Anatomy of QCD corrections: Wilson loops and amplitudes in N=4 Super Yang-Mills

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Moriond 25 March 2011

NLO cross sections

2005 Les Houches list almost completed

process wanted at NLO	background to
1. $pp \to VV +$ jet	$t\bar{t}H$, new physics Dittmaier, Kallweit, Uwer; Campbell, Ellis, Zanderighi
2. $pp \to H + 2$ jets	H in VBF Campbell, Ellis, Zanderighi; Ciccolini, Denner Dittmaier
3. $pp \to t\bar{t}b\bar{b}$	$t\bar{t}H$ Bredenstein, Denner Dittmaier, Pozzorini; Bevilacqua, Czakon, Papadopoulos, Pittau, Worek
4. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$ Bevilacqua, Czakon, Papadopoulos, Worek
5. $pp \to VVb\bar{b}$	VBF $\to H \to VV$, $t\bar{t}H$, new physics
6. $pp \to VV + 2$ jets	VBF $\to H \to VV$ VBF: Bozzi, Jäger, Oleari, Zeppenfeld
7. $pp \to V + 3$ jets	new physics Berger Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre; Ellis, Melnikov, Zanderighi
8. $pp \to VVV$	SUSY trilepton Lazopoulos, Melnikov, Petriello; Hankele, Zeppenfeld; Binoth, Ossola, Papadopoulos, Pittau
9. $pp \to b\bar{b}b\bar{b}$	Higgs, new physics GOLEM

$pp \to V + 4$ jets new physics C. Berger *et al* (BlackHat) 2010

The NLO revolution

- in the past, long time span to add one more jet to a x-section
- in the last few years, huge progress
- 2 → 2 and 2 → 3 processes:
  almost all computed and included into NLO packages
- 2 → 4 processes: a few computed

$pp \to t\,\bar{t}\,b\,\bar{b}$	Bredenstein Denner Dittmaier Pozzorini; Bevilacqua Czakon Papadopoulos Pittau Worek 2009
$pp \to Q\bar{Q}\ + 2\,\mathrm{jets}$	Bevilacqua Czakon Papadopoulos Worek 2010
$pp \to H + 3\ \mathrm{jets}$	(VBF) Figy Hankele Zeppenfeld 2007
$pp \to V + 3\ \mathrm{jets}$	Berger *et al.* (BlackHat); K. Ellis Melnikov Zanderighi 2009
$pp \to W^+W^+ + 2\ \mathrm{jets}$	Melia Melnikov Rontsch Zanderighi 2010

- 2 → 5 processes: just one

$pp \to V + 4\ \mathrm{jets}$	Berger *et al.* (BlackHat) 2010

one-loop amplitudes

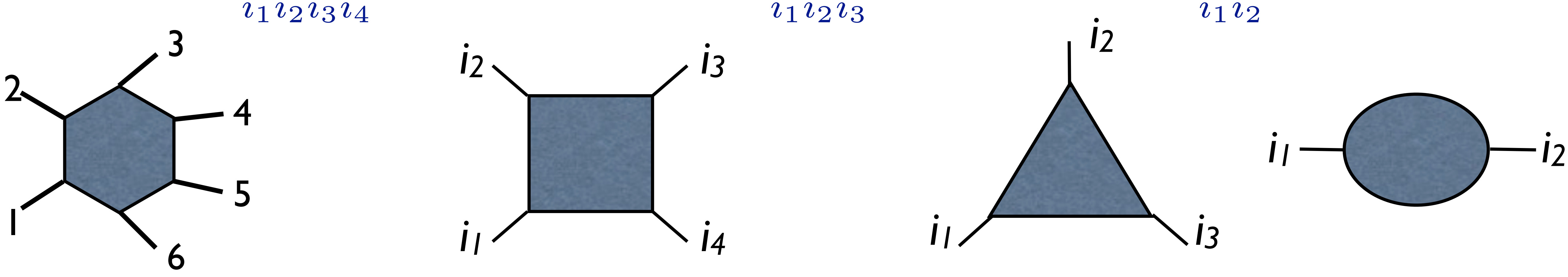
- one-loop n-point amplitudes A_n are IR divergent

↑ IR divergences are universal — Kunszt Signer Trocsanyi 1994; Catani 1998

↓ IR finite terms are process dependent:
many final-state particles → many scales → lengthy expressions

- A_n can be reduced to boxes, triangles and bubbles with rational coefficients

$$A_n = \sum_{i_1 i_2 i_3 i_4} d_{i_1 i_2 i_3 i_4}\, I^D_{i_1 i_2 i_3 i_4} + \sum_{i_1 i_2 i_3} c_{i_1 i_2 i_3}\, I^D_{i_1 i_2 i_3} + \sum_{i_1 i_2} b_{i_1 i_2}\, I^D_{i_1 i_2}$$



I: master integrals
b, c, d: rational functions of kinematic variables

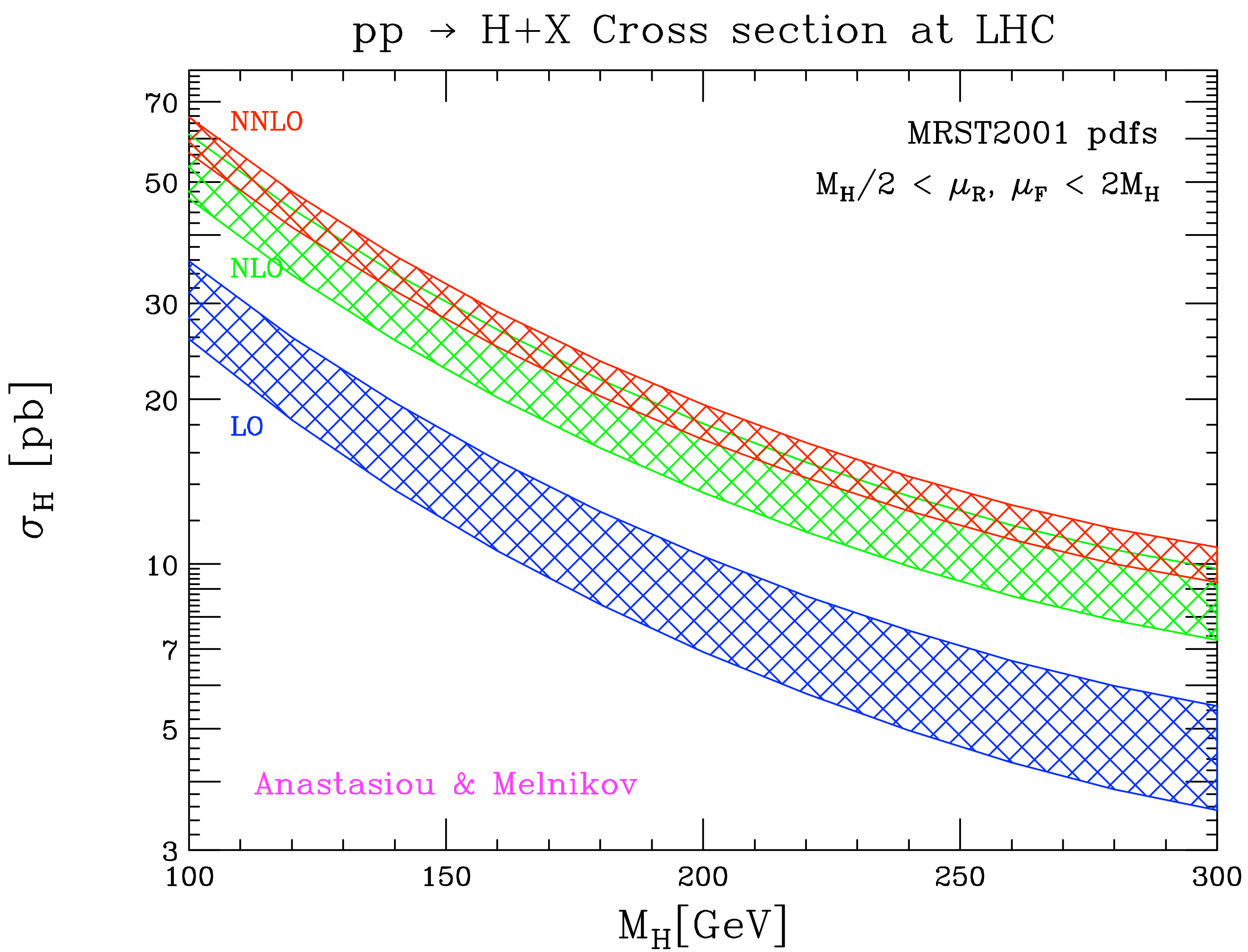
- higher polygons contribute only to $O(\varepsilon)$

One-Loop Master Integrals

although the one-loop n-point amplitudes A_n
are usually computed numerically,
it is convenient to have the one-loop master
integrals computed analytically, to be input
once and for all in your numerical routine

Is NLO enough to describe data ?

Total cross section for inclusive Higgs production at LHC



contour bands are

lower

$\mu_R = 2M_H \quad \mu_F = M_H/2$

upper

$\mu_R = M_H/2 \quad \mu_F = 2M_H$

scale uncertainty is about 10%

NNLO prediction stabilises the perturbative series

NNLO corrections may be relevant if

- NLO corrections are large:
  Higgs production from gluon fusion in hadron collisions

- NLO uncertainty bands are too large to test
  theory vs. data: *b* production in hadron collisions

- NLO is effectively leading order:
  energy distributions in jet cones

NNLO state of the art

- Drell-Yan W, Z production

  total cross section — Hamberg van Neerven Matsuura 1990; Harlander Kilgore 2002

  fully differential x-section — Melnikov Petriello 2006; Catani Cieri Ferrera de Florian Grazzini 2009

- Higgs production

  total cross section — Harlander Kilgore; Anastasiou Melnikov 2002; Ravindran Smith van Neerven 2003

  fully differential x-section — Anastasiou Melnikov Petriello 2004; Catani de Florian Grazzini 2007

- $e^+e^- \to 3\ \mathrm{jets}$

  event shapes, α_s — de Ridder Gehrmann Glover Heinrich 2007; Weinzierl 2008

$$\alpha_s(M_Z^2) = 0.1224 \pm 0.0009\,(\mathrm{stat}) \pm 0.0009\,(\mathrm{exp}) \pm 0.0012\,(\mathrm{had}) \pm 0.0035\,(\mathrm{theo})$$

Dissertori et al. 2009

$\alpha_s(M_Z^2) = 0.1224 \pm 0.0039$ combined in quadrature

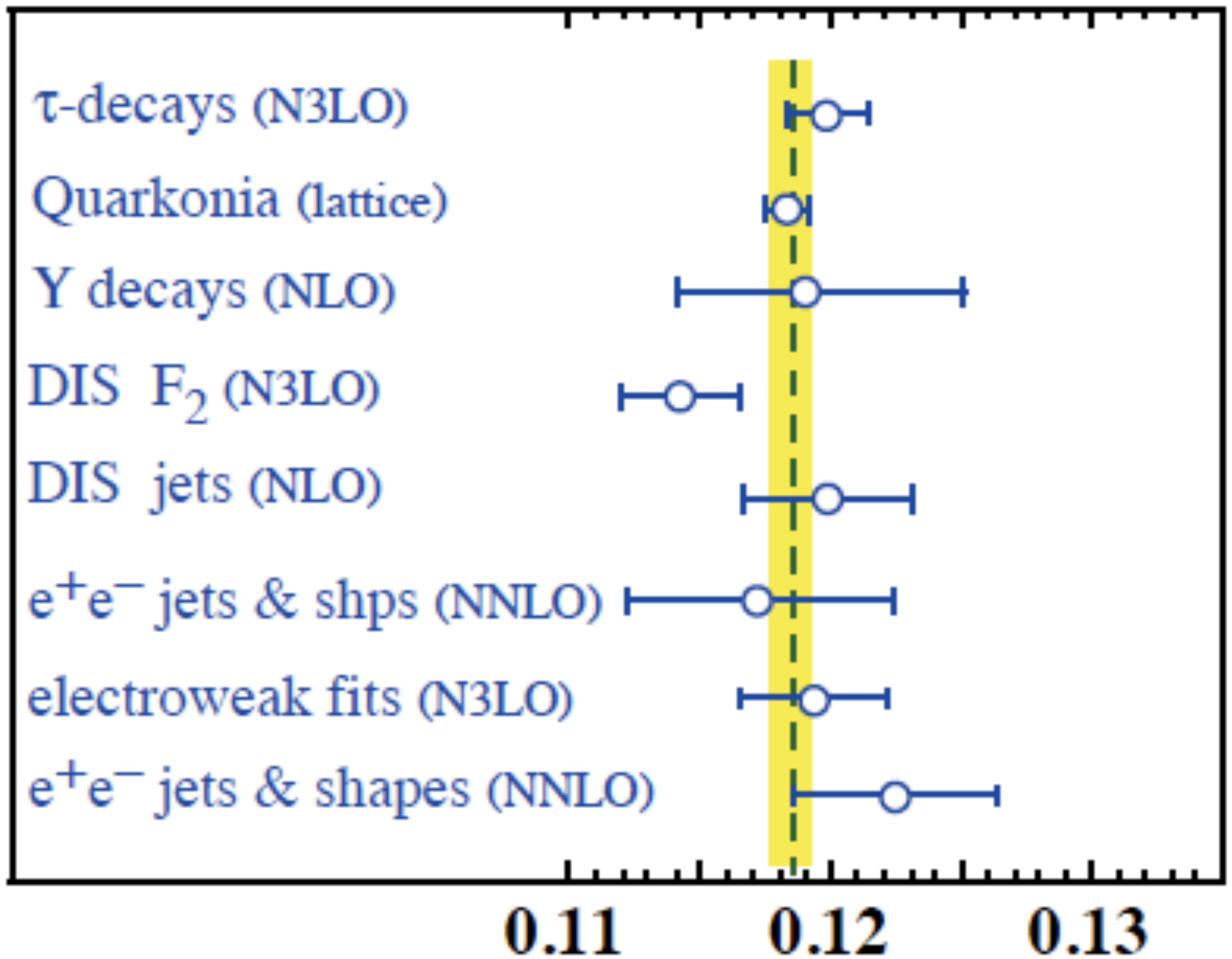
NNLO + NLL accuracy

TH uncertainty much reduced

World average of $\alpha_S(M_Z)$

$$\alpha_s(M_Z) = 0.1184 \pm 0.0007 \qquad \frac{\Delta\alpha_s}{\alpha_s} = 0.6\%$$

S. Bethke arXiv:0908.1135



- vertical line and shaded band mark the world average
- first time that shapes are included at NNLO

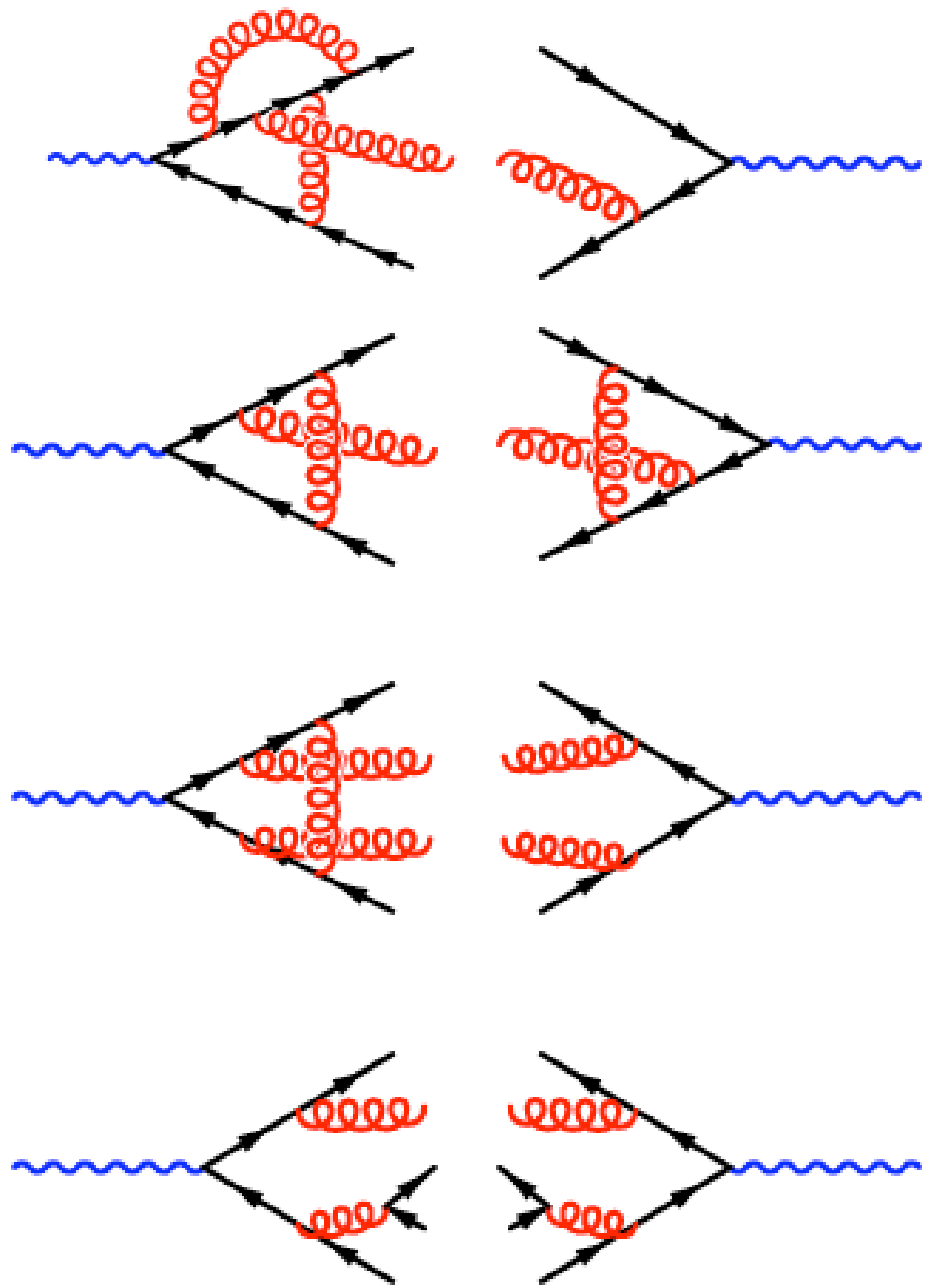
NNLO assembly kit

$e^+e^- \to 3$ jets

double virtual

real-virtual

double real



Two-loop matrix elements

- two-jet production $qq' \to qq',\ q\bar{q} \to q\bar{q},\ q\bar{q} \to gg,\ gg \to gg$

  C. Anastasiou N. Glover C. Oleari M. Tejeda-Yeomans 2000-01

  Z. Bern A. De Freitas L. Dixon 2002

- photon-pair production $q\bar{q} \to \gamma\gamma,\ gg \to \gamma\gamma$

  C. Anastasiou N. Glover M. Tejeda-Yeomans 2002

  Z. Bern A. De Freitas L. Dixon 2002

- $e^+e^- \to 3$ jets $\gamma^* \to q\bar{q}g$

  L. Garland T. Gehrmann N. Glover A. Koukoutsakis E. Remiddi 2002

- $V + 1$ jet production $q\bar{q} \to Vg$

  T. Gehrmann E. Remiddi 2002

- Drell-Yan V production $q\bar{q} \to V$

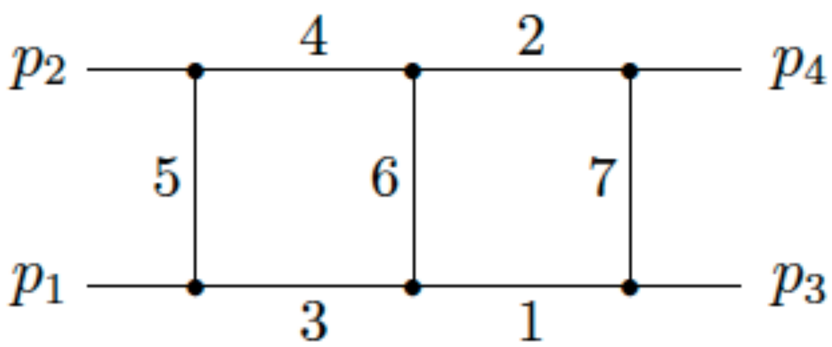
  R. Hamberg W. van Neerven T. Matsuura 1991

- Higgs production $gg \to H$ (in the $m_t \to \infty$ limit)

  R. Harlander W. Kilgore; C. Anastasiou K. Melnikov 2002

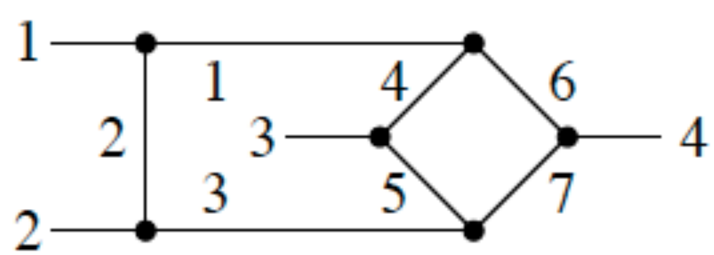
Two-Loop Master Integrals

- planar massless double box — V. Smirnov 1999



is the two-loop four-point amplitude in N=4 SYM

- non-planar massless double box — Tausk 1999



- planar one-mass double box — V. Smirnov 2000

  non-planar one-mass double box

N=4 Super Yang-Mills

- maximal supersymmetric theory (without gravity)
  conformally invariant, β fn. = 0

  - spin 1 gluon
    4 spin 1/2 gluinos
    6 spin 0 real scalars

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- AdS/CFT duality — Maldacena 97
  - large-λ limit of 4dim CFT $\leftrightarrow$ weakly-coupled string theory
    (aka weak-strong duality)

MHV amplitudes in planar N=4 SYM

- at any order in the coupling, colour-ordered MHV amplitude in N=4 SYM can be written as tree-level amplitude times a helicity-less loop coefficient $M_n^{(L)} = M_n^{(0)} m_n^{(L)}$

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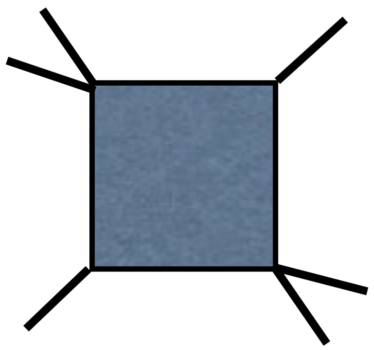
- at 1 loop

$$m_n^{(1)} = \sum_{pq} F^{2\mathrm{me}}(p, q, P, Q) \qquad\qquad n \geq 6$$



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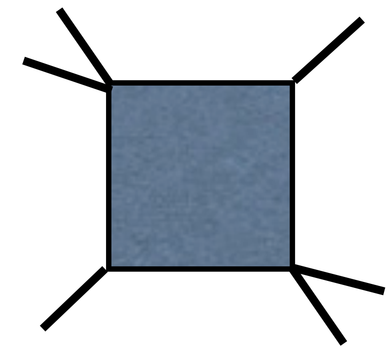

- at 2 loops, iteration formula for the *n*-pt amplitude

$$m_n^{(2)}(\epsilon) = \frac{1}{2}\left[m_n^{(1)}(\epsilon)\right]^2 + f^{(2)}(\epsilon)\, m_n^{(1)}(2\epsilon) + Const^{(2)} + R$$

Anastasiou Bern Dixon Kosower 03

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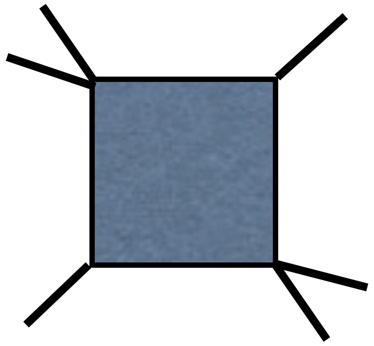

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- at all loops, ansatz for a resummed exponent

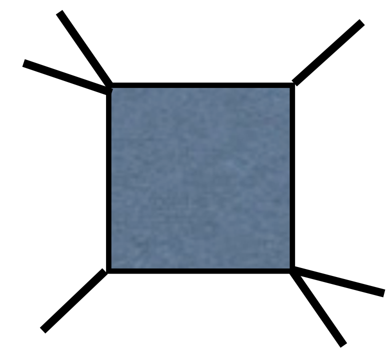
$$m_n^{(L)} = \exp\left[\sum_{l=1}^{\infty} a^l \left(f^{(l)}(\epsilon)\, m_n^{(1)}(l\epsilon) + Const^{(l)} + E_n^{(l)}(\epsilon)\right)\right] + R$$

Bern Dixon Smirnov 05

MHV amplitudes in planar *N*=4 SYM

- at any order in the coupling, colour-ordered MHV amplitude in N=4 SYM can be written as tree-level amplitude times a helicity-less loop coefficient $M_n^{(L)} = M_n^{(0)} m_n^{(L)}$

- at 1 loop



$$m_n^{(1)} = \sum_{pq} F^{2\mathrm{me}}(p, q, P, Q) \qquad n \geq 6$$

- at 2 loops, iteration formula for the *n*-pt amplitude

remainder function

$$m_n^{(2)}(\epsilon) = \frac{1}{2}\left[m_n^{(1)}(\epsilon)\right]^2 + f^{(2)}(\epsilon)\, m_n^{(1)}(2\epsilon) + Const^{(2)} + R$$

Anastasiou Bern Dixon Kosower 03

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Bern Dixon Smirnov 05

ansatz for MHV amplitudes in planar N=4 SYM

Bern Dixon Smirnov 05

$$M_n = M_n^{(0)} \left[1 + \sum_{L=1}^{\infty} a^L m_n^{(L)}(\epsilon) \right]$$

$$= M_n^{(0)} \exp \left[\sum_{l=1}^{\infty} a^l \left(f^{(l)}(\epsilon) m_n^{(1)}(l\epsilon) + Const^{(l)} + E_n^{(l)}(\epsilon) \right) \right]$$

coupling $a = \frac{\lambda}{8\pi^2}(4\pi e^{-\gamma})^\epsilon$ $\lambda = g^2 N$ 't Hooft parameter

$$f^{(l)}(\epsilon) = \frac{\hat{\gamma}_K^{(l)}}{4} + \epsilon \frac{l}{2} \hat{G}^{(l)} + \epsilon^2 f_2^{(l)} \qquad E_n^{(l)}(\epsilon) = O(\epsilon)$$

$\hat{\gamma}_K^{(l)}$ cusp anomalous dimension, known to all orders of *a*

Korchemsky Radyuskin 86
Beisert Eden Staudacher 06

$\hat{G}^{(l)}$ collinear anomalous dimension, known through O(a^4)

Bern Dixon Smirnov 05
Cachazo Spradlin Volovich 07

ansatz generalises the iteration formula for the 2-loop *n*-pt amplitude $m_n^{(2)}$

$$m_n^{(2)}(\epsilon) = \frac{1}{2}\left[m_n^{(1)}(\epsilon) \right]^2 + f^{(2)}(\epsilon)\, m_n^{(1)}(2\epsilon) + Const^{(2)} + \mathcal{O}(\epsilon)$$

MHV amplitudes ⇔ Wilson loops

- $$W[\mathcal{C}_n] = \mathrm{Tr}\,\mathcal{P}\,\exp\left[ig\oint d\tau\,\dot{x}^\mu(\tau)A_\mu(x(\tau))\right]$$

  closed contour C_n made by light-like external momenta $p_i = x_i - x_{i+1}$

  Alday Maldacena 07

- agreement between *n*-edged Wilson loop and *n*-point MHV amplitude at weak coupling (aka weak-weak duality)
  - verified for *n*-edged 1-loop Wilson loop up to 6-edged 2-loop Wilson loop

    Brandhuber Heslop Travaglini 07

    Drummond Henn Korchemsky Sokatchev 07

    Bern Dixon Kosower Roiban Spradlin Vergu Volovich 08

- *n*-edged 2-loop Wilson loops also computed (numerically)

  Anastasiou Brandhuber Heslop Khoze Spence Travaglini 09

- no amplitudes are known beyond the 6-point 2-loop amplitude

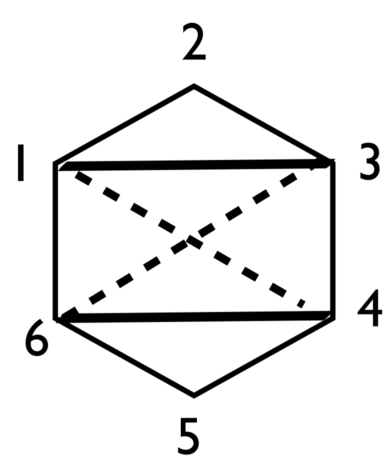
Wilson loops are easier to compute than amplitudes

Wilson loops & conformal symmetry

Drummond Henn Korchemsky Sokatchev 07

- N=4 SYM is invariant under $SO(2,4)$ conformal transformations
- the Wilson loops fulfill conformal Ward identities
- the solution of the Ward identity for special conformal boosts is given by the finite parts of the BDS ansatz + R
- for n = 4, 5, R is a constant
  for $n \geq 6$, R is a function of conformally invariant cross ratios
- for the 6-edged 2-loop Wilson loop,
  R is known analytically as a function of 3 cross ratios

$$
\begin{aligned}
u_{36} &= \frac{x_{13}^2 x_{46}^2}{x_{14}^2 x_{36}^2} = \frac{s_{12} s_{45}}{s_{123} s_{345}} \\
u_{14} &= \frac{x_{24}^2 x_{15}^2}{x_{25}^2 x_{14}^2} = \frac{s_{23} s_{56}}{s_{234} s_{123}} \\
u_{25} &= \frac{x_{35}^2 x_{26}^2}{x_{36}^2 x_{25}^2} = \frac{s_{34} s_{61}}{s_{234} s_{345}}
\end{aligned}
$$



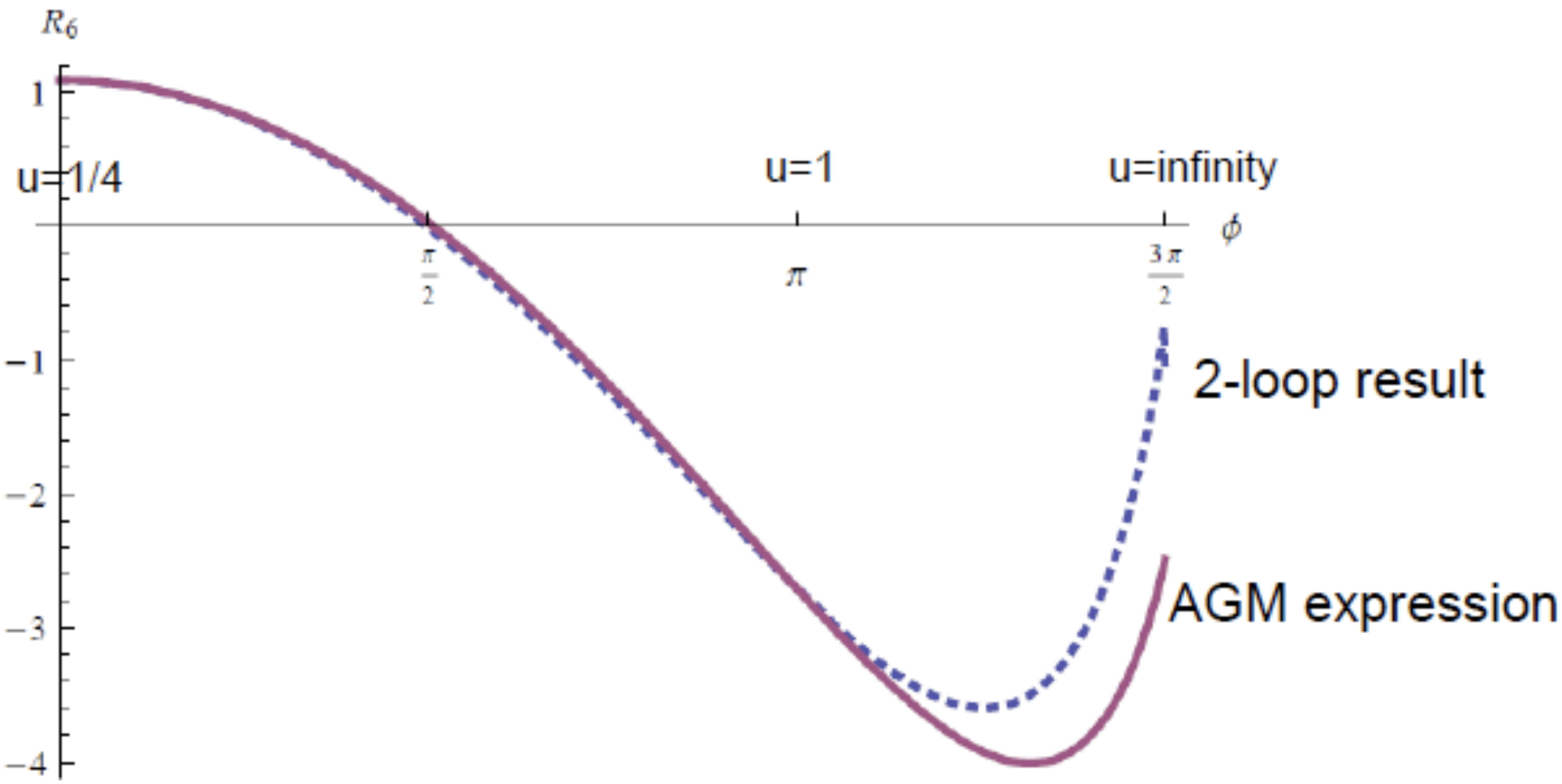
VDD Duhr Smirnov 09
Goncharov Spradlin Vergu Volovich 10

Wilson loops at weak and strong coupling

Wilson loops can also be computed at strong coupling

Alday Maldacena 07
Alday Gaiotto Maldacena 09

comparison of the 6-edged Wilson loop at weak and strong couplings



$$u = \frac{1}{4\cos^2(\phi/3)}$$

VDD Duhr Smirnov 09

Alday Gaiotto Maldacena 09
Brandhuber Heslop Khoze Travaglini 09

- the 2 curves are strikingly similar

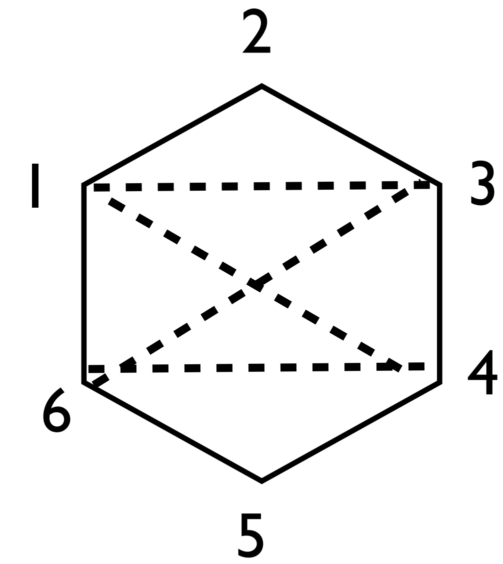
Conclusions

- an exciting period of LHC phenomenology has begun
- a lot of progress in pQCD in the last few years in NLO computations with many jets
- we expect substantial progress also for NNLO computations
- at 2 loops, the planar massless master integrals are now known up to 6 points; work is in progress to compute them for 7 or more points

Back-up slides

Z_n symmetric regular hexagons

regular hexagons are characterised by



$$x_{13}^2 = x_{24}^2 = x_{35}^2 = x_{46}^2 = x_{51}^2 = x_{62}^2; \qquad x_{14}^2 = x_{25}^2 = x_{36}^2$$

$$u_{36} = \frac{x_{13}^2 x_{46}^2}{x_{14}^2 x_{36}^2} = \frac{s_{12} s_{45}}{s_{123} s_{345}}$$

$$u_{14} = \frac{x_{24}^2 x_{15}^2}{x_{25}^2 x_{14}^2} = \frac{s_{23} s_{56}}{s_{234} s_{123}} \quad \Longrightarrow \quad u_{14} = u_{25} = u_{36} = u$$

$$u_{25} = \frac{x_{35}^2 x_{26}^2}{x_{36}^2 x_{25}^2} = \frac{s_{34} s_{61}}{s_{234} s_{345}}$$

At strong coupling, remainder function is obtained from ``minimal area surfaces in AdS$_5$ which end on a null polygonal contour at the boundary". One gets ``integral equations which determine the area as a function of the shape of the polygon. The equations are identical to those of the Thermodynamics Bethe Ansatz. The area is given by the free energy of the TBA system. The high temperature limit of the TBA system can be exactly solved"

$$R_6^{strong}(u,u,u) = \underbrace{\frac{\pi}{6} - \frac{1}{3\pi}\phi^2}_{\text{free energy}} - \frac{3}{8}\underbrace{\left(\ln^2(u) + 2\,\mathrm{Li}^2(1-u)\right)}_{\text{BDS - BDSlike}} \qquad u = \frac{1}{4\cos^2(\phi/3)}$$

Alday Gaiotto Maldacena 09

Amplitudes in twistor space

- twistors live in the fundamental irrep of $SO(2,4)$
- any point in dual space corresponds to a line in twistor space

$$x_a \leftrightarrow (Z_a, Z_{a+1})$$

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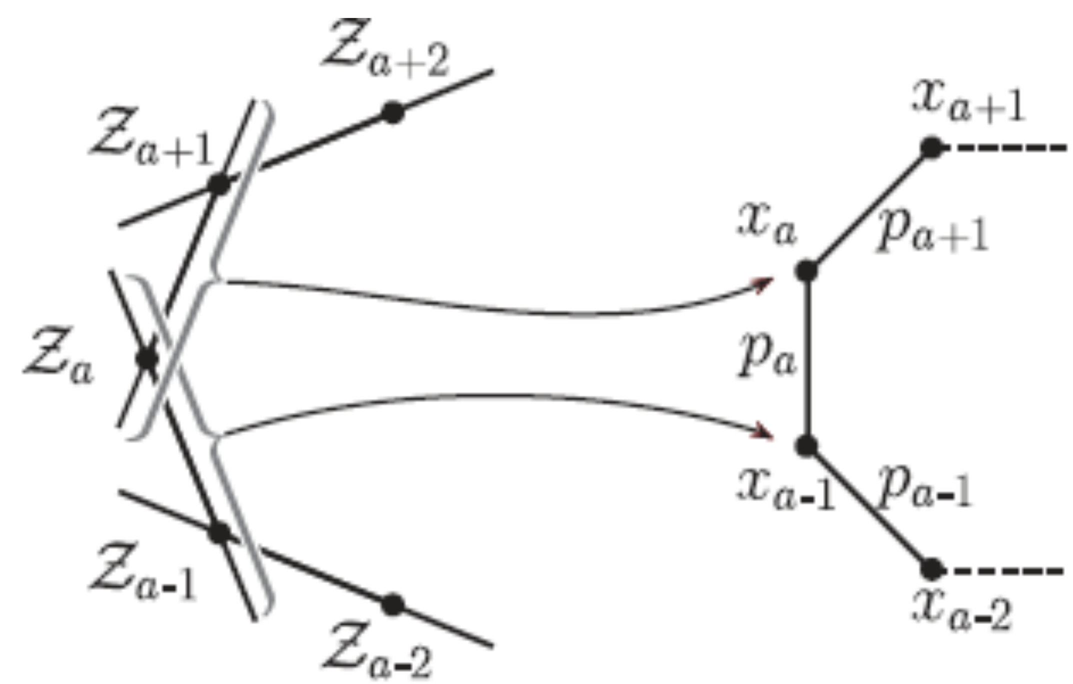
$x_a \leftrightarrow (Z_a, Z_{a+1})$

null separations in dual space correspond to intersections in twistor space



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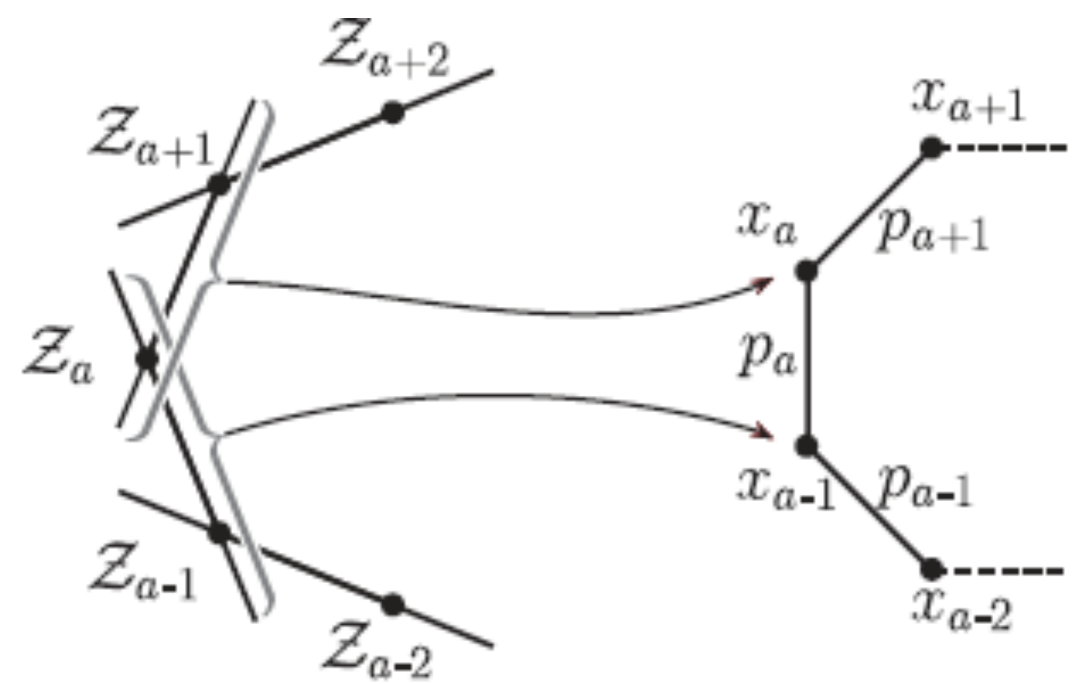
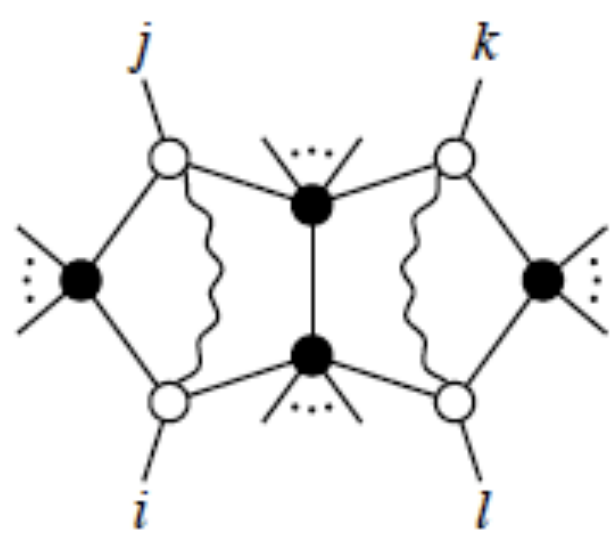


2-loop *n*-pt MHV amplitudes can be written as sum of pentaboxes in twistor space

$$m_n^{(2)} = \frac{1}{2} \sum_{i<j<k<l<i}$$



Arkani-Hamed Bourjaily Cachazo Trnka10