Cross sections
at
NNLO

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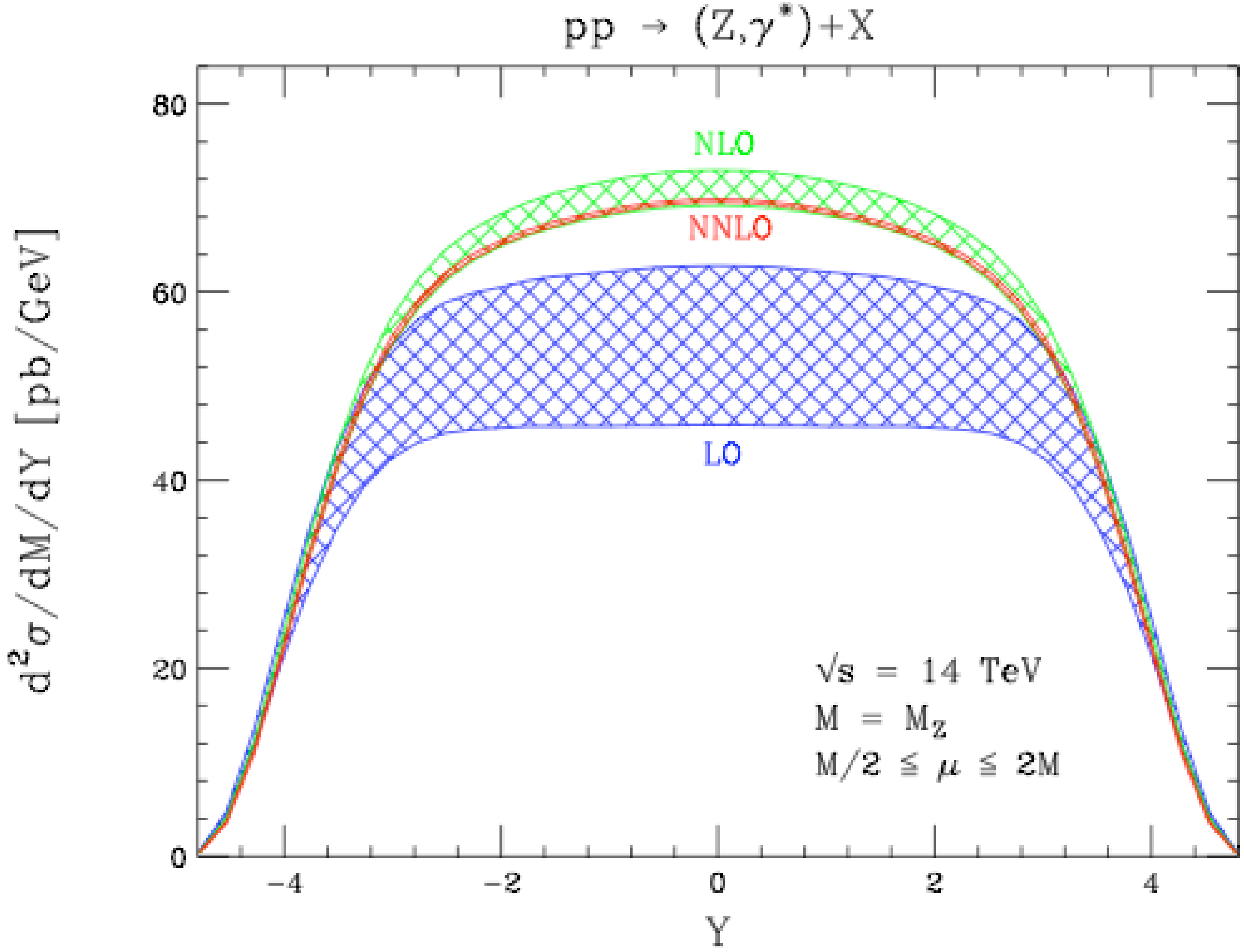
NLO features

- Jet structure: final-state collinear radiation
- PDF evolution: initial-state collinear radiation
- Opening of new channels
- Reduced sensitivity to fictitious input scales: μ_R, μ_F
  - predictive normalisation of observables
    - first step toward precision measurements
    - accurate estimate of signal and background for Higgs and new physics
- Matching with parton-shower MC's: MC@NLO

NNLO Drell-Yan Z production at LHC

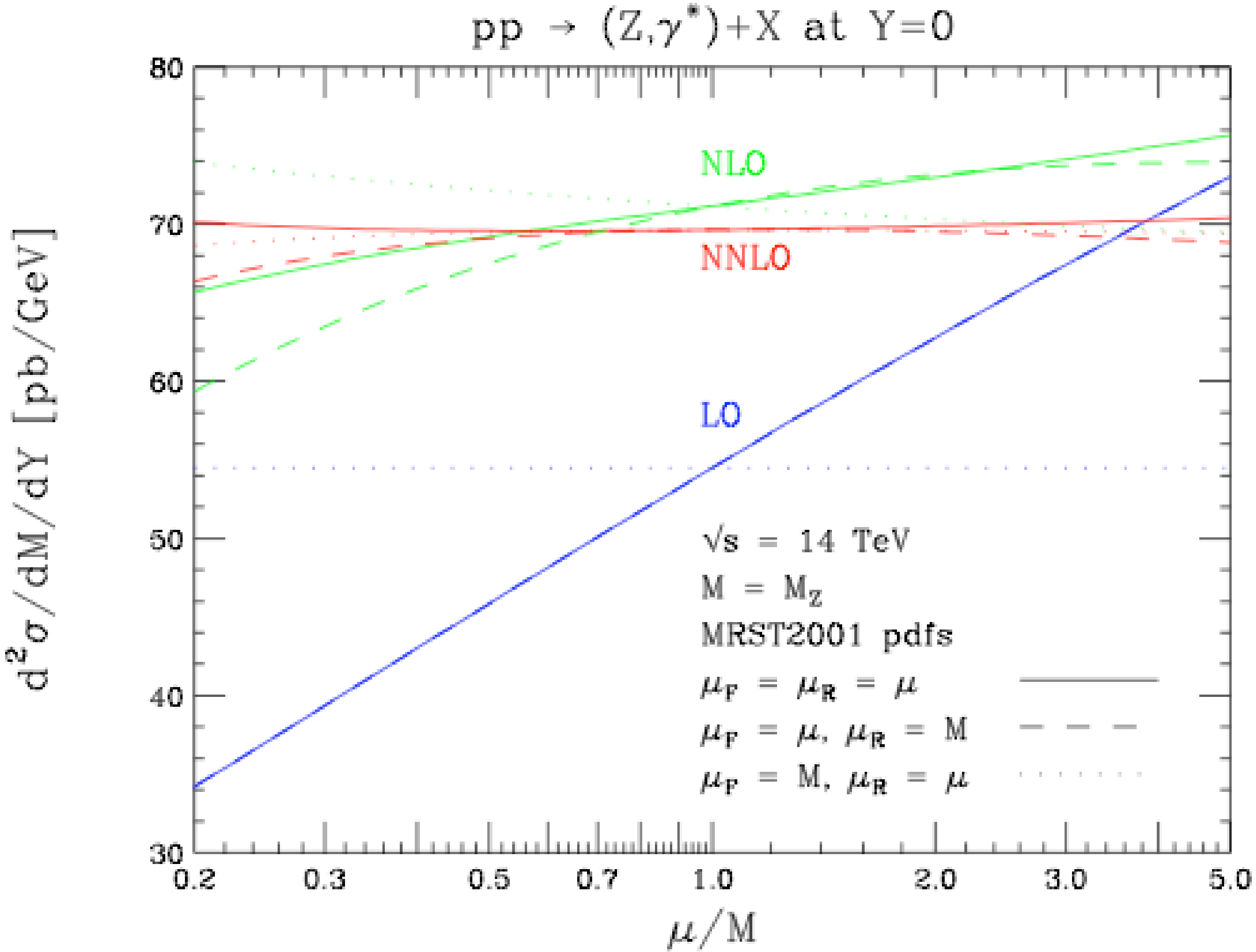


Rapidity distribution for an on-shell Z boson

- $30\%(15\%)$ NLO increase wrt to LO at central Y's (at large Y's)
  NNLO decreases NLO by $1-2\%$
- scale variation: $\approx 30\%$ at LO; $\approx 6\%$ at NLO; less than 1% at NNLO

C. Anastasiou L. Dixon K. Melnikov F. Petriello 2003

Scale variations in Drell-Yan Z production



- solid: vary μ_R and μ_F together
- dashed: vary μ_F only
- dotted: vary μ_R only

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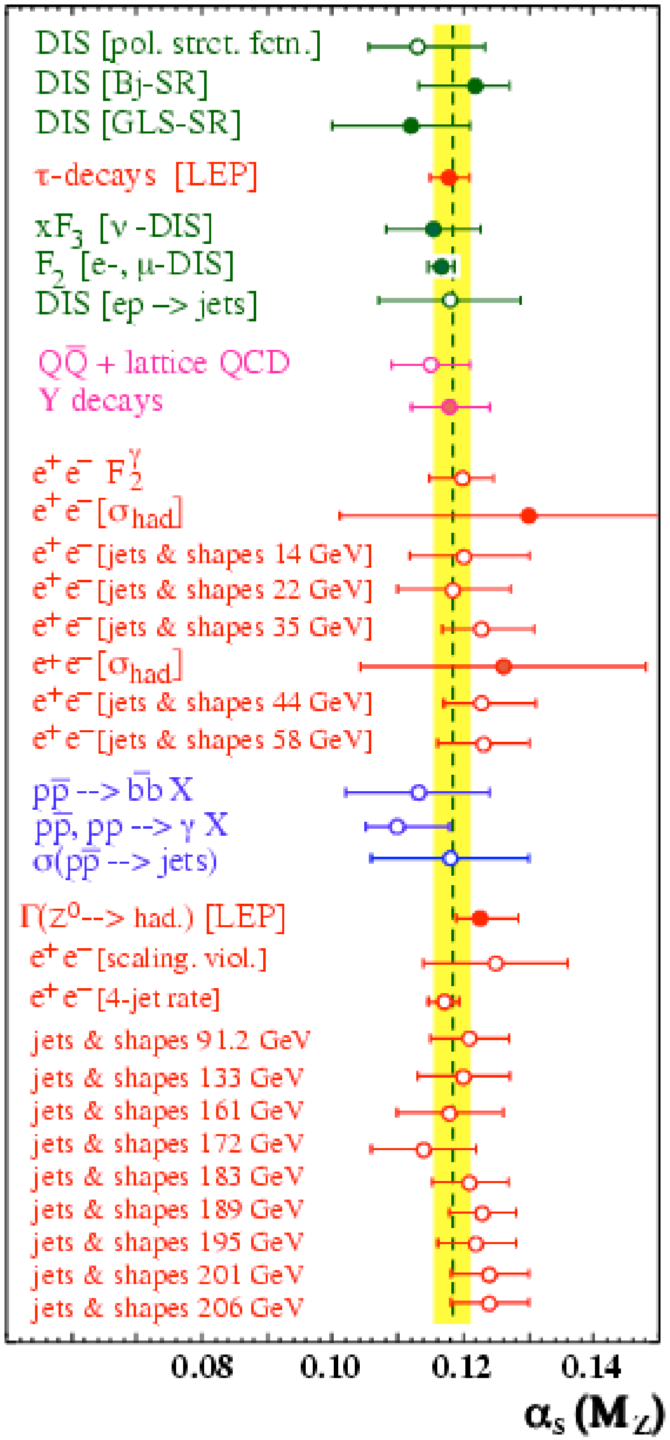
Summary of $\alpha_S(M_Z)$

S. Bethke hep-ex/0407021



world average of $\alpha_S(M_Z)$
using $\overline{\mathrm{MS}}$ and NNLO results only

$$\alpha_S(M_Z) = 0.1182 \pm 0.0027$$

(cf. 2002 $\alpha_S(M_Z) = 0.1183 \pm 0.0027$
outcome almost identical
because new entries wrt 2002
- LEP jet shape observables and
4-jet rate, and HERA jet rates
and shape variables - are NLO)

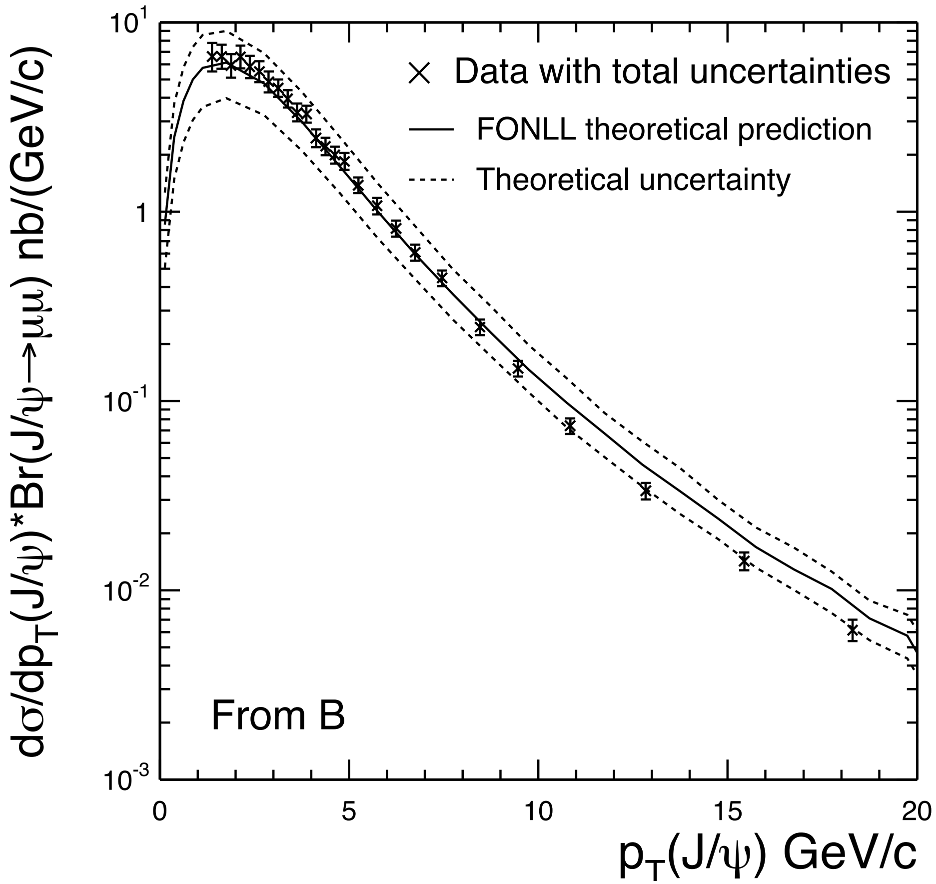
filled symbols are NNLO results

Is NLO enough to describe data ?

b cross section in $p\bar{p}$ collisions at 1.96 TeV

$$d\sigma(p\bar{p} \to H_b X, H_b \to J/\psi\ X)/dp_T(J/\psi)$$

CDF hep-ex/0412071



total x-sect is

$$19.4 \pm 0.3(stat)^{+2.1}_{-1.9}(syst)\ \text{nb}$$

FONLL = NLO + NLL

Cacciari, Frixione, Mangano, Nason, Ridolfi 2003

good agreement with data (with use of updated FF's by Cacciari & Nason)

Is NLO enough to describe data ?

Drell-Yan W acceptances at LHC with leptonic decay of the W

Cuts A $\longrightarrow |\eta^{(e)}| < 2.5,\ p_T^{(e)} > 20$ GeV, $p_T^{(\nu)} > 20$ GeV

Cuts B $\longrightarrow |\eta^{(e)}| < 2.5,\ p_T^{(e)} > 40$ GeV, $p_T^{(\nu)} > 20$ GeV

	LO		LO+HW	NLO		MC@NLO
Cuts A	0.5249	$\xrightarrow{-7.7\%}$	0.4843	0.4771	$\xrightarrow{+1.5\%}$	0.4845
	$\downarrow$5.4%			$\downarrow$7.0%		$\downarrow$6.3%
Cuts A, no spin	0.5535			0.5104		0.5151
Cuts B	0.0585	$\xrightarrow{+208\%}$	0.1218	0.1292	$\xrightarrow{+2.9\%}$	0.1329
	$\downarrow$29%			$\downarrow$16%		$\downarrow$18%
Cuts B, no spin	0.0752			0.1504		0.1570

- $|\text{MC@NLO} - \text{NLO}| = \mathcal{O}(2\%)$ S. Frixione M.L. Mangano 2004
- NNLO useless without spin correlations
- Precisely evaluated Drell-Yan W, Z cross sections could be used as ``standard candles" to measure the parton luminosity at LHC

Drell-Yan W acceptances at LHC with leptonic decay of the W

$p_\perp^{e,\min}$ (GeV)	A(NLO)	A(NNLO)
20	0.487,0.488,0.489	0.497,0.492,0.491
30	0.379,0.378,0.378	0.379,0.376,0.377
40	0.127,0.125,0.122	0.161,0.155,0.152
50	0.0312,0.0295,0.0277	0.0427,0.0397,0.0387

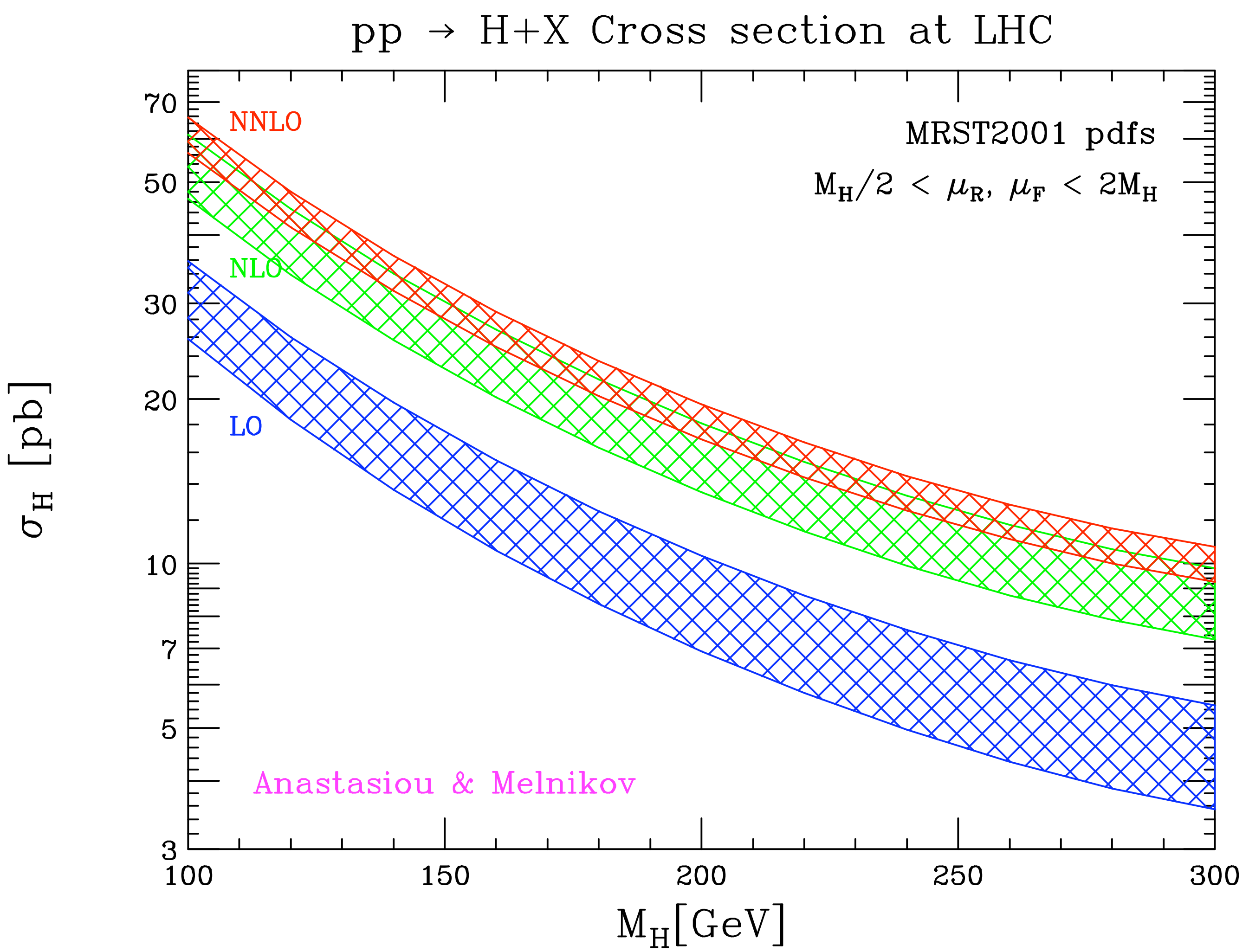
$\mu = m_W/2, m_W, 2m_W$

- At LO, $p_{e,\perp} \leq m_W/2$
- NNLO corrections are large for $p_{e,\perp} = 40, 50$ GeV
  but are at the percent level for $p_{e,\perp} = 20, 30$ GeV

K. Melnikov, F. Petriello 2006

Is NLO enough to describe data ?

Total cross section for inclusive Higgs production at LHC



contour bands are

lower

$\mu_R = 2M_H \quad \mu_F = M_H/2$

upper

$\mu_R = M_H/2 \quad \mu_F = 2M_H$

scale uncertainty is about 10%

NNLO prediction stabilises the perturbative series

NNLO corrections may be relevant if

- the main source of uncertainty in extracting info from data is due to NLO theory: α_S measurements
- NLO corrections are large:
  Higgs production from gluon fusion in hadron collisions
- NLO uncertainty bands are too large to test theory vs. data: b production in hadron collisions
- NLO is effectively leading order:
  energy distributions in jet cones

in short, NNLO is relevant where NLO fails to do its job

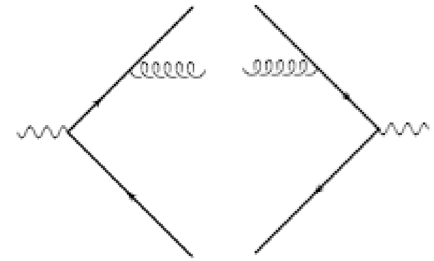
NNLO state of the art

- Drell-Yan W, Z production
  - total cross section — Hamberg, van Neerven, Matsuura 1990; Harlander, Kilgore 2002
  - fully differential cross section — Melnikov, Petriello 2006
- Higgs production
  - total cross section — Harlander, Kilgore; Anastasiou, Melnikov 2002; Ravindran, Smith, van Neerven 2003
  - fully differential cross section — Anastasiou, Melnikov, Petriello 2004
- $e^+e^- \to 3$ jets
  - the $1/N_c^2$ term — the Gehrmanns, Glover 2004-5

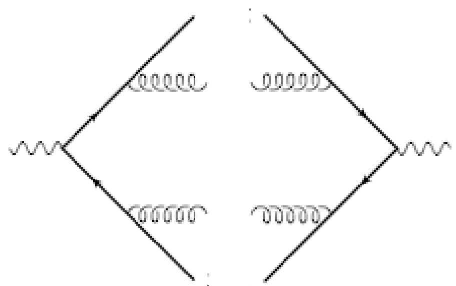
NLO assembly kit

$e^+e^- \to 3$ jets

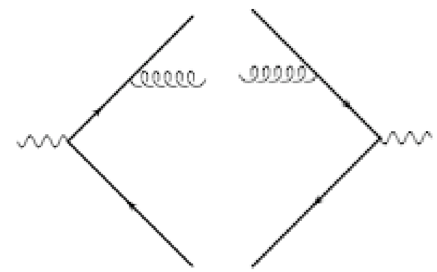
leading order $|\mathcal{M}_n^{tree}|^2$



NLO real



IR

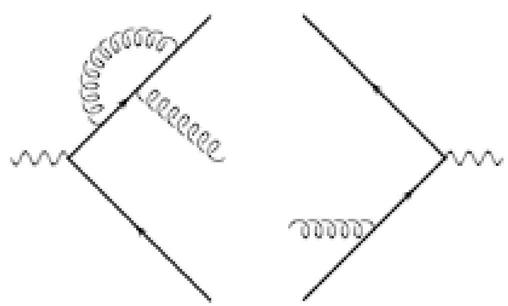


$\otimes$

$$|\mathcal{M}_{n+1}^{\text{tree}}|^2 \quad \to \quad |\mathcal{M}_n^{\text{tree}}|^2 \quad \times \quad \int dPS |P_{\text{split}}|^2$$

$$= \quad -\left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon}\right)$$

NLO virtual



$d = 4 - 2\epsilon$

$$\int d^d l \; 2(\mathcal{M}_n^{loop})^* \mathcal{M}_n^{tree} \quad = \quad \left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon}\right) |\mathcal{M}_n^{tree}|^2 + fin.$$

NLO production rates

Process-independent procedure devised in the 90's

- slicing — Giele Glover & Kosower
- subtraction — Frixione Kunszt & Signer; Nagy & Trocsanyi
  - dipole — Catani & Seymour
  - antenna — Kosower; Campbell Cullen & Glover

$$\sigma = \sigma^{\rm LO} + \sigma^{\rm NLO} = \int_m d\sigma_m^B \; J_m + \sigma^{\rm NLO}$$

$$\sigma^{\rm NLO} = \int_{m+1} d\sigma_{m+1}^{\rm R} J_{m+1} + \int_m d\sigma_m^{\rm V} J_m$$

the 2 terms on the rhs are divergent in d=4

use universal IR structure to subtract divergences

$$\sigma^{\rm NLO} = \int_{m+1} \left[d\sigma_{m+1}^{\rm R} J_{m+1} - d\sigma_{m+1}^{\rm R,A} J_m \right] + \int_m \left[d\sigma_m^{\rm V} + \int_1 d\sigma_{m+1}^{\rm R,A} \right] J_m$$

the 2 terms on the rhs are finite in d=4

NLO IR limits

collinear operator

$$\mathrm{C}_{ir}|\mathcal{M}^{(0)}_{m+2}(p_i,p_r,\ldots)|^2 \propto \frac{1}{s_{ir}}\langle\mathcal{M}_{m+1}(0)(p_{ir},\ldots)|\hat{P}^{(0)}_{f_i f_r}|\mathcal{M}_{m+1}(0)(p_{ir},\ldots)\rangle$$

soft operator

$$\mathrm{S}_r|\mathcal{M}^{(0)}_{m+2}(p_r,\ldots)|^2 \propto \frac{s_{ik}}{s_{ir}s_{rk}}\langle\mathcal{M}_{m+1}(0)(\ldots)|T_i\cdot T_k|\mathcal{M}_{m+1}(0)(\ldots)\rangle$$

counterterm

$$\sum_r\left(\sum_{i\neq r}\frac{1}{2}\mathrm{C}_{ir}+\mathrm{S}_r\right)|\mathcal{M}^{(0)}_{m+2}(p_i,p_r,\ldots)|^2$$

performs double subtraction in overlapping regions

NLO overlapping divergences

$\mathrm{C}_{ir}\mathrm{S}_r$ can be used to cancel double subtraction

$$\mathrm{C}_{ir}\left(\mathrm{S}_r - \mathrm{C}_{ir}\mathrm{S}_r\right)|\mathcal{M}^{(0)}_{m+2}|^2 = 0$$

$$\mathrm{S}_r\left(\mathrm{C}_{ir} - \mathrm{C}_{ir}\mathrm{S}_r\right)|\mathcal{M}^{(0)}_{m+2}|^2 = 0$$

the NLO counterterm

$$\mathrm{A}_1|\mathcal{M}^{(0)}_{m+2}|^2 = \sum_r \left[\sum_{i\neq r}\frac{1}{2}\mathrm{C}_{ir} + \left(\mathrm{S}_r - \sum_{i\neq r}\mathrm{C}_{ir}\mathrm{S}_r\right)\right]|\mathcal{M}^{(0)}_{m+2}(p_i,p_r,\dots)|^2$$

- has the same singular behaviour as SME, and is free of double subtractions

$$\mathrm{C}_{ir}\left(1-\mathrm{A}_1\right)|\mathcal{M}^{(0)}_{m+1}|^2 = 0 \qquad \mathrm{S}_r\left(1-\mathrm{A}_1\right)|\mathcal{M}^{(0)}_{m+1}|^2 = 0$$

- contains spurious singularities when parton $s \neq r$ becomes unresolved, but they are screened by J_m

NNLO cross sections

- **Analytic integration** — Hamberg, van Neerven, Matsuura 1990; Anastasiou Dixon Melnikov Petriello 2003
  - ↑ first method
  - ⇒ flexible enough to include a limited class of acceptance cuts by modelling cuts as ``propagators''
- **Sector decomposition** — Denner Roth 1996; Binoth Heinrich 2000; Anastasiou, Melnikov, Petriello 2004
  - ↑ flexible enough to include any acceptance cuts
  - ↑ cancellation of divergences is performed numerically
  - ⇒ can it handle many final-state partons ?
- **Subtraction**
  - ↑ process independent
  - ⇒ cancellation of divergences is semi-analytic can it be fully automatised ?

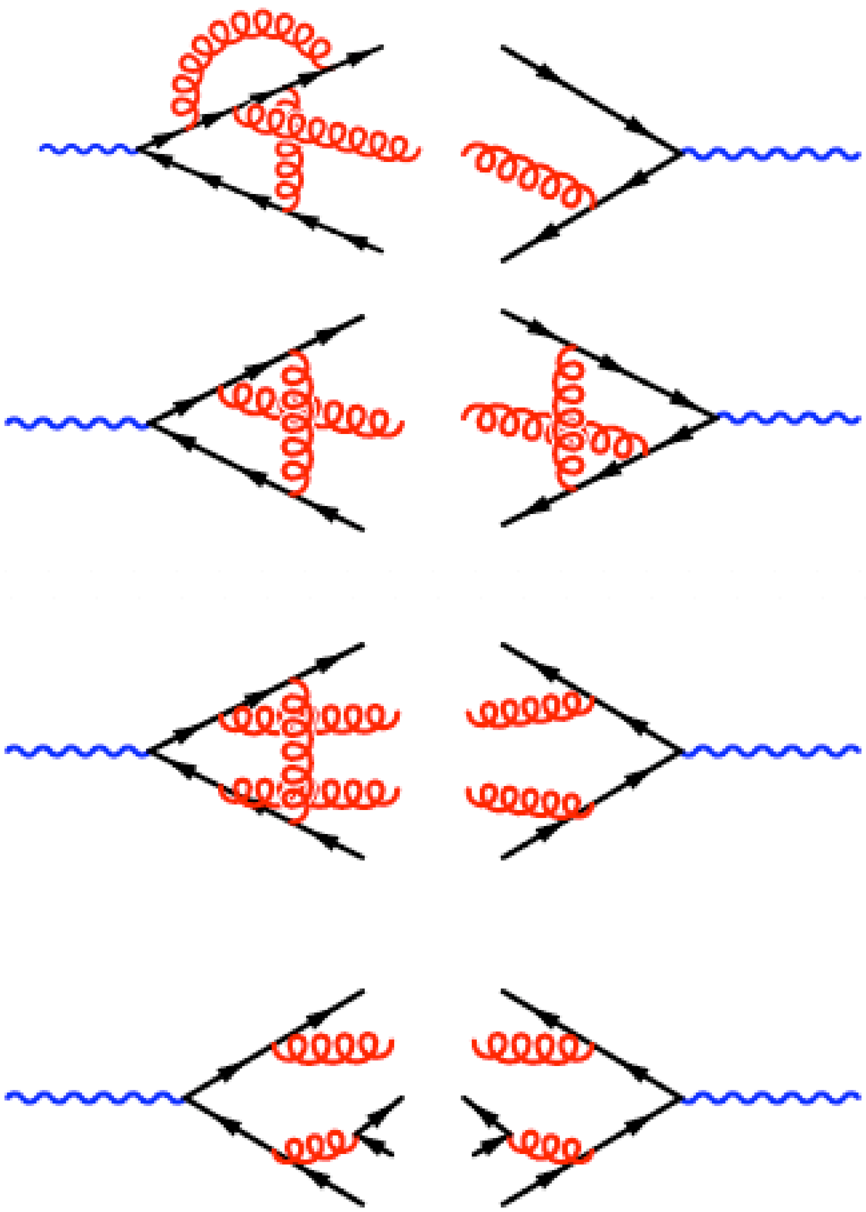
NNLO assembly kit

$e^+e^- \to 3$ jets

double virtual

real-virtual

double real



Two-loop matrix elements

- two-jet production $qq' \to qq', \ q\bar{q} \to q\bar{q}, \ q\bar{q} \to gg, \ gg \to gg$

  C. Anastasiou N. Glover C. Oleari M. Tejeda-Yeomans 2000-01

  Z. Bern A. De Freitas L. Dixon 2002

- photon-pair production $q\bar{q} \to \gamma\gamma, \ gg \to \gamma\gamma$

  C. Anastasiou N. Glover M. Tejeda-Yeomans 2002
  Z. Bern A. De Freitas L. Dixon 2002

- $e^+e^- \to 3$ jets $\gamma^* \to q\bar{q}g$

  L. Garland T. Gehrmann N. Glover A. Koukoutsakis E. Remiddi 2002

- $V + 1$ jet production $q\bar{q} \to Vg$

  T. Gehrmann E. Remiddi 2002

- Drell-Yan V production $q\bar{q} \to V$

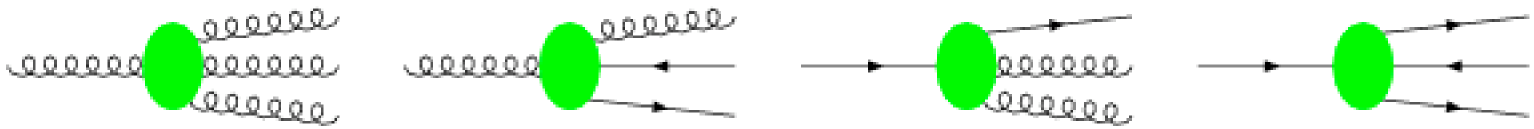
  R. Hamberg W. van Neerven T. Matsuura 1991

- Higgs production $gg \to H$ (in the $m_t \to \infty$ limit)

  R. Harlander W. Kilgore; C. Anastasiou K. Melnikov 2002

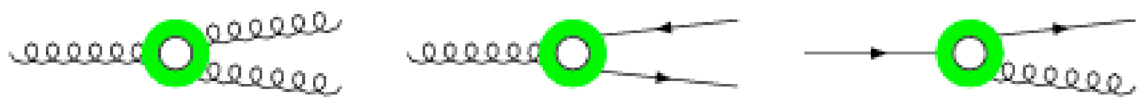
NNLO cross sections

- universal IR structure ⟹ process-independent procedure
- universal collinear and soft currents
  - 3-parton tree splitting functions



J. Campbell N. Glover 1997; S. Catani M. Grazzini 1998; A. Frizzo F. Maltoni VDD 1999; D. Kosower 2002

  - 2-parton one-loop splitting functions



Z. Bern L. Dixon D. Dunbar D. Kosower 1994; Z. Bern W. Kilgore C. Schmidt VDD 1998-99;
D. Kosower P. Uwer 1999; S. Catani M. Grazzini 1999; D. Kosower 2003

- universal subtraction counterterms
  - several ideas and works in progress

D. Kosower; S. Weinzierl; the Gehrmanns & G. Heinrich 2003
S. Frixione M. Grazzini 2004; G. Somogyi Z. Trocsanyi VDD 2005

  - but completely figured out only for $e^+e^- \to 3$ jets

the Gehrmanns & N. Glover 2005

NNLO subtraction

$$\sigma^{\mathrm{NNLO}} = \int_{m+2} d\sigma^{\mathrm{RR}}_{m+2} J_{m+2} + \int_{m+1} d\sigma^{\mathrm{RV}}_{m+1} J_{m+1} + \int_m d\sigma^{\mathrm{VV}}_m J_m$$

the 3 terms on the rhs are divergent in d=4

use universal IR structure to subtract divergences

$$\sigma^{\mathrm{NNLO}} = \int_{m+2} \left[d\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - d\sigma^{\mathrm{RR,A_2}}_{m+2} J_m \right]$$

takes care of doubly-unresolved regions,

but still divergent in singly-unresolved ones

$$+ \int_{m+1} \left[d\sigma^{\mathrm{RV}}_{m+1} J_{m+1} - d\sigma^{\mathrm{RV,A_1}}_{m+1} J_m \right]$$

still contains $1/\epsilon$ poles in regions away from 1-parton IR regions

$$+ \int_m \left[d\sigma^{\mathrm{VV}}_m + \int_2 d\sigma^{\mathrm{RR,A_2}}_{m+2} + \int_1 d\sigma^{\mathrm{RV,A_1}}_{m+1} \right] J_m$$

NNLO counterterm

- construct the 2-unresolved-parton counterterm using the IR currents

$$
\begin{aligned}
\mathbf{A}_2|\mathcal{M}^{(0)}_{m+2}|^2 = \sum_r \sum_{s\neq r} \Bigg\{ \sum_{i\neq r,s} \Bigg[& \frac{1}{6}\mathbf{C}_{irs} + \sum_{j\neq i,r,s} \frac{1}{8}\mathbf{C}_{ir;js} + \frac{1}{2}\mathbf{S}_{rs} \\
& + \frac{1}{2}\Bigg(\mathbf{CS}_{ir;s} - \mathbf{C}_{irs}\mathbf{CS}_{ir;s} - \sum_{j\neq i,r,s} \mathbf{C}_{ir;js}\mathbf{CS}_{ir;s} \Bigg) \Bigg] \\
& - \sum_{i\neq r,s} \Bigg[\mathbf{CS}_{ir;s}\mathbf{S}_{rs} + \mathbf{C}_{irs}\left(\frac{1}{2}\mathbf{S}_{rs} - \mathbf{CS}_{ir;s}\mathbf{S}_{rs} \right) \\
& + \sum_{j\neq i,r,s} \mathbf{C}_{ir;js}\left(\frac{1}{2}\mathbf{S}_{rs} - \mathbf{CS}_{ir;s}\mathbf{S}_{rs} \right) \Bigg] \Bigg\} |\mathcal{M}^{(0)}_{m+2}|^2
\end{aligned}
$$

G. Somogyi Z. Trocsanyi VDD 2005

performing double and triple subtractions in overlapping regions

$$\mathrm{C}_{irs}\,(1-\mathrm{A}_2)\,|\mathcal{M}^{(0)}_{m+2}|^2 = 0 \qquad \mathrm{S}_{rs}\,(1-\mathrm{A}_2)\,|\mathcal{M}^{(0)}_{m+2}|^2 = 0$$

$$\mathrm{C}_{ir;js}\,(1-\mathrm{A}_2)\,|\mathcal{M}^{(0)}_{m+2}|^2 = 0 \qquad \mathcal{CS}_{ir;s}\,(1-\mathrm{A}_2)\,|\mathcal{M}^{(0)}_{m+2}|^2 = 0$$

needs a NLO-type subtraction between the m+2- and the m+1-parton contributions

$$\sigma^{\rm NNLO} = \sigma^{\rm NNLO}_{\{m+2\}} + \sigma^{\rm NNLO}_{\{m+1\}} + \sigma^{\rm NNLO}_{\{m\}}$$

$$\sigma^{\rm NNLO}_{\{m+2\}} = \int_{m+2} \Big[{\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m$$

must be finite in the doubly-unresolved regions

$$- {\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} + {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \Big]_{d=4}$$

G. Somogyi Z. Trocsanyi VDD 2005

A_1 takes care of the singly-unresolved regions and A_{12} of the over-subtracting

$$\sigma^{\rm NNLO}_{\{m+1\}} = \int_{m+1} \Big[{\rm d}\sigma^{\rm RV}_{m+1} J_{m+1} - {\rm d}\sigma^{\rm RV,A_1}_{m+1} J_m + \int_1 \left({\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right) \Big]_{d=4}$$

$$\sigma^{\rm NNLO}_{\{m\}} = \int_m \Big[{\rm d}\sigma^{\rm VV}_m + \int_2 {\rm d}\sigma^{\rm RR,A_2}_{m+2} + \int_1 {\rm d}\sigma^{\rm RV,A_1}_{m+1} \Big]_{d=4} J_m$$

need to construct A_{12} such that all overlapping regions in 1-parton and 2-parton IR phase space regions are counted only once

$$\mathbf{C}_{ir}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{C}_{ir}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{S}_{r}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{S}_{r}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{C}_{irs}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{C}_{irs}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{C}_{ir;js}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{C}_{ir;js}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{CS}_{ir;s}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{CS}_{ir;s}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{S}_{rs}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{S}_{rs}|\mathcal{M}^{(0)}_{m+2}|^2$$

the definition of A_{12} is rather simple

$$\mathbf{A}_{12}|\mathcal{M}^{(0)}_{m+2}|^2 \equiv \mathbf{A}_1\mathbf{A}_2|\mathcal{M}^{(0)}_{m+2}|^2$$

but showing that it has the right properties is non trivial, and requires considering iterated singly-unresolved limits and strongly-ordered doubly-unresolved limits

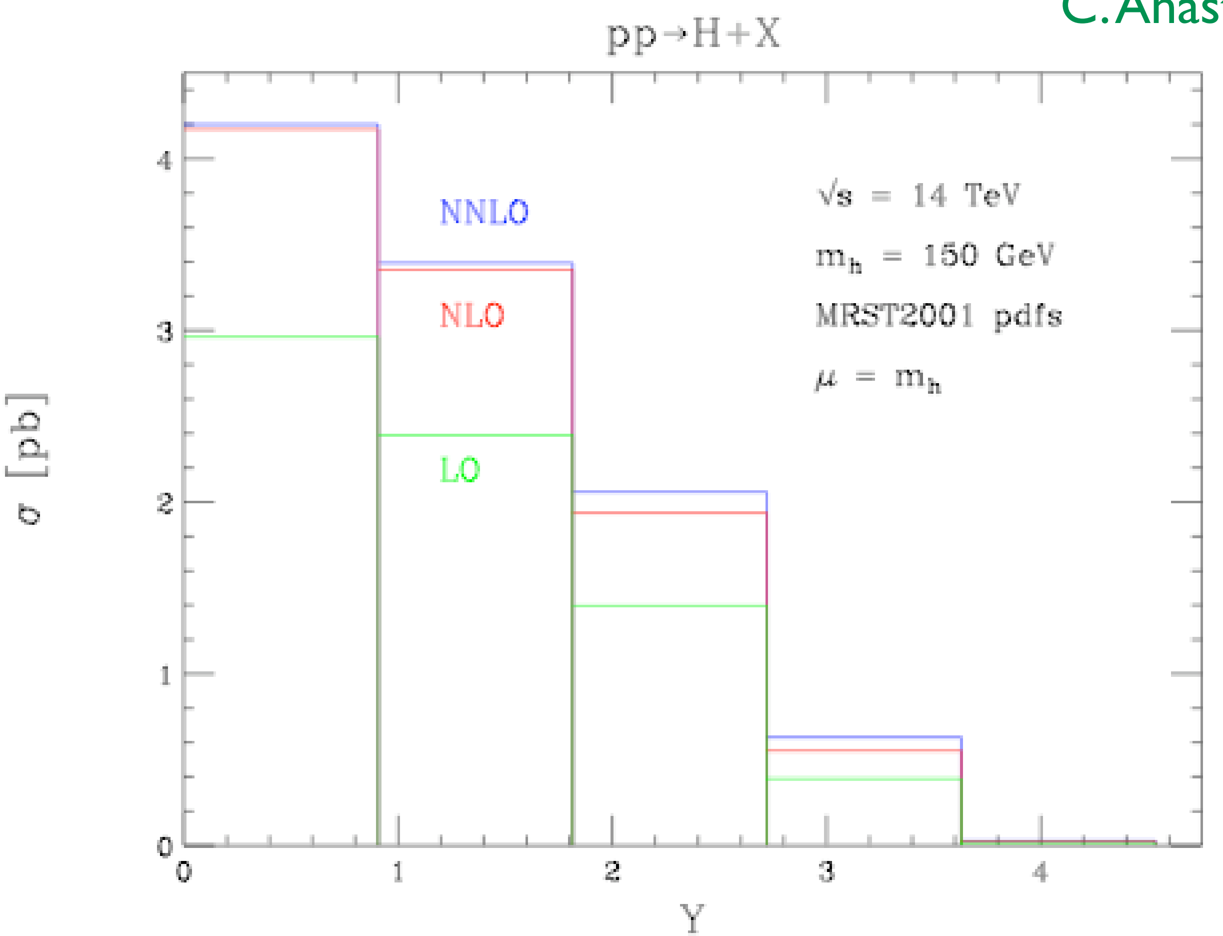
Conclusions

- need for NNLO corrections must be assessed process by process
- in the last few years, a lot of progress on the computation of NNLO cross sections
- sector decomposition is already up and running
- subtraction is making substantial progress

Higgs production at LHC

a fully differential cross section:
bin-integrated rapidity distribution, with a jet veto

C. Anastasiou K. Melnikov F. Petriello 2004



jet veto: require

$$R = 0.4$$

$$|\mathbf{p}_T^j| < p_T^{veto} = 40 \text{ GeV}$$

for 2 partons

$$R_{12}^2 = (\eta_1 - \eta_2)^2 + (\phi_1 - \phi_2)^2$$

if $R_{12} > R$

$$|\mathbf{p}_T^1|, |\mathbf{p}_T^2| < p_T^{veto}$$

if $R_{12} < R$

$$|\mathbf{p}_T^1 + \mathbf{p}_T^2| < p_T^{veto}$$

- $M_H = 150 \text{ GeV}$ (jet veto relevant in the $H \to W^+W^-$ decay channel)
- K factor is much smaller for the vetoed x-sect than for the inclusive one: average $|\mathbf{p}_T^j|$ increases from NLO to NNLO: less x-sect passes the veto

Observable (jet) functions

J_m vanishes when one parton becomes soft or collinear to another one

$$J_m(p_1, ..., p_m) \to 0\,, \quad \text{if} \quad p_i \cdot p_j \to 0$$

⇒ $d\sigma_m^{\rm B}$ is integrable over 1-parton IR phase space

J_{m+1} vanishes when two partons become simultaneously soft and/or collinear

$$J_{m+1}(p_1, ..., p_{m+1}) \to 0\,, \quad \text{if} \quad p_i \cdot p_j \text{ and } p_k \cdot p_l \to 0 \quad (i \neq k)$$

R and V are integrable over 2-parton IR phase space

observables are IR safe

$$J_{n+1}(p_1, .., p_j = \lambda q, .., p_{n+1}) \to J_n(p_1, ..., p_{n+1}) \quad \text{if} \quad \lambda \to 0$$

$$J_{n+1}(p_1, .., p_i, .., p_j, .., p_{n+1}) \to J_n(p_1, .., p, .., p_{n+1}) \quad \text{if} \quad p_i \to zp\,,\ p_j \to (1-z)p$$

for all $n \geq m$