

QCD and Gravity in the Regge limit

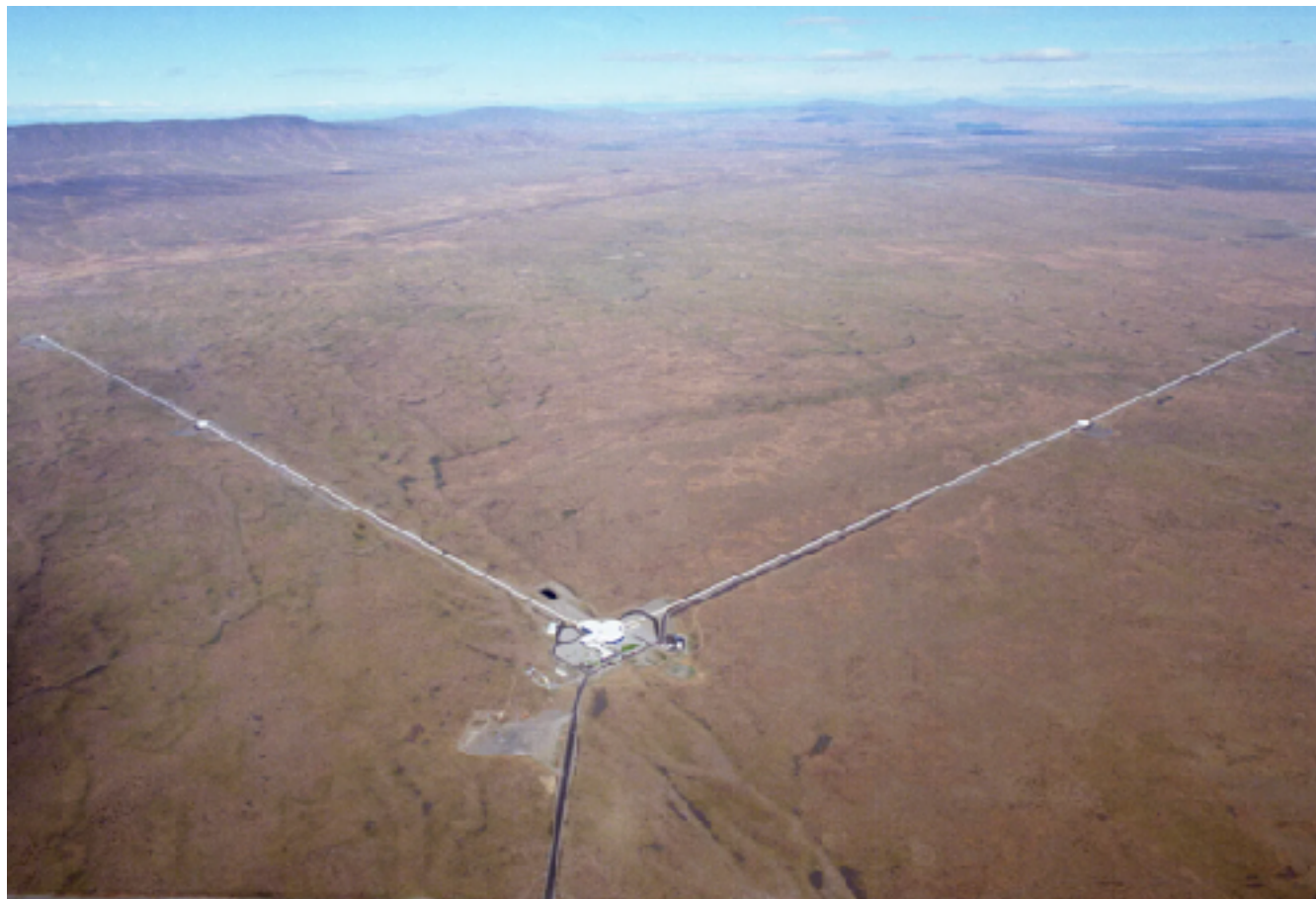
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INFN LNF

in collaboration with F.Alessio, R. Gonzo, E. Rosi, I. Rothstein, M. Saavedra

Loop Summit Cadenabbia 23 July 2025

Gravitational waves

- 1915: from GR, Einstein predicts GWs
- 2015: first GW signal, GW150914:
two black holes, each about $30 M_{\odot}$, $1.5 \cdot 10^9$ ly away



LIGO

KAGRA

$$f = 30 \text{ Hz} \rightarrow \lambda = 10^7 \text{ m}$$

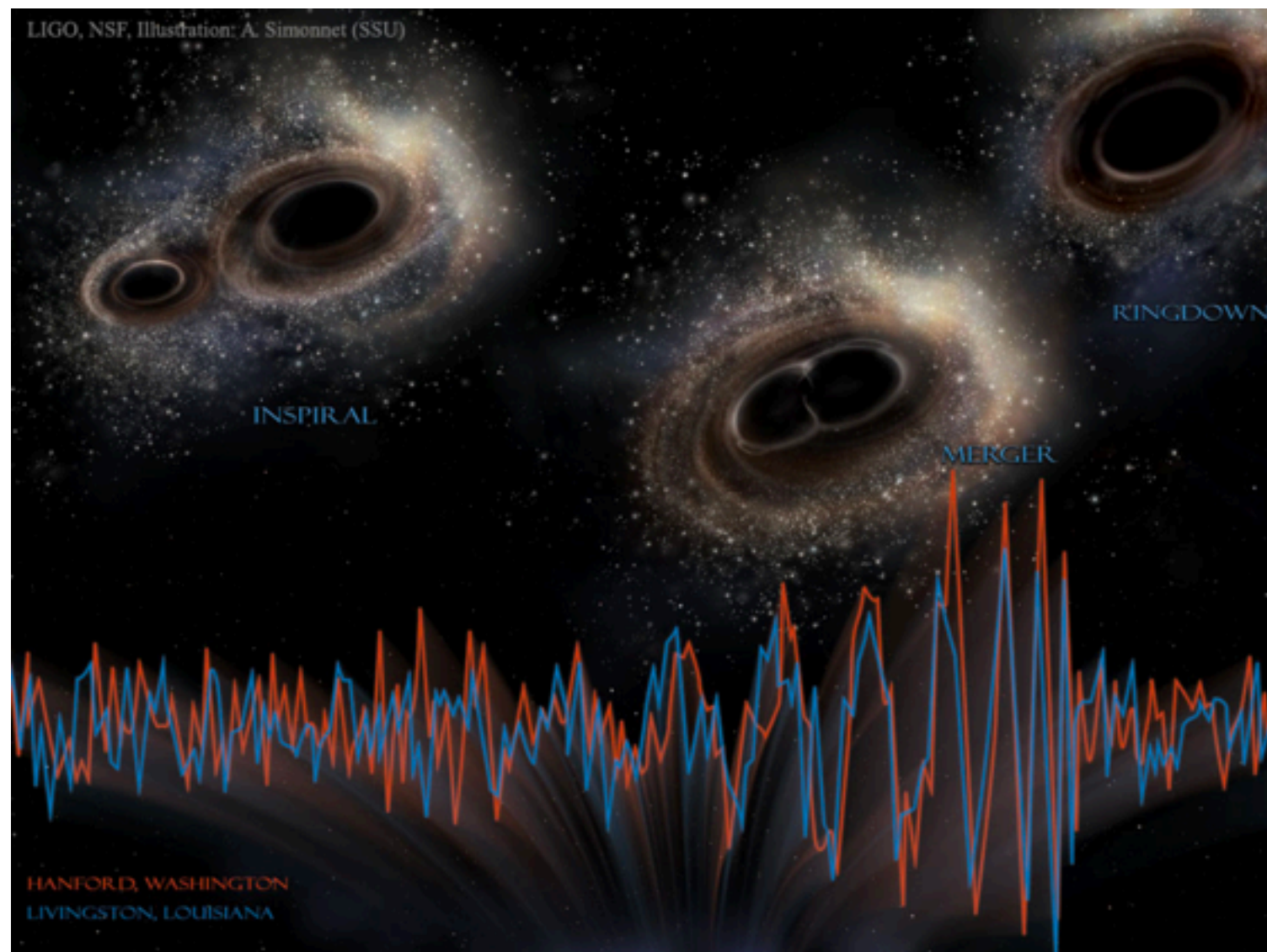
$$f = 10^4 \text{ Hz} \rightarrow \lambda = 3 \cdot 10^4 \text{ m}$$

$$\text{mass} \lesssim 200 M_{\odot}$$

$$\text{distance } z = 0.25 \sim 3.3 \cdot 10^9 \text{ ly}$$

Virgo





GW150914

inspiral

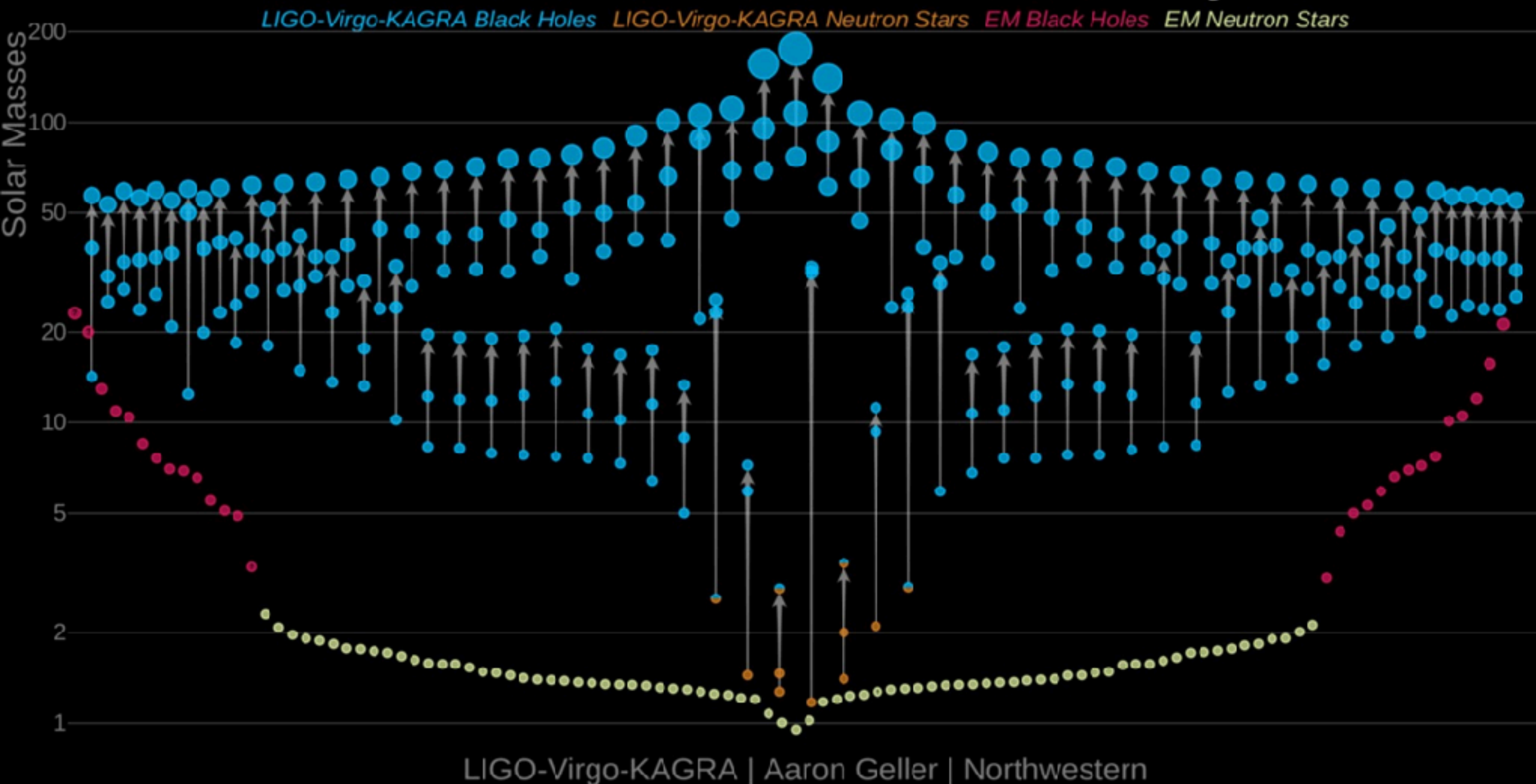
merger ringdown

PN, PM, SF expansions

NR

BHPT

Masses in the Stellar Graveyard

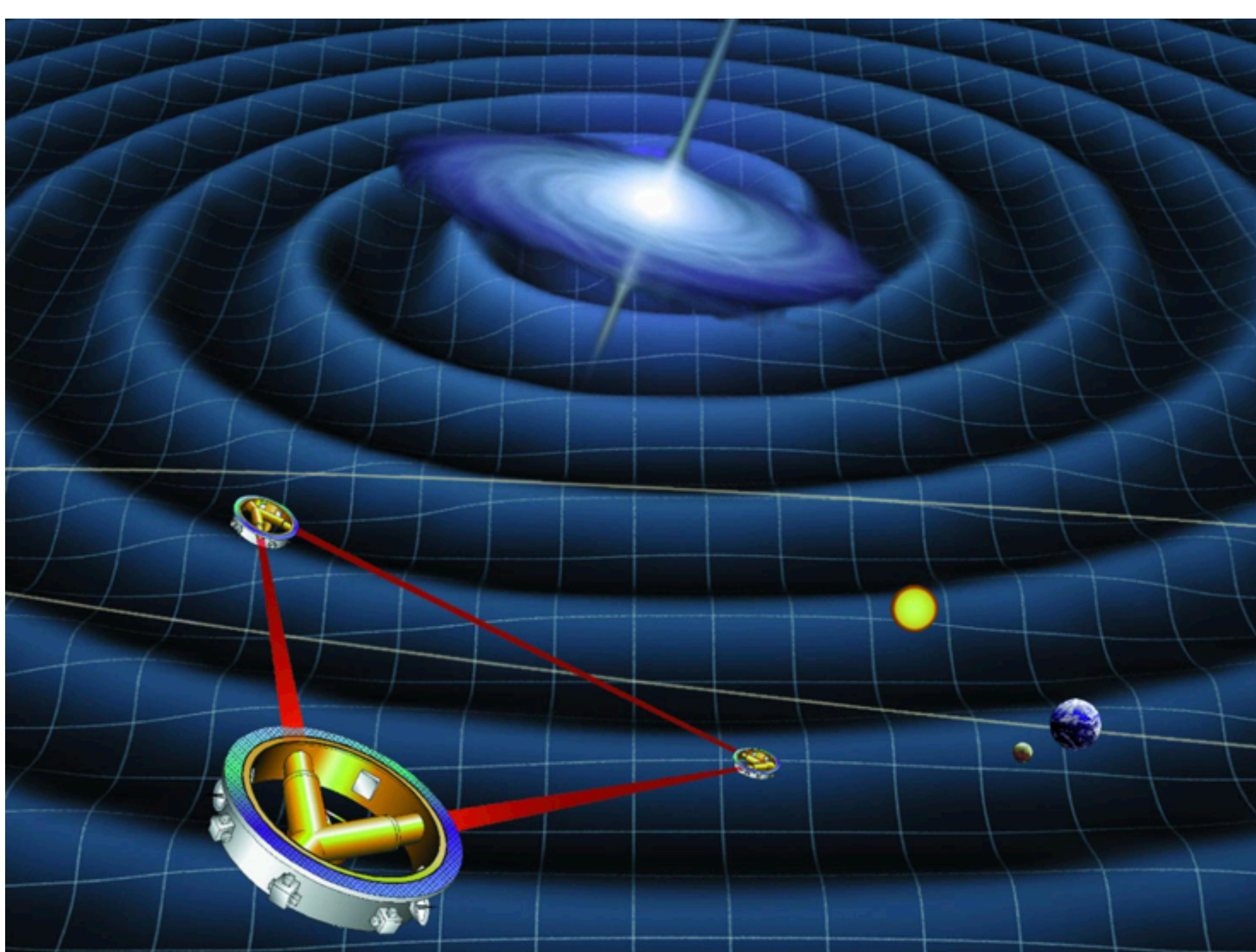


observed for $< 200 M_{\odot}$ so far

taxonomy from models, not from GWs

LISA

launch ~ 2035



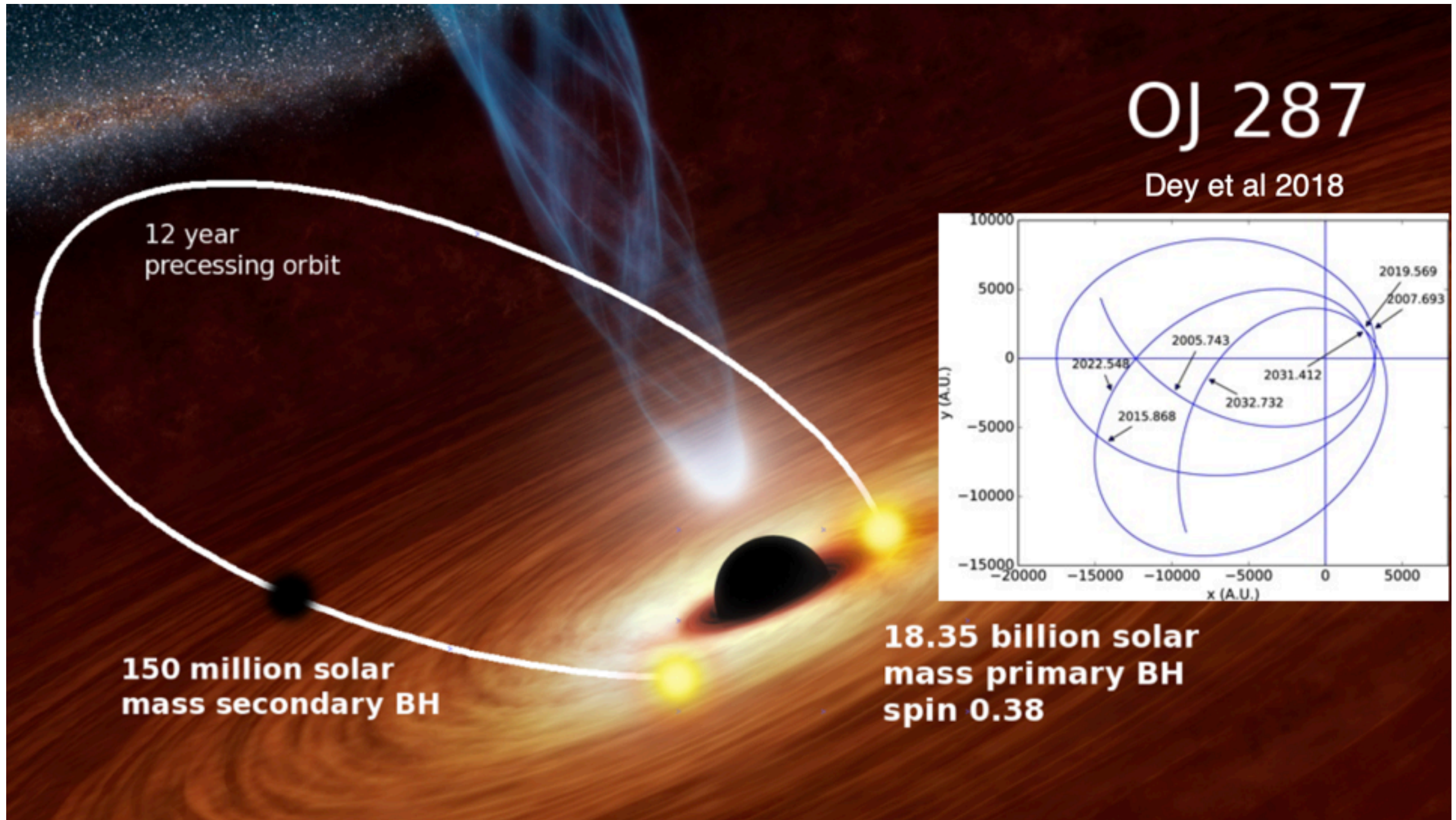
$$f = 0.1 \text{ Hz} \rightarrow \lambda = 3 \cdot 10^9 \text{ m}$$

$$f = 10^{-4} \text{ Hz} \rightarrow \lambda = 3 \cdot 10^{12} \text{ m} \quad \text{mass} \sim 10^4 - 10^7 M_{\odot} \text{ (MBH)} \quad \text{distance } z = 10 \sim 13.4 \cdot 10^9 \text{ ly}$$

$$M_1 \sim 10 M_{\odot} \quad M_2 \sim 10^5 M_{\odot} \text{ (EMRI)} \quad \text{distance } z = 4 \sim 12 \cdot 10^9 \text{ ly}$$

inspiral with up to $\sim 10^4$ cycles

Binary system of SMBH



Waveform

- interested in modelling inspired phase with analytic expansions

$$r_s \ll b$$

GWs carry info about the potential of a BH binary system

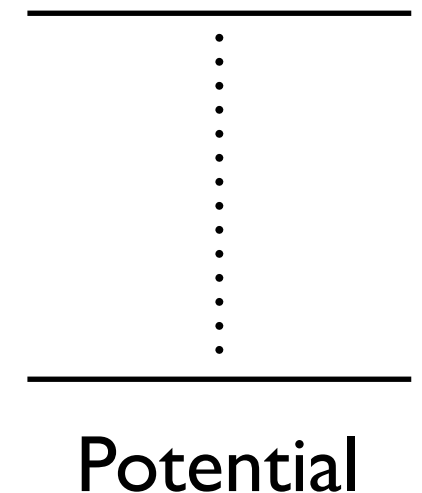
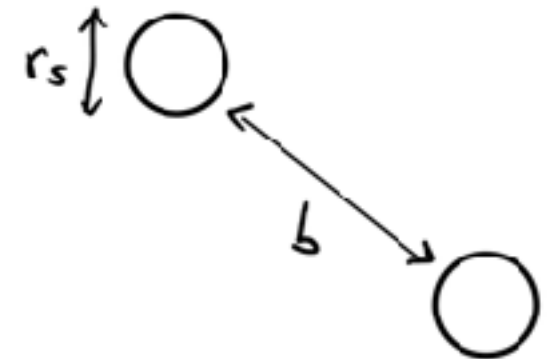
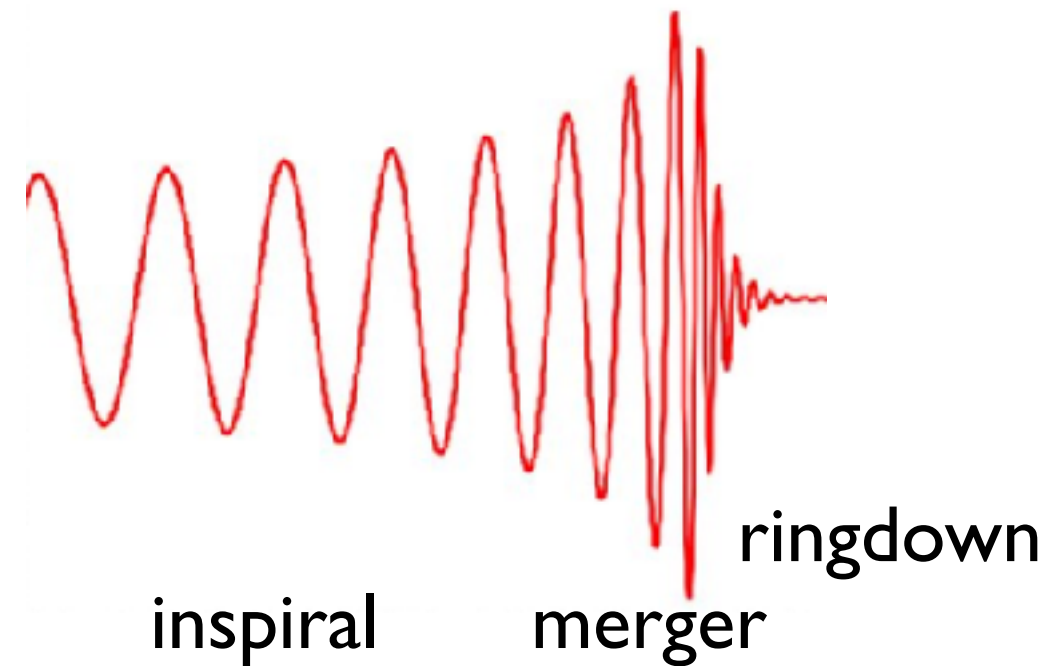
- Post Newtonian (PN) expansion: v^2/c^2
(deals with 3-dim integrals in config. space)

1 PN: Lorentz Droste 1917; Einstein Infeld Hoffmann 1938

...

6 PN: Bini Damour Geralico 2020-2021

Blümlein Maier Marquard Schäfer 2020-2021



Caveat:

GR deals with bodies as finite-size objects
analytic expansions deal with point-like objects

🌟 EFT of point particles coupled to gravity

$r_s \ll b$ [Goldberger Rothstein hep-th/0409156](#)

use RG methods to organise $\log(r_s/b)$

🌟 Post Minkowskian (PM) expansion: G/r

ideal for eccentric orbits

matches loop expansion of amplitude:

1 PM = $O(G/r)$ = tree level

2 PM = $O(G^2/r^2)$ = one loop, etc.

2 PM: [Westpfahl Goller 1979](#) ... [Cheung Rothstein Solon 1808.02489](#)

potential coefficients are obtained by matching EFT and full theory amplitudes
order by order in κ (because EFT and full theory describe same IR dynamics)

from the gravitational dynamics that governs the orbital evolution and radiation emission. Using the EFT formulation, it is easy to show that the divergences which arise at v^6 in the PN expansion can be attributed to the existence of new operators in the effective point particle description. However, these operators can be removed via a point transformation of the metric tensor and thus never contribute to physical quantities. This leads to the conclusion that there are no finite size effects at order v^6 . Practically, this means that *whenever one encounters a log divergent integral at order v^6 in the potential, one may simply set it to zero. Its value cannot affect physical predictions.*

How classical is a quantum loop?

put $\hbar$ back

$\hbar$ counting $1/\hbar$ from vertex $\exp\left(\frac{i}{\hbar} \int d^4x \mathcal{L}_{int}(\varphi)\right)$

$\hbar$ from (massless) propagator $[\varphi(\vec{x}), \pi(\vec{y})] = i\hbar\delta^3(\vec{x} - \vec{y})$

get $\hbar^{I-V+1} = \hbar^L$ $\langle 0|T\varphi(x)\varphi(y)|0\rangle = \int \frac{d^4k}{(2\pi)^4} \frac{i\hbar e^{ik(x-y)}}{k^2 - i\varepsilon}$

Note that k is wavenumber, with $p=\hbar k$

with masses Klein-Gordon is $\left(\square + \frac{m^2}{\hbar^2}\right)\varphi(x) = 0$

$$\langle 0|T\varphi(x)\varphi(y)|0\rangle = \int \frac{d^4k}{(2\pi)^4} \frac{i\hbar e^{ik(x-y)}}{k^2 - \frac{m^2}{\hbar^2} - i\varepsilon}$$

so, effectively

$$e \rightarrow e/\sqrt{\hbar} \quad \kappa \rightarrow \kappa/\sqrt{\hbar}$$

massless momenta $p \rightarrow \hbar p$

Boulware Deser 1975

Gupta Radford 1980

....

Donoghue Holstein hep-th/0405239

Kosower Maybee O'Connell 1811.10950

High-energy gravitational scattering and the general relativistic two-body problem

Thibault Damour^{*}

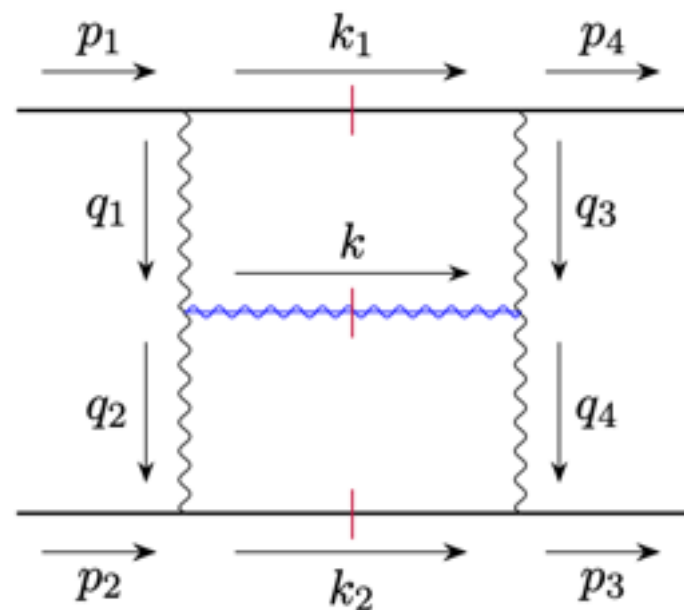
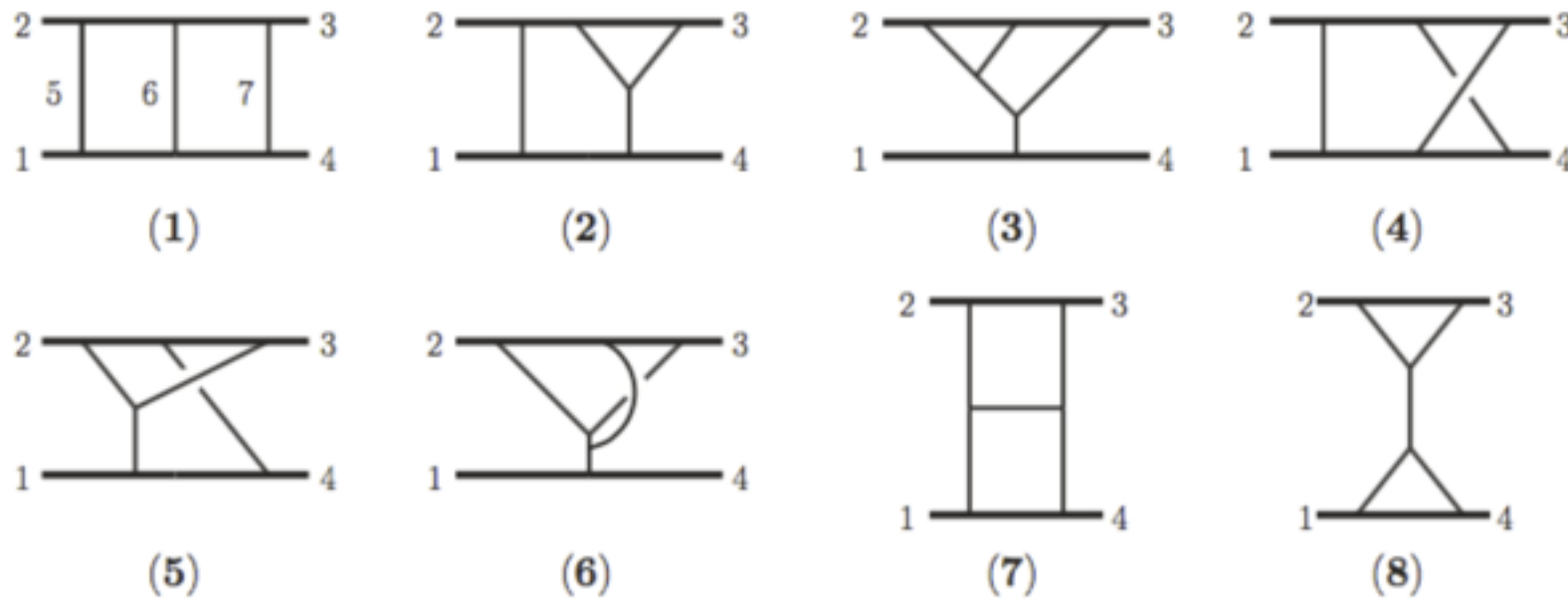
Institut des Hautes Etudes Scientifiques, 35 route de Chartres, 91440 Bures-sur-Yvette, France



(Received 29 October 2017; published 26 February 2018)

A technique for translating the classical scattering function of two gravitationally interacting bodies into a corresponding (effective one-body) Hamiltonian description has been recently introduced [[Phys. Rev. D **94**, 104015 \(2016\)](#)]. Using this technique, we derive, for the first time, to second-order in Newton's constant (i.e. one classical loop) the Hamiltonian of two point masses having an arbitrary (possibly relativistic) relative velocity. The resulting (second post-Minkowskian) Hamiltonian is found to have a tame high-energy structure which we relate both to gravitational self-force studies of large mass-ratio binary systems, and to the ultra high-energy quantum scattering results of Amati, Ciafaloni and Veneziano. We derive several consequences of our second post-Minkowskian Hamiltonian: (i) the need to use special phase-space gauges to get a tame high-energy limit; and (ii) predictions about a (rest-mass independent) linear Regge trajectory behavior of high-angular-momenta, high-energy circular orbits. Ways of testing these predictions by dedicated numerical simulations are indicated. We finally indicate a way to connect our classical results to the quantum gravitational scattering amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the two-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian.

3 PM = $O(G^3/r^3)$: Bern Cheung Roiban Shen Solon Zeng I90I.04424 & I908.0I493



H diagram

Amati Ciafaloni Veneziano I990

Regge limit $s \gg |t|$

$$\text{Im}A^{(2)}(s, t) \simeq G^3 s^3 \log(s/t) \cdot (\text{poles in } \varepsilon)$$

Di Vecchia Heissenberg Russo Veneziano 2008.I2743

$$\text{Re}A^{(2)}(s, t) \simeq \frac{1}{\log(s/t)} \text{Im}A^{(2)}(s, t)$$

Heavy effective field theory (HEFT)

Brandhuber Chen Travaglini Wen 2108.04216

2 PM: $A^{(1)}(s, t) \simeq G^2 m_1^2 m_2^2 (m_1 + m_2) y^2 (1 + O(1/y^2))$

$$y = \frac{s - m_1 - m_2}{2m_1 m_2}$$

(probe limit)

3 PM, 0 SF:

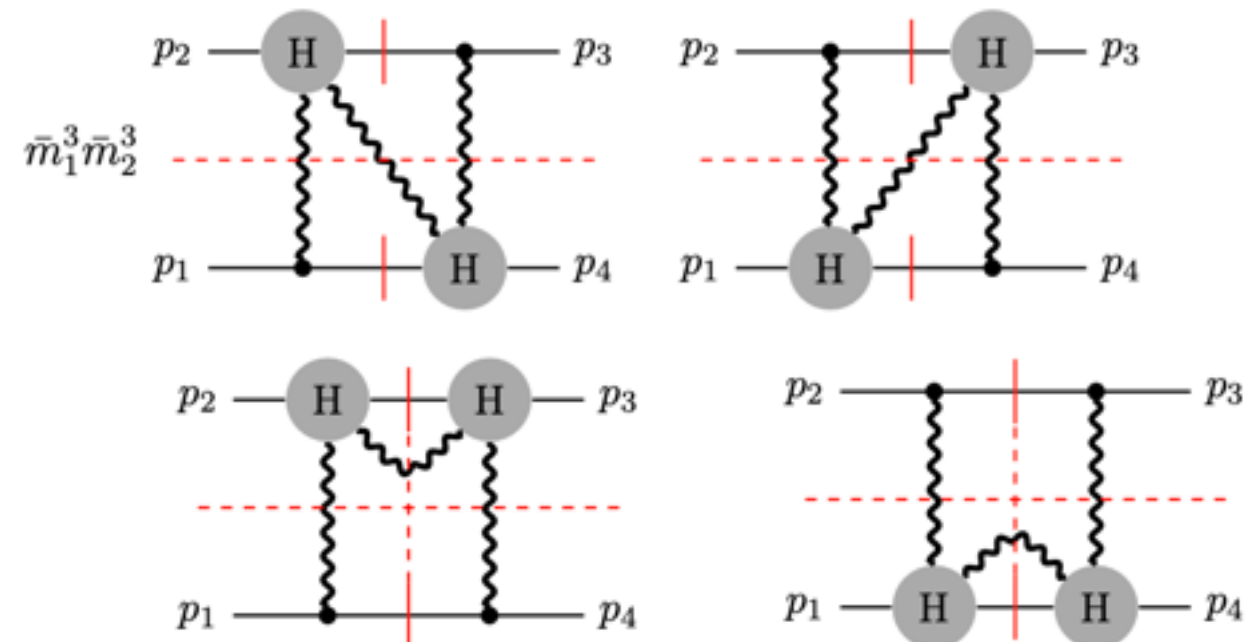
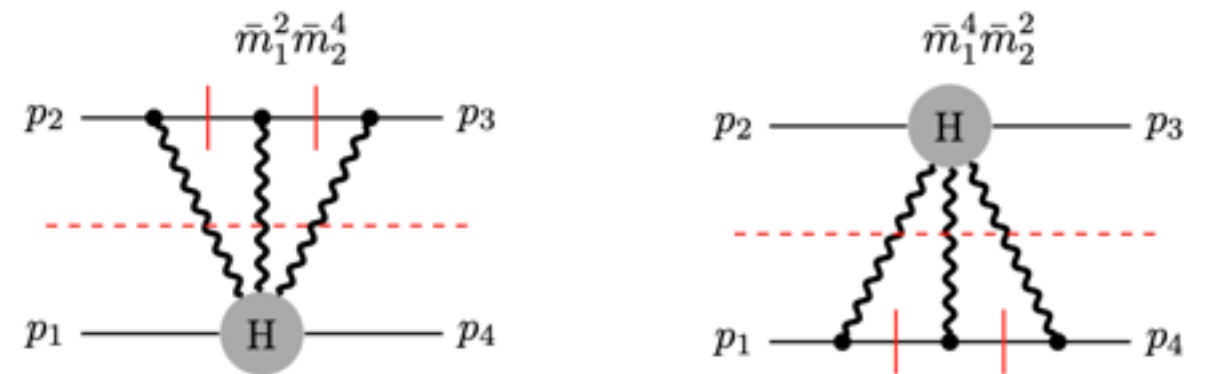
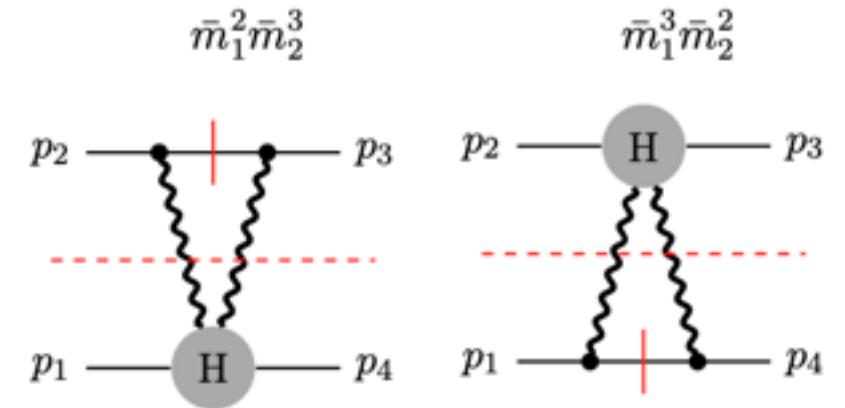
$$A_{\text{0SF}}^{(2)}(s, t) \simeq G^3 m_1^2 m_2^2 (m_1^2 + m_2^2) y^2 (1 + O(1/y^2))$$

3 PM, 1 SF: (beyond the probe)

$$\text{Re} A_{\text{1SF}}^{(2)}(s, t) \simeq G^3 m_1^3 m_2^3 y^3 (1 + O(1/y))$$

$$\text{Im} A_{\text{1SF}}^{(2)}(s, t) \simeq G^3 m_1^3 m_2^3 y^3 (\log(2y) + O(1/y))$$

radiation reaction needed
to cancel additional power of $\log(s/|t|)$

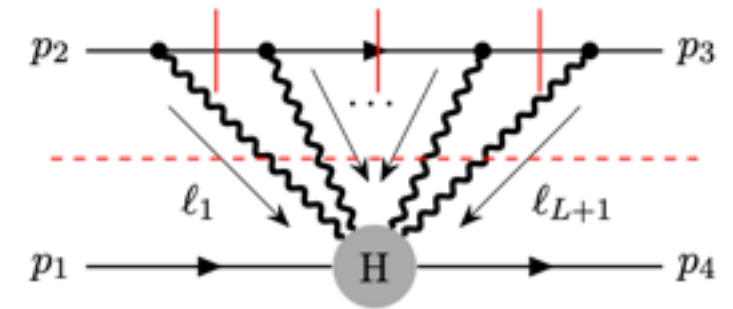


(radiation reaction)

Probe limit

$(L+1)$ -PM, 0 SF: $A_{\text{0SF}}^{(L)}(s, t) \simeq G^{L+1} m_1^2 m_2^{L+2} y^2 (1 + O(1/y^2))$

$1 + O(1/y^2)$ = $2(L+1)$ -degree polynomial in $1/y^2$



Brandhuber Chen Travaglini Wen 2108.04216 conjectured polynomial form, with unknown coefficients

Sasank Chava (2023, MSc thesis unpublished)

obtained polynomial coefficients leveraging geometric info
from geodesic eq. for test particle in a Schwarzschild background

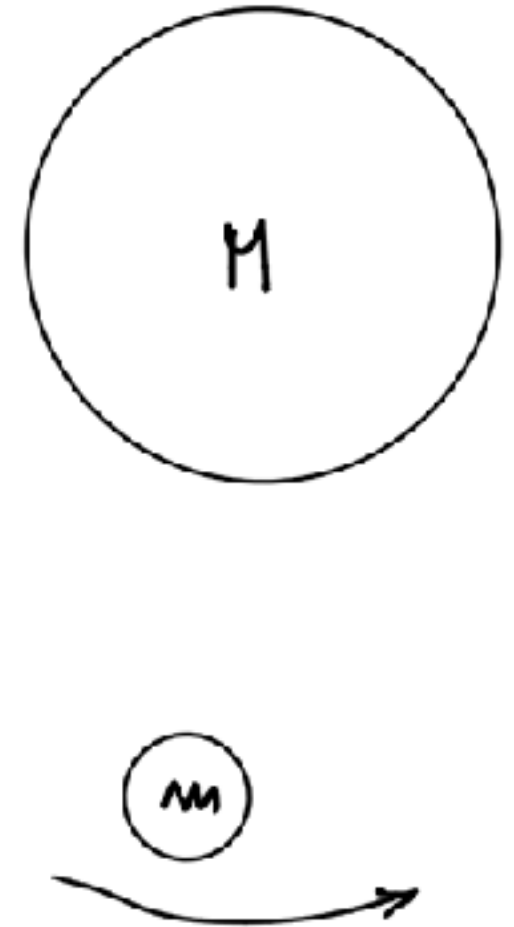
Cheung Shah Solon 2010.08568
Damour 1710.10599

next-to-probe

an analytic resummation of next-to-probe, or 1 SF, terms is not yet known

Self force

- Probe limit corresponds to Schwarzschild geometry: static (infinitely heavy) black hole
 - in Self Force (SF) expansion, one expands Einstein's equation in powers of m/M and solves them (numerically)
- 0 SF = Probe limit = Schwarzschild
1 SF is known for generic orbits
2 SF is known for specific cases (e.g. quasi circular orbits)



4 PM, 0 SF: $A_{0\text{SF}}^{(3)}(s, t) \simeq G^4 m_1^2 m_2^2 (m_1^3 + m_2^3) y^2 (1 + O(1/y^2))$

4 PM, 1 SF: $A_{1\text{SF}}^{(3)}(s, t) \simeq G^4 m_1^3 m_2^3 (m_1 + m_2) y^3 (\log(2y) + O(1/y))$

Bern Parra-Martinez Roiban Ruf Shen Solon Zeng 2101.07254 & 2112.10750
Dlpa Kälin Liu Neef Porto 2106.08276, 2112.11296 & 2210.05541

5 PM, 0 SF: $A_{0\text{SF}}^{(4)}(s, t) \simeq G^5 m_1^2 m_2^2 (m_1^4 + m_2^4) y^2 (1 + O(1/y^2))$

5 PM, 1 SF: $A_{1\text{SF}}^{(4)}(s, t) \simeq G^5 m_1^3 m_2^3 (m_1^2 + m_2^2) y^3 (\log(2y) + O(1/y))$

Driesse Jakobsen Mogull Plefka Sauer Usovitsch 2403.07781 & 2411.11846
Dlpa Kälin Liu Porto 2506.20665

mostly through world-line formulation

$$S = - \sum_{i=1}^2 \frac{m_i}{2} \int d\tau_i g_{\mu\nu}(x_i(\tau_i)) v_i^\mu(\tau_i) v_i^\nu(\tau_i)$$

5 PM, 2 SF: $A_{2\text{SF}}^{(4)}(s, t) \simeq G^5 m_1^4 m_2^4 y^4 (\log^2(2y) + O(1/y)) \quad ?$

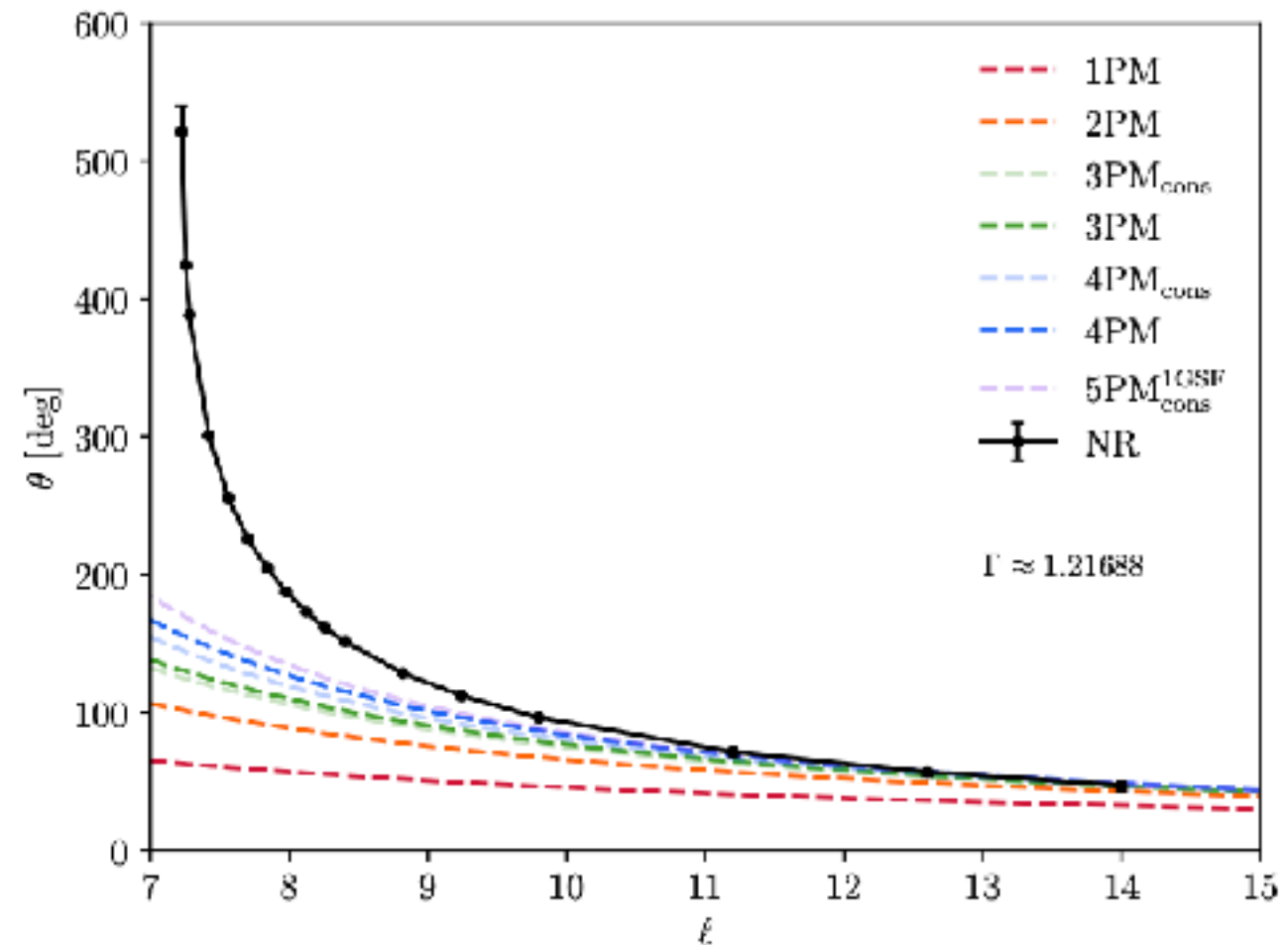
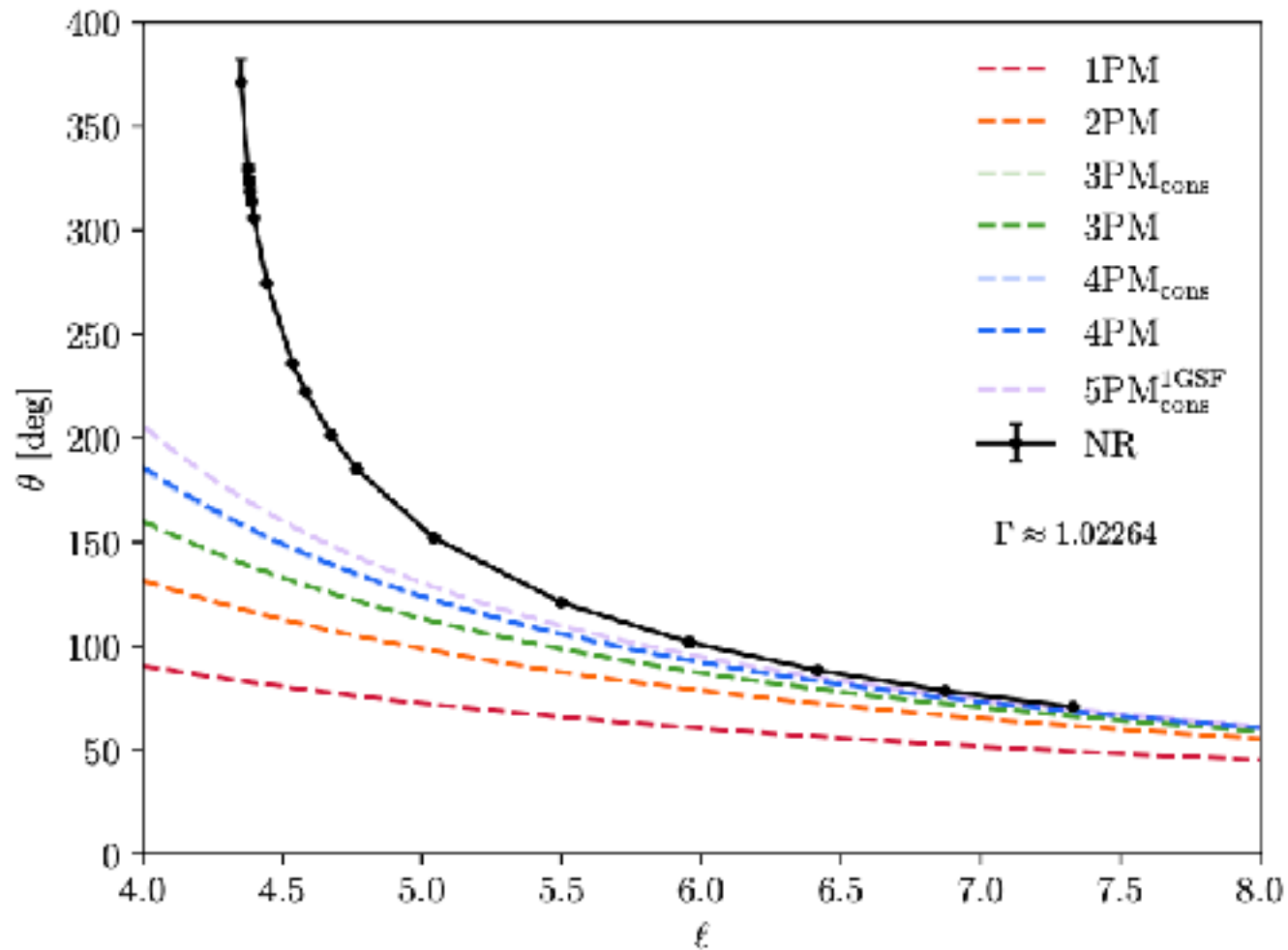
unknown yet



Comparison to Numerical Relativity

Pratten Schmidt Swain 2411.09652

scattering angle from NR data vs. PM



Γ : highest and lowest energy in the NR simulation



comparison worsens as we approach strong-field regime

Regge limit - Foreword

- In the Regge limit, radiative corrections to $2 \rightarrow 2$ amplitudes display iterative patterns of the evolution in rapidity $y \simeq \log(s/|t|)$, either if the evolution occurs in the t channel or in the s channel

- In the Regge limit of QCD, the leading radiative corrections are associated to a gluon ladder exchanged in the t channel, the Reggeised or Glauber gluon.

BFKL 1976-77

t channel two-gluon ladder and s channel terms are logarithmically suppressed

$$\mathcal{A} \simeq \exp \left(i\pi \mathbf{T}_s^2 + \mathbf{T}_t^2 \log \left(\frac{s}{-t} \right) \right)$$

- In the Regge limit of gravity, the leading radiative corrections are due to the eikonal phase terms (Weinberg's soft gravitons).

t channel one-graviton ladder is power suppressed in t/s

Bartels Lipatov Sabio-Vera 1208.3423

Melville Naculich Schnitzer White 1306.6019

colour-kinematics duality

$$\mathbf{T}_s^2 \rightarrow s \quad \mathbf{T}_t^2 \rightarrow t$$

- s channel ladders win over t channel ladders

Glauber EFT of gravity



Mimicking Glauber EFT of QCD

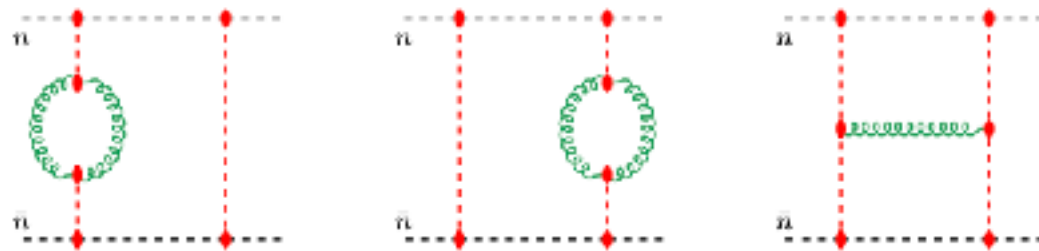
Rothstein Stewart 1601.04695

$$\mathcal{M}_{2 \rightarrow 2} = i \sum_M J_{(M)} \otimes S_{(M)} \otimes \bar{J}_{(M)}$$

Rothstein Saavedra 2412.04428 showed that the exchange of a t -channel two-graviton ladder is ruled by a rapidity RGE, whose anomalous dimension is Lipatov gravity (BFKL-like) kernel with graviton trajectory and graviton central-emission vertex (CEV)

Lipatov 1982

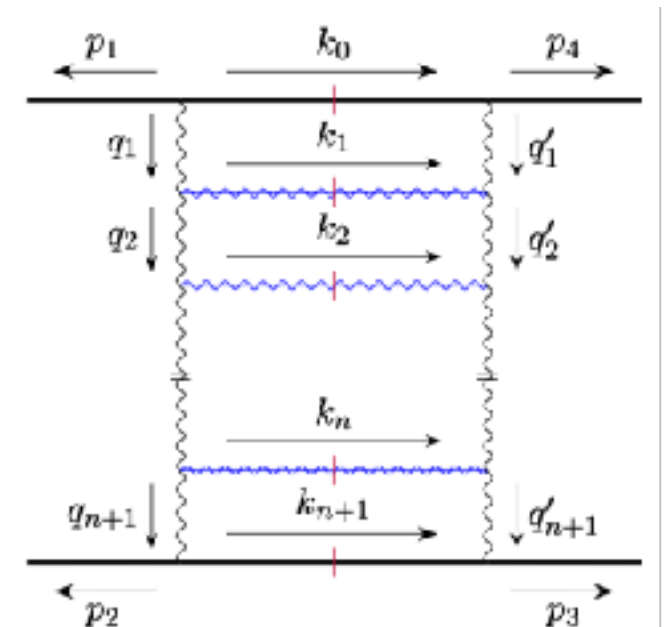
$$\nu \frac{d}{d\nu} S_{(N)} = -\gamma_{(N)}^\nu \otimes S_{(N)} - S_{(N)} \otimes \gamma_{(N)}^\nu$$



$$\gamma_{(M)}^\nu \sim \sum_j \omega_G(q_i) I_{\perp(M-1)} + \sum_{\text{Pairs } i,j} \mathcal{K}^{\text{GR}}(q_i, q_j; q) I_{\perp(M-2)}$$

through $\hbar$ counting, easy to see that trajectory is quantum, in fact the whole two-graviton ladder is quantum, except for one graviton CEV emission: the **H diagram**

$$\text{Im } \mathcal{M}_{2 \rightarrow 2}^{(n+1)}(s, q^2) \sim s^3 \log \left(\frac{s}{-t} \right)^n \frac{G}{q_\perp^2} G q_\perp^n (G q_\perp)^n$$



Glauber EFT of gravity

Rothstein Saavedra 2412.04428 showed that the exchange of a t -channel multi-graviton ladder is ruled by a rapidity RGE, whose anomalous dimension is a convolution of gravity BFKL-like kernels



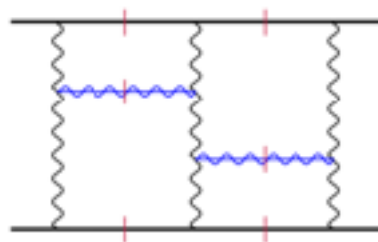
Expanding the rapidity RGE to second order in G

$$S_{(3,3)}^{(2)} \sim \text{[diagram 1]} + \text{[diagram 2]}$$

(note these are $G^2 s$ l.u.s corrections to $S_{(2,2)}^{(1)}$)

the first is classical, the second is quantum

through $\hbar$ counting, easy to see that the whole three-graviton ladder is quantum, except for the convolution of two graviton CEVs



$$\text{Im}A^{(4)}(s, t) \simeq G^5 s^4 \log^2(s/t) \cdot (\text{poles in } \varepsilon)$$

In fact, a t -channel ladder with $(n+2)$ gravitons in the s channel features a classical term of $(2n+3)$ -PM order, and provides a correction of $\mathcal{O}((G^2 s \log(s/|t|))^n)$ to the H diagram

s-channel discontinuities

Revisiting the **H diagram** Amati Ciafaloni Veneziano 1990

in MRK, the light-cone dof's decouple from the transverse dof's

$$\text{Disc}_s \mathcal{M}_{2 \rightarrow 2}^{(2)}(s, q^2) \simeq \frac{\kappa^6 s^3}{8\pi} \log(s) H_1(q_\perp^2),$$

$$H_1(q_\perp^2) \equiv \int_{q_{1\perp}} \int_{q_{2\perp}} \frac{\mathcal{K}^{\text{GR}}(q_1, q_2; q)}{q_{1\perp}^2 q_{2\perp}^2 (q - q_1)_\perp^2 (q - q_2)_\perp^2}$$

gravity (BFKL-like) kernel

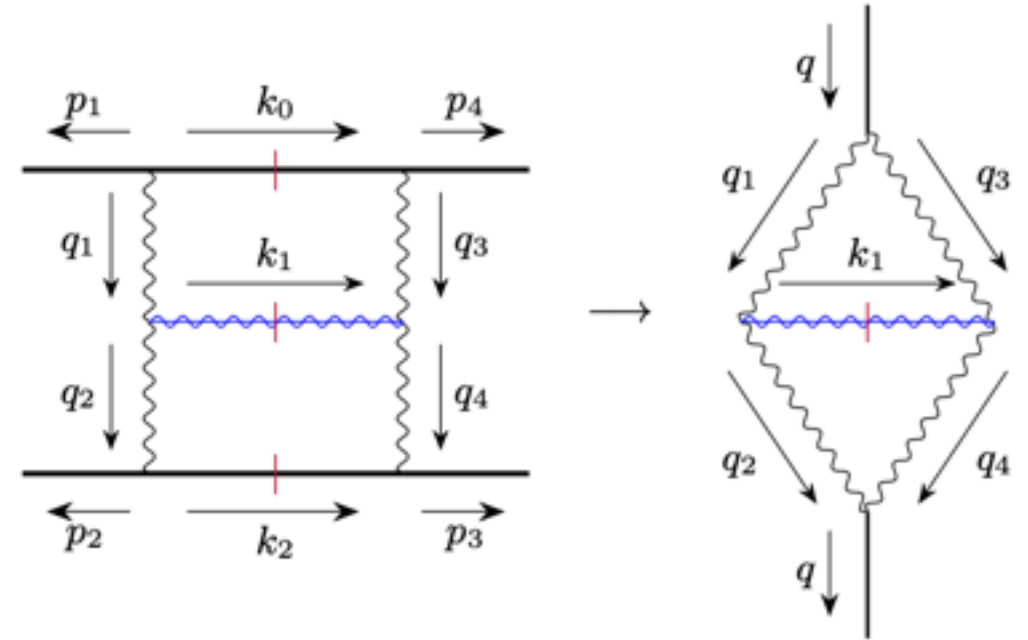
$$\begin{aligned} \mathcal{K}^{\text{GR}}(q_1, q_2; q_3, q_4) = & \left[(q_{1\perp} + q_{3\perp}) \cdot (q_{2\perp} + q_{4\perp}) - \frac{q_{2\perp}^2 q_{3\perp}^2 + q_{1\perp}^2 q_{4\perp}^2}{k_\perp^2} \right]^2 \\ & + \frac{4}{k_\perp^4} \left[q_{1\perp}^2 q_{2\perp}^2 q_{3\perp}^2 q_{4\perp}^2 - q_{3\perp}^2 q_{4\perp}^2 (q_{1\perp} \cdot q_{2\perp})^2 - q_{1\perp}^2 q_{2\perp}^2 (q_{3\perp} \cdot q_{4\perp})^2 \right] \end{aligned}$$

in momentum space $\text{Disc } \mathcal{M}_{2 \rightarrow 2}^{(2)}(s, q^2) = 8G^3 s^3 \log(s) \left(-\frac{1}{\epsilon^2} + \frac{2}{\epsilon} + \zeta_2 + \mathcal{O}(\epsilon^0) \right)$

Fourier transforming in impact parameter space

$$\bar{\mathcal{M}}_{2 \rightarrow 2}(s, b) \simeq \frac{1}{2s} \int d^d q_\perp e^{ib \cdot q_\perp} \mathcal{M}_{2 \rightarrow 2}(s, -q_\perp^2) \equiv \text{F.T.}[\mathcal{M}_{2 \rightarrow 2}](s, b)$$

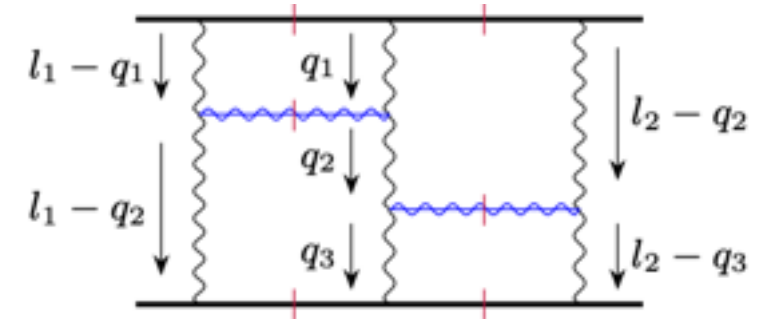
$$\text{Disc } \tilde{\mathcal{M}}_{2 \rightarrow 2}^{(2)}(s, b) = \frac{8G^3 s^2}{b^2} \log(s) \left(-\frac{1}{\epsilon} + 2 + \mathcal{O}(\epsilon^0) \right)$$



H^2 diagram

through iterated s-channel discontinuities

$$\text{Disc}_s^2 \mathcal{M}_{2 \rightarrow 2}^{(4)}(s, q^2) \simeq c \frac{\kappa^{10} s^4}{64\pi^2} \log(s)^2 H_2(q_\perp^2)$$

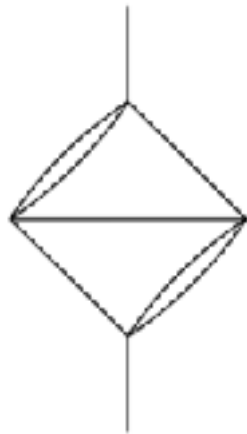


with H_2 the convolution of two gravity kernels

the result is reduced to bubbles and a kite integral

in momentum space

$$\text{Disc}_s^2 \mathcal{M}_{2 \rightarrow 2}^{(4)}(s, q^2) \simeq c \frac{\kappa^{10} s^4}{32\pi^2} \log(s)^2 \frac{1}{(q^2)^3} \left(\frac{q^2}{4\pi} \right)^d \left[-\frac{3}{\epsilon^3} - \frac{3}{\epsilon^2} - \frac{27}{\epsilon} + \frac{6}{\epsilon} \zeta_2 + \frac{6}{\epsilon} \zeta_3 + O(1) \right]$$



i.e. an $O(G^5 s^4 \log(s/|t|)^2)$ term

Alessio VDD Gonzo Rosi Rothstein Saavedra 25xx.yyyyyy

in impact parameter space

$$\text{Disc}_s^2 \tilde{\mathcal{M}}_{2 \rightarrow 2}^{(4)}(s, b^2) \simeq c \frac{\kappa^{10} s^3}{128\pi^2} \log(s)^2 \frac{(b^2)^3}{(\pi b^2)^{5d/2}} e^{9\gamma\epsilon} \left[\frac{3}{8\epsilon^2} - \frac{3}{\epsilon} + \frac{15}{2} + \frac{15}{16} \zeta_2 - \frac{3}{4} \zeta_3 + O(\epsilon) \right]$$

Conclusions

- “It has been shown that the non-resummed PM-expanded scattering angles demonstrate poor convergence towards NR. This motivates the exploration of *resummation* strategies...
... in particular we focus on the approach to the high-energy limit for equal-mass non-spinning binaries, which proves to be challenging for all resummation schemes considered ...”
Pratten Schmidt Swain 2411.09652
- In the Regge limit, at leading logarithmic accuracy, there is an s channel sequence of classical corrections of $O((G^2 s \log(s/|t|))^n)$ to the *H diagram*
- Those terms are $(2n+3)$ -PM and $(n+1)$ -SF order, i.e. maximal SF order within a fixed PM order
- Using EFT and iterated s-channel discontinuities, we computed the *H² diagram*, which is an $O(G^5 s^4 \log(s/|t|)^2)$ term, i.e. a 5 PM, 2 SF term and represents the first correction to the *H diagram*