A few Facts on the High Energy Limit of QCD

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GOAL to analyse the QCD dynamics in the $s \gg |t|$ limit: the high energy limit (HEL)

FACT in HEL the scattering processes are dominated by sub-processes with gluon exchange in the t channel

BFKL theory resums multiple gluon radiation out of the gluon exchanged in the t channel

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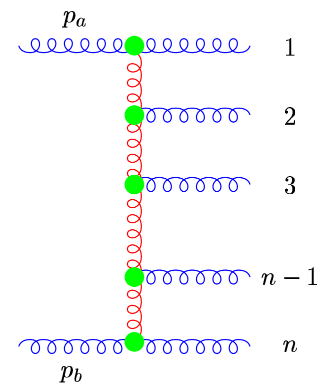
PHENOM. Process-dependent questions:

- ☛ does a fixed-order expansion in α_S suffice to describe the data ?
- ☛ can the data be described in terms of other, e.g. soft gluon, resummations ?
- ☛ in phase space, where do sub-processes with gluon exchange in the t channel dominate over the other sub-processes ?

BFKL RESUMMATION

- in any scattering process with $s \gg |t|$ gluon exchange in the t channel dominates



- BFKL is a resummation of multiple gluon radiation out of the gluon exchanged in the t channel

- for $s \gg |t|$ BFKL resums the Leading Log (and Next-to-Leading Log) contributions, in $\log(s/t)$, of the radiative corrections to the gluon propagator in the t channel, to all orders in α_S

- the LL terms are obtained in the approximation of strong rapidity ordering $(y_1 \gg y_2 \gg \ldots \gg y_n)$ and no k_t ordering of the emitted gluons

- the NLL terms are universal

- the resummation yields a 2-dim integral equation for the evolution of the gluon propagator in the t channel

BFKL PHENOMENOLOGY

- in principle, the BFKL resummation can be applied to any scattering process with $s \gg |t|$, where t is a typical (squared) transverse energy scale

  - ☛ in $p\, p$ collisions $\left\{ \begin{array}{c} \text{dijet} \\ V,\, H + 2 \text{ jet} \\ \text{heavy diquark} \end{array} \right\}$ production at large rapidities

  - ☛ in DIS $\left\{ \begin{array}{c} F_2 \text{ scaling violations} \\ \text{forward jet production} \end{array} \right.$

  - ☛ in e^+e^-, $\gamma^*\gamma^* \to$ hadrons at large Y

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- in HEL, the partonic cross section is $\hat{\sigma}(AB \to j_1 j_2) \sim \mathcal{I}(j_1)\mathcal{F}_{BFKL}\mathcal{I}(j_2)$

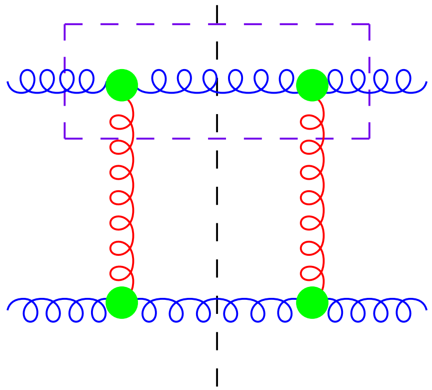
- the BFKL ladder $\mathcal{F}_{BFKL}$ is universal

- the impact factors $\mathcal{I}(j) \sim |C^{g;g}|^2$ are process dependent

IMPACT FACTORS

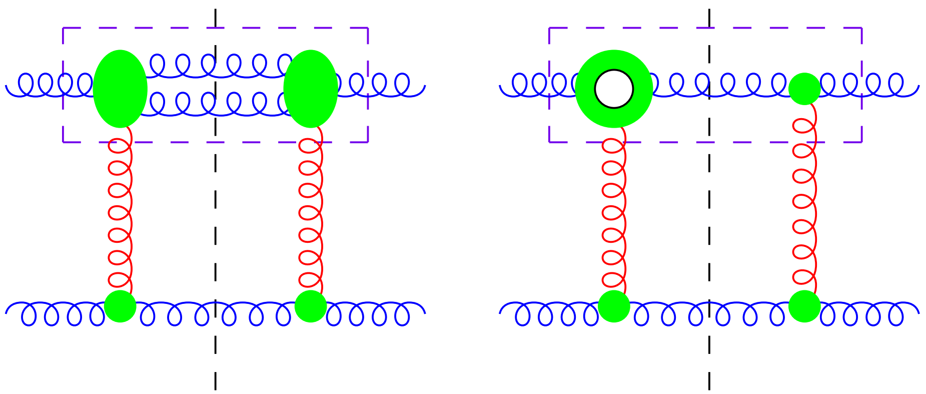
LO IMPACT FACTOR

$g\, g^* \to g$:



☛ at LO the impact factors are known for all the processes of interest

NLO IMPACT FACTOR



☛ at NLO the impact factors are known for $q\, g^* \to q$, $g\, g^* \to g$ and $\gamma^* g^* \to q\bar{q}$

Bartels, Colferai, Gieseke, Vacca 2001-02

Drawbacks of the BFKL ladder

- energy and longitudinal momentum are not conserved:
  in dijet production, the exact x's are

$$x_a = \frac{e^{y_{j_1}}}{\sqrt{S}} \left(|p_{j_1\perp}| + |p_{j_2\perp}| e^{-\Delta y} + \sum_{i=1}^{n} p_{i\perp} e^{y_i - y_{j_1}} \right)$$

$$x_b = \frac{e^{-y_{j_2}}}{\sqrt{S}} \left(|p_{j_2\perp}| + |p_{j_1\perp}| e^{-\Delta y} + \sum_{i=1}^{n} p_{i\perp} e^{-y_i + y_{j_2}} \right)$$

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BFKL Monte Carlo

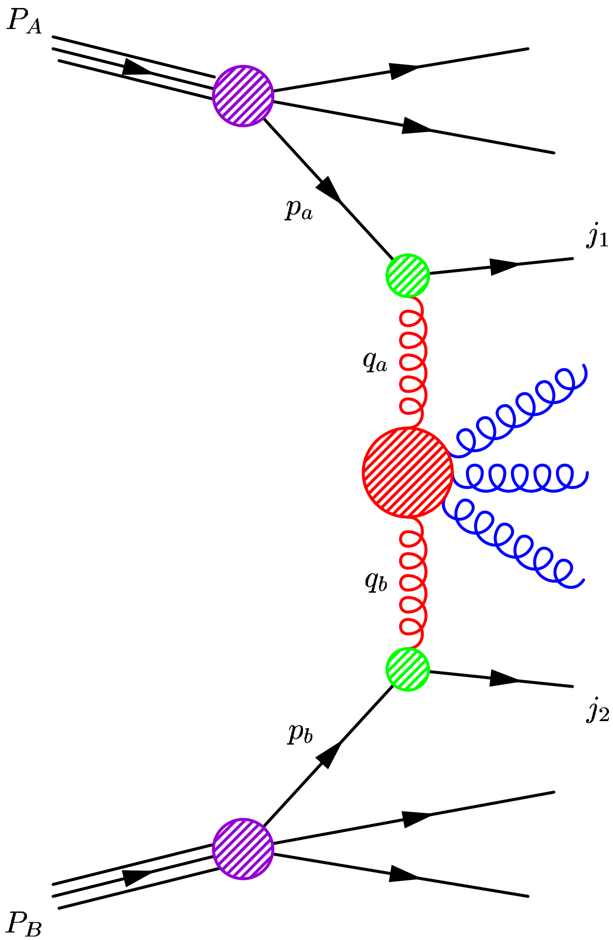
- an iterative solution of the BFKL equation can account for
  - running of α_S
  - energy and longitudinal momentum conservation

Schmidt 1996; Orr, Stirling 1997

- an iterative solution of the NLL kernel is now available

Andersen, Sabio-Vera 2003

DIJET PRODUCTION IN $p\,p$ COLLISIONS



KINEMATICS

$p_a = x_a P_A \quad p_b = x_b P_B$:

incoming parton momenta

S : hadron c.m. energy

$s = x_a x_b S$: parton c.m. energy

$E_{j_{1,2\perp}}$: jet transverse energy

$Q^2 = -t$: typical momentum transfer

➥ $Q^2 \sim E_{j\perp}^2$

$\Delta y = |y_{j_1} - y_{j_2}|$:

rapidity difference between the jets

❋ $$\ln \frac{S}{Q^2} = \ln \frac{1}{x_a} + \ln \frac{s}{Q^2} + \ln \frac{1}{x_b}$$

❋ $x_{a,b} = \mathcal{O}(1) \qquad \ln \frac{s}{Q^2} \simeq \Delta y \gg 1$

➥ physics of large rapidity intervals,

and not small-x physics

DIJET PRODUCTION IN HEL

- in an event with two or more jets, tag the most forward and the most backward jets

$$\Delta y = |y_{j_1} - y_{j_2}| \simeq \ln \frac{x_a x_b S}{E_{j_1\perp} E_{j_2\perp}}$$

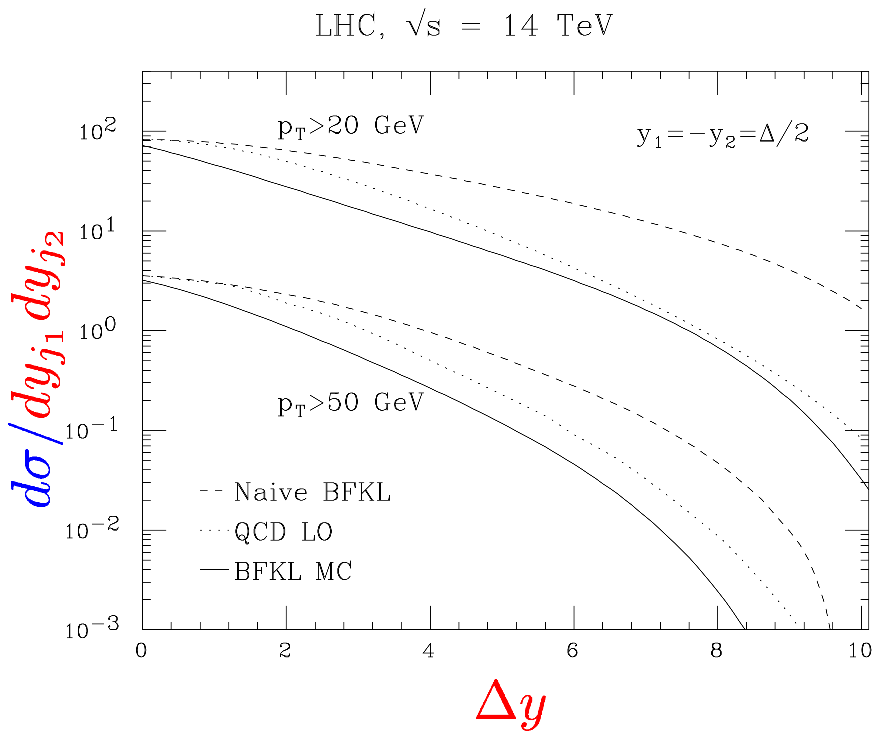
- minimise the jet transverse energy
- maximise $s = x_a x_b S$
  - in a collider with ramping-up energy S, fix $= x_{a,b}$

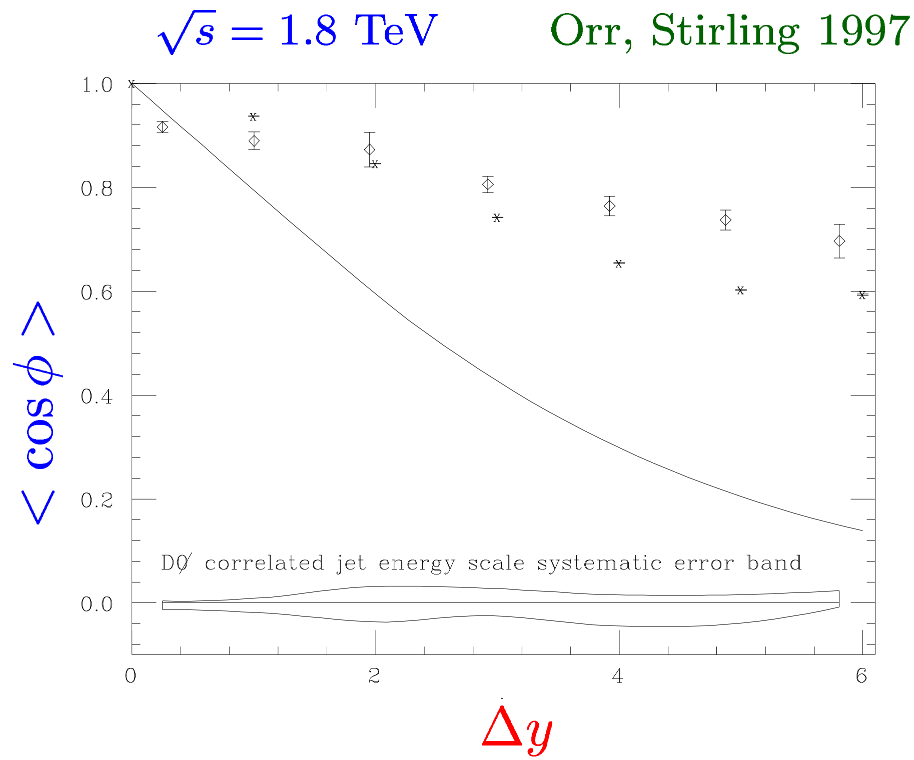
    analyse $\frac{d\sigma}{dx_a dx_b}$ for different values of S Mueller, Navelet 1987
  - in a fixed energy S collider, increase $= x_{a,b}$

    analyse $\begin{cases} \frac{d\sigma}{d\Delta y} & \text{for different values of } \Delta y \\ \frac{d\sigma}{d\Delta y\, d\phi} & \phi: \text{ azimuthal angle between tagged jets} \end{cases}$

Schmidt, VDD; Stirling 1993-95

Dijet Production – Phenomenology





- in $\dfrac{d\sigma}{dy_{j_1}dy_{j_2}}$ the BFKL Monte Carlo yields a depletion rather than an enhancement, both for Tevatron & LHC, due to the falling parton luminosities

- $< \cos\phi >$ shows too much azimuthal decorrelation wrt Tevatron D0 data, while it is well described by a parton-shower Monte Carlo (HERWIG)

Caveat $< \cos\phi >$ is dominated by soft gluon (Sudakov) effects

Mueller-Navelet Jets

Mueller-Navelet proposal for colliders with ramping-up energy S:

- take the cross section for dijet production in HEL at fixed x's:

$$\frac{d\sigma}{dx_a^0 dx_b^0} = \int d^2p_{j_{1\perp}} d^2p_{j_{2\perp}} f_{\rm eff}(x_a^0, \mu_F^2)\, f_{\rm eff}(x_b^0, \mu_F^2)\, \frac{d\hat\sigma_{gg}}{d^2p_{j_{1\perp}} d^2p_{j_{2\perp}}}$$

- use approximate x's: $x_a^{MN} = \frac{E_\perp}{\sqrt{S}} e^{y_{j_{1\perp}}}$ $\quad x_b^{MN} = \frac{E_\perp}{\sqrt{S}} e^{-y_{j_{2\perp}}}$
  - ➥ the x's are in a one-to-one correspondence with the rapidities
  - ➥ Δy is fixed at its max: $\Delta y = \ln \frac{x_a^{MN} x_b^{MN} S}{E_\perp^2}$
- integrate out transverse energies above a threshold $E_\perp$: $|p_{j_{1,2\perp}}| \geq E_\perp$
  - ➥ the Mueller-Navelet gluon-gluon cross section is

$$\hat\sigma_{gg} = \frac{9\pi\alpha_S^2}{2E_\perp^2} \frac{e^{4C_A \ln 2\alpha_S \Delta y/\pi}}{\sqrt{7\zeta_3 C_A \alpha_S \Delta y/2}}$$

- compute the cross section at different c.m. energies S

Mueller-Navelet Jets – D0 Analysis

D0 implementation of Mueller-Navelet: D0 Collaboration 1999

- exact LO x's:
$$\begin{cases} x_1 = \dfrac{2|p_{j_1\perp}|}{\sqrt{S}} e^{\bar{y}} \cosh \dfrac{\Delta y}{2} \\ x_2 = \dfrac{2|p_{j_2\perp}|}{\sqrt{S}} e^{-\bar{y}} \cosh \dfrac{\Delta y}{2} \end{cases} \qquad \bar{y} = \frac{y_{j_1\perp} + y_{j_2\perp}}{2}$$

- acceptance cuts:
$$\begin{cases} |p_{j_{1,2}\perp}| \geq 20 \text{ GeV} \qquad Q^2 = |p_{j_1\perp} p_{j_2\perp}| \leq Q^2_{MAX} = 1000 \text{ GeV}^2 \\ |y_{j_{1,2}\perp}| \leq 3 \qquad \Delta y \geq 2 \end{cases}$$

- measure the cross section at $\sqrt{S_A} = 1800$ GeV and $\sqrt{S_B} = 630$ GeV in 6 (x_1, x_2) bins, with $0.06 \leq x_1, x_2 \leq 0.22$

- using the Mueller-Navelet cross section, compute the ratio $R = \dfrac{\sigma(S_A)}{\sigma(S_B)}$

➥ get the BFKL intercept

$$\alpha_{BFKL} = 1.65 \pm 0.07$$

Our Mueller-Navelet/D0 Analysis

❋ D0 uses an upper bound on Q^2, such that $E_\perp^2/Q^2_{MAX} = 0.4$

❋ in HEL $\begin{cases} x_1 \to x_a^0 \neq x_a^{MN} \\ x_2 \to x_b^0 \neq x_b^{MN} \end{cases}$

$x_{a,b}^{MN}$ are not good approximations to $x_{1,2}$

❋ the rapidity interval can be written as

$$\Delta y = Y + \ln \frac{E_\perp^2}{p_{j_{1\perp}} p_{j_{2\perp}}} \quad \text{with} \quad Y = \ln \frac{x_a^0 x_b^0 S}{E_\perp^2}$$

$\Delta y \geq 2$ ☛ $Q^2_{MAX} = E_\perp^2 e^{(Y-2)}$

➥ an effective maximum momentum transfer

$Q^2_{MAX} = \min(1000\,\mathrm{GeV}^2,\ E_\perp^2 e^{(Y-2)})$

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we analysed Mueller-Navelet/D0 with

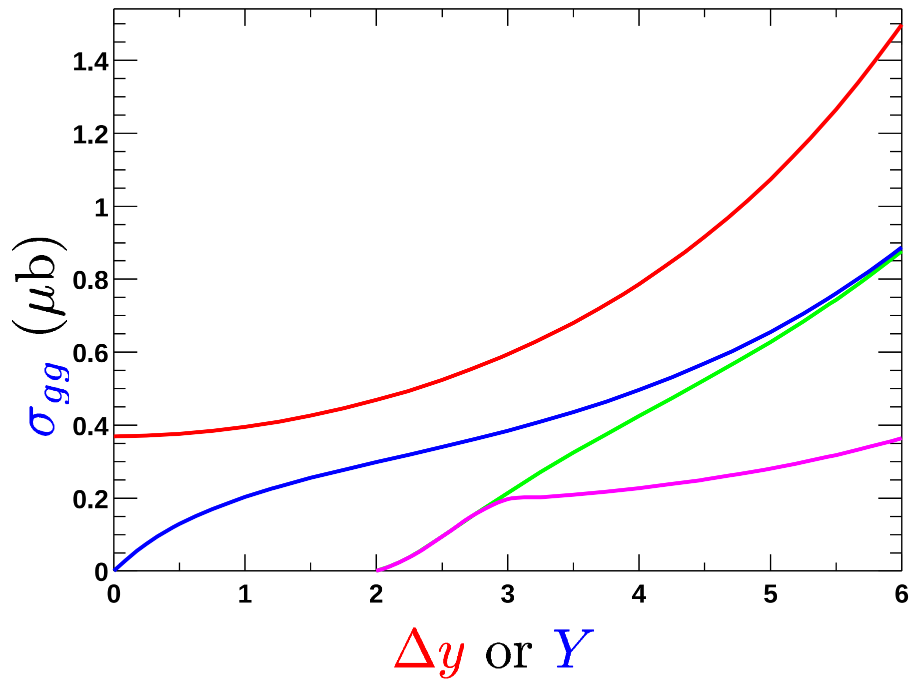
☛ analytic BFKL — Andersen, Frixione, Schmidt, Stirling, VDD 2001

☛ BFKL Monte Carlo

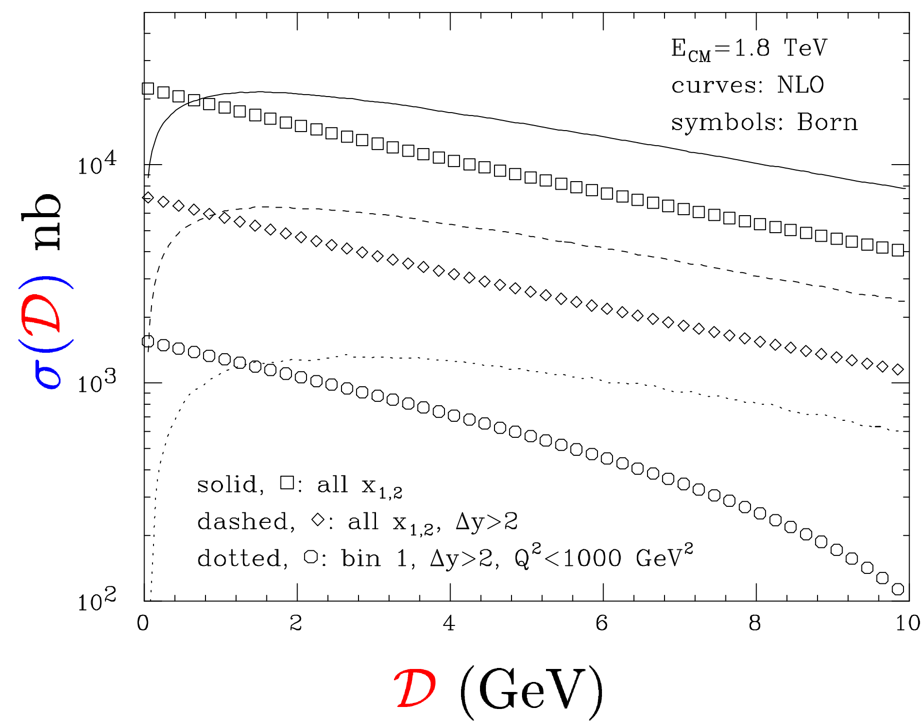
☛ general-purpose NLO partonic Monte Carlo

Our Mueller-Navelet/D0 Analysis

analytic BFKL



NLO partonic Monte Carlo



red: Mueller-Navelet (at fixed $x_{a,b}^{MN}$)

blue: at fixed $x_{a,b}^{0}$

green: at fixed $x_{a,b}^{0}$ with $\Delta y \geq 2$

magenta: green + $p_{j_1\perp} p_{j_2\perp} \leq 1000\,\text{GeV}^2$

☛ in σ_{gg} all curves have asymptotically the same shape: sub-leading terms are important

$\mathcal{D} = |\min(p_{j_{1\perp}}) - \min(p_{j_{2\perp}})|$

symbols: LO

curves: NLO

Caveat

soft gluon effects at $\mathcal{D} = 0$

CONCLUSIONS

THEORY

- in the high energy limit, scattering amplitudes factorise into a gluon ladder and impact factors
- the BFKL resummation is known at NLL accuracy
- impact factors are known at LO accuracy for all processes of interest, and at NLO for the most topical cases
- a BFKL Monte Carlo at LL accuracy is available;
  work is in progress on a NLL Monte Carlo

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Dijet Production in $p\,p$ collisions (at Tevatron)

- **Azimuthal decorrelation** No evidence of BFKL radiation has been found. Data are described well by parton shower generators (HERWIG)
- **Mueller-Navelet Jets** Sub-leading effects forbid the extraction of the BFKL intercept
- **Caveat** Both analyses above are contaminated by soft gluon effects