

# The high-energy limit of $H + 2$ jet production via gluon fusion

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At the Large Hadron Collider (LHC), the main production channels of a Higgs boson are gluon fusion and weak-boson fusion (WBF) [1, 2]. The WBF process,  $qq \rightarrow qqH$ , occurs through the exchange of a  $W$  or a  $Z$  boson in the  $t$  channel, and is characterized by the production of two forward quark jets [3]. Even though it is smaller than the gluon fusion channel by about a factor of 5 for an intermediate mass Higgs boson, it is interesting because it is expected to provide information on Higgs boson couplings [4]. In this respect,  $H + 2$  jet production via gluon-gluon fusion, which has a larger production rate before cuts, can be considered a background; it has the same final-state topology, and thus may hide the features of the WBF process.

In Higgs production via gluon fusion, the Higgs boson is produced mostly via a top quark loop. The computation of  $H + 2$  jet production involves up to pentagon quark loops [5]. However, if the Higgs mass is smaller than the threshold for the creation of a top-quark pair,  $M_H \lesssim 2M_t$ , the coupling of the Higgs to the gluons via a top-quark loop can be replaced by an effective coupling [6]: this is called the *large- $M_t$  limit*. It simplifies the calculation, because it reduces the number of loops in a given diagram by one. In  $H + 2$  jet production, the large- $M_t$  limit yields a good approximation to the exact calculation if, in addition to the condition  $M_H \lesssim 2M_t$ , we require that the jet transverse energies are smaller than the top-quark mass,  $p_\perp \lesssim M_t$  [5]. However, the large  $M_t$  approximation is quite insensitive to the value of the Higgs-jet and/or dijet invariant masses. The last issue is not academic, because Higgs production via WBF, to which we should like to compare, features typically two forward quark jets, and thus a large dijet invariant mass.

In this contribution, we consider  $H + 2$  jet production when Higgs-jet and/or dijet invariant masses become much larger than the typical momentum transfers in the scattering. We term these conditions the *high-energy limit*. In this limit the scattering amplitude

factorizes into *impact factors* connected by a gluon exchanged in the  $t$  channel. Assembling together different impact factors, the amplitudes for different sub-processes can be obtained. Thus the high-energy factorization constitutes a stringent consistency check on any amplitude for the production of a Higgs plus one or more jets.

In the high-energy limit of  $H + 2$  jet production, the relevant (squared) energy scales are the parton center-of-mass energy  $s$ , the Higgs mass  $M_H^2$ , the dijet invariant mass  $s_{j_1 j_2}$ , and the jet-Higgs invariant masses  $s_{j_1 H}$  and  $s_{j_2 H}$ . At leading order they are related through momentum conservation,

$$s = s_{j_1 j_2} + s_{j_1 H} + s_{j_2 H} - M_H^2. \quad (1)$$

There are two possible high-energy limits to consider:  $s_{j_1 j_2} \gg s_{j_1 H}, s_{j_2 H} \gg M_H^2$  and  $s_{j_1 j_2}, s_{j_2 H} \gg s_{j_1 H}, M_H^2$ . In the first case the Higgs boson is centrally located in rapidity between the two jets, and very far from either jet. In the second case the Higgs boson is close to one jet, say to jet  $j_1$ , in rapidity, and both of these are very far from jet  $j_2$ . In both cases the amplitudes will factorize, and the relevant Higgs vertex in case 1 and the Higgs–gluon and Higgs–quark impact factors in case 2 can be obtained from the amplitudes for  $q Q \rightarrow q Q H$  and  $q g \rightarrow q g H$  scattering.

## The high-energy limit $s_{j_1 j_2} \gg s_{j_1 H}, s_{j_2 H} \gg M_H^2$

We consider the production of two partons of momenta  $p_1$  and  $p_3$  and a Higgs boson of momentum  $p_H$ , in the scattering between two partons of momenta  $p_2$  and  $p_4$ , where all momenta are taken as outgoing. We consider the limit in which the Higgs boson is produced centrally in rapidity, and very far from either jet,  $s_{j_1 j_2} \gg s_{j_1 H}, s_{j_2 H} \gg M_H^2$ , which is equivalent to require that

$$p_1^+ \gg p_H^+ \gg p_3^+, \quad p_1^- \ll p_H^- \ll p_3^-, \quad (2)$$

where we have introduced the light-cone coordinates  $p^\pm = p_0 \pm p_z$ , and complex transverse coordinates  $p_\perp = p^x + ip^y$ . In the limit (2), the amplitudes are dominated by gluon exchange in the  $t$  channel, with emission of the Higgs boson from the  $t$ -channel gluon. We can write the amplitude for  $q Q \rightarrow q Q H$  scattering in the high-energy limit as [7]

$$\begin{aligned} & i \mathcal{M}^{qq \rightarrow Hqq}(p_2^{-\nu_1}, p_1^{\nu_1} | H | p_3^{\nu_3}, p_4^{-\nu_3}) \\ &= 2s [g T_{a_1 \bar{a}_2}^c C^{\bar{q};q}(p_2^{-\nu_1}; p_1^{\nu_1})] \frac{1}{t_1} [\delta^{cc'} C^H(q_1, p_H, q_2)] \frac{1}{t_2} [g T_{a_3 \bar{a}_4}^{c'} C^{\bar{q};q}(p_4^{-\nu_3}; p_3^{\nu_3})], \quad (3) \end{aligned}$$

where  $q_1 = -(p_1 + p_2)$ ,  $q_2 = p_3 + p_4$ ,  $t_i \simeq -|q_{i\perp}|^2$ ,  $i = 1, 2$ , and the  $\nu$ 's are the quark helicities. In Eq. (3) we have made explicit the helicity conservation along the massless quark lines. The effective vertex  $C^{\bar{q};q}$  for the production of a quark jet,  $q g^* \rightarrow q$ , contributes a phase factor [8]: its square is 1. The effective vertex for Higgs production along the gluon ladder,  $g^* g^* \rightarrow H$ , with  $g^*$  an off-shell gluon, is

$$C^H(q_1, p_H, q_2) = 2 \frac{g^2 M_t^2}{v} \left( m_{H_\perp}^2 A_1(q_1, q_2) - 2A_2(q_1, q_2) \right). \quad (4)$$

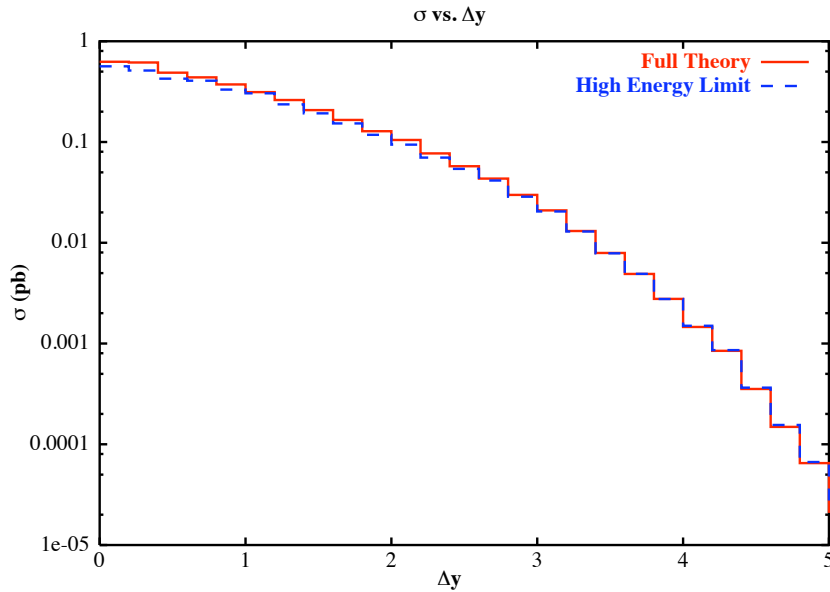


Figure 1: Cross section in  $H + 2$  jet production in  $pp$  collisions at the LHC energy  $\sqrt{s} = 14$  TeV as a function of  $\Delta y$ , with  $M_H = 120$  GeV and  $M_t = 175$  GeV. The dijet invariant mass fulfills the constraint  $\sqrt{s_{j_1 j_2}} \geq 600$  GeV. The rapidity interval  $\Delta y$  is defined as  $\Delta y = \min(|y_{j_1} - y_H|, |y_{j_2} - y_H|)$ , with the kinematical constraint  $y_{j_1} > y_H > y_{j_2}$ . The solid line is the exact production rate; the dashed line is the rate in the high-energy limit.

The scalar coefficients of the triangle vertex with two off-shell gluons,  $A_{1,2}$ , are defined in terms of the form factors  $F_T$  and  $F_L$  of Ref. [5] as

$$A_1 = \frac{i}{(4\pi)^2} F_T, \quad A_2 = \frac{i}{(4\pi)^2} (F_T q_1 \cdot q_2 + F_L q_1^2 q_2^2). \quad (5)$$

We have checked analytically that the amplitude for  $qg \rightarrow qgH$  scattering can also be written as Eq. (3), provided we perform on one of the two effective vertices  $C^{\bar{q}q}$  the substitution (for the sake of illustration, we display it here for the lower vertex)

$$ig f^{bb'c} C^{g;g}(p_b^{\nu_b}; p_{b'}^{\nu_{b'}}) \leftrightarrow g T_{b'\bar{b}}^c C^{\bar{q};q}(p_b^{-\nu_{b'}}; p_{b'}^{\nu_{b'}}), \quad (6)$$

and use the effective vertices  $g^*g \rightarrow g$  for the production of a gluon jet [8] (which contribute a phase factor as well). The same check on the (squared) amplitude for  $gg \rightarrow ggH$  scattering has been performed numerically. Thus, in the high-energy limit (2), the amplitudes for  $qQ \rightarrow qQH$ ,  $qg \rightarrow qgH$  and  $gg \rightarrow ggH$  scattering only differ by the color strength in the jet-production vertex. Therefore, in a production rate it is enough to consider one of them and include the others through the effective parton distribution function [9],  $f_{\text{eff}}(x, \mu_F^2) = G(x, \mu_F^2) + (C_F/C_A) \sum_f [Q_f(x, \mu_F^2) + \bar{Q}_f(x, \mu_F^2)]$ , where  $x$  is the momentum fraction of the incoming parton,  $\mu_F^2$  is the collinear factorization scale, and where the sum is over the quark flavors.

In Fig. 1 we plot the cross section in  $H + 2$  jet production in  $pp$  collisions at the LHC energy  $\sqrt{s} = 14$  TeV, as a function of  $\Delta y$ , which is defined as the smallest rapidity difference between the Higgs and the jets,  $\Delta y = \min(|y_{j_1} - y_H|, |y_{j_2} - y_H|)$ , with the kinematical constraint  $y_{j_1} > y_H > y_{j_2}$ . The solid line is the exact production rate, with the amplitudes evaluated in Ref. [5]; the dashed line is the rate in the high-energy limit (2), with the amplitudes evaluated using Eqns. (3)–(6). It is apparent that the high-energy limit works very well over the whole  $\Delta y$  spectrum. However, in the evaluation of the effective vertex (4), we used the exact value of the scalar coefficients  $A_{1,2}$ . A more conservative statement is to say that when any kinematic quantity involved in the amplitude (3) is evaluated in the limit (2), we expect the high-energy limit to represent a good approximation of the exact calculation when  $\Delta y \gtrsim 2$ .

## The high-energy limit $s_{j_1 j_2}, s_{j_2 H} \gg s_{j_1 H}, M_H^2$

Next, we consider the limit in which the Higgs boson is produced forward in rapidity, and close to one of the jets, say to jet  $j_1$ , and both are very far from jet  $j_2$ , *i.e.*  $s_{j_1 j_2}, s_{j_2 H} \gg s_{j_1 H}, M_H^2$ . This limit implies that

$$p_1^+ \simeq p_H^+ \gg p_3^+, \quad p_1^- \simeq p_H^- \ll p_3^- . \quad (7)$$

In this limit, the amplitudes are again dominated by gluon exchange in the  $t$  channel, and factorize into an effective vertex for the production of a jet and another for the production of a Higgs plus a jet. For example, in the limit (7) the amplitude for  $q g \rightarrow q g H$  scattering [5] with the incoming gluon (quark) of momentum  $p_2$  ( $p_4$ ), can be written as [7]

$$\begin{aligned} & i \mathcal{M}^{gq \rightarrow gHq}(p_2^{\nu_2}; p_1^{\nu_1}, H | p_3^{\nu_3}; p_4^{-\nu_3}) \\ &= 2s [ig f^{a_2 a_1 c} C^{g;Hg}(p_2^{\nu_2}; p_1^{\nu_1}, p_H)] \frac{1}{t} [ig T_{a_3 \bar{a}_4}^c C^{\bar{q};q}(p_4^{-\nu_3}; p_3^{\nu_3})] , \end{aligned} \quad (8)$$

where  $C^{g;Hg}(p_2^{\nu_2}; p_1^{\nu_1}, p_H)$  is the effective vertex for the production of a Higgs boson and a gluon jet,  $g^* g \rightarrow gH$ . It has two independent helicity configurations, which we can take to be  $C^{g;Hg}(p_2^-; p_1^+, p_H)$  and  $C^{g;Hg}(p_2^+; p_1^+, p_H)$  [7]. High-energy factorization also implies that the amplitude for  $g g \rightarrow g g H$  scattering can be put in the form (8), up to replacing the incoming quark with a gluon via the substitution (6). Likewise, the amplitude for  $q Q \rightarrow q Q H$  scattering can be written as

$$\begin{aligned} & i \mathcal{M}^{qQ \rightarrow qHQ}(p_2^{-\nu_1}; p_1^{\nu_1}, p_H | p_3^{\nu_3}; p_4^{-\nu_3}) \\ &= 2s [g T_{a_1 \bar{a}_2}^c C^{\bar{q};Hq}(p_2^{-\nu_1}; p_1^{\nu_1}, p_H)] \frac{1}{t} [g T_{a_3 \bar{a}_4}^{c'} C^{\bar{q};q}(p_4^{-\nu_3}; p_3^{\nu_3})] , \end{aligned} \quad (9)$$

where  $C^{\bar{q};Hq}(p_2^{-\nu_1}; p_1^{\nu_1}, p_H)$  is the effective vertex for the production of a Higgs and a quark jet,  $g^* q \rightarrow qH$ . There is only one independent helicity configuration, which we can take to be  $C^{\bar{q};Hq}(p_2^-; p_1^+, p_H)$ , and its expression is given in Ref. [7], where an analysis of the limit (7) with the kinematic parameters of Fig. 1 can also be found.

In conclusion, we have considered  $H + 2$  jet production via gluon fusion, when either one of the Higgs-jet or the dijet invariant masses become much larger than the typical momentum transfers in the scattering. These limits also occur naturally in Higgs production via WBF. We have shown that we can write the scattering amplitudes in accordance to high-energy factorization, Eqns. (3), (8) and (9). The corresponding effective vertices, whose squares are the impact factors, can be found in Ref. [7].

## References

- [1] G. L. Bayatian *et al.*, CMS Technical Proposal, report CERN/LHCC/94-38x (1994); R. Kinnunen and D. Denegri, *Expected SM/SUSY Higgs observability in CMS*, CMS NOTE 1997/057; R. Kinnunen and A. Nikitenko, *Study of  $H_{SUSY} \rightarrow \tau\tau \rightarrow l^\pm + h^\mp + E_t^{miss}$  in CMS*, CMS TN/97-106; R. Kinnunen and D. Denegri, *The  $H_{SUSY} \rightarrow \tau\tau \rightarrow h^\pm + h^\mp + X$  channel, its advantages and potential instrumental drawbacks*, [hep-ph/9907291].
- [2] ATLAS Collaboration, ATLAS TDR, report CERN/LHCC/99-15 (1999).
- [3] Y. L. Dokshitzer, S. I. Troian and V. A. Khoze, *Collective QCD Effects In The Structure Of Final Multi - Hadron States*, Sov. J. Nucl. Phys. **46** (1987) 712.  
J. D. Bjorken, *Rapidity gaps and jets as a new physics signature in very high-energy hadron-hadron collisions*, Phys. Rev. **D47** (1993) 101.
- [4] D. Zeppenfeld, R. Kinnunen, A. Nikitenko and E. Richter-Was, *Measuring Higgs boson couplings at the LHC*, Phys. Rev. D **62** (2000) 013009 [hep-ph/0002036].
- [5] V. Del Duca, W. Kilgore, C. Oleari, C. Schmidt and D. Zeppenfeld,  *$H + 2$  jets via gluon fusion*, Phys. Rev. Lett. **87** (2001) 122001 [hep-ph/0105129].  
V. Del Duca, W. Kilgore, C. Oleari, C. Schmidt and D. Zeppenfeld, *Gluon-fusion contributions to  $H + 2$  jet production*, Nucl. Phys. B **616** (2001) 367 [hep-ph/0108030].
- [6] M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, *Low-Energy Theorems For Higgs Boson Couplings To Photons*, Sov. J. Nucl. Phys. **30** (1979) 711.  
J. Ellis, M. K. Gaillard and D. V. Nanopoulos, *A Phenomenological Profile Of The Higgs Boson*, Nucl. Phys. **B106** (1976) 292.
- [7] V. Del Duca, W. Kilgore, C. Oleari, C. Schmidt and D. Zeppenfeld, in preparation.
- [8] V. Del Duca, A. Frizzo and F. Maltoni, *Factorization of tree QCD amplitudes in the high-energy limit and in the collinear limit*, Nucl. Phys. **B568** (2000) 211 [hep-ph/9909464].
- [9] B. L. Combridge and C. J. Maxwell, *Untangling Large  $P(T)$  Hadronic Reactions*, Nucl. Phys. **B239** (1984) 429.