Recent developments in QCD

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QCD

- an unbroken Yang-Mills gauge field theory featuring asymptotic freedom and confinement

- in non-perturbative regime (low Q^2) many approaches: lattice, Regge theory, χ PT, large N_c, HQET

- in perturbative regime (high Q^2) QCD is a precision toolkit for exploring Higgs & BSM physics

QCD at LHC

Precise determination of

- strong coupling constant α_s
- parton distributions
- LHC parton luminosity

Precise prediction for

- Higgs production
- new physics processes
- their backgrounds

Goal: to make theoretical predictions of signals and backgrounds as accurate as the LHC data

LHC: present & future

- calibrate the detectors, and re-discover the SM
  i.e. measure known cross sections: jets, $W, Z, t\bar{t}$

- understand the EWSB/find New-Physics signals
  (ranging from Z' to leptons, to gluinos in SUSY
  decay chains, to finding the Higgs boson)

- constrain and model the New-Physics theories

in all the steps above (except probably Z' to leptons)

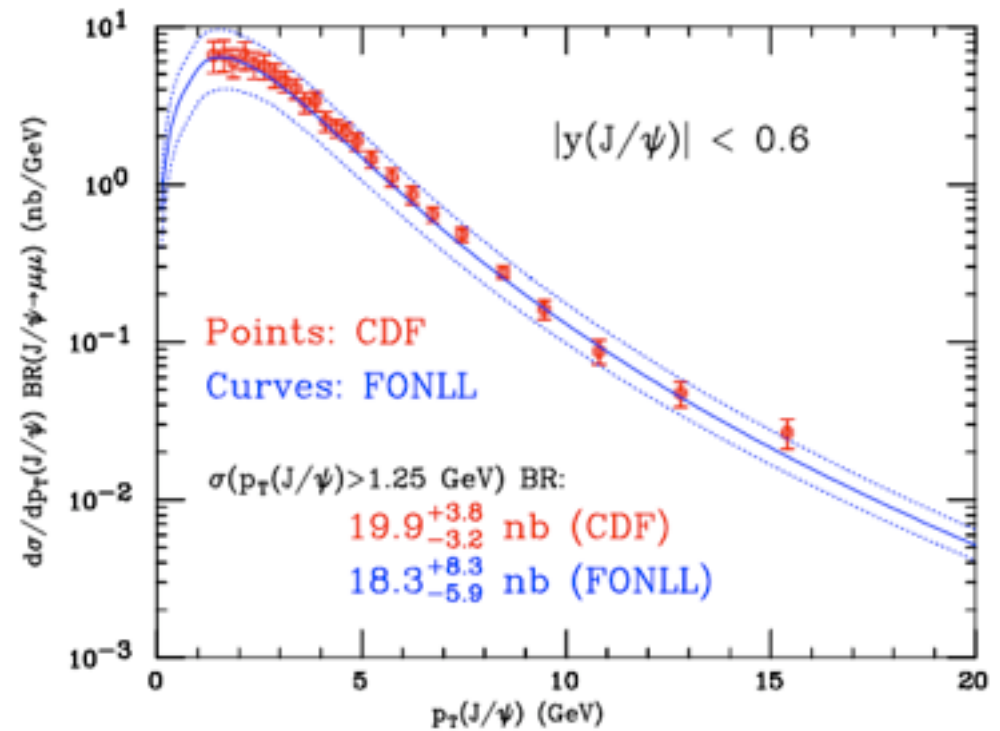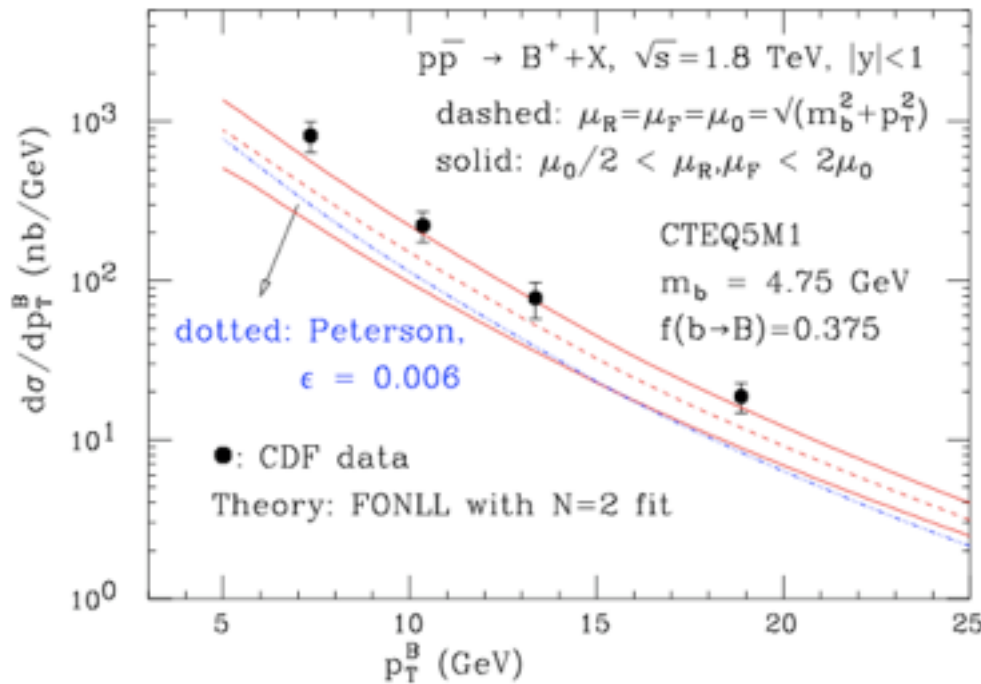precise QCD predictions play a crucial role

Tales from the past

B production in the 90's
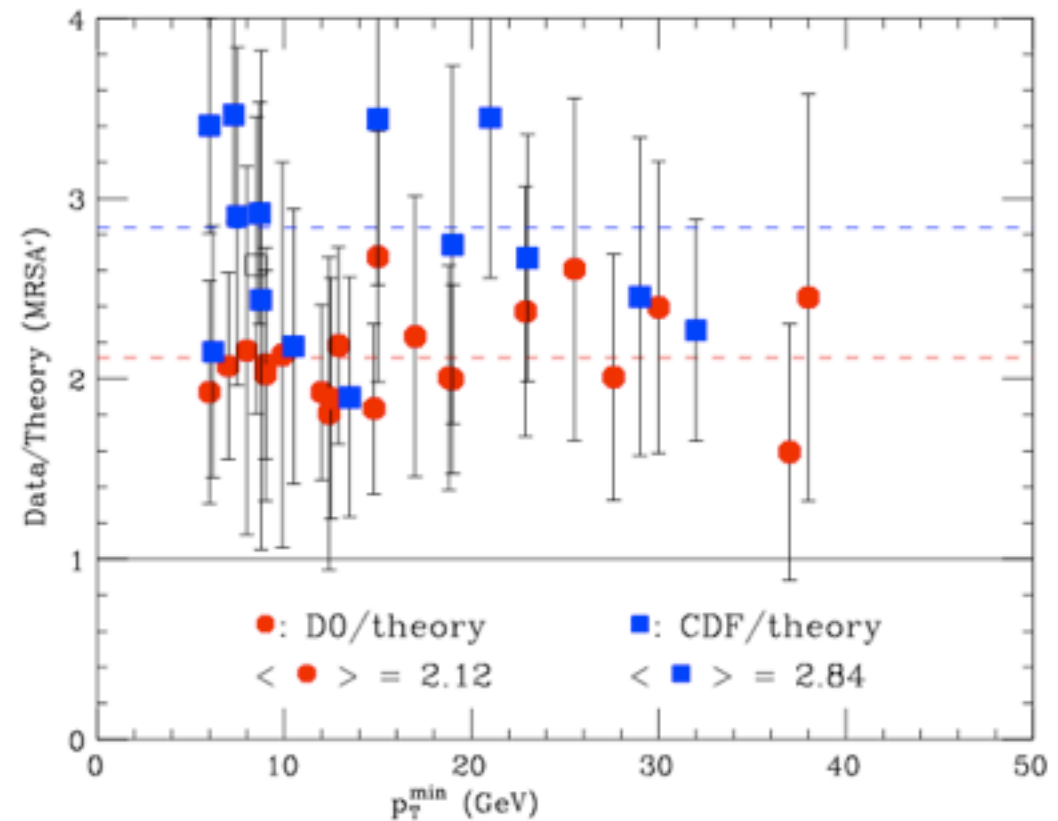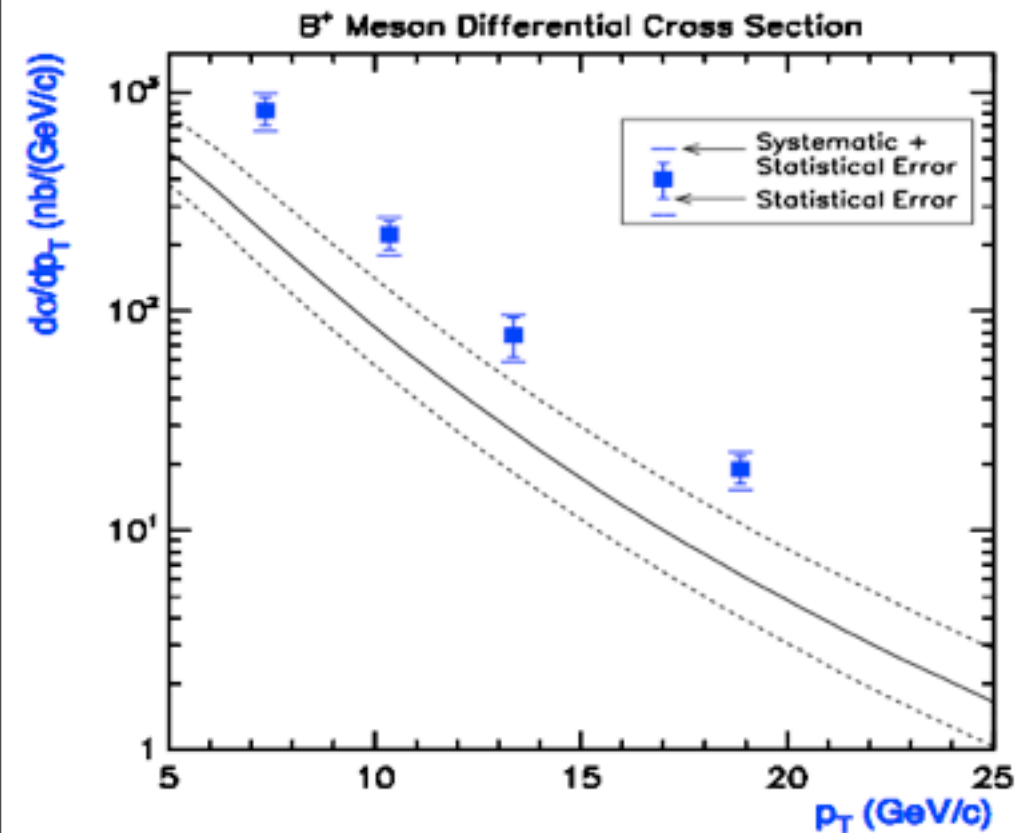
discrepancy between Tevatron data and NLO prediction

B cross section in $p\bar{p}$ collisions at 1.96 TeV

$$d\sigma(p\bar{p} \to H_b X, H_b \to J/\psi \ X)/dp_T(J/\psi)$$

FONLL = NLO + NLL   total x-sect is $19.4 \pm 0.3(stat)^{+2.1}_{-1.9}(syst)$ nb

Cacciari, Frixione, Mangano, Nason, Ridolfi 2003   CDF hep-ex/0412071

use of updated fragmentation functions by Cacciari & Nason
and resummation

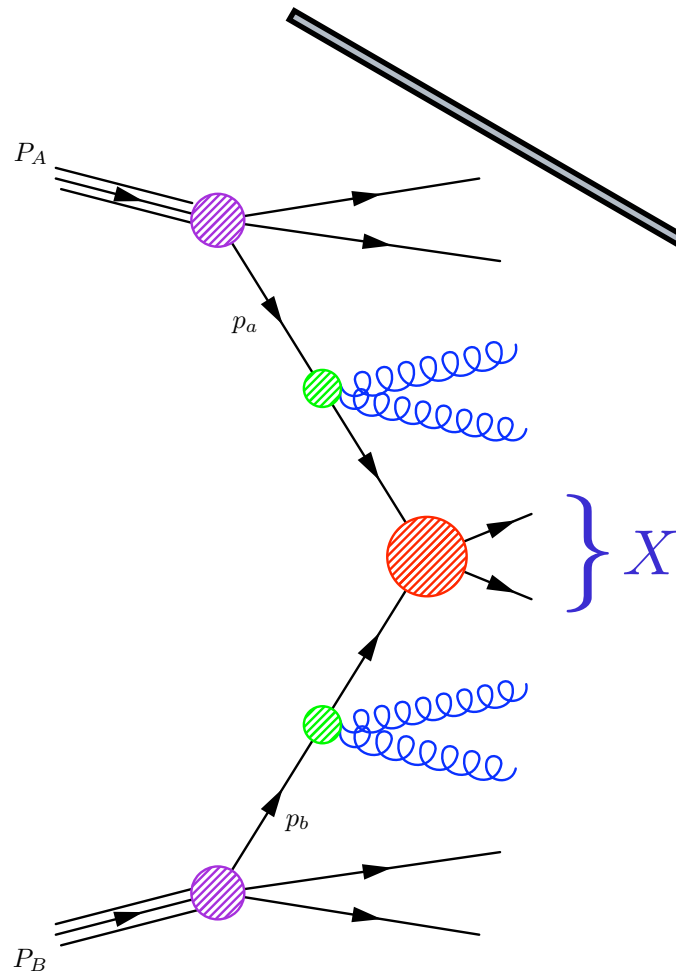good agreement with data   no New Physics

QCD at high Q^2

- Parton model

- Perturbative QCD

  - factorisation

  - universality of IR behaviour

  - cancellation of IR singularities

  - IR safe observables: inclusive rates

    - jets

    - event shapes

Factorisation

computed in pQCD

extracted from data
evolved through DGLAP

is the separation between
the short- and the long-range interactions

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_{a/A}(x_1, \mu_F^2) \, f_{b/B}(x_2, \mu_F^2)$$

$$\times \quad \hat{\sigma}_{ab \to X} \left(x_1, x_2, \{p_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_F^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2} \right)$$

$$\} X$$

$$X = W, Z, H, Q\bar{Q}, \text{high-}E_T \text{jets}, \dots$$

$\hat{\sigma}$ is known as a fixed-order expansion in α_S

$$\hat{\sigma} = C\alpha_S^n (1 + c_1 \alpha_S + c_2 \alpha_S^2 + \dots)$$

$c_1 = $ NLO $\qquad c_2 = $ NNLO
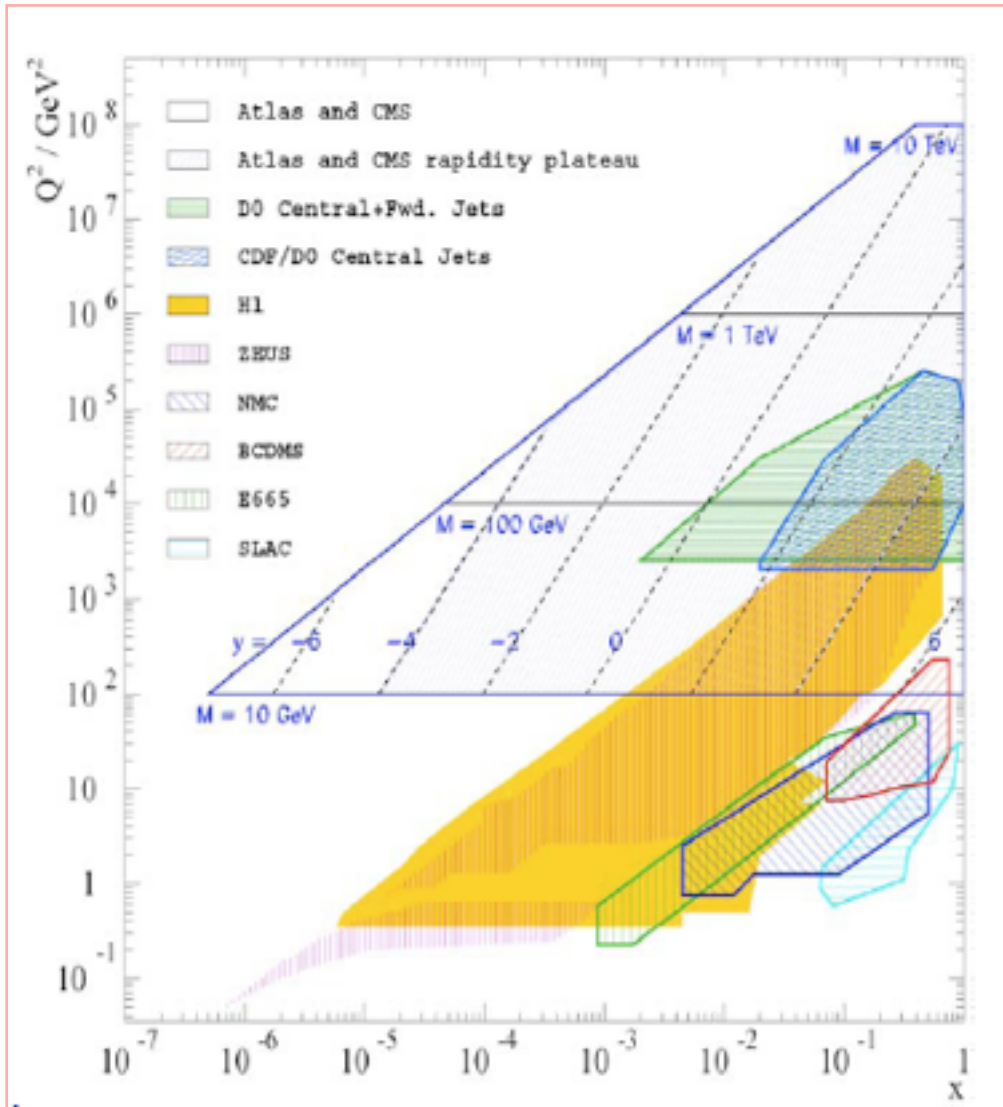
or as an all-order resummation

$$\hat{\sigma} = C\alpha_S^n [1 + (c_{11}L + c_{10})\alpha_S + (c_{22}L^2 + c_{21}L + c_{20})\alpha_S^2 + \dots]$$

where $\quad L = \ln(M/q_T), \ln(1-x), \ln(1/x), \ln(1-T), \dots$

$c_{11}, c_{22} = $ LL $\qquad c_{10}, c_{21} = $ NLL $\qquad c_{20} = $ NNLL

LHC kinematic reach

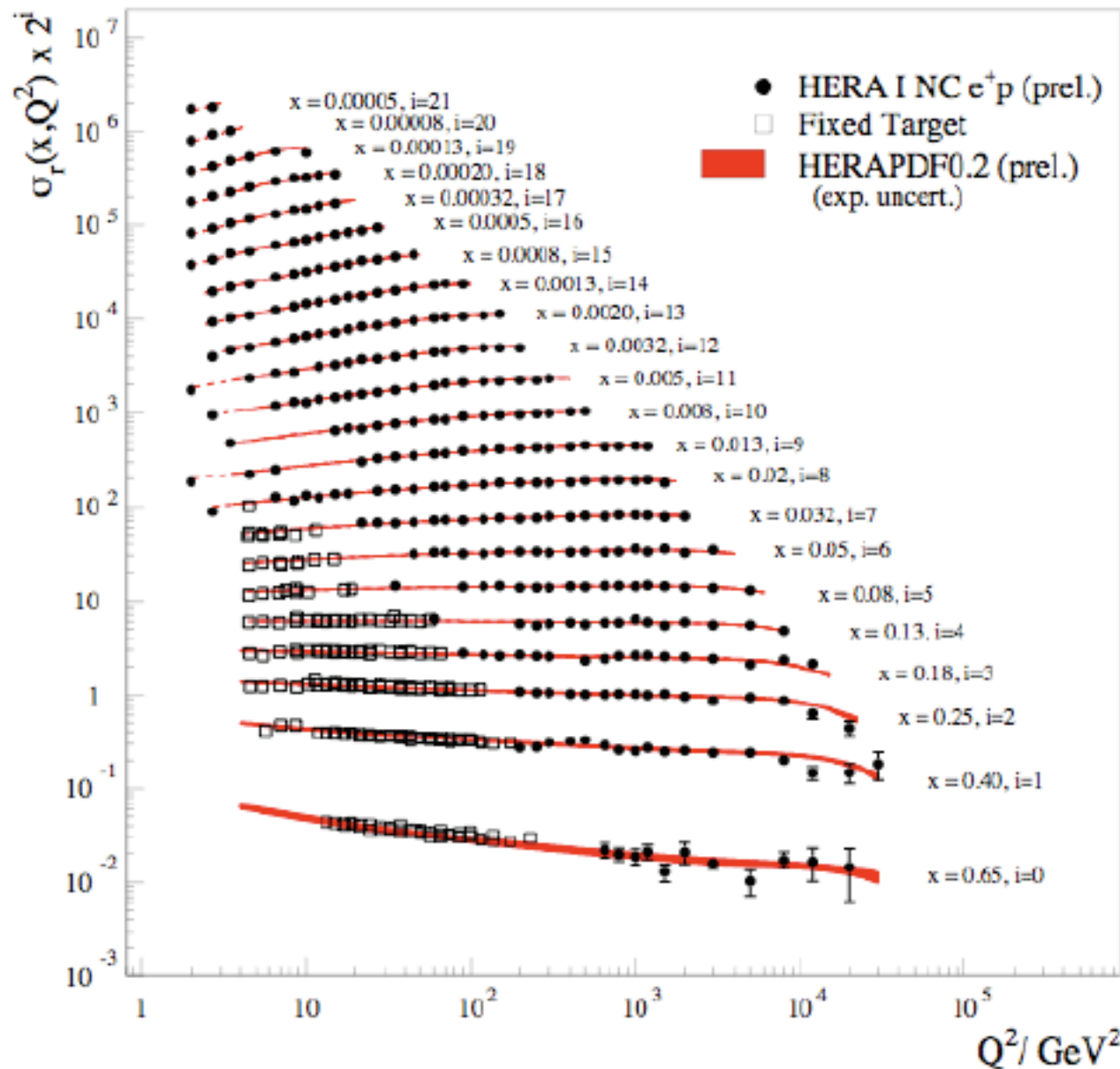LHC opens up a
new kinematic range

x range covered by HERA
but Q^2 range must be provided
by DGLAP evolution

100-200 GeV physics is
large x physics (valence quarks)
at Tevatron, but smaller x physics
(gluons & sea quarks) at the LHC

rapidity distributions
span widest x range

Feynman x's for the production of a particle of mass M $x_{1,2} = \dfrac{M}{14\,\text{TeV}}\, e^{\pm y}$

HERA F_2

DIS09

large violations at small x

small violations at large x
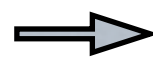
horizontal lines $\Longrightarrow$ Bjorken scaling

straight (non-horizontal) lines $\Longrightarrow$ scaling violations, logarithmic in Q^2
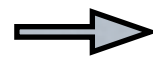
PDF global fits

J. Stirling, KITP collider conf 2004

global fits

MRST, MSTW: Martin *et al.*

CTEQ: Pumplin *et al.*

Alekhin (DIS data only)

method

Perform fit by minimising χ^2 to all data, including both statistical and systematic errors

Start evolution at some Q_0^2, where PDF's are parametrised with functional form, e.g.
$$xf(x, Q_0^2) = (1-x)^\eta (1 + \epsilon x^{0.5} + \gamma x)x^\delta$$

Cut data at $Q^2 > Q_{min}^2$ and at $W^2 > W_{min}^2$ to avoid higher twist contamination

Allow $\bar{u} \neq \bar{d}$ as implied by E866 Drell-Yan asymmetry data

accuracy

NNLO evolution

H1, ZEUS $F_2^{e^+p}(x, Q^2)$, $F_2^{e^-p}(x, Q^2)$

BCDMS $F_2^{\mu p}(x, Q^2)$, $F_2^{\mu d}(x, Q^2)$

NMC $F_2^{\mu p}(x, Q^2)$, $F_2^{\mu d}(x, Q^2)$, $(F_2^{\mu n}(x, Q^2)/F_2^{\mu p}(x, Q^2))$

SLAC $F_2^{\mu p}(x, Q^2)$, $F_2^{\mu d}(x, Q^2)$

E665 $F_2^{\mu p}(x, Q^2)$, $F_2^{\mu d}(x, Q^2)$

CCFR $F_2^{\nu(\bar{\nu})p}(x, Q^2)$, $F_3^{\nu(\bar{\nu})p}(x, Q^2)$

$\rightarrow q, \bar{q}$ at all x and g at medium, small x

H1, ZEUS $F_{2,c}^{e^+p}(x, Q^2) \rightarrow c$

E605, E772, E866 Drell-Yan $pN \rightarrow \mu\bar{\mu} + X \rightarrow \bar{q}\,(g)$

E866 Drell-Yan p,n asymmetry $\rightarrow \bar{u}, \bar{d}$

CDF W rapidity asymmetry $\rightarrow u/d$ ratio at high x

CDF, D0 Inclusive jet data $\rightarrow g$ at high x

CCFR, NuTeV Dimuon data constrains strange sea $s, \bar{s}$

no prompt photon data included in the fits

PDF: recent developments

- more systematic treatment of uncertainties in global fits

  MSTW, CTEQ, Alekhin
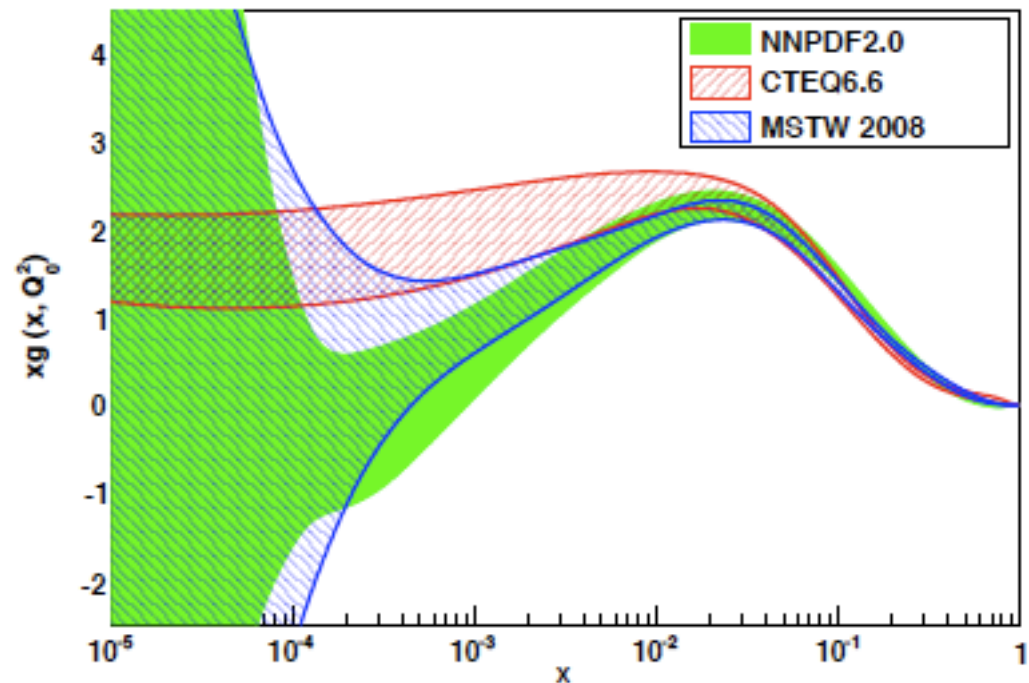
- more accurate treatment of heavy flavours in the vicinity of the quark mass (few % effect on Drell-Yan at the LHC)

- PDF's from neural network global fit (NNPDF), based on unbiased priors
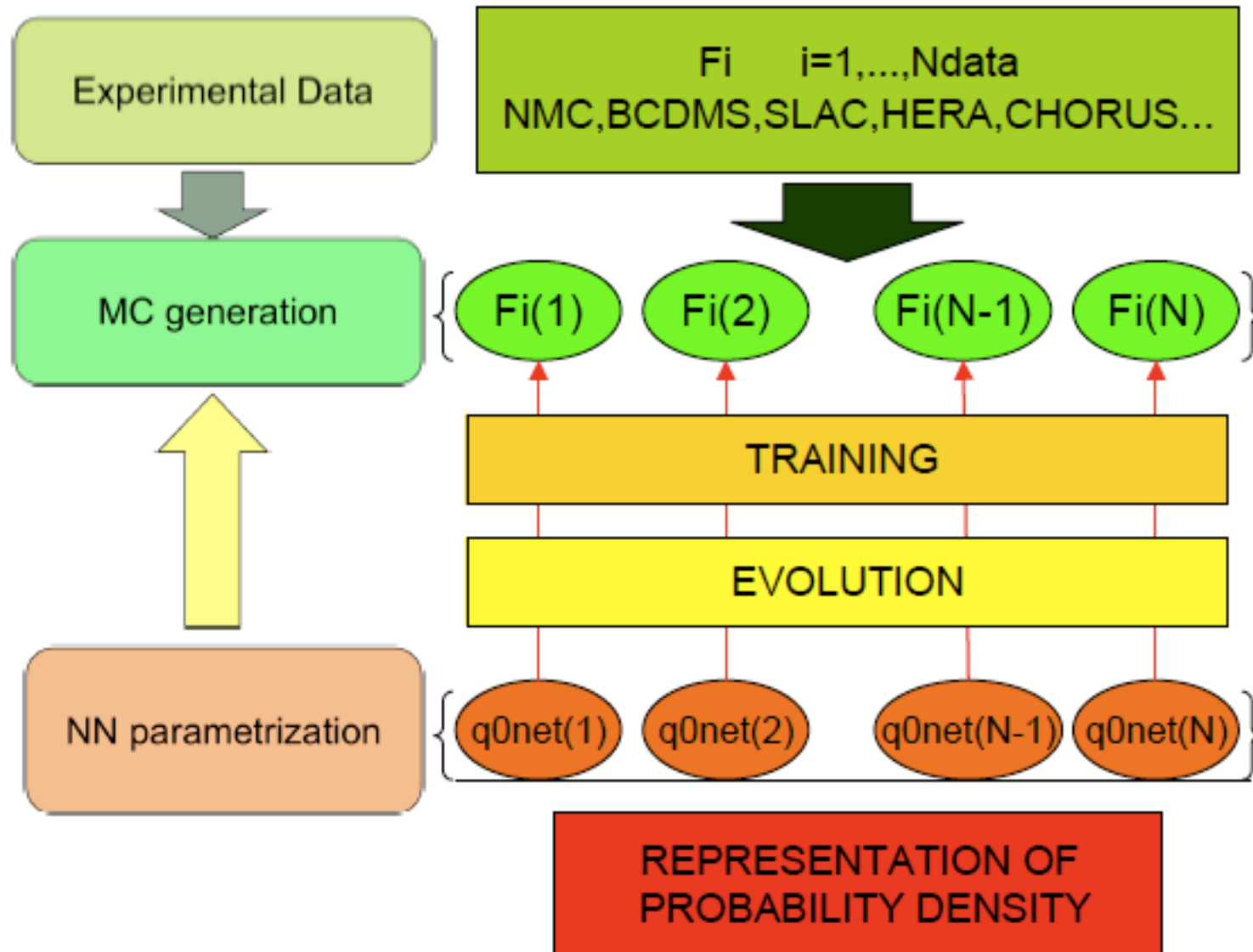
NNPDF2.0 arXiv:1002.4407

gluon distribution

note the larger uncertainty in NNPDF at small x

NNPDF

Ubiali (NNPDF Coll.) DIS09

stop before over-training: NN fitting of statistical fluctuations

NNPDF

Ubiali (NNPDF Coll.) DIS09

Determination of unbiased PDFs with faithful estimation of their uncertainties.

$$\langle \mathcal{F}[f_i(x)] \rangle = \int [\mathcal{D}f_i] \, \mathcal{F}[f_i(x)] \mathcal{P}[f_i(x)] \quad \rightarrow \quad \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} \mathcal{F}[f_i^{(k)(\text{net})}(x)]$$

$$\sigma_{\mathcal{F}[f(x)]} = \sqrt{\langle \mathcal{F}[f(x)]^2 \rangle - \langle \mathcal{F}[f(x)] \rangle^2}$$

* The measure $\mathcal{P}[f_i(x)]$ in space of PDFs is determined with MC method.
* Use all information contained in experiments.
* Redundant parametrization of PDFs: reduce bias.
* Statistic estimators to assess errors, correlations, stability and size of systematics.

sort of path-integral method in the space of PDFs

Parton cross section
3 complementary approaches to $\hat{\sigma}$

	matrix-elem MC's	fixed-order x-sect	shower MC's
final-state description	hard-parton jets. Describes geometry, correlations, ...	limited access to final-state structure	full information available at the hadron level
higher-order effects: loop corrections	hard to implement: must introduce negative probabilities	straightforward to implement (when available)	included as vertex corrections (Sudakov FF's)
higher-order effects: hard emissions	included, up to high orders (multijets)	straightforward to implement (when available)	approximate, incomplete phase space at large angles
resummation of large logs	?	feasible (when available)	unitarity implementation (i.e. correct shapes but not total rates)

M.L. Mangano KITP collider conf 2004

Parton shower MonteCarlo generators
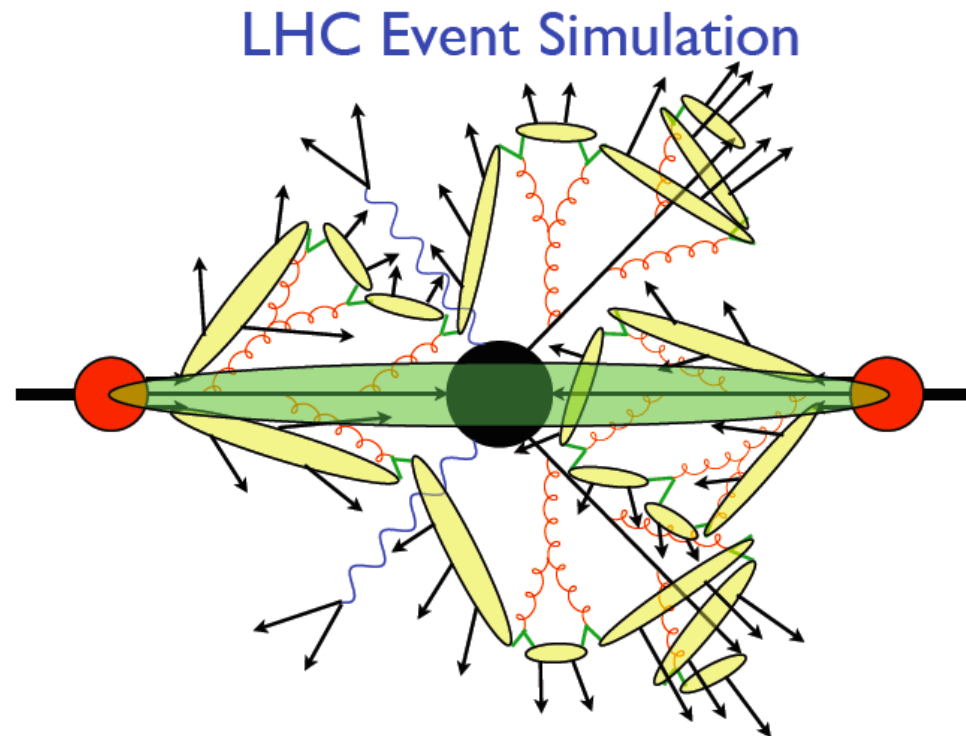
HERWIG   B. Webber *et al.* 1992

re-written as a C++ code (HERWIG++)

PYTHIA   T. Sjostrand 1994   (also re-written as a C++ code)

SHERPA   F. Krauss *et al.* 2003

model parton showering and hadronisation

LHC Event Simulation

Matrix-element MonteCarlo generators

⬆ several automated codes to yield
large number of (up to 8-9) final-state partons

⬆ can be straightforwardly interfaced to parton-shower MC's

⬆ ideal to scout new territory

⬇ large dependence on ren/fact scales
example: Higgs (via gluon fusion) + 2 jets is $\alpha_s^4(Q^2)$

➡ unreliable for precision calculations

Matrix-element MonteCarlo generators

- multi-parton LO generation: processes with many jets (or V/H bosons)

  - ALPGEN   M.L.Mangano M. Moretti F. Piccinini R. Pittau A. Polosa 2002

  - MADGRAPH/MADEVENT   W.F. Long F. Maltoni T. Stelzer 1994/2003

  - COMPHEP   A. Pukhov et al. 1999

  - GRACE/GR@PPA   T. Ishikawa *et al.* K. Sato *et al.* 1992/2001

  - HELAC   C. Papadopoulos *et al.* 2000

- processes with 6 final-state fermions

  - PHASE   E. Accomando A. Ballestrero E. Maina 2004

- merged with parton showers

  - all of the above, merged with HERWIG or PYTHIA

MonteCarlo interfaces

CKKW   S. Catani F. Krauss R. Kuhn B. Webber 2001
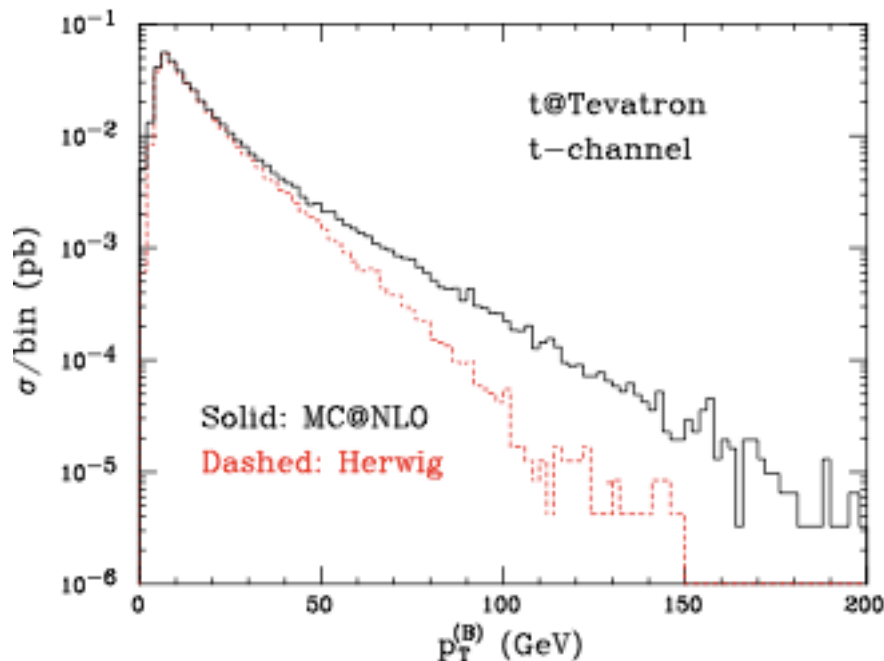
MLM   L. Lonnblad 2002   M.L. Mangano 2005

procedures to interface parton subprocesses with
a different number of final states to parton-shower MC's

MC@NLO   S. Frixione B. Webber 2002

POWHEG   P. Nason 2004

procedures to interface NLO computations to parton-shower MC's

Single top in MC@NLO

Frixione Laenen Motylinski Webber 2005

at low p_T, parton shower
models collinear radiation

at high p_T, NLO
models hard radiation

Matrix-element MonteCarlo generator at NLO

- desirable to have a multi-parton NLO generator
  interfaced to a parton shower:
  a sort of MadGraph cum MC@NLO

- a step in this direction: automation of subtraction of IR divergences

  Frederix Frixione Maltoni Stelzer 2009

  MadGraph provides real amplitude
  user inputs virtual amplitude
  procedure provides subtraction counterterms

Next to Leading Order

- Jet structure: final-state collinear radiation

- PDF evolution: initial-state collinear radiation

- Opening of new channels

- Reduced sensitivity to fictitious input scales: μ_R, μ_F

  - predictive normalisation of observables

    - first step toward precision measurements
    - accurate estimate of signal and background for Higgs and new physics

- Matching with parton-shower MC's:
  MC@NLO   POWHEG

NLO cross sections: experimenter's wishlist

2005 Les Houches

Table 42: The LHC "priority" wishlist for which a NLO computation seems now feasible.

process $(V \in \{Z, W, \gamma\})$	relevant for
1. $pp \to V V$ jet	$t\bar{t}H$, new physics
2. $pp \to t\bar{t}\, b\bar{b}$	$t\bar{t}H$
3. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
4. $pp \to V V\, b\bar{b}$	VBF$\to H \to VV$, $t\bar{t}H$, new physics
5. $pp \to V V + 2$ jets	VBF$\to H \to VV$
6. $pp \to V + 3$ jets	various new physics signatures
7. $pp \to V V V$	SUSY trilepton

High (EXP) demand for cross sections of Y + n jets
Y = vector boson(s), Higgs, heavy quark(s), ...
Big TH community effort

To compute the cross section of Y + n jets, we need:
1) tree-level amplitude for Y + (n+3) partons
2) one-loop amplitude for Y + (n+2) partons
3) a method to cancel the IR divergences
   and so to compute the cross section

3: until the mid 90's, we did not have systematic methods
   to cancel the IR divergences
2: until 2007-8, we did not have systematic methods
   to compute the one-loop amplitudes

NLO history of final-state distributions

- $e^+e^- \rightarrow 3$ jets   K. Ellis Ross Terrano 1981

    $e^+e^- \rightarrow 4$ jets   Bern *et al.*; Glover *et al.*; Nagy Trocsanyi 1996-97

- $e^+e^- \rightarrow 4$ fermions   Denner Dittmaier Roth Wieders 2005

- $pp \rightarrow 1, 2$ jets   K. Ellis Sexton 1986   Giele Glover Kosower 1993

    $pp \rightarrow 3$ jets   Bern Dixon Kosower; Kunszt Signer Trocsanyi 1993-1995

                               Nagy 2001

- $pp \rightarrow V + 1$ jet   Giele Glover Kosower 1993

    $pp \rightarrow V + 2$ jet   Bern Dixon Kosower Weinzierl; Campbell Glover Miller 1996-97

                                 Campbell K. Ellis 2003

    $pp \rightarrow V + 3$ jet   Berger *et al.* (BlackHat); K. Ellis Melnikov Zanderighi 2009

- $pp \rightarrow V b\bar{b}$ Campbell K. Ellis 2003

- $pp \rightarrow VV$ Ohnemus Owens, Baur *et al.* 1991-96; Dixon *et al.* 2000

    $pp \rightarrow VV +$ jet   Dittmaier Kallweit Uwer; Campbell K. Ellis Zanderighi 2007

- $pp \rightarrow VVV$ Lazopoulos Melnikov Petriello; Hankele Zeppenfeld 2007

                   Binoth Ossola Papadopoulos Petriello 2008

- $pp \rightarrow \gamma\gamma$ Bailey *et al.* 1992; Binoth *et al.* 1999

    $pp \rightarrow \gamma\gamma + 1$ jet   Bern *et al.* 1994   Del Duca Maltoni Nagy Trocsanyi 2003

NLO history of final-state distributions

$pp \rightarrow Q\bar{Q}$ — Dawson K. Ellis Nason 1989 Mangano Nason Ridolfi 1992

$pp \rightarrow Q\bar{Q} + 1$ jet — Brandenburg Dittmaier Uwer Weinzierl 2005-7
Dittmaier Kallweit Uwer 2007-8

$pp \rightarrow Q\bar{Q} + 2$ jets — van Hameren Papadopoulos Pittau 2009
Bevilacqua Czakon Papadopoulos Worek 2010

$pp \rightarrow t\,\bar{t}\,Z$ — Lazopoulos McElmurry Melnikov Petriello 2008

$pp \rightarrow t\,\bar{t}\,b\,\bar{b}$ — Bredenstein Denner Dittmaier Pozzorini 2009

$pp \rightarrow t\,(+\,W)$ — Harris Laenen Phaf Sullivan Weinzierl 2002
Campbell K. Ellis Tramontano 2004; Cao Yuan et al. 2004-5
W: Campbell Tramontano 2005

$pp \rightarrow H + 1$ jet — (GGF; $\infty\ m_t$) C. Schmidt 1997 De Florian Grazzini Kunszt 1999

$pp \rightarrow H + 2$ jets — (GGF; $\infty\ m_t$) Campbell K. Ellis Zanderighi 2006
(WBF) Campbell K. Ellis; Figy Oleari Zeppenfeld 2003
Ciccolini Denner Dittmaier 2007 (includes s channel)

$pp \rightarrow H + 3$ jets — (WBF) Figy Hankele Zeppenfeld 2007

$pp \rightarrow HQ\bar{Q}$ — Beenakker *et al.* ; Dawson *et al.* 2001

$pp \rightarrow H\,Q$ — t: Maltoni Paul Stelzer Willenbrock 2001
b: Campbell, K. Ellis Maltoni Willenbrock 2002

Tuesday, March 2, 2010

… summarising

- in the past, long time span to add one more jet to a x-section

- $2 \to 2$ and $2 \to 3$ processes:
  almost all computed and included into NLO packages

- $2 \to 4$ processes: very few computed

  $e^+ e^- \to 4 \text{ fermions}$ Denner Dittmaier Roth Wieders 2005

  $pp \to t\bar{t}b\bar{b}$ Bredenstein Denner Dittmaier Pozzorini 2009

  $pp \to V + 3 \text{ jet}$ Berger *et al.* (BlackHat); K. Ellis Melnikov Zanderighi 2009

  $pp \to Q\bar{Q} + 2 \text{ jets}$ Bevilacqua Czakon Papadopoulos Worek 2010

- $2 \to 5$ processes: none

$pp \rightarrow t\,\bar{t} + 2\,\text{jets}$ at NLO

Scale dependence of total x-sect
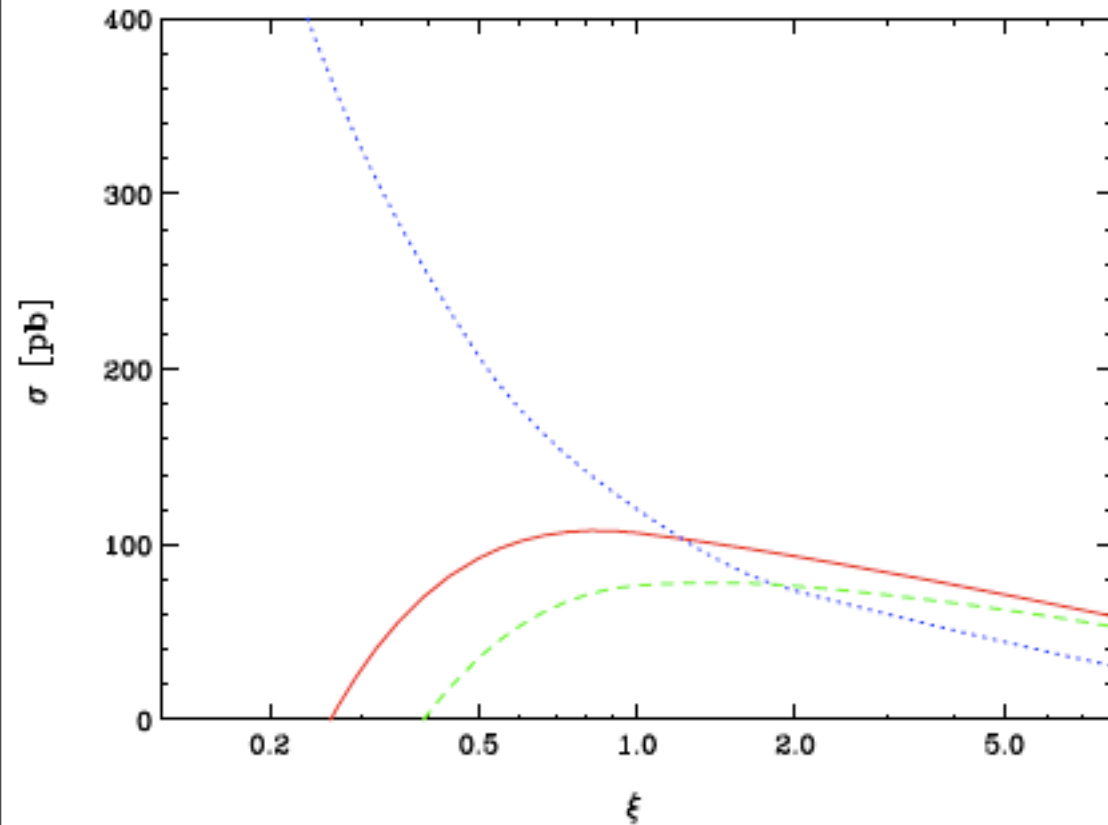
$$\mu_R = \mu_F = \xi\,\mu_0 \quad \text{with} \quad \mu_0 = m_t$$

dots: LO
solid: NLO
dash: NLO with jet veto of 50 GeV

- reducible background to $pp \rightarrow H\,t\,\bar{t}$

- NLO/LO $\equiv$ K factor = 0.89
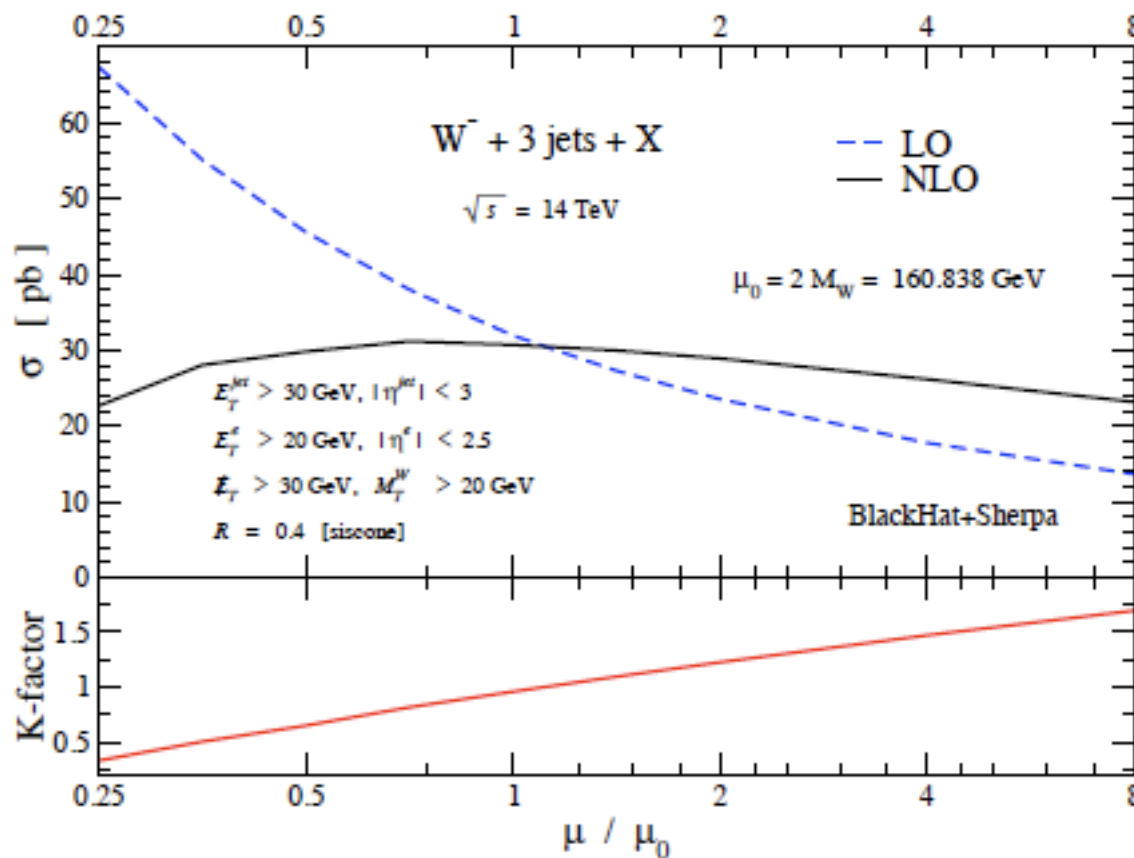
- Reduced theoretical error: 40-70% at LO; 12-13% at NLO

W + 3-jet cross section at NLO

A C++ code based on generalised unitarity, and on-shell recursion for the rational parts

computes
- one-loop 6-gluon (and MHV 7- and 8-gluon) amplitudes
- one-loop *W* + 5-parton amplitudes
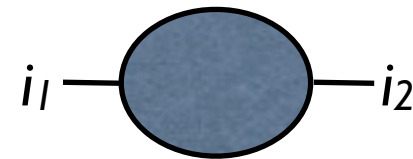- NLO *W* + 3-jet cross section

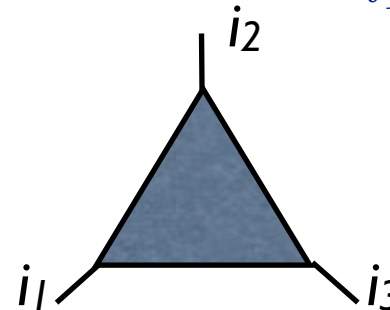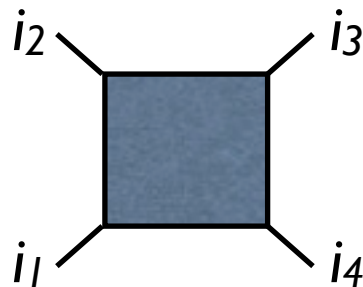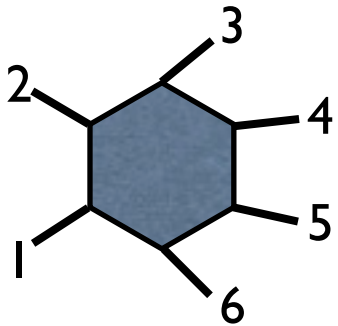# one-loop amplitudes

- one-loop *n*-point amplitudes A_n are IR divergent

  ↑ IR divergences are universal   Kunszt Signer Trocsanyi 1994; Catani 1998

  ↓ IR finite terms are process dependent:
  many final-state particles → many scales → lengthy expressions

- A_n can be reduced to boxes, triangles and bubbles
  with rational coefficients

$$A_n = \sum_{i_1 i_2 i_3 i_4} d_{i_1 i_2 i_3 i_4}\, I^D_{i_1 i_2 i_3 i_4} + \sum_{i_1 i_2 i_3} c_{i_1 i_2 i_3}\, I^D_{i_1 i_2 i_3} + \sum_{i_1 i_2} b_{i_1 i_2}\, I^D_{i_1 i_2}$$

I: master integrals
b, c, d: rational functions of kinematic variables

- higher polygons contribute only to $O(\varepsilon)$

NLO progress: unitarity method

- use unitarity cuts: Cutkovsky rule $\dfrac{1}{p^2 + i0} \to 2\pi i\, \delta^+(p^2)$

  to factorise coefficients of A_n into products of tree amplitudes

- compute b, c, d with $D=4$: cut-constructible terms
  $O(\varepsilon)$ part of b, c, d: rational term R_n Bern Dixon Dunbar Kosower 1994

  $$A_n = \sum_{i_1 i_2 i_3 i_4} d_{i_1 i_2 i_3 i_4}\, I^4_{i_1 i_2 i_3 i_4} + \sum_{i_1 i_2 i_3} c_{i_1 i_2 i_3}\, I^4_{i_1 i_2 i_3} + \sum_{i_1 i_2} b_{i_1 i_2}\, I^4_{i_1 i_2} + R_n$$

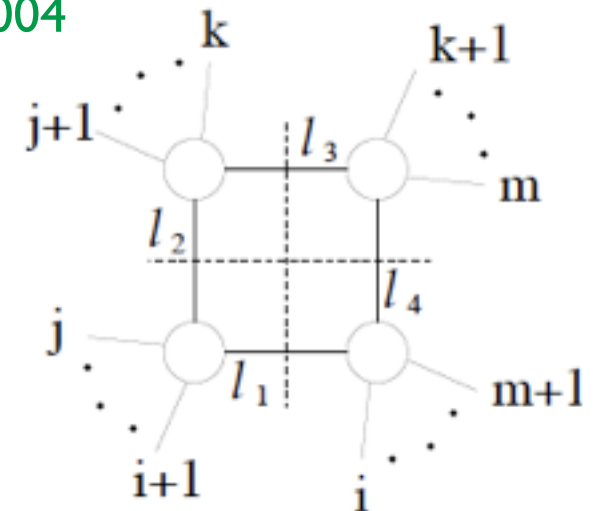- R_n computable through on-shell recursion

  Berger Bern Dixon Forde Kosower 2006

- generalized unitarity: Britto Cachazo Feng 2004
  quadruple cuts with complex momenta

  box coefficient d_i determined by
  product of 4 tree amplitudes

  however, c_i, b_i still difficult to extract from triple
  and double cuts because terms already included
  in d_i must be subtracted

NLO progress: OPP method

- reduction of one-loop amplitude at integrand level

$$A(q') = \frac{N(q)}{\bar{D}_1 \cdots \bar{D}_m} \qquad \bar{D}_i = (q' + p_i)^2 - m_i^2 \qquad q' = q + \tilde{q} \qquad q \cdot \tilde{q} = 0$$

q' lives in D dimensions; q lives in 4 dimensions
ε part of numerator treated separately

- partial fraction the numerator into terms
  with 4, 3, 2, 1 denominator factors

$$N(q) = \sum_{i_1 i_2 i_3 i_4} (d_{i_1 i_2 i_3 i_4} + \tilde{d}_{i_1 i_2 i_3 i_4}) \prod_{i \neq i_1 i_2 i_3 i_4} D_i$$

$$+ \sum_{i_1 i_2 i_3} (c_{i_1 i_2 i_3} + \tilde{c}_{i_1 i_2 i_3}) \prod_{i \neq i_1 i_2 i_3} D_i + \sum_{i_1 i_2} (b_{i_1 i_2} + \tilde{b}_{i_1 i_2}) \prod_{i \neq i_1 i_2} D_i$$

$\tilde{d}, \tilde{c}, \tilde{b}$ vanish upon integration

- reduced to problem of fitting d, c, b by evaluating $N(q)$ at different values of q,
  e.g. singling out choices of q such that 4, 3, 2, 1 among all D_i vanish, and then
  inverting the system. First find all possible 4-pt functions, then 3-pt functions, etc.

- for quadruple cuts, similar to BCF, but algorithmic: can be automated

NLO progress: integer dimensions

Giele Kunszt Melnikov 2008

- in D dimensions the one-loop amplitude A_n can be reduced to pentagons, boxes, triangles and bubbles

$$A_n = \sum_{i_1 i_2 i_3 i_4 i_5} e_{i_1 i_2 i_3 i_4 i_5} I^D_{i_1 i_2 i_3 i_4 i_5} + \sum_{i_1 i_2 i_3 i_4} d_{i_1 i_2 i_3 i_4} I^D_{i_1 i_2 i_3 i_4}$$
$$+ \sum_{i_1 i_2 i_3} c_{i_1 i_2 i_3} I^D_{i_1 i_2 i_3} + \sum_{i_1 i_2} b_{i_1 i_2} I^D_{i_1 i_2}$$

take D_s = # of spin states of internal particles

in D_s dimensions, gluons have D_s-2 spin states, quarks have $2^{(D_s-2)/2}$ spin states

$$A_n^{(D, D_s)} = \int d^D q \frac{N^{(D_s)}(q)}{d_1 \cdots d_n} \qquad \text{with } D \leq D_s$$

- dependence of A on D_s is linear $N^{(D_s)}(q) = N_0(q) + (D_s - 4)N_1(q)$

  - compute N_0, N_1 numerically separately through 2 different integer values of D_s
  - on D_s-dimension cuts, spin density matrix is well defined
  - choose basis of master integrals with no explicit D dependence in the coefficients
  - after reduction to master integrals, continue D to 4-2ε
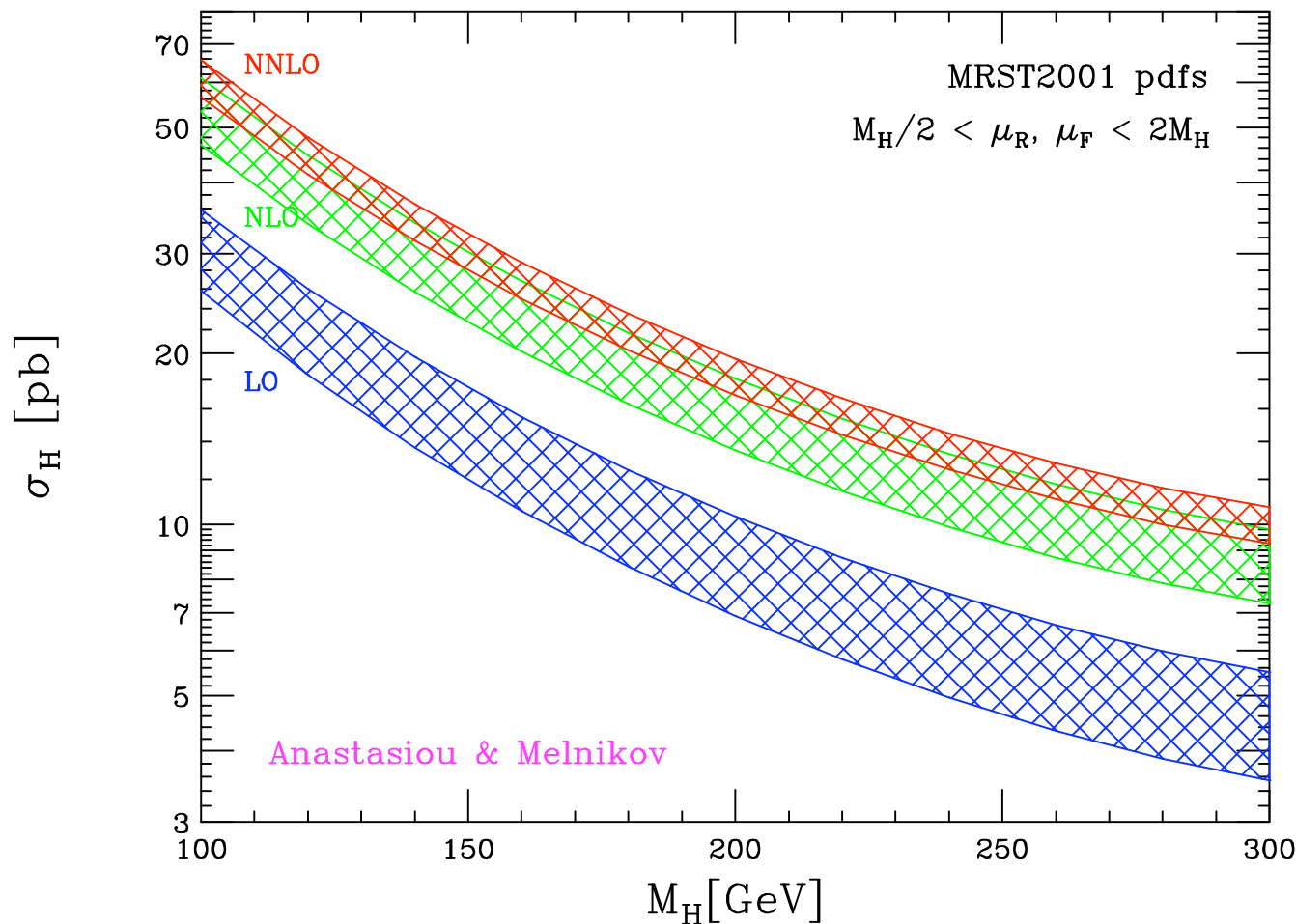
- procedure can be completely automated

Is NLO enough to describe data ?

Total cross section for inclusive Higgs production at LHC

$pp \rightarrow H+X$ Cross section at LHC

MRST2001 pdfs

$M_H/2 < \mu_R,\ \mu_F < 2M_H$

NNLO

NLO

LO

σ_H [pb]

Anastasiou & Melnikov

M_H[GeV]

contour bands are lower

$\mu_R = 2M_H \quad \mu_F = M_H/2$
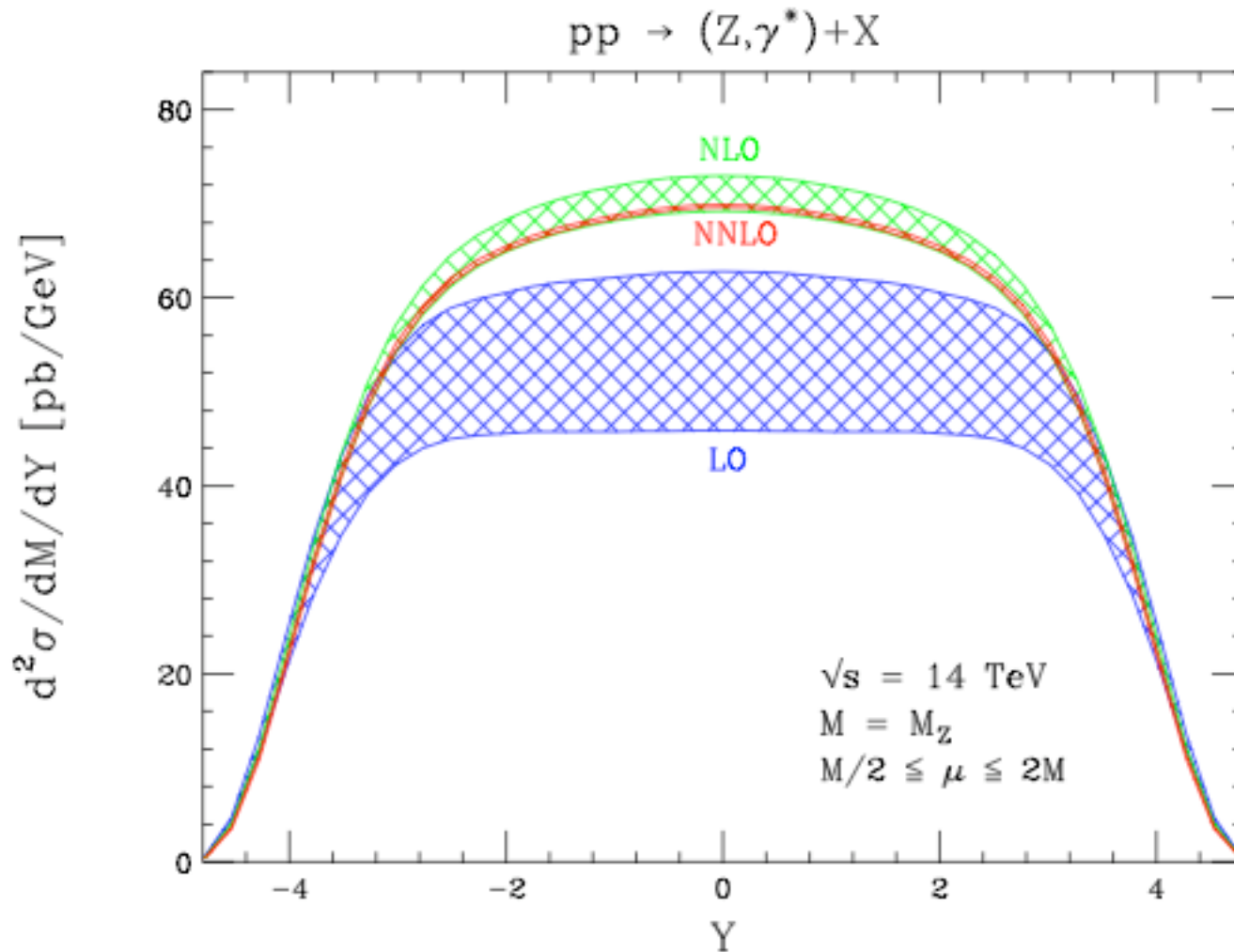
upper

$\mu_R = M_H/2 \quad \mu_F = 2M_H$

scale uncertainty is about 10%

NNLO prediction stabilises the perturbative series
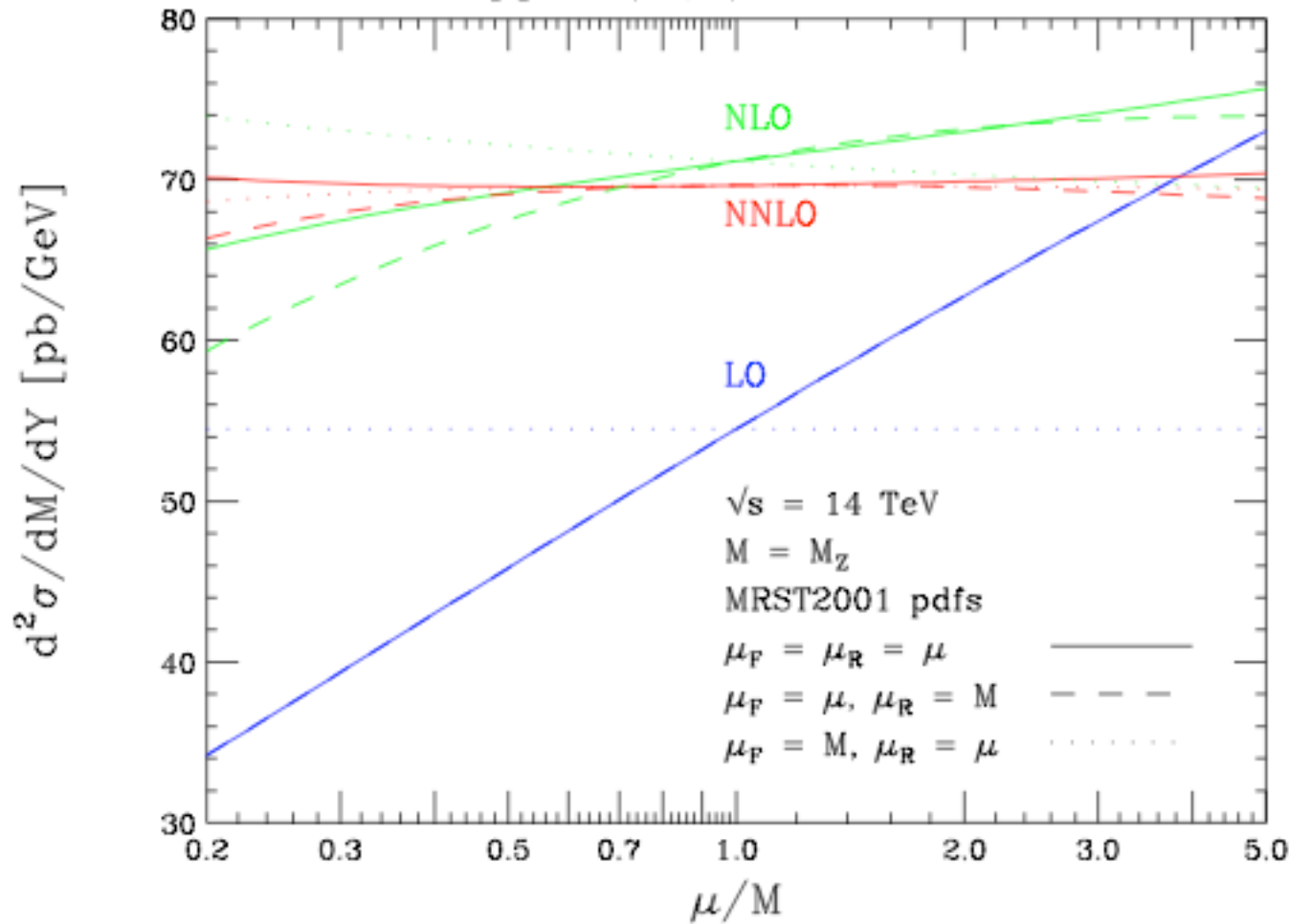
Drell-Yan Z production at LHC

$pp \rightarrow (Z, \gamma^*) + X$

Rapidity distribution for an on-shell Z boson

NLO

NNLO

LO

$\sqrt{s} = 14$ TeV
$M = M_Z$
$M/2 \leq \mu \leq 2M$

30%(15%) NLO increase wrt to LO at central Y's (at large Y's)
NNLO decreases NLO by $1 - 2\%$

scale variation: $\approx 30\%$ at LO; $\approx 6\%$ at NLO; less than 1% at NNLO

C. Anastasiou L. Dixon K. Melnikov F. Petriello 2003
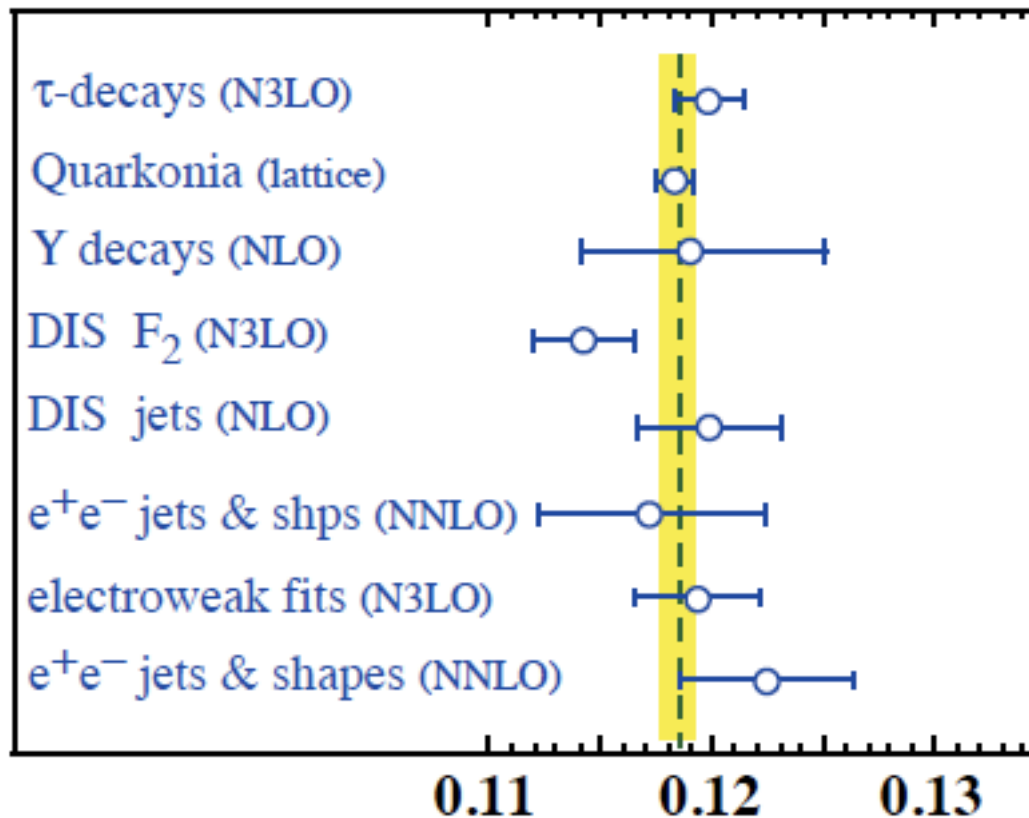
Scale variations in Drell-Yan Z production

solid: vary μ_R and μ_F together

dashed: vary μ_F only

dotted: vary μ_R only

C. Anastasiou L. Dixon K. Melnikov F. Petriello 2003

World average of $\alpha_S(M_Z)$

$$\alpha_s(M_Z) = 0.1184 \pm 0.0007$$

S. Bethke arXiv:0908.1135

- vertical line and shaded band mark the world average

- first time that shapes are included at NNLO

NNLO corrections may be relevant if

- the main source of uncertainty in extracting info from data is due to NLO theory: α_S measurements

- NLO corrections are large:
  Higgs production from gluon fusion in hadron collisions

- NLO uncertainty bands are too large to test
  theory vs. data: b production in hadron collisions

- NLO is effectively leading order:
  energy distributions in jet cones

NNLO state of the art

- **Drell-Yan W, Z production**

  total cross section

  Hamberg van Neerven Matsuura 1990
  Harlander Kilgore 2002

  fully differential x-section

  Melnikov Petriello 2006
  Catani Cieri Ferrera de Florian Grazzini 2009

- **Higgs production**

  total cross section

  Harlander Kilgore; Anastasiou Melnikov 2002
  Ravindran Smith van Neerven 2003

  fully differential x-section

  Anastasiou Melnikov Petriello 2004
  Catani de Florian Grazzini 2007

- $e^+ e^- \to 3 \text{ jets}$

  event shapes, α_s

  de Ridder Gehrmann Glover Heinrich 2007
  Weinzierl 2008

$$\alpha_s(M_Z^2) = 0.1224 \pm 0.0009 \, (\text{stat}) \pm 0.0009 \, (\text{exp}) \pm 0.0012 \, (\text{had}) \pm 0.0035 \, (\text{theo})$$

Dissertori et al. 2009

$\alpha_s(M_Z^2) = 0.1224 \pm 0.0039$ combined in quadrature

NNLO + NLL accuracy
TH uncertainty much reduced

NNLO cross sections

 Analytic integration

Hamberg, van Neerven, Matsuura 1990
Anastasiou Dixon Melnikov Petriello 2003

⬆ first method

➡ flexible enough to include a limited class of acceptance cuts by modelling cuts as ``propagators''

 Sector decomposition

Denner Roth 1996; Binoth Heinrich 2000
Anastasiou, Melnikov, Petriello 2004

⬆ flexible enough to include any acceptance cuts

⬆ cancellation of divergences is performed numerically

➡ can it handle many final-state partons ?

 Subtraction

Kosower 1998, 2003; Weinzierl 2003
Frixione Grazzini 2004
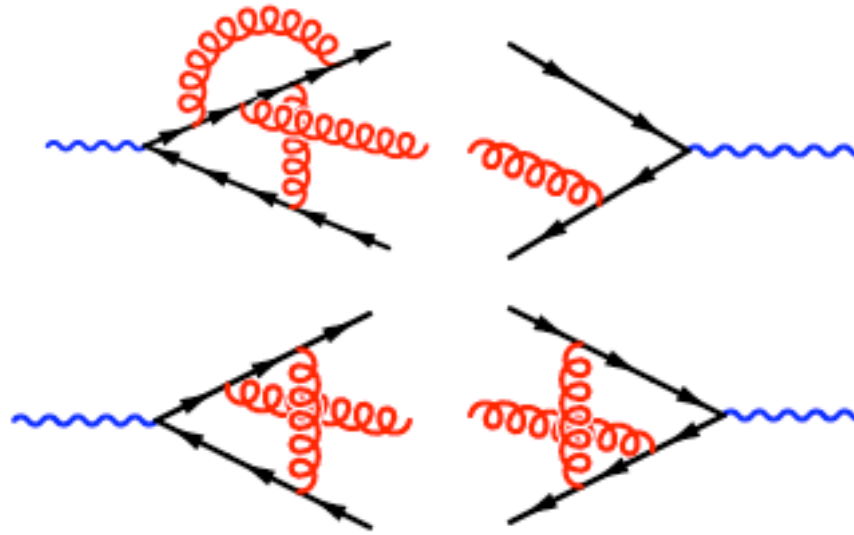de Ridder Gehrmann Glover 2004-5
Somogyi Trocsanyi VDD 2005-6

⬆ process independent

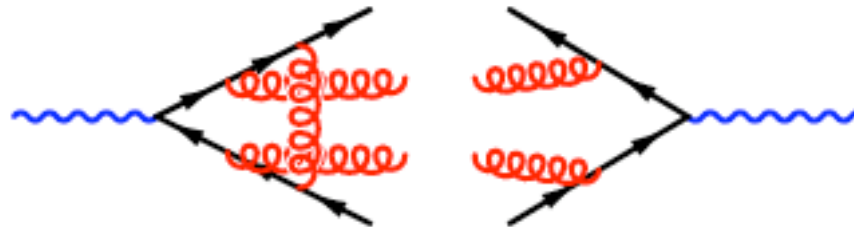➡ cancellation of divergences is semi-analytic can it be fully automatised ?
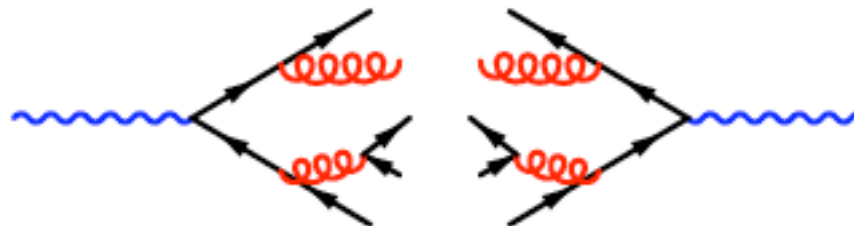
NNLO assembly kit

$e^+e^- \to 3$ jets

double virtual

real-virtual

double real

Two-loop matrix elements

- two-jet production $\quad qq' \to qq', \; q\bar{q} \to q\bar{q}, \; q\bar{q} \to gg, \; gg \to gg$

  C. Anastasiou N. Glover C. Oleari M. Tejeda-Yeomans 2000-01

  Z. Bern A. De Freitas L. Dixon 2002

- photon-pair production $\quad q\bar{q} \to \gamma\gamma, \; gg \to \gamma\gamma$

  C. Anastasiou N. Glover M. Tejeda-Yeomans 2002

  Z. Bern A. De Freitas L. Dixon 2002

- $e^+e^- \to 3 \text{ jets} \qquad \gamma^* \to q\bar{q}g$

  L. Garland T. Gehrmann N. Glover A. Koukoutsakis E. Remiddi 2002

- $V + 1 \text{ jet}$ production $\quad q\bar{q} \to Vg$

  T. Gehrmann E. Remiddi 2002

- Drell-Yan V production $\quad q\bar{q} \to V$

  R. Hamberg W. van Neerven T. Matsuura 1991

- Higgs production $\quad gg \to H \qquad$ (in the $m_t \to \infty$ limit)

  R. Harlander W. Kilgore; C. Anastasiou K. Melnikov 2002

Resummations: Higgs production from gluon fusion
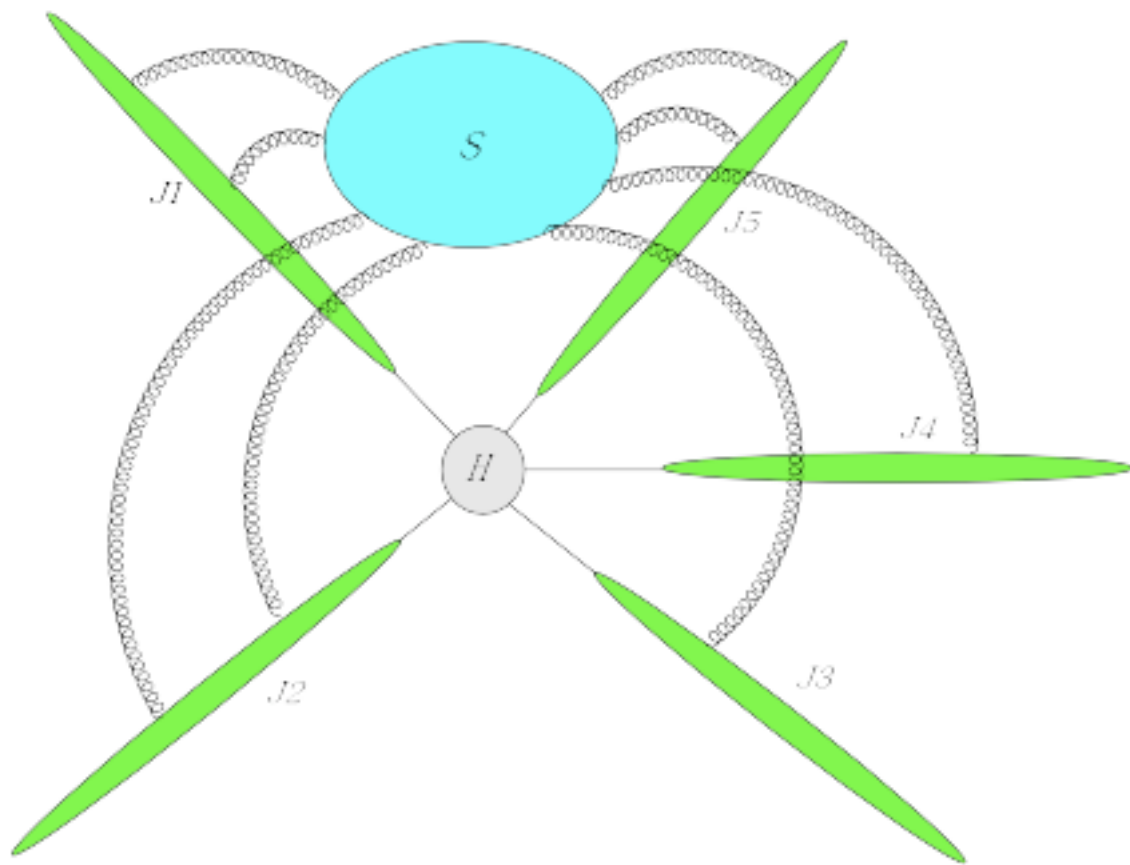
- gluon density is rapidly increasing as $x \to 0$
  $\Rightarrow$ Higgs production occurs near partonic threshold

- total energy of gluons in the final state is $E_X = \dfrac{\hat{s} - m_H^2}{2 m_H} \to 0$
  need to resum soft-gluon emission

Higgs q_T distribution   Bozzi Catani de Florian Grazzini 2005
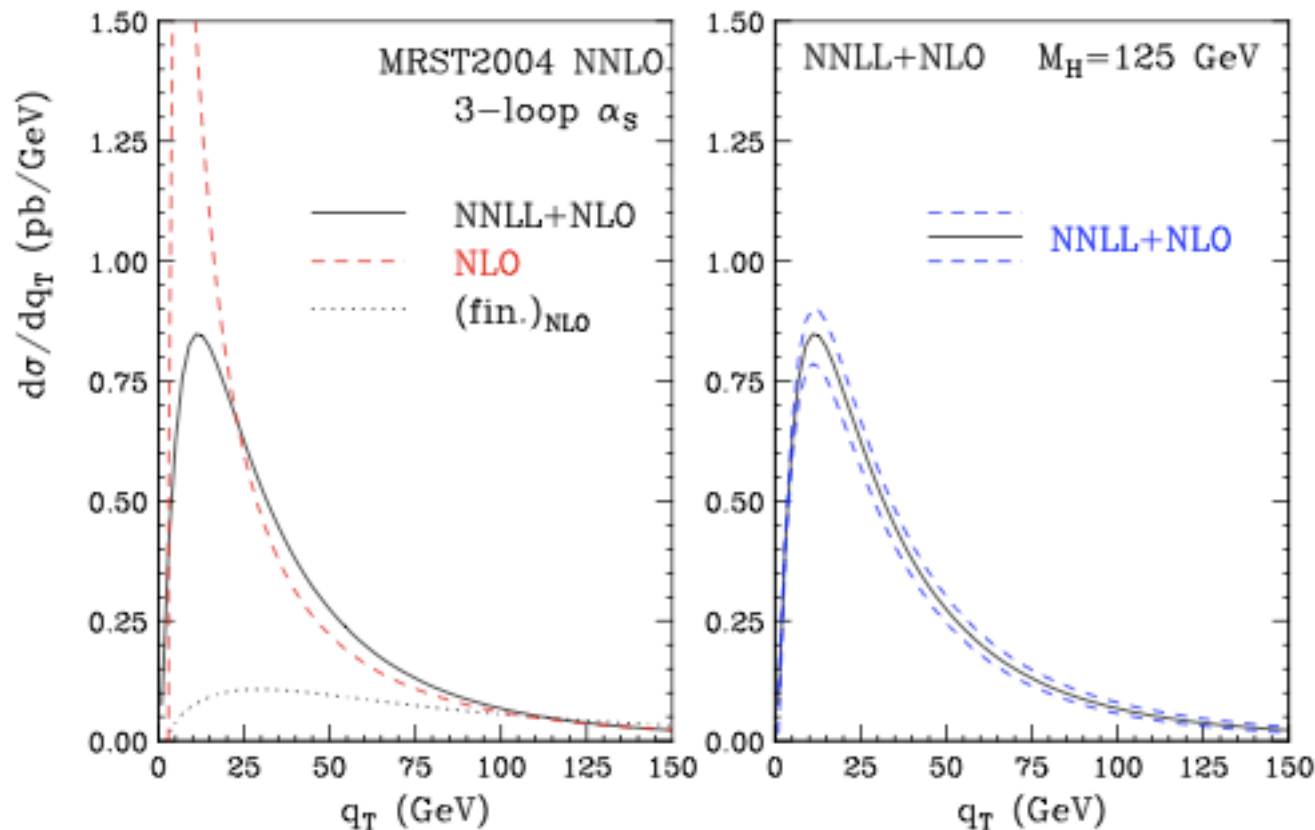
At small q_T,
NLO blows up

Factorisation of a multi-leg amplitude

Mueller 1981
Sen 1983
Botts Sterman 1987
Kidonakis Oderda Sterman 1998
Catani 1998
Tejeda-Yeomans Sterman 2002
Kosower 2003
Aybat Dixon Sterman 2006
Becher Neubert 2009
Gardi Magnea 2009

$$\mathcal{M}_N(p_i/\mu, \epsilon) = \sum_L \mathcal{S}_{NL}(\beta_i \cdot \beta_j, \epsilon) \, H_L \left(\frac{2p_i \cdot p_j}{\mu^2}, \frac{(2p_i \cdot n_i)^2}{n_i^2 \mu^2} \right) \prod_i \frac{J_i \left(\frac{(2p_i \cdot n_i)^2}{n_i^2 \mu^2}, \epsilon \right)}{\mathcal{J}_i \left(\frac{2(\beta_i \cdot n_i)^2}{n_i^2}, \epsilon \right)}$$

$p_i = \beta_i Q_0/\sqrt{2}$ value of Q_0 is immaterial in S, J

to avoid double counting of soft-collinear region (IR double poles),
$\mathcal{J}_i$ removes eikonal part from J_i, which is already in S
$J_i/\mathcal{J}_i$ contains only single collinear poles

Soft anomalous dimension

evolution equation for reduced soft anomalous dimension

$$\sum_{j \neq i} \frac{\partial}{\partial \ln \rho_{ij}} \Gamma^{\bar{\mathcal{S}}}(\rho_{ij}, \alpha_s) = \frac{1}{4} \gamma_K^{(i)}(\alpha_s)$$

Becher Neubert; Gardi Magnea 2009

solution

$$\Gamma^{\bar{\mathcal{S}}}(\rho_{ij}, \alpha_s) = -\frac{1}{8} \hat{\gamma}_K(\alpha_s) \sum_{i \neq j} \ln(\rho_{ij}) \, T_i \cdot T_j + \frac{1}{2} \hat{\delta}_{\bar{\mathcal{S}}}(\alpha_s) \sum_{i=1}^{n} C_i$$

with

$$\gamma_K^{(i)} = C_i \hat{\gamma}_K(\alpha_s) \qquad \hat{\gamma}_K = 2 \frac{\alpha_s(\mu^2)}{\pi} + K \left(\frac{\alpha_s(\mu^2)}{\pi} \right)^2 + K^{(2)} \left(\frac{\alpha_s(\mu^2)}{\pi} \right)^3 + \cdots$$

- only 2-eikonal-line correlations

- generalises 2-loop solution

  Are there any 3(or more)-line correlations ?
  Gardi Magnea say maybe; Becher Neubert say no

Conclusions

- QCD is an extensively developed and tested gauge theory

- a lot of progress in the last few years in

  - MonteCarlo generators
  - NLO computations with many jets
  - NNLO computations
  - resummations

- better and better approximations of signal and background for Higgs and New Physics