

A SMUGGLED TECHNICAL NOTE
ON
MUELLER NAVELET JETS
FROM
THE FELLOWS' GARDEN

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GOAL

to analyse the QCD dynamics in the $s \gg |t|$ limit:
the high energy limit (HEL)

FACT

in HEL the scattering processes are dominated by
sub-processes with gluon exchange in the t channel

BFKL

theory resums multiple gluon radiation out of
the gluon exchanged in the t channel

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PHENOM.

Process-dependent questions:

- * does a fixed-order expansion in α_s suffice to describe the data ?
- * can the data be described in terms of other, e.g. Sudakov, resummations ?
- * in phase space, where do sub-processes with gluon exchange in the t channel dominate over the other sub-processes ?

DIJET PRODUCTION IN pp COLLISIONS

KINEMATICS

$$p_a = x_a P_A \quad p_b = x_b P_B :$$

incoming parton momenta

S : hadron c.m. energy

$s = x_a x_b S$: parton c.m. energy

$E_{j_{1,2\perp}}$: jet transverse energy

$Q^2 = -t$: typical momentum transfer

$$\hat{\mathcal{O}} \quad Q^2 \sim E_{j\perp}^2$$

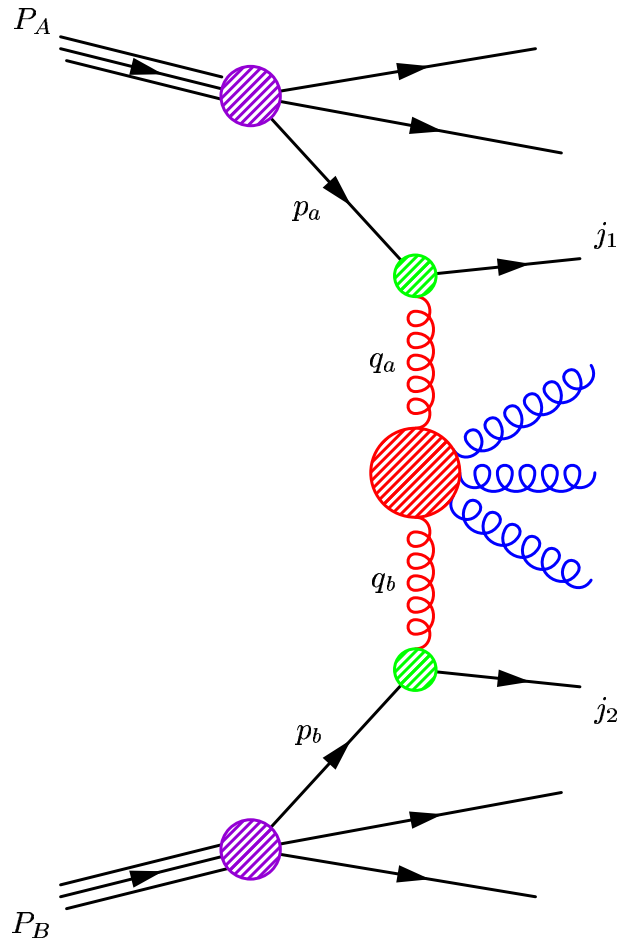
$$\Delta y = |y_{j_1} - y_{j_2}| :$$

rapidity difference between the jets

$$\ln \frac{S}{Q^2} = \ln \frac{1}{x_a} + \ln \frac{s}{Q^2} + \ln \frac{1}{x_b}$$

$$x_{a,b} = \mathcal{O}(1) \quad \ln \frac{s}{Q^2} \simeq \Delta y \gg 1$$

$\hat{\mathcal{O}}$ physics of large rapidity intervals,
and not small- x physics



DIJET PRODUCTION IN HEL

the cross section for dijet production in HEL:

$$\frac{d\sigma}{d^2p_{j1\perp} d^2p_{j2\perp} dy_{j1} dy_{j2}} = x_a^0 f_{\text{eff}}(x_a^0, \mu_F^2) x_b^0 f_{\text{eff}}(x_b^0, \mu_F^2) \frac{d\hat{\sigma}_{gg}}{d^2p_{j1\perp} d^2p_{j2\perp}}$$

the parton momentum fractions in HEL:

$$x_a^0 = \frac{|p_{j1\perp}|}{\sqrt{S}} e^{y_{j1\perp}}, \quad x_b^0 = \frac{|p_{j2\perp}|}{\sqrt{S}} e^{-y_{j2\perp}}$$

the effective p.d.f.

$$f_{\text{eff}}(x, \mu_F^2) = G(x, \mu_F^2) + \frac{4}{9} \sum_f [Q_f(x, \mu_F^2) + \bar{Q}_f(x, \mu_F^2)]$$

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parton cross section in terms of BFKL ladder f and impact factors:

$$\frac{d\hat{\sigma}_{gg}}{d^2p_{j1\perp} d^2p_{j2\perp}} = \left[\frac{C_A \alpha_s}{|p_{j1\perp}|^2} \right] f(q_{a\perp}, q_{b\perp}, \Delta y) \left[\frac{C_A \alpha_s}{|p_{j2\perp}|^2} \right]$$

asymptotically, the LL BFKL ladder is:

$$\lim_{\Delta y \gg 1} f(q_{a\perp}, q_{b\perp}, \Delta y) \sim \frac{e^{4C_A \ln 2 \alpha_s \Delta y / \pi}}{\sqrt{7\zeta_3 C_A \alpha_s \Delta y / 2}}$$

BFKL MONTE CARLO

DRAWBACKS OF THE BFKL LADDER

- the (N)LL BFKL resummation is performed at fixed α_s
 \hat{O} any variation in the scale of α_s occurs in the (N)LL terms
- energy and longitudinal momentum are not conserved:
in dijet production, the exact x 's are

$$x_a = \frac{e^{y_{j_1}}}{\sqrt{S}} \left(|p_{j_{1\perp}}| + |p_{j_{2\perp}}| e^{-\Delta y} + \sum_{i=1}^n p_{i\perp} e^{y_i - y_{j_1}} \right)$$
$$x_b = \frac{e^{-y_{j_2}}}{\sqrt{S}} \left(|p_{j_{2\perp}}| + |p_{j_{1\perp}}| e^{-\Delta y} + \sum_{i=1}^n p_{i\perp} e^{-y_i + y_{j_2}} \right)$$

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- a Monte Carlo solution of the BFKL equation can account for
 - running of α_s
 - energy and longitudinal momentum conservation

Schmidt 1996

Orr, Stirling 1997

DIJET PRODUCTION IN HEL

k in an event with two or more jets, tag the most forward and the most backward jets

$$\Delta y = |y_{j_1} - y_{j_2}| \simeq \ln \frac{x_a x_b S}{E_{j_{1\perp}} E_{j_{2\perp}}}$$

k minimise the jet transverse energy

k maximise $s = x_a x_b S$

¶ in a collider with ramping-up energy S , fix $x_{a,b}$

analyse $\frac{d\sigma}{dx_a dx_b}$ for different values of S Mueller, Navelet 1987

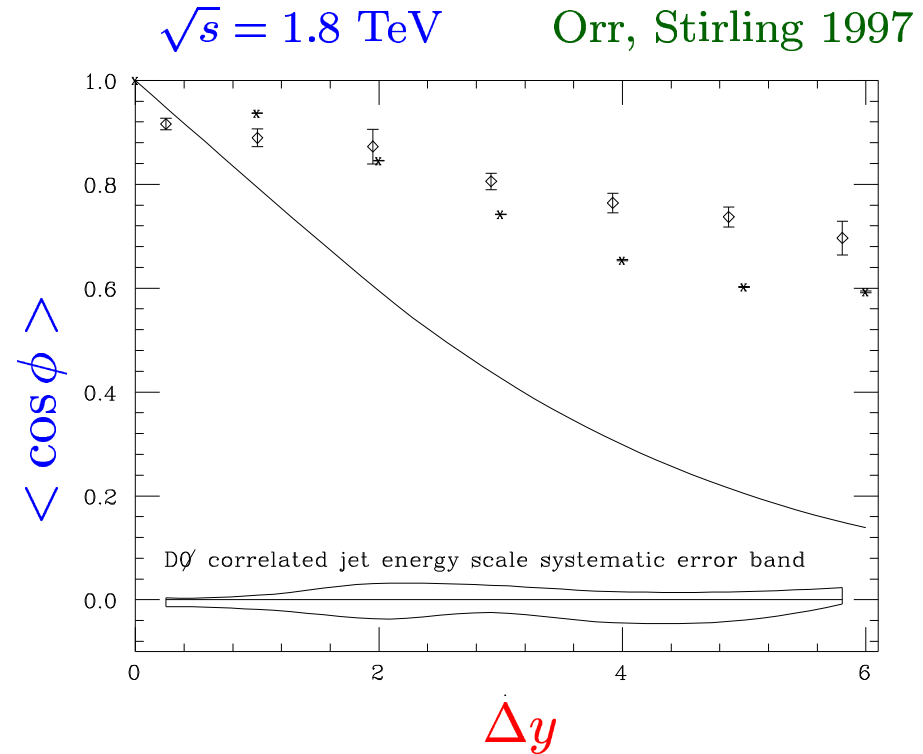
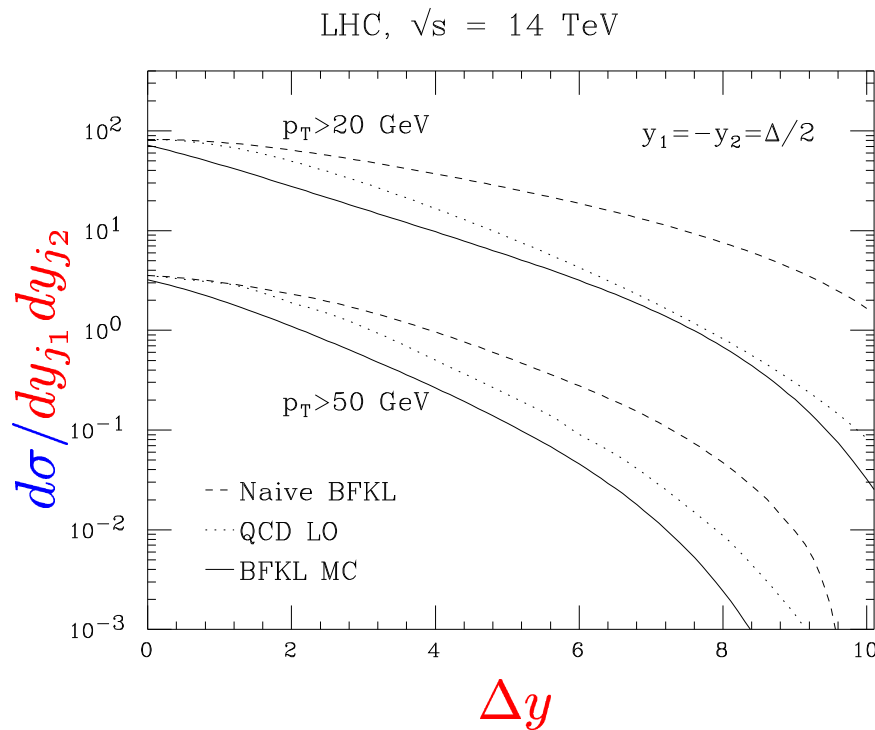
• in a fixed energy S collider, increase $x_{a,b}$

analyse $\left\{ \begin{array}{l} \frac{d\sigma}{d\Delta y} \\ \frac{d\sigma}{d\Delta y d\phi} \end{array} \right.$ for different values of Δy

ϕ : azimuthal angle between tagged jets

Schmidt, VDD; Stirling 1993-95

DIJET PRODUCTION – PHENOMENOLOGY



\hat{O} in $\frac{d\sigma}{dy_{j1} dy_{j2}}$ the **BFKL Monte Carlo** yields a depletion rather than an enhancement, both for **Tevatron** & **LHC**, due to the **falling parton luminosities**

\hat{O} $\langle \cos \phi \rangle$ shows too much azimuthal decorrelation wrt **Tevatron D0** data, while it is well described by a parton-shower **Monte Carlo (HERWIG)**

CAVEAT $\langle \cos \phi \rangle$ is dominated by **soft gluon (Sudakov)** effects

MUELLER-NAVELET JETS

Mueller-Navelet proposal for colliders with ramping-up energy S :

k take the cross section for dijet production in HEL at fixed x 's:

$$\frac{d\sigma}{dx_a^0 dx_b^0} = \int d^2 p_{j1\perp} d^2 p_{j2\perp} f_{\text{eff}}(x_a^0, \mu_F^2) f_{\text{eff}}(x_b^0, \mu_F^2) \frac{d\hat{\sigma}_{gg}}{d^2 p_{j1\perp} d^2 p_{j2\perp}}$$

k use approximate x 's: $x_a^{MN} = \frac{E_\perp}{\sqrt{S}} e^{y_{j1\perp}}$ $x_b^{MN} = \frac{E_\perp}{\sqrt{S}} e^{-y_{j2\perp}}$

Ô the x 's are in a one-to-one correspondence with the rapidities

Ô Δy is fixed at its max: $\Delta y = \ln \frac{x_a^{MN} x_b^{MN} S}{E_\perp^2}$

k integrate out transverse energies above a threshold E_\perp : $|p_{j1,2\perp}| \geq E_\perp$

Ô the Mueller-Navelet gluon-gluon cross section is

$$\hat{\sigma}_{gg} = \frac{9\pi\alpha_s^2}{2E_\perp^2} \frac{e^{4C_A \ln 2\alpha_s \Delta y / \pi}}{\sqrt{7\zeta_3 C_A \alpha_s \Delta y / 2}}$$

k compute the cross section at different c.m. energies S

MUELLER-NAVELET JETS – D0 ANALYSIS

D0 implementation of Mueller-Navelet:

D0 Collaboration 1999

* exact LO x 's:
$$\begin{cases} x_1 = \frac{2|p_{j1\perp}|}{\sqrt{S}} e^{\bar{y}} \cosh \frac{\Delta y}{2} \\ x_2 = \frac{2|p_{j2\perp}|}{\sqrt{S}} e^{-\bar{y}} \cosh \frac{\Delta y}{2} \end{cases}$$

$$\bar{y} = \frac{y_{j1\perp} + y_{j2\perp}}{2}$$

* acceptance cuts:

$$\begin{cases} |p_{j1,2\perp}| \geq 20 \text{ GeV} & Q^2 = |p_{j1\perp} p_{j2\perp}| \leq Q_{MAX}^2 = 1000 \text{ GeV}^2 \\ |y_{j1,2\perp}| \leq 3 & \Delta y \geq 2 \end{cases}$$

* measure the cross section at $\sqrt{S_A} = 1800 \text{ GeV}$ and $\sqrt{S_B} = 630 \text{ GeV}$ in 6 (x_1, x_2) bins, with $0.06 \leq x_1, x_2 \leq 0.22$

* using the Mueller-Navelet cross section, compute the ratio $R = \frac{\sigma(S_A)}{\sigma(S_B)}$

\hat{O} get the BFKL intercept

$$\alpha_{BFKL} = 1.65 \pm 0.07$$

OUR MUELLER-NAVELET / D0 ANALYSIS

* D0 uses an upper bound on Q^2 , such that $E_{\perp}^2 / Q_{MAX}^2 = 0.4$

* in HEL $\begin{cases} x_1 \rightarrow x_a^0 \neq x_a^{MN} \\ x_2 \rightarrow x_b^0 \neq x_b^{MN} \end{cases}$

$x_{a,b}^{MN}$ are not good approximations to $x_{1,2}$

* the rapidity interval can be written as

$$\Delta y = Y + \ln \frac{E_{\perp}^2}{p_{j1\perp} p_{j2\perp}} \quad \text{with} \quad Y = \ln \frac{x_a^0 x_b^0 S}{E_{\perp}^2}$$

$$\Delta y \geq 2 \quad \hat{O} \quad Q_{MAX}^2 = E_{\perp}^2 e^{(Y - 2)}$$

\hat{O} an effective maximum momentum transfer

$$Q_{MAX}^2 = \min(1000 \text{ GeV}^2, E_{\perp}^2 e^{(Y - 2)})$$

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we analysed Mueller-Navelet/D0 with

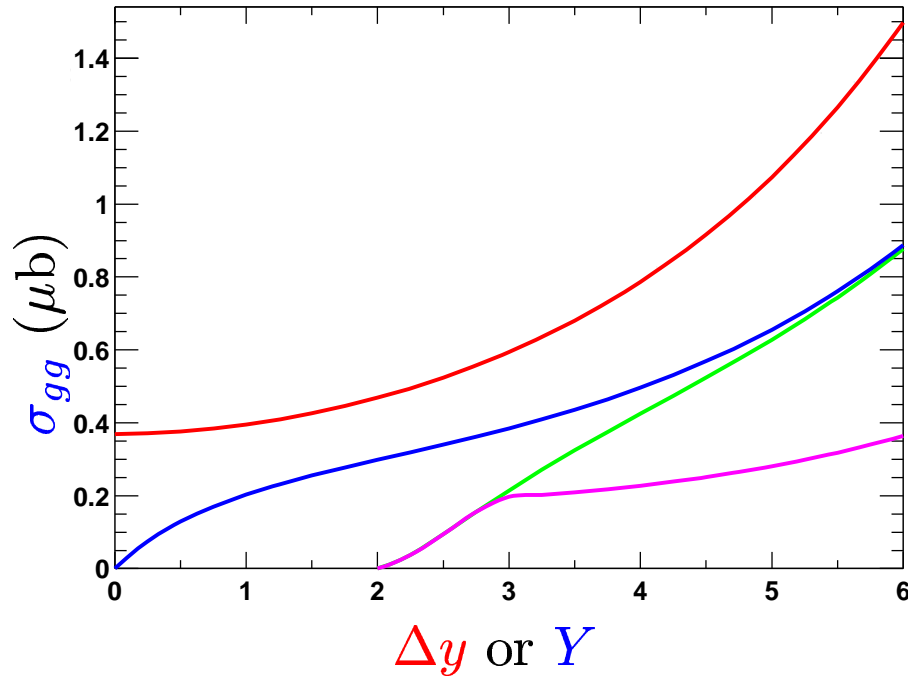
k analytic BFKL Andersen, Frixione, Schmidt, Stirling, VDD 2001

k BFKL Monte Carlo

k general-purpose NLO partonic Monte Carlo

OUR MUELLER-NAVELET / DO ANALYSIS

analytic BFKL



red: Mueller-Navelet (at fixed $x_{a,b}^{MN}$)

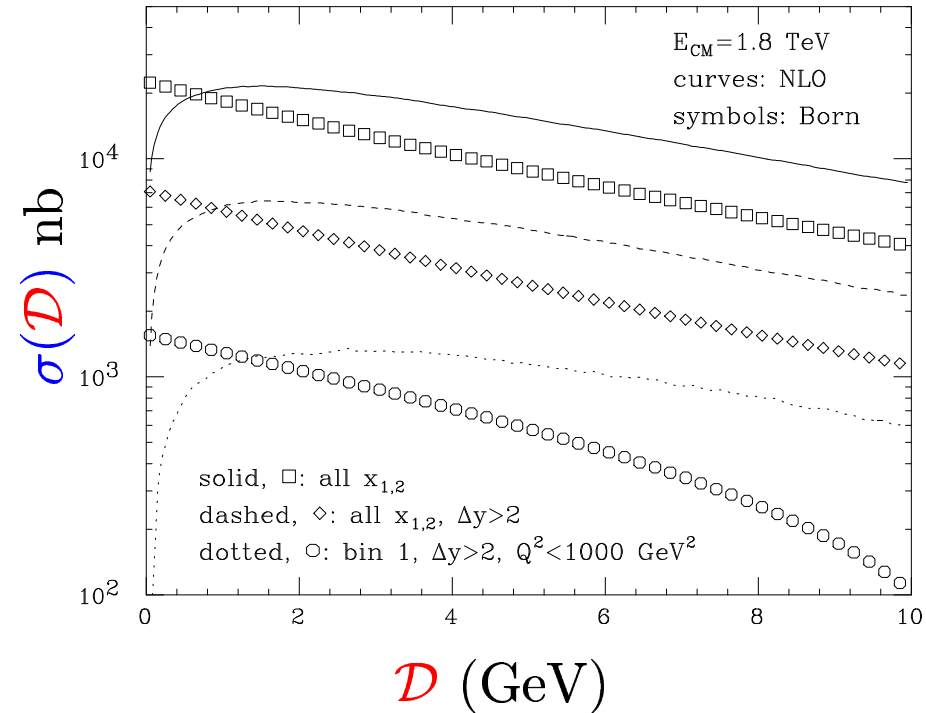
blue: at fixed $x_{a,b}^0$

green: at fixed $x_{a,b}^0$ with $\Delta y \geq 2$

magenta: green + $p_{j1\perp} p_{j2\perp} \leq 1000 \text{ GeV}^2$

k in σ_{gg} all curves have asymptotically the same shape $\hat{\sigma}$ sub-leading terms are important

NLO partonic Monte Carlo



$$\mathcal{D} = |\min(p_{j1\perp}) - \min(p_{j2\perp})|$$

symbols: LO

curves: NLO

CAVEAT

soft gluon effects at $\mathcal{D} = 0$

CONCLUSIONS

- * **AZIMUTHAL DECORRELATION** No evidence of **BFKL** radiation has been found. Data are described well by parton shower generators (**HERWIG**)
- * **MUELLER-NAVELET JETS** Sub-leading effects forbid the extraction of the **BFKL** intercept
- * **CAVEAT** Both analyses above are **contaminated** by **soft gluon** (**Sudakov**) effects