QCD at LHC

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Napoli 15 ottobre 2004
an unbroken Yang-Mills gauge field theory featuring asymptotic freedom → confinement

in non-perturbative regime (low $Q^2$) many approaches: lattice, Regge theory, $\chi$ PT, large $N_c$, HQET

in perturbative regime (high $Q^2$) QCD is a precision toolkit for exploring Higgs & BSM physics

LEP was an electroweak machine

Tevatron & LHC are QCD machines
Precise determination of

- strong coupling constant $\alpha_s$
- parton distributions
- electroweak parameters
- LHC parton luminosity

Precise prediction for

- Higgs production
- new physics processes
- their backgrounds
Summary of $\alpha_S(M_Z)$

S. Bethke hep-ex/0407021

world average of $\alpha_S(M_Z)$

using $\overline{\text{MS}}$ and NNLO results only

$\alpha_S(M_Z) = 0.1182 \pm 0.0027$

(cf. 2002 $\alpha_S(M_Z) = 0.1183 \pm 0.0027$

outcome almost identical

because new entries wrt 2002

- LEP jet shape observables and

4-jet rate, and HERA jet rates

and shape variables - are NLO)

filled symbols are NNLO results
Strong interactions at high $Q^2$

- Parton model
- Perturbative QCD
- Factorisation
- Universality of IR behaviour
- Cancellation of IR singularities
- IR safe observables: inclusive rates
  - Jets
  - Event shapes
Factorisation is the separation between the short- and the long-range interactions

\[ \sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_a/A(x_1, \mu_F^2) f_b/B(x_2, \mu_F^2) \times \hat{\sigma}_{ab \to X} \left( x_1, x_2, \{p_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_F^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2} \right) \] 

\[ X = W, Z, H, Q\bar{Q}, \text{high-}\text{ET jets}, \ldots \]

\( \hat{\sigma} \) is known as a fixed-order expansion in \( \alpha_S \)

\[ \hat{\sigma} = C \alpha_S^n (1 + c_1 \alpha_S + c_2 \alpha_S^2 + \ldots) \]

\( c_1 = \text{NLO} \quad c_2 = \text{NNLO} \)

or as an all-order resummation

\[ \hat{\sigma} = C \alpha_S^n [1 + (c_{11} L + c_{10}) \alpha_S + (c_{22} L^2 + c_{21} L + c_{20}) \alpha_S^2 + \ldots] \]

where \( L = \ln(M/q_T), \ln(1-x), \ln(1/x), \ln(1-T), \ldots \)

\( c_{11}, c_{22} = \text{LL} \quad c_{10}, c_{21} = \text{NLL} \quad c_{20} = \text{NNLL} \)
Evolution

factorisation scale $\mu_F$ is arbitrary

cross section cannot depend on $\mu_F$

$$\mu_F \frac{d\sigma}{d\mu_F} = 0$$

implies DGLAP equations

$$\mu_F \frac{df_a(x, \mu^2_F)}{d\mu_F} = P_{ab}(x, \alpha_S(\mu^2_F)) \otimes f_b(x, \mu^2_F) + O\left(\frac{1}{Q^2}\right)$$

$$\mu_F \frac{d\hat{\sigma}_{ab}(Q^2/\mu^2_F, \alpha_S(\mu^2_F))}{d\mu_F} = -P_{ac}(x, \alpha_S(\mu^2_F)) \otimes \hat{\sigma}_{cb}(Q^2/\mu^2_F, \alpha_S(\mu^2_F)) + O\left(\frac{1}{Q^2}\right)$$

$P_{ab}(x, \alpha_S(\mu^2_F))$ is calculable in pQCD
Factorisation-breaking contributions

- underlying event (see Rick Field’s studies at CDF)
- power corrections
  - MC’s and theory modelling of power corrections laid out and tested at LEP where they provide an accurate determination of $\alpha_S$
  - models still need be tested in hadron collisions (see e.g. Tevatron studies at different $\sqrt{s}$)
- double-parton scattering
  - observed by Tevatron CDF in the inclusive sample $p\bar{p} \to \gamma + 3$ jets
  - potentially important at LHC $\sigma_D \propto \sigma^2_S$
- diffractive events
Power corrections at Tevatron

Ratio of inclusive jet cross sections at 630 and 1800 GeV

\[ \sigma(630 \text{ GeV})/\sigma(1800 \text{ GeV}), \text{ with:} \]
\[ \sigma(\sqrt{s}) = \sigma(\sqrt{s})_{\text{NLO}} (E_T \rightarrow E_T + \Lambda) \]

In the ratio the dependence on the pdf’s cancels

Bjorken-scaling variable

\[ x_T = \frac{2E_T}{\sqrt{s}} \]

M.L. Mangano
KITP collider conf 2004

Solid: \( \Lambda = 2.8 \text{ GeV} \)
Dashes: \( \Lambda = 0 \)
\( \ast \): CDF

dashes: theory prediction with no power corrections
solid: best fit to data with free power-correction parameter \( \Lambda \) in the theory
Factorisation in diffraction ??

diffraction in DIS

no proof of factorisation in diffractive events

data do not support it

double pomeron exchange in $p\bar{p}$
## 3 complementary approaches to $\hat{\sigma}$

<table>
<thead>
<tr>
<th></th>
<th>matrix-elem MC’s</th>
<th>fixed-order x-sect</th>
<th>shower MC’s</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>final-state description</strong></td>
<td>hard-parton jets. Describes geometry, correlations, ...</td>
<td>limited access to final-state structure</td>
<td>full information available at the hadron level</td>
</tr>
<tr>
<td><strong>higher-order effects:</strong></td>
<td>higher-orders (multijets)</td>
<td>straightforward to implement (when available)</td>
<td>included as vertex corrections (Sudakov FF’s)</td>
</tr>
<tr>
<td><strong>loop corrections</strong></td>
<td></td>
<td></td>
<td>approximate, incomplete phase space at large angles</td>
</tr>
<tr>
<td><strong>hard emissions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>resummation of large logs</strong></td>
<td>?</td>
<td>feasible (when available)</td>
<td>unitarity implementation (i.e. correct shapes but not total rates)</td>
</tr>
</tbody>
</table>

M.L. Mangano KITP collider conf 2004
Matrix-element MonteCarlo generators

- efficient multi-parton generation: up to 2 $\rightarrow$ 9 jets subprocesses

  - **ALPGEN**  
  
  - **MADGRAPH/MADEVENT**  

  - **COMPHEP**  
    A. Pukhov et al. 1999

  - **GRACE/GR@PPA**  

  - **HELAC**  
    C. Papadopoulos et al. 2000

merged with parton showers

all of the above, merged with HERWIG or PYTHIA

- **SHERPA**  
  F. Krauss et al. 2003

[→ talk di Frixione]
Shower **MonteCarlo** generators

**HERWIG**  B. Webber et al. 1992

being re-written as a C++ code (HERWIG++)

**PYTHIA**  T. Sjostrand 1994

and more

**CKKW**  S. Catani F. Krauss R. Kuhn B. Webber 2001

a procedure to interface parton subprocesses with a different number of final states to parton showers

**MC@NLO**  S. Frixione B. Webber 2002

a procedure to interface NLO computations to shower MC's

→  talk di Frixione
NLO features

- Jet structure: final-state collinear radiation
- PDF evolution: initial-state collinear radiation
- Opening of new channels
- Reduced sensitivity to fictitious input scales: $\mu_R, \mu_F$
  - Predictive normalisation of observables
    - First step toward precision measurements
    - Accurate estimate of signal and background for Higgs and new physics
- Matching with parton-shower MC’s: \textsc{MC@NLO}
Jet structure

the jet non-trivial structure shows up first at NLO leading order

NLO

NNLO
Somebody’s wishlist

Dear Santa Claus,

I’d like to have the following cross sections at **NLO**

<table>
<thead>
<tr>
<th>Single boson</th>
<th>Diboson</th>
<th>Triboson</th>
<th>Heavy flavour</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W + \leq 5j$</td>
<td>$WW + \leq 5j$</td>
<td>$WWW + \leq 3j$</td>
<td>$\bar{t}\bar{t} + \leq 3j$</td>
</tr>
<tr>
<td>$W + bb + \leq 3j$</td>
<td>$WW + bb + \leq 3j$</td>
<td>$WWW + bb + \leq 3j$</td>
<td>$\bar{t}\bar{t} + \gamma + \leq 2j$</td>
</tr>
<tr>
<td>$W + car{c} + \leq 3j$</td>
<td>$WW + car{c} + \leq 3j$</td>
<td>$WWW + \gamma\gamma + \leq 3j$</td>
<td>$\bar{t}\bar{t} + W + \leq 2j$</td>
</tr>
<tr>
<td>$Z + \leq 5j$</td>
<td>$ZZ + \leq 5j$</td>
<td>$Z\gamma\gamma + \leq 3j$</td>
<td>$\bar{t}\bar{t} + Z + \leq 2j$</td>
</tr>
<tr>
<td>$Z + bb + \leq 3j$</td>
<td>$ZZ + bb + \leq 3j$</td>
<td>$WZZ + \leq 3j$</td>
<td>$\bar{t}\bar{t} + H + \leq 2j$</td>
</tr>
<tr>
<td>$Z + car{c} + \leq 3j$</td>
<td>$ZZ + car{c} + \leq 3j$</td>
<td>$ZZZ + \leq 3j$</td>
<td>$b\bar{b} + \leq 2j$</td>
</tr>
<tr>
<td>$\gamma + \leq 5j$</td>
<td>$\gamma\gamma + \leq 5j$</td>
<td>$\gamma\gamma + \leq 5j$</td>
<td></td>
</tr>
<tr>
<td>$\gamma + bb + \leq 3j$</td>
<td>$\gamma\gamma + bb + \leq 3j$</td>
<td>$\gamma\gamma + bb + \leq 3j$</td>
<td></td>
</tr>
<tr>
<td>$\gamma + car{c} + \leq 3j$</td>
<td>$\gamma\gamma + car{c} + \leq 3j$</td>
<td>$\gamma\gamma + car{c} + \leq 3j$</td>
<td></td>
</tr>
<tr>
<td>$WZ + \leq 5j$</td>
<td>$WZ + \bar{b}\bar{b} + \leq 3j$</td>
<td>$WZ + \leq 5j$</td>
<td></td>
</tr>
<tr>
<td>$WZ + car{c} + \leq 3j$</td>
<td>$WZ + car{c} + \leq 3j$</td>
<td>$WZ + \leq 3j$</td>
<td></td>
</tr>
<tr>
<td>$W\gamma + \leq 3j$</td>
<td>$W\gamma + \leq 3j$</td>
<td>$W\gamma + \leq 3j$</td>
<td></td>
</tr>
<tr>
<td>$Z\gamma + \leq 3j$</td>
<td>$Z\gamma + \leq 3j$</td>
<td>$Z\gamma + \leq 3j$</td>
<td></td>
</tr>
</tbody>
</table>
NLO history

\[
\begin{align*}
    e^+e^- & \rightarrow 3 \text{ jets} & \text{K. Ellis, D. Ross, A. Terrano 1981} \\
    e^+e^- & \rightarrow 4 \text{ jets} & \text{Z. Bern et al., N. Glover et al., Z. Nagy Z. Trocsanyi 1996-97} \\
    pp & \rightarrow 1, 2 \text{ jets} & \text{K. Ellis J. Sexton 1986, W. Giele N. Glover D. Kosower 1993} \\
    pp & \rightarrow 3 \text{ jets} & \text{Z. Bern et al., Z. Kunszt et al. 1993-1995, Z. Nagy 2001} \\
    pp & \rightarrow V + 1 \text{ jet} & \text{W. Giele N. Glover & D. Kosower 1993} \\
    pp & \rightarrow V + 2 \text{ jet} & \text{Bern et al., Glover et al. 1996-97, K. Ellis & Campbell 2003} \\
    pp & \rightarrow Vb\bar{b} & \text{K. Ellis & J. Campbell 2003} \\
    pp & \rightarrow Vb\bar{b} + 1 \text{ jet} & \text{??} \\
    pp & \rightarrow VV & \text{Ohnemus & Owens, Baur et al. 1991-96, Dixon et al. 2000} \\
    pp & \rightarrow VV + 1 \text{ jet} & \text{??} \\
    pp & \rightarrow \gamma\gamma & \text{B. Bailey et al 1992, T. Binoth et al 1999} \\
    pp & \rightarrow \gamma\gamma + 1 \text{ jet} & \text{Z. Bern et al. 1994, V. Del Duca et al. 2003} \\
    pp & \rightarrow Q\bar{Q} & \text{Dawson K. Ellis Nason 1989, Mangano Nason Ridolfi 1992} \\
    pp & \rightarrow Q\bar{Q} + 1 \text{ jet} & \text{A. Brandenburg et al. 2005 ?}
\end{align*}
\]
NLOJET++

Author(s): Z. Nagy

http://www.ippp.dur.ac.uk/~nagyz/nlo++.html

Multi-purpose C++ library for calculating jet cross-sections in $e^+e^-$ annihilation, DIS and hadron-hadron collisions.

$k_\perp$ algorithm

\[
e^+e^- \rightarrow \leq 4 \text{ jets}
\]

\[
e p \rightarrow (\leq 3 + 1) \text{ jets}
\]

\[
p\bar{p} \rightarrow \leq 3 \text{ jets}
\]

hep-ph/0110315
**MCFM**

Author(s): JC, R. K. Ellis

http://mcfm.fnal.gov

Fortran package for calculating a number of processes involving vector bosons, Higgs, jets and heavy quarks at hadron colliders.

\[ p\bar{p} \rightarrow V + \leq 2 \text{jets} \]

\[ p\bar{p} \rightarrow V + b\bar{b} \]

with \( V = W, Z \).
AYLEN/EMILIA

Author(s): L. Dixon, Z. Kunszt, A. Signer, D. de Florian

http://www.itp.phys.ethz.ch/staff/dflorian/codes.html

Fortran implementation of gauge boson pair production at hadron colliders, including full spin and decay angle correlations.

\[ p\bar{p} \rightarrow VV' \quad \text{and} \quad p\bar{p} \rightarrow V\gamma \quad \text{with} \quad V, V' = W, Z \]

Anomalous triple gauge boson couplings at the LHC:

hep-ph/0002138
DIPHOX/EPHOX

Author(s): P. Aurenche, T. Binoth, M. Fontannaz, J. Ph. Guillet, G. Heinrich, E. Pilon, M. Werlen

http://wwwlapp.in2p3.fr/lapth/PHOX_FAMILY/main.html

Fortran code to compute processes involving photons, hadrons and jets in DIS and hadron colliders.

\[ p\bar{p} \rightarrow \gamma^+ \leq 1 \text{ jet} \]

\[ p\bar{p} \rightarrow \gamma\gamma \]

\[ \gamma p \rightarrow \gamma^+ \text{ jet} \]

Preliminary H1 data,

Heavy quark production

Author(s): M. L. Mangano, P. Nason and G. Ridolfi
http://www.ge.infn.it/~ridolfi/hvqlibx.tgz
Fortran code for the calculation of heavy quark cross-sections and distributions in a fully differential manner

- Based on the more inclusive calculations of Dawson et al, Beenakker et al.
- Does not include multiple gluon radiation, $\log(p_T/m_b)$ (FONLL)
  Cacciari et al., hep-ph/9803400
- These are the same matrix elements that are incorporated into MC@NLO
  Frixione et al., hep-ph/0305252

\[ |y(J/\psi)| < 0.6 \]

\[ \sigma(p_T(J/\psi)>1.25 \text{ GeV}): \]
\[ \text{Points: CDF, } 19.9^{+3.8}_{-3.2} \text{ nb} \]
\[ \text{Solid: FONLL, } 19.0^{+8.4}_{-6.0} \text{ nb} \]
\[ \text{Dashes: MC@NLO, } 17.2 \text{ nb} \]

hep-ph/0312132
**NLO assembly kit**

\[ e^+ e^- \rightarrow 3 \text{ jets} \]

**leading order** \[ |M_{n}^{\text{tree}}|^2 \]

**NLO real**

\[ |M_{n+1}^{\text{tree}}|^2 \rightarrow |M_{n}^{\text{tree}}|^2 + \int dPS |P_{\text{split}}|^2 \]

= \[- \left( \frac{A}{\epsilon^2} + \frac{B}{\epsilon} \right) \]

**NLO virtual**

\[ d = 4 - 2\epsilon \]

\[ \int d^d l \ 2(\mathcal{M}_{n}^{\text{loop}})^* \mathcal{M}_{n}^{\text{tree}} = \left( \frac{A}{\epsilon^2} + \frac{B}{\epsilon} \right) |\mathcal{M}_{n}^{\text{tree}}|^2 + \text{fin.} \]
\[ \hat{\sigma} = \sigma_{\text{LO}} + \sigma_{\text{NLO}} = \int_n d\sigma^B + \sigma_{\text{NLO}} \]

\[ \sigma_{\text{NLO}} = \int_{n+1} d\sigma^R + \int_n d\sigma^V \]

the 2 terms on the rhs are divergent in \( d=4 \)

use universal IR structure to subtract divergences

\[ \sigma_{\text{NLO}} = \int_{n+1} \left[ (d\sigma^R)_{\epsilon=0} - (d\sigma^A)_{\epsilon=0} \right] + \int_n \left( d\sigma^V + \int_1 d\sigma^A \right)_{\epsilon=0} \]

the 2 terms on the rhs are finite in \( d=4 \)
NLO complications

- Loop integrals are involved and process-dependent.
- More particles → many scales → lengthy analytic expressions.
- Even though it is known how to compute loop integrals with \(2 \to n\) particles,
  no integrals with \(n > 3\) have been computed analytically (numerically).
- No numeric methods yet for hadron collisions.
- Counterterms are subtracted analytically.
Is **NLO** enough to describe data?

$b$ cross section in $p\bar{p}$ collisions at 1.96 TeV

$$d\sigma(p\bar{p} \rightarrow H_b X, H_b \rightarrow J/\psi X)/dp_T(J/\psi)$$

**NLO + NLL**

perfect agreement with data (with use of updated FF’s by Cacciari & Nason)

Cacciari, Frixione, Mangano, Nason, Ridolfi 2003
Is NLO enough to describe data?

Inclusive jet $p_T$ cross section at Tevatron

good agreement between NLO and data over several orders of magnitude

constrains the gluon distribution at high $x$
Is NLO enough to describe data?

di-lepton rapidity distribution for \((Z, \gamma^*)\) production vs. Tevatron Run I data

LO and NLO curves are for the MRST PDF set
no spin correlations
Is NLO enough to describe data?

Drell-Yan $W$ cross section at LHC with leptonic decay of the $W$

Cuts A $\longrightarrow |\eta^{(e)}| < 2.5$, $p_T^{(e)} > 20$ GeV, $p_T^{(\nu)} > 20$ GeV

Cuts B $\longrightarrow |\eta^{(e)}| < 2.5$, $p_T^{(e)} > 40$ GeV, $p_T^{(\nu)} > 20$ GeV

<table>
<thead>
<tr>
<th></th>
<th>LO</th>
<th>LO+HW</th>
<th>NLO</th>
<th>MC@NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cuts A</strong></td>
<td>0.5249</td>
<td>0.4843</td>
<td>0.4771</td>
<td>0.4845</td>
</tr>
<tr>
<td></td>
<td>↓5.4%</td>
<td></td>
<td>↓7.0%</td>
<td>↓6.3%</td>
</tr>
<tr>
<td><strong>Cuts A, no spin</strong></td>
<td>0.5535</td>
<td></td>
<td>0.5104</td>
<td>0.5151</td>
</tr>
<tr>
<td><strong>Cuts B</strong></td>
<td>0.0585</td>
<td>0.1218</td>
<td>0.1292</td>
<td>0.1329</td>
</tr>
<tr>
<td></td>
<td>↓29%</td>
<td></td>
<td>↓16%</td>
<td>↓18%</td>
</tr>
<tr>
<td><strong>Cuts B, no spin</strong></td>
<td>0.0752</td>
<td>0.1504</td>
<td>0.1570</td>
<td></td>
</tr>
</tbody>
</table>

$|MC@NLO - NLO| = \mathcal{O}(2\%)$

S. Frixione M.L. Mangano 2004

NNLO useless without spin correlations

Precisely evaluated Drell-Yan $W$, $Z$ cross sections could be used as "standard candles" to measure the parton luminosity at LHC
Is NLO enough to describe data?

Total cross section for inclusive Higgs production at LHC

\[ \mu_R = 2M_H \quad \mu_F = M_H / 2 \]

contour bands are lower
\[ \mu_R = M_H / 2 \quad \mu_F = 2M_H \]

upper

scale uncertainty is about 10%

NNLO prediction stabilises the perturbative series
NNLO state of the art

Drell-Yan $W, Z$ production

- **total cross section**: Hamberg, van Neerven, Matsuura 1990
  Harlander, Kilgore 2002

- **rapidity distribution**: Anastasiou et al. 2003

Higgs production

- **total cross section**: Harlander, Kilgore; Anastasiou, Melnikov 2002

- **fully differential cross section**: Anastasiou, Melnikov, Petriello 2004

$e^+e^- \rightarrow 3 \text{ jets}$

- **the $C_F^2$ term**: the Gehrmanns, Glover 2004
Drell-Yan $Z$ production at LHC

Rapidity distribution for an on-shell $Z$ boson

30% (15%) NLO increase wrt to LO at central $Y$'s (at large $Y$'s)
NNLO decreases NLO by $1 - 2\%$

scale variation: $\approx 30\%$ at LO; $\approx 6\%$ at NLO; less than 1% at NNLO

C. Anastasiou L. Dixon K. Melnikov F. Petriello 2003
Scale variations in Drell-Yan $Z$ production

$pp \rightarrow (Z, \gamma^*) + X$ at $Y=0$

- solid: vary $\mu_R$ and $\mu_F$ together
- dashed: vary $\mu_F$ only
- dotted: vary $\mu_R$ only

$\sqrt{s} = 14$ TeV
$M = M_Z$
MRST2001 pdfs
$\mu_F = \mu_R = \mu$
$\mu_F = \mu, \mu_R = M$
$\mu_F = M, \mu_R = \mu$

C. Anastasiou L. Dixon K. Melnikov F. Petriello 2003
Drell-Yan $W$ production at LHC

Rapidity distribution for an on-shell $W^-$ boson (left) $W^+$ boson (right)

distributions are symmetric in $Y$

NNLO scale variations are $1\% (3\%)$ at central (large) $Y$

C. Anastasiou L. Dixon K. Melnikov F. Petriello 2003
Higgs production at LHC

a fully differential cross section:
bin-integrated rapidity distribution, with a jet veto

\[ R = 0.4 \]
\[ |p_T^j| < p_T^{veto} = 40 \text{ GeV} \]

for 2 partons
\[ R_{12}^2 = (\eta_1 - \eta_2)^2 + (\phi_1 - \phi_2)^2 \]

if \( R_{12} > R \)
\[ |p_T^1|, |p_T^2| < p_T^{veto} \]

if \( R_{12} < R \)
\[ |p_T^1 + p_T^2| < p_T^{veto} \]

\( M_H = 150 \text{ GeV} \) (jet veto relevant in the \( H \rightarrow W^+W^- \) decay channel)

K factor is much smaller for the vetoed x-sect than for the inclusive one:
average \( |p_T^j| \) increases from NLO to NNLO; less x-sect passes the veto
NNLO assembly kit

e^+e^- \rightarrow 3 \text{ jets}

double virtual

real-virtual

double real
Two-loop matrix elements

two-jet production \( qq' \rightarrow qq', \) \( q\bar{q} \rightarrow q\bar{q}, \) \( q\bar{q} \rightarrow gg, \) \( gg \rightarrow gg \)

C. Anastasiou N. Glover C. Oleari M. Tejeda-Yeomans 2000-01
Z. Bern A. De Freitas L. Dixon 2002

photon-pair production \( q\bar{q} \rightarrow \gamma\gamma, \) \( gg \rightarrow \gamma\gamma \)

C. Anastasiou N. Glover M. Tejeda-Yeomans 2002
Z. Bern A. De Freitas L. Dixon 2002

e^+e^- \rightarrow 3 \text{jets} \quad \gamma^* \rightarrow q\bar{q}g


\( V + 1 \text{ jet} \) production \( q\bar{q} \rightarrow Vg \)

T. Gehrmann E. Remiddi 2002

Drell-Yan \( V \) production \( q\bar{q} \rightarrow V \)

R. Hamberg W. van Neerven T. Matsuura 1991

Higgs production \( gg \rightarrow H \) (in the \( m_t \rightarrow \infty \) limit)

R. Harlander W. Kilgore; C. Anastasiou K. Melnikov 2002
**NNLO cross sections**

**universal IR structure** → **process-independent procedure**

**universal** collinear and soft currents

3-parton tree splitting functions


2-parton one-loop splitting functions

Z. Bern W. Kilgore C. Schmidt VDD 1998-99; D. Kosower P. Uwer 1999; D. Kosower 2003

**universal** subtraction counterterms

several ideas and works in progress
but so far not yet completely figured out

D. Kosower; S. Weinzierl; the Gehrmanns & G. Heinrich 2003
LHC parton kinematics

$x_{1,2} = (M/14 \text{ TeV}) \exp(\pm y)$

$Q = M$

$Q^2 (\text{GeV}^2)$

$y = 6 4 2 0 2 4 6$

$M = 10 \text{ TeV}$

$M = 1 \text{ TeV}$

$M = 100 \text{ GeV}$

$M = 10 \text{ GeV}$

HERA

fixed target

J. Stirling
Parton distribution functions (PDF)

factorisation for the structure functions (e.g. $F_{2}^{ep}$, $F_{L}^{ep}$)

$$F_{i}(x, \mu^{2}_{F}) = C_{ij} \otimes q_{j} + C_{ig} \otimes g$$

with the convolution

$$[a \otimes b](x) \equiv \int_{x}^{1} \frac{dy}{y} a(y) b\left(\frac{x}{y}\right)$$

$C_{ij}, C_{ig}$ coefficient functions

$q_{i}(x, \mu^{2}_{F})$ $g(x, \mu^{2}_{F})$ PDF's

DGLAP evolution equations

$$\frac{d}{d \ln \mu^{2}_{F}} \begin{pmatrix} q_{i} \\ g \end{pmatrix} = \begin{pmatrix} P_{q_{i}q_{j}} & P_{q_{i}g} \\ P_{gq_{j}} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q_{j} \\ g \end{pmatrix}$$

perturbative series

$$P_{ij} \approx \alpha_{s} P_{ij}^{(0)} + \alpha_{s}^{2} P_{ij}^{(1)} + \alpha_{s}^{3} P_{ij}^{(2)}$$

anomalous dimension

$$\gamma_{ij}(N) = - \int_{0}^{1} dx \ x^{N-1} \ P_{ij}(x)$$
PDF's

general structure of the quark-quark splitting functions

\[ P_{q_i q_k} = P_{q_i \bar{q}_k} = \delta_{ik} P_{qq}^v + P_{qq}^s \]
\[ P_{q_i \bar{q}_k} = P_{\bar{q}_i q_k} = \delta_{ik} P_{q\bar{q}}^v + P_{q\bar{q}}^s \]

non-singlet

flavour asymmetry

\[ q_{ns,ik}^\pm = q_i \pm \bar{q}_i - (q_k \pm \bar{q}_k) \]
\[ P_{ns}^\pm = P_{qq}^v \pm P_{q\bar{q}}^v \]

sum of valence distributions of all flavours

\[ q_{ns}^v = \sum_{r=1}^{n_f} (q_r - \bar{q}_r) \]
\[ P_{ns}^v = P_{qq}^v - P_{q\bar{q}}^v + n_f (P_{qq}^s - P_{q\bar{q}}^s) \]

singlet

\[ q_s = \sum_{i=1}^{n_f} (q_i + \bar{q}_i) \]
\[ \frac{d}{d \ln \mu_F^2} \begin{pmatrix} q_s \\ g \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q_s \\ g \end{pmatrix} \]

with

\[ P_{qq} = P_{ns}^+ + n_f (P_{qq}^s + P_{q\bar{q}}^s) \]
\[ P_{qg} = n_f P_{qig}, \quad P_{gq} = P_{gq_i} \]
PDF history

leading order (or one-loop)
anomalous dim/splitting functions
  Gross Wilczek 1973; Altarelli Parisi 1977

NLO (or two-loop)
  $F_2, F_L$
  anomalous dim/splitting functions
  Bardeen Buras Duke Muta 1978
  Curci Furmanski Petronzio 1980

NNLO (or three-loop)
  $F_2, F_L$
  anomalous dim/splitting functions
  Zijlstra van Neerven 1992; Moch Vermaseren 1999
  Moch Vermaseren Vogt 2004

the calculation of the three-loop anomalous dimension is
the toughest calculation ever performed in perturbative QCD!

one-loop $\gamma_{ij}^{(0)}/P_{ij}^{(0)}$ $\Rightarrow$ 18 Feynman diagrams

two-loop $\gamma_{ij}^{(1)}/P_{ij}^{(1)}$ $\Rightarrow$ 350 Feynman diagrams

three-loop $\gamma_{ij}^{(2)}/P_{ij}^{(2)}$ $\Rightarrow$ 9607 Feynman diagrams

20 man-year-equivalents, $10^6$ lines of dedicated algebra code
exact NNLO results, estimates from fixed moments and leading small-\(x\) term
Bjorken-scaling violations

H1, ZEUS: ongoing fits for PDF’s; so far NNLO not included
PDF global fits

MRST: Martin Roberts Stirling Thorne
CTEQ: Pumplin et al.
Alekhin (DIS data only)

method
Perform fit by minimising $\chi^2$ to all data, including both statistical and systematic errors

Start evolution at some $Q_0^2$, where PDF's are parametrised with functional form, e.g.

$$xf(x, Q_0^2) = (1 - x)^\eta (1 + \epsilon x^{0.5} + \gamma x) x^\delta$$

Cut data at $Q^2 > Q_{\text{min}}^2$ and at $W^2 > W_{\text{min}}^2$ to avoid higher twist contamination

Allow $\bar{u} \neq \bar{d}$ as implied by E866 Drell-Yan asymmetry data

accuracy
NLO evolution and fixed moments of NNLO

no prompt photon data included in the fits
MRST 2001 PDF's

![Graph showing PDF's for different quark types and gluons with x f(x, Q^2) on the y-axis and x on the x-axis. The graph includes curves for up, down, anti-up, anti-down, strange, charm, and gluon distributions. The MRST2001 dataset is used with Q^2 = 10 GeV^2.]
PDF uncertainties

**direct effect on Tevatron & LHC cross section predictions**

**various approaches being used, most notably**

Hessian (error matrix) approach (H1, ZEUS, CTEQ, Alekhin)

\[
\chi^2 - \chi^2_{\text{min}} \equiv \Delta \chi^2 = \sum_{i,j} (a_i - a_i^{(0)})H_{ij}(a_j - a_j^{(0)})
\]

\(H\) is related to the covariance matrix of the parameters

\[
C_{ij}(a) = \Delta \chi^2(H^{-1})_{ij}
\]

diagonalise \(H_{ij}\) and define PDF sets \(S_i^\pm\) displaced along the eigenvector direction by \(\Delta \chi^2 = \sum_i z_i^2\). Then uncertainty on physical quantity is given by

\[
(\Delta F)^2 = \frac{1}{2} \sum_i (F(S_i^+) - F(S_i^-))^2
\]

Lagrange multiplier method (CTEQ, MRST)

perform fit while constraining value of some physical quantity \(F\). Minimise

\[
\Psi(\lambda, a) = \chi^2_{\text{global}}(a) + \lambda F(a)
\]

for various values of \(\lambda\) and parton parameters \(\{a\}\). Gives set of best fits for particular values of parameter \(F(a)\). Uncertainty then determined by deciding allowed range of \(\Delta \chi^2\). Can also see which data sets in global fit most directly influenced by variation in \(F(a)\)

want to know more? see e.g. R. Thorne et al hep-ex/0205233
Error on up distribution at $Q^2 = 10000 \, \text{GeV}^2$

from MRST2001E (see hep-ph/0211080)

- Hessian method used
- error is about 2%
parton luminosity uncertainties at LHC (MRST2001E)

for \( q\bar{q} \) (relevant for Drell-Yan production)

for \( gg \) (relevant for Higgs production)
**$W, Z$ total cross sections**

- **MRST2001**
- **NNLO:** only few fixed moments
- current best (MRST) estimate
  \[ \delta \sigma^{\text{NNLO}}_{W,Z} (\text{total pdf}) = \pm 4\% \]
  (expt. pdf error is 2%)

- larger uncertainty in the NLO prediction, because of problems at small $x$ in the global fit to DIS data and because large rapidity $W, Z$‘s sample small $x$
PDF uncertainty on $W, WH$ cross sections at LHC

- Use $\sigma(W), \sigma(Z)$ as "standard candles", i.e. to calibrate other cross sections, e.g. $\sigma(WH)$
- $\sigma(WH)$ more precisely predicted because it samples quark PDF's at higher $x$ than $\sigma(W)$
Photons at fixed-target experiments

probe the gluon distribution at high $x$

at $\sqrt{s} = 1800$ GeV, $p_{T,jet} = 180$ GeV $\Rightarrow x_T = 0.2$

data are not consistent with theory, and (even more worrisome) are not consistent with each other

currently they are not used in PDF fits

P. Aurenhé et al. 1998
Photons at the Tevatron at 1800 GeV and 630 GeV

Data are not consistent with theory (but D0 is better off than CDF)

Problems? TH: Narrow isolation cones used by experiments
Photons as a background to Higgs searches

Higgs production

\[ pp \rightarrow \gamma\gamma + \text{jet} \quad \text{at LHC} \]

di-photon invariant mass distribution

Di-photon decay important in the low-mass Higgs searches

Isolation cone \( R_\gamma = 0.4 \)

Hadronic energy allowed inside cone is \( E_{T,\max} = \epsilon \, pT_\gamma \)

Used Frixione's photon isolation criterion (avoids use of fragmentation functions)

\[ E_T \leq E_{T,\max} \left( \frac{1 - \cos r}{1 - \cos R_\gamma} \right)^n \]

\( K \) factor is > 2

F. Maltoni Z. Nagy Z. Trocsanyi VDD 2003
Conclusions

QCD is an extensively developed and tested gauge theory

a lot of progress in the last 4-5 years in

- MonteCarlo generators
- NLO cross sections with one more jet
- NNLO computations

better and better approximations of signal and background for Higgs and New Physics

new formal developments (I didn’t discuss):

- QCD as a string theory in twistor space  
  E. Witten 2003

- novel ways of computing (analytically) 
  tree multi-parton matrix elements and 
  (N=4) loop matrix elements  
  F. Cachazo P. Svrcek E. Witten 2004