploring Higgs & BSM physics

LEP was an electroweak machine

Tevatron & LHC are QCD machines
Precise determination of

- strong coupling constant $\alpha_s$
- parton distributions
- electroweak parameters
- LHC parton luminosity

Precise prediction for

- Higgs production
- new physics processes
- their backgrounds
Strong interactions at high $Q^2$

- Parton model
- Perturbative QCD
- Factorisation
- Universality of IR behaviour
- Cancellation of IR singularities
- IR safe observables: inclusive rates
  - Jets
  - Event shapes
Factorisation

is the separation between the short- and the long-range interactions

\[ \sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_{a/A}(x_1, \mu_F^2) \, f_{b/B}(x_2, \mu_F^2) \times \hat{\sigma}_{ab \to X} \left( x_1, x_2, \{ p_i^\mu \}; \alpha_S(\mu_R^2), \alpha(\mu_F^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2} \right) \]

\( X = W, Z, H, Q\bar{Q}, \text{high-}E_T \text{jets}, \ldots \)

\( \hat{\sigma} \) is known as a fixed-order expansion in \( \alpha_S \)

\[ \hat{\sigma} = C \alpha_S^n (1 + c_1 \alpha_S + c_2 \alpha_S^2 + \ldots) \]

\( c_1 = \text{NLO} \quad c_2 = \text{NNLO} \)

or as an all-order resummation

\[ \hat{\sigma} = C \alpha_S^n [1 + (c_{11} L + c_{10}) \alpha_S + (c_{22} L^2 + c_{21} L + c_{20}) \alpha_S^2 + \ldots] \]

where

\[ L = \ln(M/q_T), \ln(1-x), \ln(1/x), \ln(1-T), \ldots \]

\( c_{11}, c_{22} = \text{LL} \quad c_{10}, c_{21} = \text{NLL} \quad c_{20} = \text{NNLL} \)
Factorisation-breaking contributions

underlying event

power corrections

MC’s and theory modelling of power corrections laid out and tested at LEP where they provide an accurate determination of $\alpha_s$ models still need be tested in hadron collisions (see e.g. Tevatron studies at different $\sqrt{s}$)

diffractive events

Is double-parton scattering breaking factorisation?

observed by Tevatron CDF in the inclusive sample

$p\bar{p} \rightarrow \gamma + 3$ jets

potentially important at LHC $\sigma_D \propto \sigma_S^2$
Power corrections at Tevatron

Ratio of inclusive jet cross sections at 630 and 1800 GeV

\[ \frac{\sigma(630 \text{ GeV})}{\sigma(1800 \text{ GeV})} \text{, with:} \]
\[ \sigma(\sqrt{s}) = \sigma(\sqrt{s})_{\text{NLO}} \left( E_T \to E_T + \Lambda \right) \]

Bjorken-scaling variable

\[ x_T = \frac{2E_T}{\sqrt{s}} \]

In the ratio the dependence on the pdf’s cancels

- Dashes: theory prediction with no power corrections
- Solid: best fit to data with free power-correction parameter \( \Lambda \) in the theory

M.L. Mangano
KITP collider conf 2004
Factorisation in diffraction ??

diffraction in DIS  
double pomeron exchange in $p\bar{p}$

no proof of factorisation in diffractive events

data do not support it
Summary of $\alpha_S(M_Z)$

S. Bethke hep-ex/0407021

world average of $\alpha_S(M_Z)$ using $\overline{\text{MS}}$ and NNLO results only

$\alpha_S(M_Z) = 0.1182 \pm 0.0027$

(cf. 2002 $\alpha_S(M_Z) = 0.1183 \pm 0.0027$

outcome almost identical because new entries wrt 2002 - LEP jet shape observables and 4-jet rate, and HERA jet rates and shape variables - are NLO)

filled symbols are NNLO results
### 3 Complementary Approaches to \( \hat{\sigma} \)

<table>
<thead>
<tr>
<th></th>
<th>Matrix-Elem MC’s</th>
<th>Fixed-Order X-Sect</th>
<th>Shower MC’s</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Final-State Description</strong></td>
<td>hard-parton jets. Describes geometry, correlations, ...</td>
<td>limited access to final-state structure</td>
<td>full information available at the hadron level</td>
</tr>
<tr>
<td><strong>Higher-Order Effects: Loop Corrections</strong></td>
<td>hard to implement: must introduce negative probabilities</td>
<td>straightforward to implement (when available)</td>
<td>included as vertex corrections (Sudakov FF’s)</td>
</tr>
<tr>
<td><strong>Higher-Order Effects: Hard Emissions</strong></td>
<td>included, up to high orders (multijets)</td>
<td>straightforward to implement (when available)</td>
<td>approximate, incomplete phase space at large angles</td>
</tr>
<tr>
<td><strong>Resummation of Large Logs</strong></td>
<td>?</td>
<td>feasible (when available)</td>
<td>unitarity implementation (i.e. correct shapes but not total rates)</td>
</tr>
</tbody>
</table>

M.L. Mangano KITP collider conf 2004
Matrix-element MonteCarlo generators

- **ALPGEN** M.L. Mangano M. Moretti F. Piccinini R. Pittau A. Polosa 2002
- **COMPHEP** A. Pukhov et al. 1999
- **GRACE/GR@PPA** T. Ishikawa et al. K. Sato et al. 1992/2001
- **HELAC** C. Papadopoulos et al. 2000

Processes with 6 final-state fermions

- **PHASE** E. Accomando A. Ballestrero E. Maina 2004

Merged with parton showers

- All of the above, merged with HERWIG or PYTHIA

- **SHERPA** F. Krauss et al. 2003
Shower MonteCarlo generators

**HERWIG**  
B. Webber et al. 1992  
being re-written as a C++ code (HERWIG++)

**PYTHIA**  
T. Sjostrand 1994

and more

**CKKW**  
S. Catani F. Krauss R. Kuhn B. Webber 2001  
a procedure to interface parton subprocesses with a different number of final states to parton showers

**MC@NLO**  
S. Frixione B. Webber 2002  
a procedure to interface NLO computations to shower MC’s
NLO features

- Jet structure: final-state collinear radiation
- PDF evolution: initial-state collinear radiation
- Opening of new channels
- Reduced sensitivity to fictitious input scales: $\mu_R, \mu_F$
  - predictive normalisation of observables
  - first step toward precision measurements
  - accurate estimate of signal and background for Higgs and new physics
- Matching with parton-shower MC’s: MC@NLO
Jet structure

the jet non-trivial structure shows up first at NLO

leading order

NLO

NNLO
**Desired NLO cross sections**

<table>
<thead>
<tr>
<th>Single boson</th>
<th>Diboson</th>
<th>Triboson</th>
<th>Heavy flavour</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W + \leq 5j$</td>
<td>$WW + \leq 5j$</td>
<td>$WWW + \leq 3j$</td>
<td>$t\bar{t} + \leq 3j$</td>
</tr>
<tr>
<td>$W + b\bar{b} + \leq 3j$</td>
<td>$WW + b\bar{b} + \leq 3j$</td>
<td>$WWW + b\bar{b} + \leq 3j$</td>
<td>$t\bar{t} + \gamma + \leq 2j$</td>
</tr>
<tr>
<td>$W + c\bar{c} + \leq 3j$</td>
<td>$WW + c\bar{c} + \leq 3j$</td>
<td>$WWW + \gamma\gamma + \leq 3j$</td>
<td>$t\bar{t} + W + \leq 2j$</td>
</tr>
<tr>
<td>$Z + \leq 5j$</td>
<td>$ZZ + \leq 5j$</td>
<td>$Z\gamma\gamma + \leq 3j$</td>
<td>$t\bar{t} + Z + \leq 2j$</td>
</tr>
<tr>
<td>$Z + b\bar{b} + \leq 3j$</td>
<td>$ZZ + b\bar{b} + \leq 3j$</td>
<td>$WZZ + \leq 3j$</td>
<td>$t\bar{t} + H + \leq 2j$</td>
</tr>
<tr>
<td>$Z + c\bar{c} + \leq 3j$</td>
<td>$ZZ + c\bar{c} + \leq 3j$</td>
<td>$ZZZ + \leq 3j$</td>
<td>$t\bar{b} + \leq 2j$</td>
</tr>
<tr>
<td>$\gamma + \leq 5j$</td>
<td>$\gamma\gamma + \leq 5j$</td>
<td></td>
<td>$b\bar{b} + \leq 3j$</td>
</tr>
<tr>
<td>$\gamma + b\bar{b} + \leq 3j$</td>
<td>$\gamma\gamma + b\bar{b} + \leq 3j$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma + c\bar{c} + \leq 3j$</td>
<td>$\gamma\gamma + c\bar{c} + \leq 3j$</td>
<td>$WZ + \leq 5j$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$WZ + b\bar{b} + \leq 3j$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$WZ + c\bar{c} + \leq 3j$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$W\gamma + \leq 3j$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$Z\gamma + \leq 3j$</td>
<td></td>
</tr>
</tbody>
</table>
NLO history of final-state distributions

- $e^+e^- \rightarrow 3$ jets
  - K. Ellis, D. Ross, A. Terrano 1981
- $e^+e^- \rightarrow 4$ jets
  - Z. Bern et al., N. Glover et al., Z. Nagy Z. Trocsanyi 1996-97
- $pp \rightarrow 1, 2$ jets
  - K. Ellis J. Sexton 1986, W. Giele N. Glover D. Kosower 1993
- $pp \rightarrow 3$ jets
- $pp \rightarrow \gamma\gamma$
- $pp \rightarrow \gamma\gamma + 1$ jet
  - Z. Bern et al. 1994, V. Del Duca et al. 2003
- $pp \rightarrow V + 1$ jet
  - W. Giele N. Glover & D. Kosower 1993
- $pp \rightarrow V + 2$ jet
  - Bern et al., Glover et al. 1996-97, K. Ellis & Campbell 2003
- $pp \rightarrow Vb\bar{b}$
  - K. Ellis & J. Campbell 2003
- $pp \rightarrow VV$
- $pp \rightarrow Q\bar{Q}$
- $pp \rightarrow Q\bar{Q} + 1$ jet
  - A. Brandenburg et al. 2005-6
- $pp \rightarrow H + 1$ jet
  - C. Schmidt 1997, D. De Florian M. Grazzini Z. Kunszt 1999
- $pp \rightarrow H + 2$ jets
  - (WBF) Campbell, K. Ellis; Figy, Oleari, Zeppenfeld 2003
- $pp \rightarrow HQ\bar{Q}$
  - W. Beenakker et al.; S. Dawson et al. 2001
- $e^+e^- \rightarrow 4$ fermions
  - Denner Dittmaier Roth Wieders 2005
**NLOJET++**

Author(s): Z. Nagy

http://www.ippp.dur.ac.uk/~nagyz/nlo++.html

Multi-purpose C++ library for calculating jet cross-sections in $e^+e^-$ annihilation, DIS and hadron-hadron collisions.

- $e^+e^- \rightarrow \leq 4$ jets
- $ep \rightarrow (\leq 3 + 1)$ jets
- $p\bar{p} \rightarrow \leq 3$ jets

hep-ph/0110315
**MCFM**

Author(s): JC, R. K. Ellis

http://mcfm.fnal.gov

Fortran package for calculating a number of processes involving vector bosons, Higgs, jets and heavy quarks at hadron colliders.

\[ p\bar{p} \rightarrow V + \leq 2 \text{ jets} \]

\[ p\bar{p} \rightarrow V + b\bar{b} \]

with \( V = W, Z \).
**NLO assembly kit**

\[e^+ e^- \rightarrow 3 \text{ jets}\]

**leading order**

\[|M_{tree}^{tree}|^2\]

**NLO real**

\[|M_{n+1}^{tree}|^2 \rightarrow |M_n^{tree}|^2 \times \int dP S |P_{split}|^2 = -\left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon}\right)\]

**NLO virtual**

\[d = 4 - 2\epsilon\]

\[\int d^d l 2(M_{loop}^{loop})^* M_{tree}^{tree} = \left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon}\right) |M_n^{tree}|^2 + \text{fin.}\]
NLO production rates

Process-independent procedure devised in the 90's

- slicing: Giele Glover & Kosower
- subtraction: Frixione Kunszt & Signer; Nagy & Trocsanyi
- dipole: Catani & Seymour
- antenna: Kosower; Campbell Cullen & Glover

\[ \sigma = \sigma^{\text{LO}} + \sigma^{\text{NLO}} = \int d\sigma^B_m J_m + \sigma^{\text{NLO}} \]

\[ \sigma^{\text{NLO}} = \int_{m+1} d\sigma^R_{m+1} J_{m+1} + \int_m d\sigma^V_m J_m \]

the 2 terms on the rhs are divergent in d=4

use universal IR structure to subtract divergences

\[ \sigma^{\text{NLO}} = \int_{m+1} \left[ d\sigma^R_{m+1} J_{m+1} - d\sigma^R,A_{m+1} J_m \right] + \int_m \left[ d\sigma^V_m + \int_1 d\sigma^R,A_{m+1} \right] J_m \]

the 2 terms on the rhs are finite in d=4
**Observable (jet) functions**

\( J_m \) vanishes when one parton becomes soft or collinear to another one

\[
J_m(p_1, \ldots, p_m) \to 0, \quad \text{if} \quad p_i \cdot p_j \to 0
\]

\( J_{m+1} \) vanishes when two partons become simultaneously soft and/or collinear

\[
J_{m+1}(p_1, \ldots, p_{m+1}) \to 0, \quad \text{if} \quad p_i \cdot p_j \text{ and } p_k \cdot p_l \to 0 \quad (i \neq k)
\]

\( d\sigma^B_m \) is integrable over 1-parton IR phase space

\( d\sigma^B_m \) is integrable over 1-parton IR phase space

\( R \) and \( V \) are integrable over 2-parton IR phase space

**observables are IR safe**

\[
J_{n+1}(p_1, \ldots, p_j = \lambda q, \ldots, p_{n+1}) \to J_n(p_1, \ldots, p_{n+1}) \quad \text{if} \quad \lambda \to 0
\]

\[
J_{n+1}(p_1, \ldots, p_i, \ldots, p_j, \ldots, p_{n+1}) \to J_n(p_1, \ldots, p, \ldots, p_{n+1}) \quad \text{if} \quad p_i \to zp, \quad p_j \to (1-z)p
\]

for all \( n \geq m \)
NLO complications

- Loop integrals are involved and process-dependent.
- More particles $\rightarrow$ many scales $\rightarrow$ lengthy analytic expressions.
- Even though it is known how to compute loop integrals with $2 \rightarrow n$ particles, no one-loop amplitudes with $n > 4$ have been computed (either analytically or numerically).
- No fully numeric methods yet for hadron collisions.
- Counterterms are subtracted analytically.
Is NLO enough to describe data?

$b$ cross section in $p\bar{p}$ collisions at 1.96 TeV

$$d\sigma(p\bar{p} \to H_b X, H_b \to J/\psi X)/dp_T(J/\psi)$$

From B

CDF hep-ex/0412071

total x-sect is

$$19.4 \pm 0.3^{+2.1}_{-1.9} \text{(syst)} \text{ nb}$$

FONLL = NLO + NLL

Cacciari, Frixione, Mangano, Nason, Ridolfi 2003

good agreement with data (with use of updated FF's by Cacciari & Nason)
Is NLO enough to describe data?

di-lepton rapidity distribution for \((Z, \gamma^*)\) production vs. Tevatron Run I data

\[
\bar{p}p \rightarrow (Z, \gamma^*) + X
\]

LO and NLO curves are for the MRST PDF set

no spin correlations

C. Anastasiou, L. Dixon, K. Melnikov, F. Petriello 2003
Is **NLO** enough to describe data?

Drell-Yan $W$ cross section at LHC with leptonic decay of the $W$

Cuts A $\rightarrow$ $|\eta^{(e)}| < 2.5, p_T^{(e)} > 20 \text{ GeV}, p_T^{(\nu)} > 20 \text{ GeV}$

Cuts B $\rightarrow$ $|\eta^{(e)}| < 2.5, p_T^{(e)} > 40 \text{ GeV}, p_T^{(\nu)} > 20 \text{ GeV}$

<table>
<thead>
<tr>
<th></th>
<th>LO</th>
<th>LO+HW</th>
<th>NLO</th>
<th>MC@NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cuts A</strong></td>
<td>0.5249</td>
<td>0.4843</td>
<td>0.4771</td>
<td>0.4845</td>
</tr>
<tr>
<td></td>
<td>↓5.4%</td>
<td>↓5.4%</td>
<td>↓7.0%</td>
<td>↓6.3%</td>
</tr>
<tr>
<td><strong>Cuts A, no spin</strong></td>
<td>0.5535</td>
<td>0.5104</td>
<td>0.5151</td>
<td></td>
</tr>
<tr>
<td><strong>Cuts B</strong></td>
<td>0.0585</td>
<td>0.1218</td>
<td>0.1292</td>
<td>0.1329</td>
</tr>
<tr>
<td></td>
<td>↓29%</td>
<td>↓16%</td>
<td>↓18%</td>
<td></td>
</tr>
<tr>
<td><strong>Cuts B, no spin</strong></td>
<td>0.0752</td>
<td>0.1504</td>
<td>0.1570</td>
<td></td>
</tr>
</tbody>
</table>

$|MC@NLO - NLO| = \mathcal{O}(2\%)$

**NNLO** useless without spin correlations

Precisely evaluated Drell-Yan $W, Z$ cross sections could be used as `standard candles` to measure the parton luminosity at LHC.
Is NLO enough to describe data?

Total cross section for inclusive Higgs production at LHC

\[ \sigma_H \] vs \( M_H [\text{GeV}] \)

- **NNLO prediction stabilises the perturbative series**
- Contour bands are
  - lower \( \mu_R = 2M_H \quad \mu_F = M_H/2 \)
  - upper \( \mu_R = M_H/2 \quad \mu_F = 2M_H \)
- Scale uncertainty is about 10%
NNLO corrections may be relevant if

- the main source of uncertainty in extracting info from data is due to NLO theory: $\alpha_s$ measurements
- NLO corrections are large: Higgs production from gluon fusion in hadron collisions
- NLO uncertainty bands are too large to test theory vs. data: $b$ production in hadron collisions
- NLO is effectively leading order: energy distributions in jet cones

in short, NNLO is relevant where NLO fails to do its job
NNLO state of the art

Drell-Yan $W, Z$ production
- total cross section
  - Hamberg, van Neerven, Matsuura 1990
  - Harlander, Kilgore 2002
- rapidity distribution
  - Anastasiou, Dixon, Melnikov, Petriello 2003

Higgs production
- total cross section
  - Harlander, Kilgore; Anastasiou, Melnikov 2002
  - Ravindran, Smith, van Neerven 2003
- fully differential cross section
  - Anastasiou, Melnikov, Petriello 2004

$e^+ e^- \rightarrow 3$ jets
- the $1/N_c^2$ term
  - the Gehrmanns, Glover 2004-5
NNLO cross sections

**Analytic integration**

- first method
- flexible enough to include a limited class of acceptance cuts by modelling cuts as "propagators"

**Sector decomposition**

- flexible enough to include any acceptance cuts
- cancellation of divergences is performed numerically
- can it handle many final-state partons?

**Subtraction**

- process independent
- cancellation of divergences is semi-analytic
- can it be fully automatised?
Drell-Yan $Z$ production at LHC

Rapidity distribution for an on-shell $Z$ boson

\[ \frac{d^2\sigma}{dM/dY} \text{ [pb/GeV]} \]

$\sqrt{s} = 14$ TeV

- $M = M_Z$
- $M/2 \leq \mu \leq 2M$

30%(15%) NLO increase wrt to LO at central $Y$'s (at large $Y$'s)

NNLO decreases NLO by $1 - 2\%$

scale variation: $\approx 30\%$ at LO; $\approx 6\%$ at NLO; less than $1\%$ at NNLO

C. Anastasiou L. Dixon K. Melnikov F. Petriello 2003
Scale variations in Drell-Yan $Z$ production

$pp \to (Z,\gamma^*) + X$ at $Y=0$

- solid: vary $\mu_R$ and $\mu_F$ together
- dashed: vary $\mu_F$ only
- dotted: vary $\mu_R$ only

$\sqrt{s} = 14$ TeV
$M = M_Z$
MRST2001 pdfs
$\mu_F = \mu_R = \mu$
$\mu_F = \mu, \, \mu_R = M$
$\mu_F = M, \, \mu_R = \mu$
Drell-Yan $W$ production at LHC

Rapidity distribution for an on-shell $W^-$ boson (left) $W^+$ boson (right)

distributions are symmetric in $Y$

NNLO scale variations are $1\% (3\%)$ at central (large) $Y$

C. Anastasiou L. Dixon K. Melnikov F. Petriello 2003
Higgs production at LHC

a fully differential cross section:
bin-integrated rapidity distribution, with a jet veto

C. Anastasiou K. Melnikov F. Petriello 2004

jet veto: require
\[ R = 0.4 \]
\[ |p_T^j| < p_T^{veto} = 40 \text{ GeV} \]

for 2 partons
\[ R_{12}^2 = (\eta_1 - \eta_2)^2 + (\phi_1 - \phi_2)^2 \]

if \( R_{12} > R \)
\[ |p_T^1|, |p_T^2| < p_T^{veto} \]

if \( R_{12} < R \)
\[ |p_T^1 + p_T^2| < p_T^{veto} \]

\[ M_H = 150 \text{ GeV} \] (jet veto relevant in the \( H \rightarrow W^+W^- \) decay channel)

K factor is much smaller for the vetoed x-sect than for the inclusive one: average \( |p_T^j| \) increases from NLO to NNLO: less x-sect passes the veto
NNLO assembly kit

$e^+e^- \rightarrow 3$ jets

double virtual

real-virtual

double real
Two-loop matrix elements

two-jet production \( qq' \rightarrow qq', q\bar{q} \rightarrow q\bar{q}, q\bar{q} \rightarrow gg, \ gg \rightarrow gg \)

C. Anastasiou N. Glover C. Oleari M. Tejeda-Yeomans 2000-01
Z. Bern A. De Freitas L. Dixon 2002

photon-pair production \( q\bar{q} \rightarrow \gamma\gamma, \ \gg \rightarrow \gamma\gamma \)

C. Anastasiou N. Glover M. Tejeda-Yeomans 2002
Z. Bern A. De Freitas L. Dixon 2002

\( e^+e^- \rightarrow 3 \ \text{jets} \quad \gamma^* \rightarrow q\bar{q}g \)


\( V + 1 \ \text{jet} \ \text{production} \quad q\bar{q} \rightarrow Vg \)

T. Gehrmann E. Remiddi 2002

Drell-Yan \( V \ \text{production} \quad q\bar{q} \rightarrow V \)

R. Hamberg W. van Neerven T. Matsuura 1991

Higgs production \( gg \rightarrow H \) (in the \( m_t \rightarrow \infty \) limit)

R. Harlander W. Kilgore; C. Anastasiou K. Melnikov 2002
**NNLO cross sections**

**universal IR structure** \[\rightarrow\] **process-independent procedure**

**universal** collinear and soft currents

3-parton tree splitting functions


2-parton one-loop splitting functions

Z. Bern L. Dixon D. Dunbar D. Kosower 1994; Z. Bern W. Kilgore C. Schmidt VDD 1998-99; D. Kosower P. Uwer 1999; D. Kosower 2003

**universal** subtraction counterterms

several ideas and works in progress

D. Kosower; S. Weinzierl; the Gehrmanns & G. Heinrich 2003
S. Frixione M. Grazzini 2004; G. Somogyi Z. Trocsanyi VDD 2005

but completely figured out only for \(e^+e^- \rightarrow 3\) jets

the Gehrmanns & N. Glover 2005
LHC parton kinematics

\[ x_{1,2} = (M/14 \text{ TeV}) \exp(\pm y) \]

\[ Q = M \]

\[ y = 6, 4, 2, 0, 2, 4, 6 \]

\[ M = 10 \text{ TeV} \]

\[ M = 1 \text{ TeV} \]

\[ M = 100 \text{ GeV} \]

\[ M = 10 \text{ GeV} \]

HERA

fixed target

J. Stirling
Evolution

factorisation scale $\mu_F$ is arbitrary

cross section cannot depend on $\mu_F$

$$\mu_F \frac{d\sigma}{d\mu_F} = 0$$

implies DGLAP equations

$$\mu_F \frac{df_a(x, \mu_F^2)}{d\mu_F} = P_{ab}(x, \alpha_S(\mu_F^2)) \otimes f_b(x, \mu_F^2) + O\left(\frac{1}{Q^2}\right)$$

$$\mu_F \frac{d\hat{\sigma}_{ab}(Q^2/\mu_F^2, \alpha_S(\mu_F^2))}{d\mu_F} = -P_{ac}(x, \alpha_S(\mu_F^2)) \otimes \hat{\sigma}_{cb}(Q^2/\mu_F^2, \alpha_S(\mu_F^2)) + O\left(\frac{1}{Q^2}\right)$$

$P_{ab}(x, \alpha_S(\mu_F^2))$ is calculable in pQCD

V. Gribov L. Lipatov; Y. Dokshitzer
G. Altarelli G. Parisi
Parton distribution functions (PDF)

factorisation for the structure functions (e.g. $F_{2}^{ep}$, $F_{L}^{ep}$)

$\mathcal{F}_i(x, \mu_F^2) = C_{ij} \otimes q_j + C_{ig} \otimes g$

with the convolution $[a \otimes b](x) \equiv \int_{x}^{1} \frac{dy}{y} a(y) b\left(\frac{x}{y}\right)$

$C_{ij}$, $C_{ig}$ coefficient functions

$q_i(x, \mu_F^2)$, $g(x, \mu_F^2)$ PDF’s

DGLAP evolution equations

$$\frac{d}{d \ln \mu_F^2} \begin{pmatrix} q_i \\ g \end{pmatrix} = \begin{pmatrix} P_{q_i q_j} & P_{q_j g} \\ P_{g q_i} & P_{g g} \end{pmatrix} \otimes \begin{pmatrix} q_j \\ g \end{pmatrix}$$

perturbative series $P_{ij} \approx \alpha_s P_{ij}^{(0)} + \alpha_s^2 P_{ij}^{(1)} + \alpha_s^3 P_{ij}^{(2)}$

anomalous dimension $\gamma_{ij}(N) = -\int_{0}^{1} dx x^{N-1} P_{ij}(x)$
PDF’s

General structure of the quark-quark splitting functions

\[
P_{q_i q_k} = P_{\bar{q}_i \bar{q}_k} = \delta_{ik} P_{qq}^v + P_{qq}^s
\]

\[
P_{q_i \bar{q}_k} = P_{\bar{q}_i q_k} = \delta_{ik} P_{q\bar{q}}^v + P_{q\bar{q}}^s
\]

Flavour non-singlet

Flavour asymmetry

\[
q_{ns,ik}^\pm = q_i \pm \bar{q}_i - (q_k \pm \bar{q}_k)
\]

\[
P_{ns}^\pm = P_{qq}^v \pm P_{q\bar{q}}^v
\]

Sum of valence distributions of all flavours

\[
q_{ns}^v = \sum_{r=1}^{n_f} (q_r - \bar{q}_r)
\]

\[
P_{ns}^v = P_{qq}^v - P_{q\bar{q}}^v + n_f (P_{qq}^s - P_{q\bar{q}}^s)
\]

Flavour singlet

\[
q_s = \sum_{i=1}^{n_f} (q_i + \bar{q}_i)
\]

\[
\frac{d}{d \ln \mu_F^2} \begin{pmatrix} q_s \\ g \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q_s \\ g \end{pmatrix}
\]

With

\[
P_{qq} = P_{ns}^+ + n_f (P_{qq}^s + P_{q\bar{q}}^s)
\]

\[
P_{qg} = n_f P_{qig} \quad P_{gq} = P_{gqi}
\]
PDF history

leading order (or one-loop) anomalous dim/splitting functions

Gross Wilczek 1973; Altarelli Parisi 1977

NLO (or two-loop)

\[ F_2, F_L \]

anomalous dim/splitting functions

Bardeen Buras Duke Muta 1978
Curci Furmanski Petronzio 1980

NNLO (or three-loop)

\[ F_2, F_L \]

anomalous dim/splitting functions

Zijlstra van Neerven 1992; Moch Vermaseren 1999
Moch Vermaseren Vogt 2004

the calculation of the three-loop anomalous dimension is the toughest calculation ever performed in perturbative QCD!

one-loop \[ \gamma_{ij}^{(0)} / P_{ij}^{(0)} \] 18 Feynman diagrams

two-loop \[ \gamma_{ij}^{(1)} / P_{ij}^{(1)} \] 350 Feynman diagrams

three-loop \[ \gamma_{ij}^{(2)} / P_{ij}^{(2)} \] 9607 Feynman diagrams

20 man-year-equivalents, \( 10^6 \) lines of dedicated algebra code
Numerical examples

exact NNLO results, estimates from fixed moments and leading small-$x$ term
HERA $F_2$

Bjorken-scaling violations

H1, ZEUS: ongoing fits for PDF’s; so far **NNLO** not included
PDF global fits

global fits

MRST: Martin Roberts Stirling Thorne

CTEQ: Pumplin et al.

Alekhin (DIS data only)

method

Perform fit by minimising $\chi^2$ to all data, including both statistical and systematic errors.

Start evolution at some $Q_0^2$, where PDF's are parametrised with functional form, e.g.

$$xf(x, Q_0^2) = (1 - x)^\eta (1 + \epsilon x^{0.5} + \gamma x) x^\delta$$

Cut data at $Q^2 > Q_{\text{min}}^2$ and at $W^2 > W_{\text{min}}^2$ to avoid higher twist contamination.

Allow $\bar{u} \neq \bar{d}$ as implied by

- E866 Drell-Yan asymmetry data

accuracy

NLO evolution and fixed moments of NNLO

- no prompt photon data included in the fits

\[\begin{align*}
\text{H1, ZEUS} & \quad F_2^{ep}(x, Q^2), \quad F_2^{e-}(x, Q^2) \\
\text{BCDMS} & \quad F_2^{\mu p}(x, Q^2), \quad F_2^{\mu d}(x, Q^2) \\
\text{NMC} & \quad F_2^{ep}(x, Q^2), \quad F_2^{e-}(x, Q^2), \quad (F_2^{\mu}(x, Q^2) / F_2^{\mu p}(x, Q^2)) \\
\text{SLAC} & \quad F_2^{\mu p}(x, Q^2), \quad F_2^{\mu d}(x, Q^2) \\
\text{E665} & \quad F_2^{\mu p}(x, Q^2), \quad F_2^{\mu d}(x, Q^2) \\
\text{CCFR} & \quad F_2^{\nu (\bar{\nu}) p}(x, Q^2), \quad F_3^{\nu (\bar{\nu}) p}(x, Q^2) \\
\end{align*}\]
Conclusions

QCD is an extensively developed and tested gauge theory

a lot of progress in the last 4-5 years in

- MonteCarlo generators
- NLO cross sections with one more jet
- NNLO computations

better and better approximations of signal and background for Higgs and New Physics