QCD Radiative Corrections to $\gamma^*\gamma^* \to$ Hadrons

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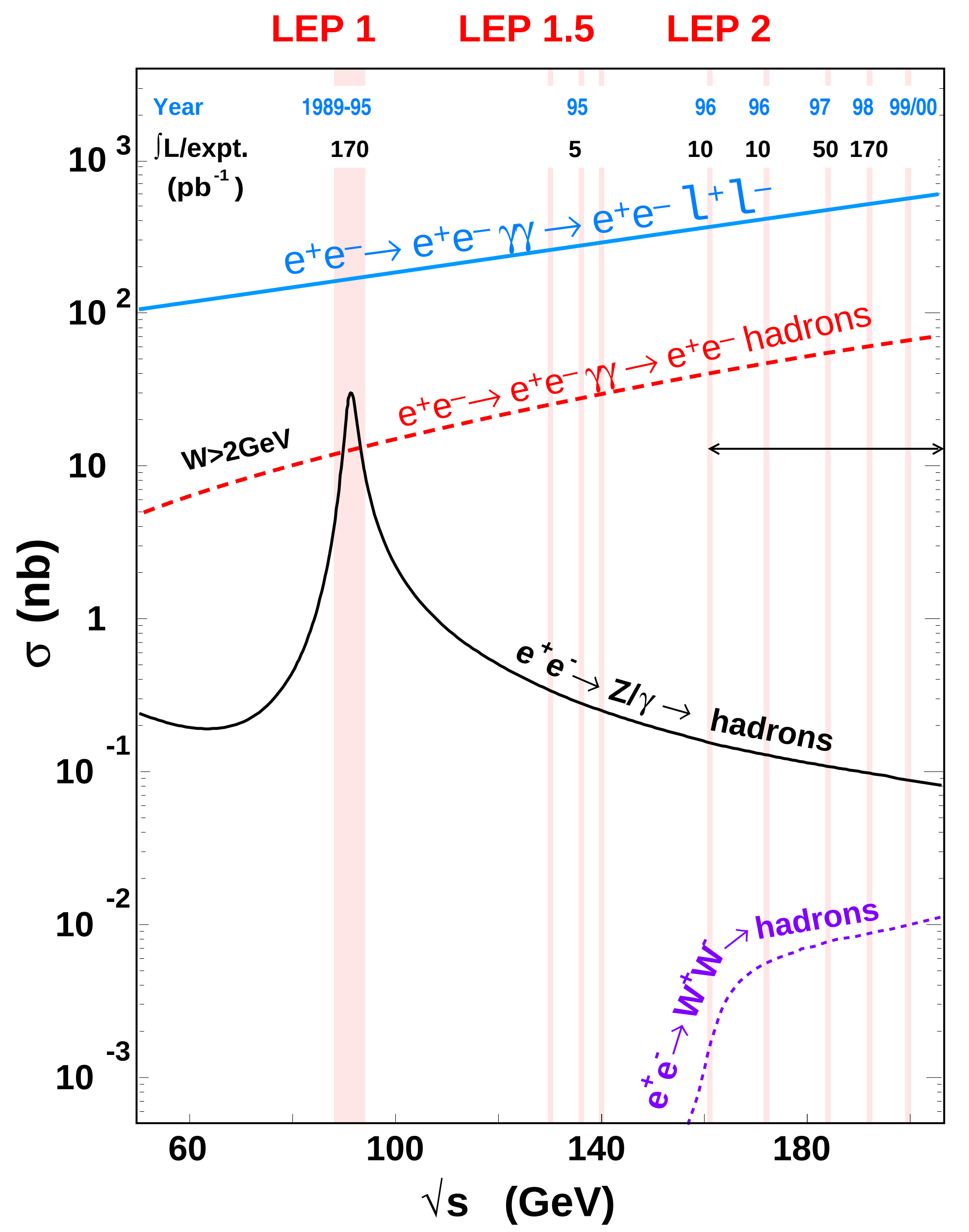
THE LEP CROSS SECTIONS



GOAL to analyse the QCD dynamics in the $s \gg |t|$ limit: the high energy limit (HEL)

FACT in HEL the scattering processes are dominated by sub-processes with gluon exchange in the t channel

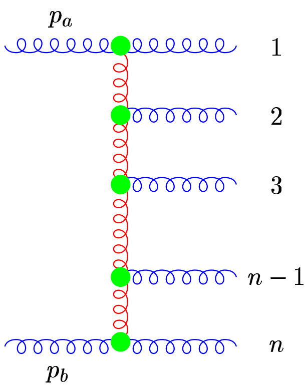
BFKL theory resums multiple gluon radiation out of the gluon exchanged in the t channel

PHENOM. Process-dependent questions:

- ☛ does a fixed-order expansion in α_S suffice to describe the data ?
- ☛ can the data be described in terms of other, e.g. Sudakov, resummations ?
- ☛ in phase space, where do sub-processes with gluon exchange in the t channel dominate over the other sub-processes ?

BFKL RESUMMATION

- in any scattering process with $s \gg |t|$ gluon exchange in the t channel dominates



- BFKL is a resummation of multiple gluon radiation out of the gluon exchanged in the t channel

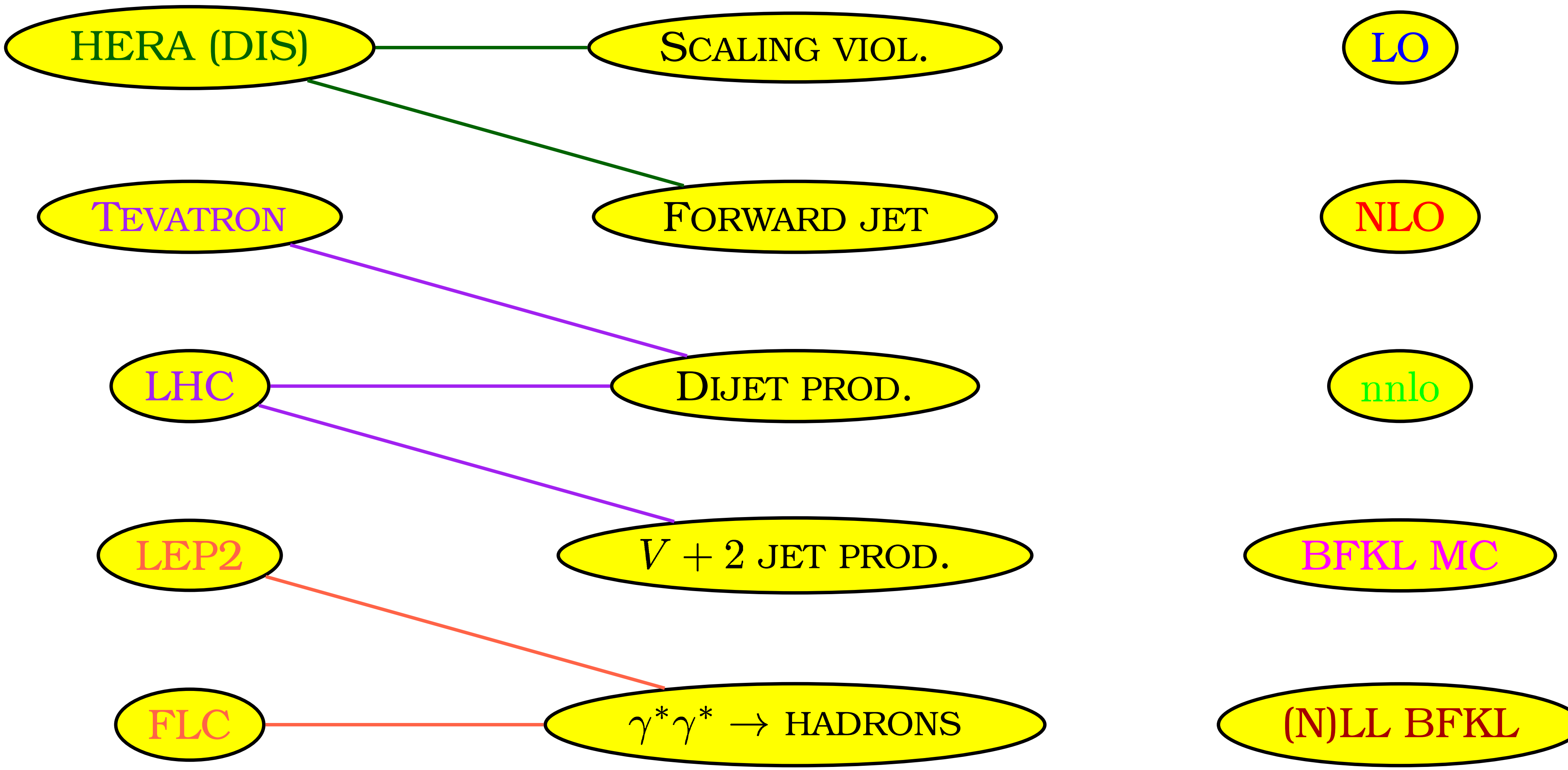
- for $s \gg |t|$ BFKL resums the Leading Log (and Next-to-Leading Log) contributions, in $\log(s/t)$, of the radiative corrections to the gluon propagator in the t channel, to all orders in α_S

- the LL terms are obtained in the approximation of strong rapidity ordering $(y_1 \gg y_2 \gg \ldots \gg y_n)$ and no k_t ordering of the emitted gluons

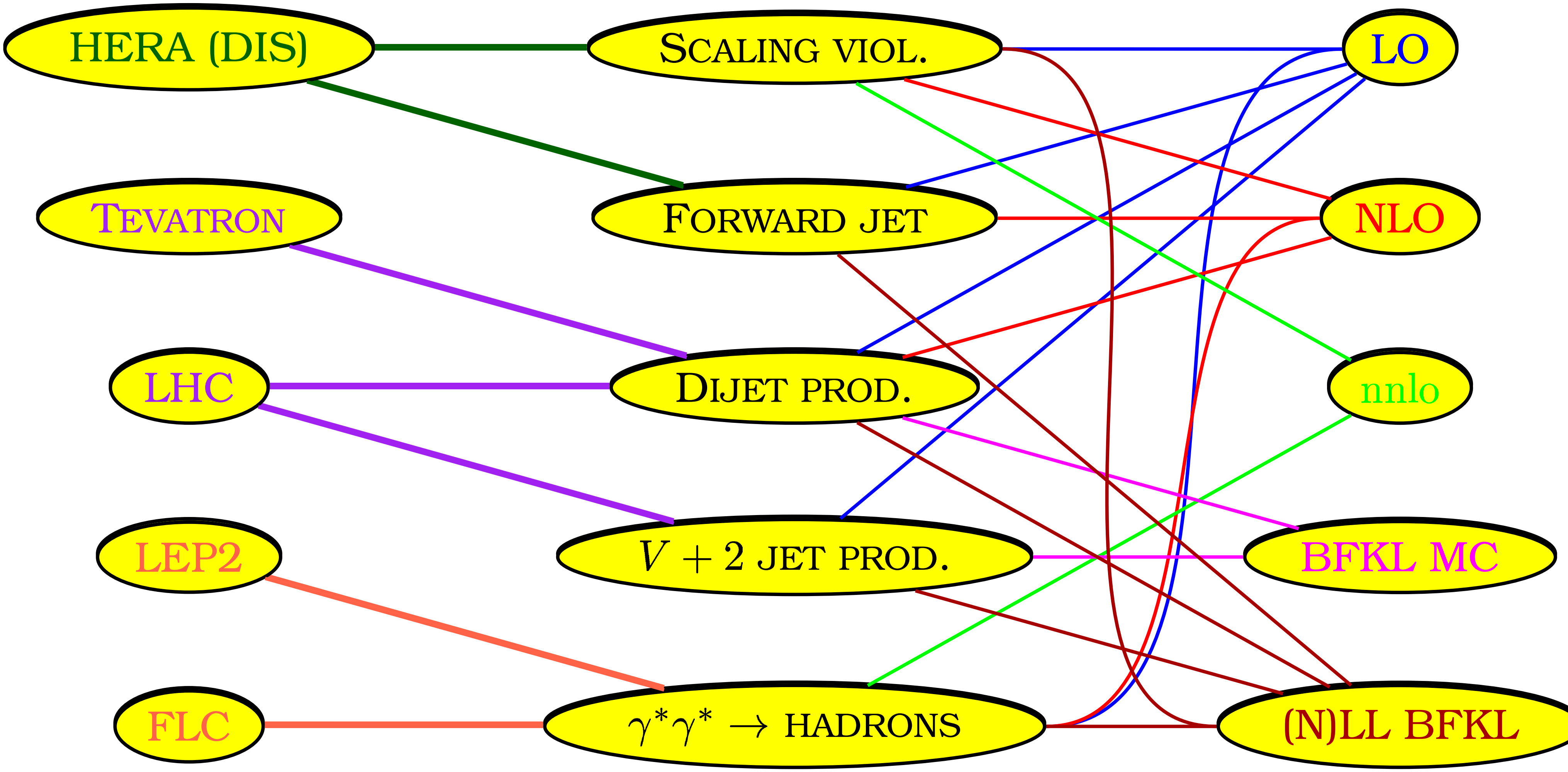
- the NLL terms are universal

- the resummation yields a 2-dim integral equation for the evolution of the gluon propagator in the t channel

Status of BFKL Analyses

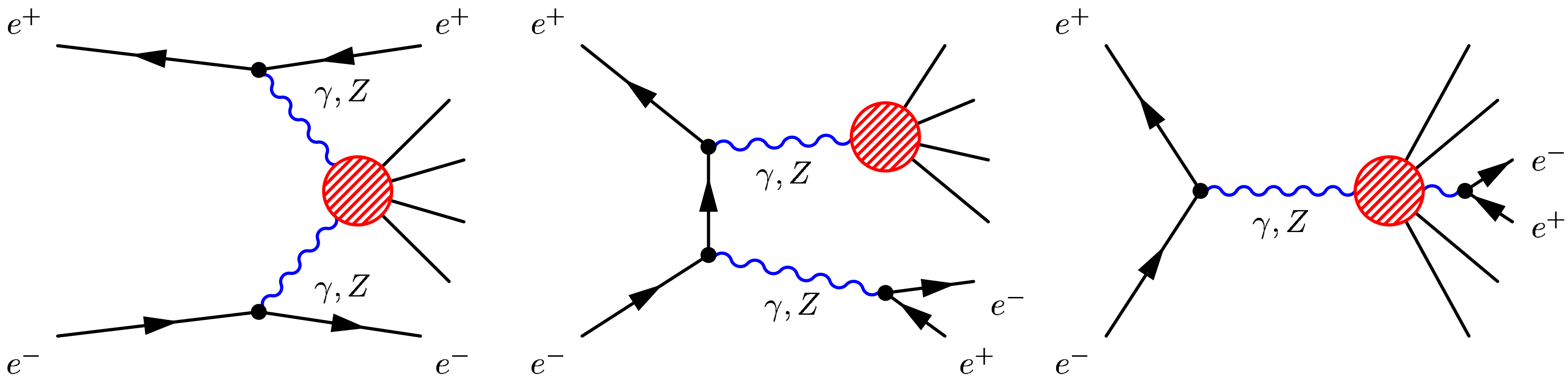


Status of BFKL Analyses



$e^+e^- \rightarrow e^+e^-$ HADRONS

Several (> 100) diagrams contribute to $e^+e^- \rightarrow e^+e^-$ hadrons



(a) (b) (c)

L3 & OPAL: small scattering angles of the outgoing leptons make:

☞ annihilation processes negligible

☞ photon virtualities small → Z contribution negligible

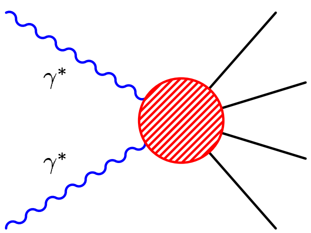
thus, only diagrams of type (a) survive:

$$e^+e^- \longrightarrow e^+e^- + \underbrace{\gamma^*\gamma^*}$$

→ hadrons

$\gamma^*\gamma^* \to$ HADRONS

The fixed order expansion in α_S



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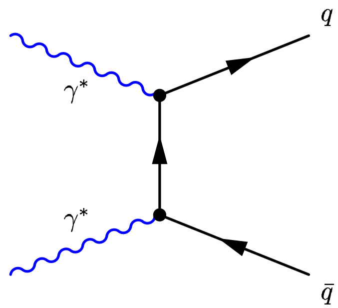
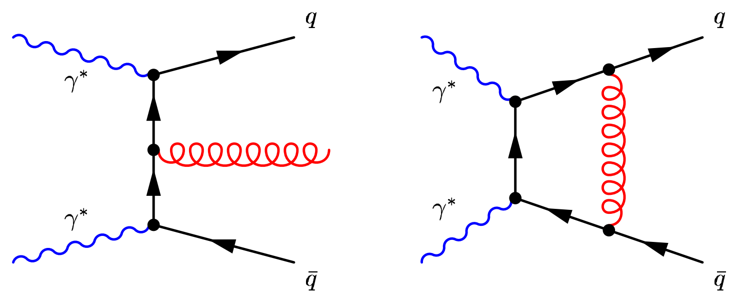
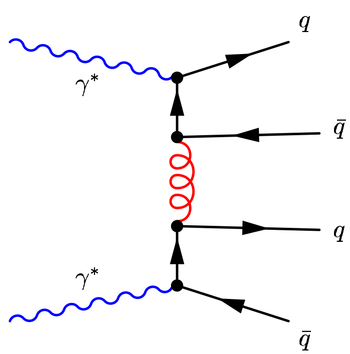


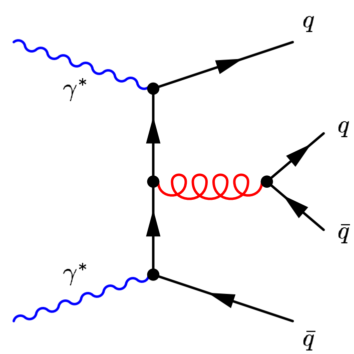


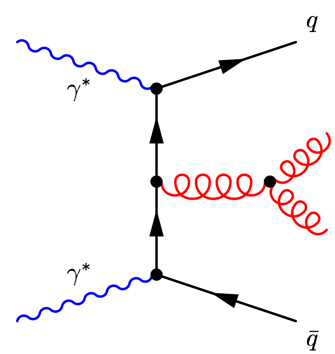
LO

NLO

Cacciari, Frixione, Trocsanyi, VDD, hep-ph/0011368







BFKL (Born)

NNLO (4-partons)

Maltoni, Trocsanyi, VDD, hep-ph/0202237

Theoretical framework

$\gamma^*\gamma^*$ cross section as fixed-order expansion + resummation

$$\sigma_{\gamma^*\gamma^*} \sim \sum_{j=0}^{\infty} a_{0j}\alpha_S^j + a_1\alpha_S^2 \sum_{j=0}^{\infty} \left(\alpha_S \log\left(\frac{W^2}{\mu_W^2}\right)\right)^j + \cdots$$

- W: hadronic energy μ_W: transverse energy scale
- 2^{nd} sum collects terms which feature only gluon exchange in the t channel, and resums leading log (LL) corrections
- ellipses refer to log corrections beyond LL
- 1^{st} sum is a fixed-order expansion in α_S starting at $\mathcal{O}(\alpha_S^0)$, and collects the contributions which do not feature only gluon exchange in the t channel
- a_{0j} behave like $1/W^2$ (or eventually like $1/(W\mu_W)$)
- a_{00} is LO term; a_{01} is NLO term; $a_{02} + a_1(j = 0)$ is NNLO term
- a_1 behaves like $1/\mu_W^2$

PHASE SPACE factorises into leptonic & hadronic parts

KINEMATICS we use the variable $Y = \log \frac{y_1 y_2 s}{\sqrt{Q_1^2 Q_2^2}}$

$y_i \propto$ the light-cone momentum fraction of the virtual photons

$$y_i = \frac{q_i^0 + q_i^3}{\sqrt{s}} = 1 - \frac{2E_i}{\sqrt{s}} \cos^2 \frac{\theta_i}{2}, \qquad i = 1, 2$$

$E_i,\ \theta_i$: energies and scattering angles of the leptons in the e^+e^- frame

❋ IN HEL $\quad y_1 y_2 s \simeq W^2$

➥ $Y \simeq \log \frac{W^2}{\sqrt{Q_1^2 Q_2^2}}$ becomes BFKL–like variable

NLO CALCULATION

- we used a general-purpose NLO partonic Monte Carlo (subtraction)
- we took the massless limit of the outgoing quarks

μ_R SCALE the calculation

- is LO in $\alpha_{\rm em}$

  $\alpha_{\rm em}(\mu_R^2)$: we choose $\alpha_{\rm em}^2(Q_1^2)\alpha_{\rm em}^2(Q_2^2)$ (one-loop $\overline{\rm MS}$ running)

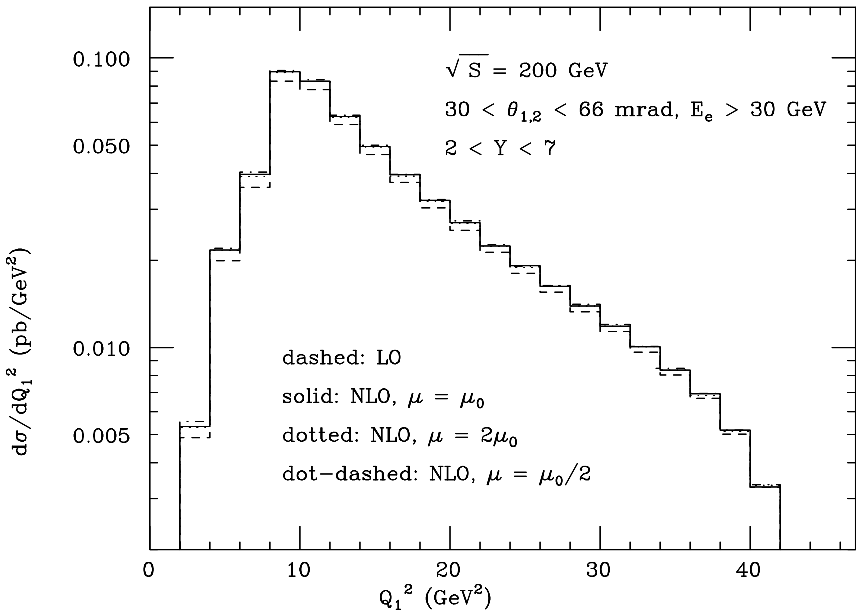
- has α_S occurring first at NLO

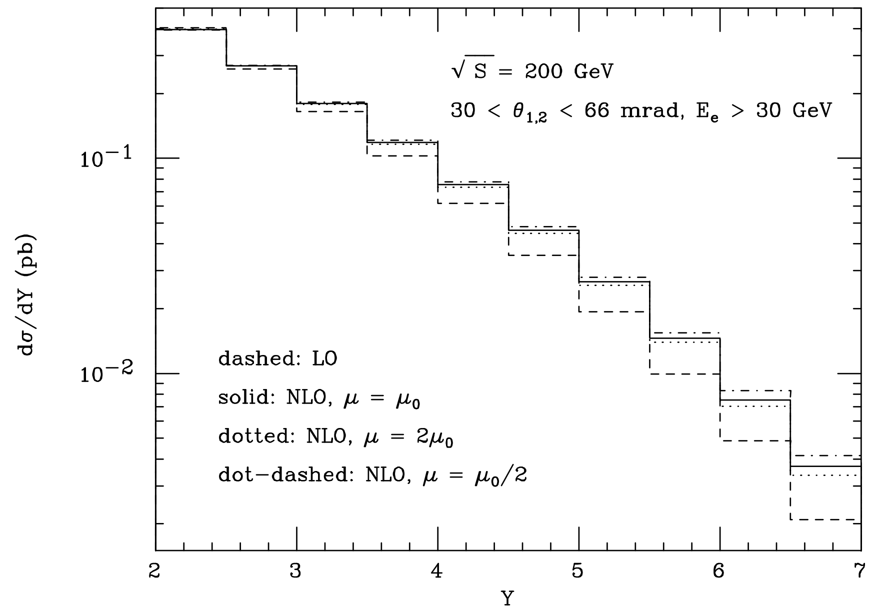
  $\alpha_S(\mu_R^2)$: $$\mu_0^2 = \frac{Q_1^2 + Q_2^2}{2} + \left(\frac{\sum_i p_{i_\perp}}{2}\right)^2$$ i=1,2,3

  $p_{i_\perp}$: transverse momenta in the $\gamma^*\gamma^*$ frame

NLO SCALE UNCERTAINTIES

- Q^2 & Y distributions
- we varied $\frac{\mu_0}{2} < \mu_R < 2\mu_0$





L3 CUTS $E_i \geq 30$ GeV $\quad 30 \leq \theta_i \leq 66$ mrad $\quad 2 \leq Y \leq 7$

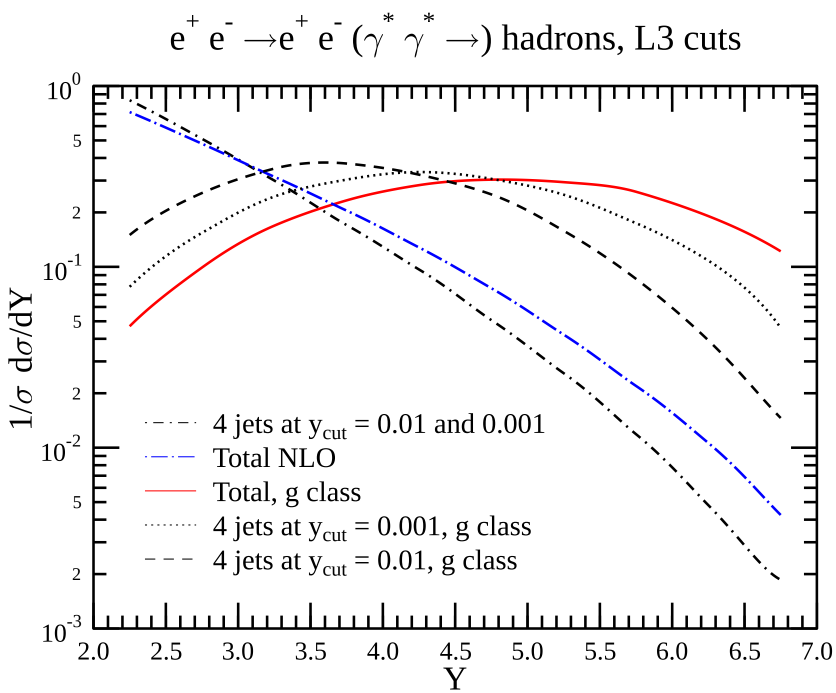
L3 Collaboration '99

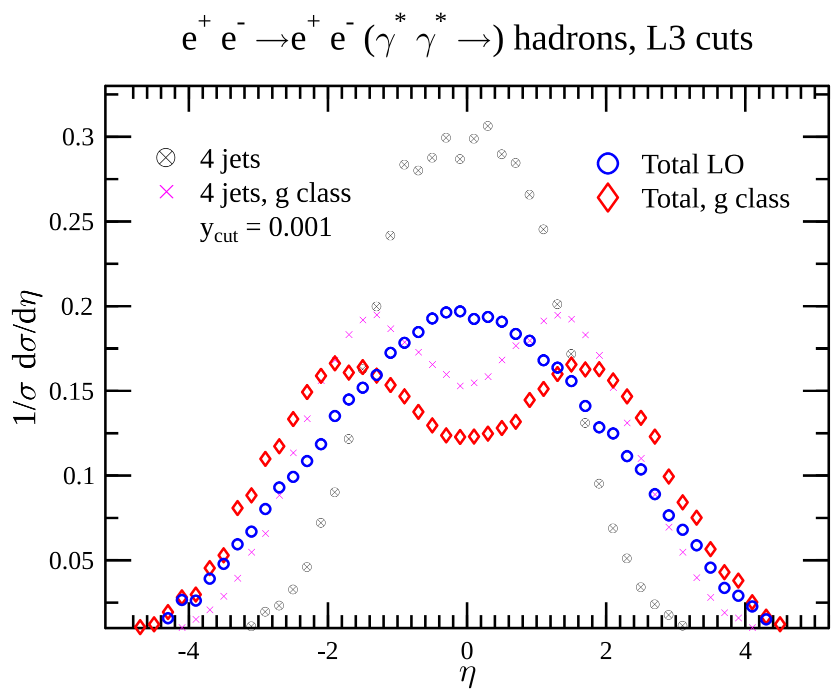
- uncertainty related to μ is smaller than NLO corrections
- effect of NLO corrections is small, except at large Y, where they induce a 50% increase

NNLO 4-PARTON FINAL STATES

diagrams with exchange in the t channel of a

- ❋ gluon (g class) are gauge invariant & infrared finite
- ❋ quark (f class) display infrared divergences

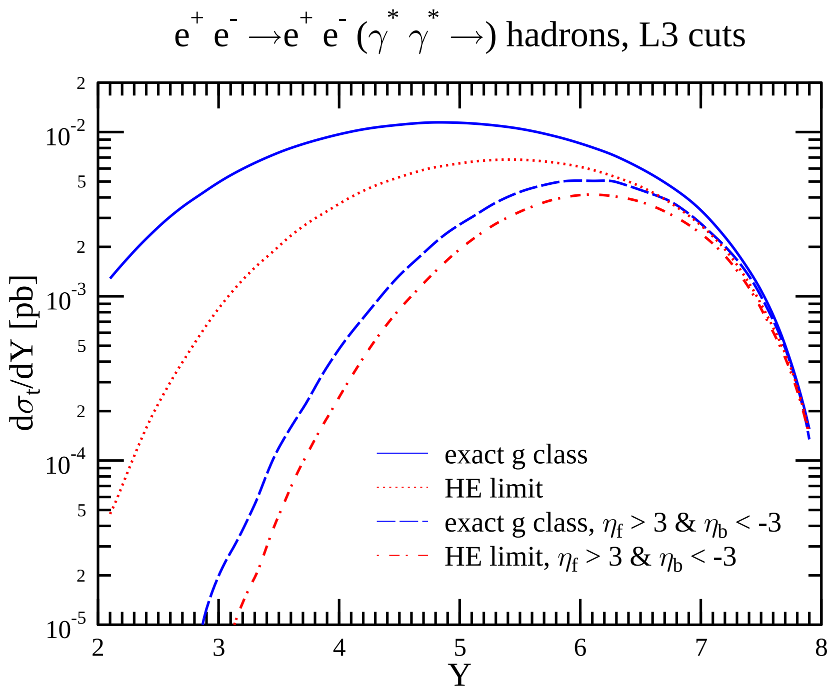


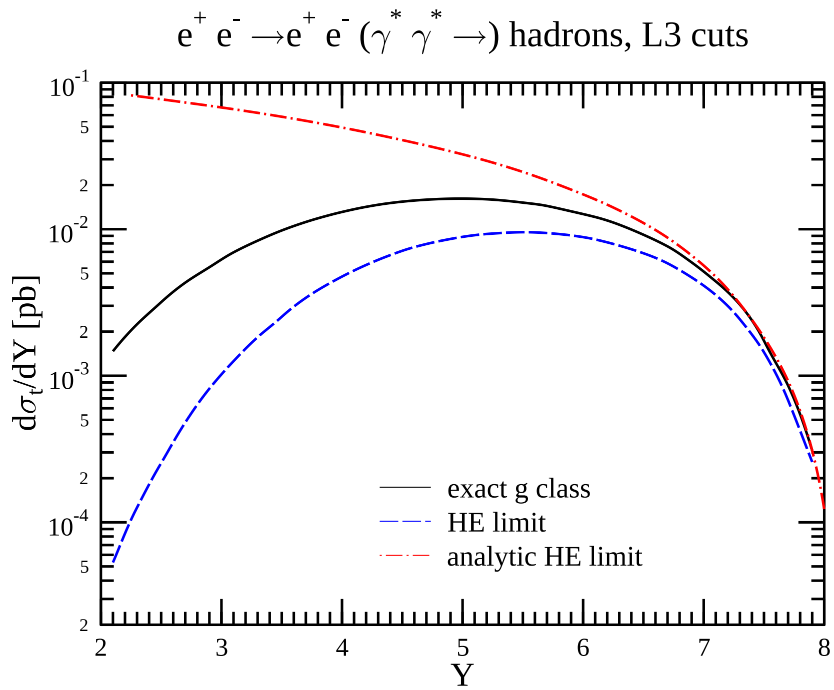


✤ the g class and f class have different shapes in the Y and η distributions: the g class contributes more at large Y and its quarks are more forward

THE HIGH ENERGY LIMIT (HEL)

✻ in HEL the scattering amplitude of the g class and the 4-quark phase space factorise into two impact factors connected by the t channel gluon





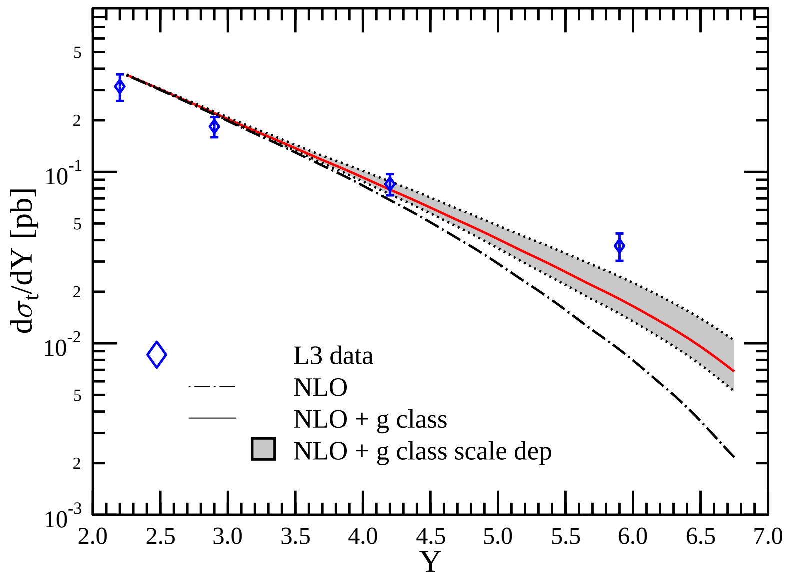
➥ HEL yields errors smaller than 20% only for $Y > 7$

➥ requiring the quarks forward underestimates exact g class

➥ analytic HEL is much larger than exact g class

PHENOMENOLOGY

☛ L3 data versus NLO or NLO + g class



$\Delta W_{\gamma\gamma}$ (GeV)	L3 data $d\sigma_{ee}/dW_{\gamma\gamma}$ (pb/GeV)	NLO $d\sigma_{ee}/dW_{\gamma\gamma}$ (pb/GeV)	NLO + g class $d\sigma_{ee}/dW_{\gamma\gamma}$ (pb/GeV)
5– 10	$0.0747 \pm 0.0096 \pm 0.0067$	$0.0883^{+0.0004}_{-0.0027}$	$0.0885^{+0.0003}_{-0.0027}$
10– 20	$0.0263 \pm 0.0024 \pm 0.0024$	$0.0300^{+0.0001}_{-0.0001}$	$0.0305^{+0.0003}_{-0.0002}$
20– 40	$0.0062 \pm 0.0007 \pm 0.0006$	$0.0057^{+0.0001}_{-0.0003}$	$0.0064^{+0.0006}_{-0.0003}$
40–100	$0.0014 \pm 0.0002 \pm 0.0001$	$0.0004^{+0.0001}_{-0.0000}$	$0.0007^{+0.0002}_{-0.0001}$

ΔY	L3 data $d\sigma_{ee}/dY$ (pb)	NLO $d\sigma_{ee}/dY$ (pb)	NLO + g class $d\sigma_{ee}/dY$ (pb)
2.0–2.5	$0.315 \pm 0.048 \pm 0.028$	$0.366^{+0.001}_{-0.001}$	$0.368^{+0.002}_{-0.002}$
2.5–3.5	$0.184 \pm 0.018 \pm 0.017$	$0.203^{+0.002}_{-0.001}$	$0.208^{+0.004}_{-0.003}$
3.5–5.0	$0.085 \pm 0.009 \pm 0.008$	$0.070^{+0.002}_{-0.002}$	$0.080^{+0.008}_{-0.005}$
5.0–7.0	$0.037 \pm 0.006 \pm 0.003$	$0.010^{+0.001}_{-0.001}$	$0.018^{+0.006}_{-0.003}$

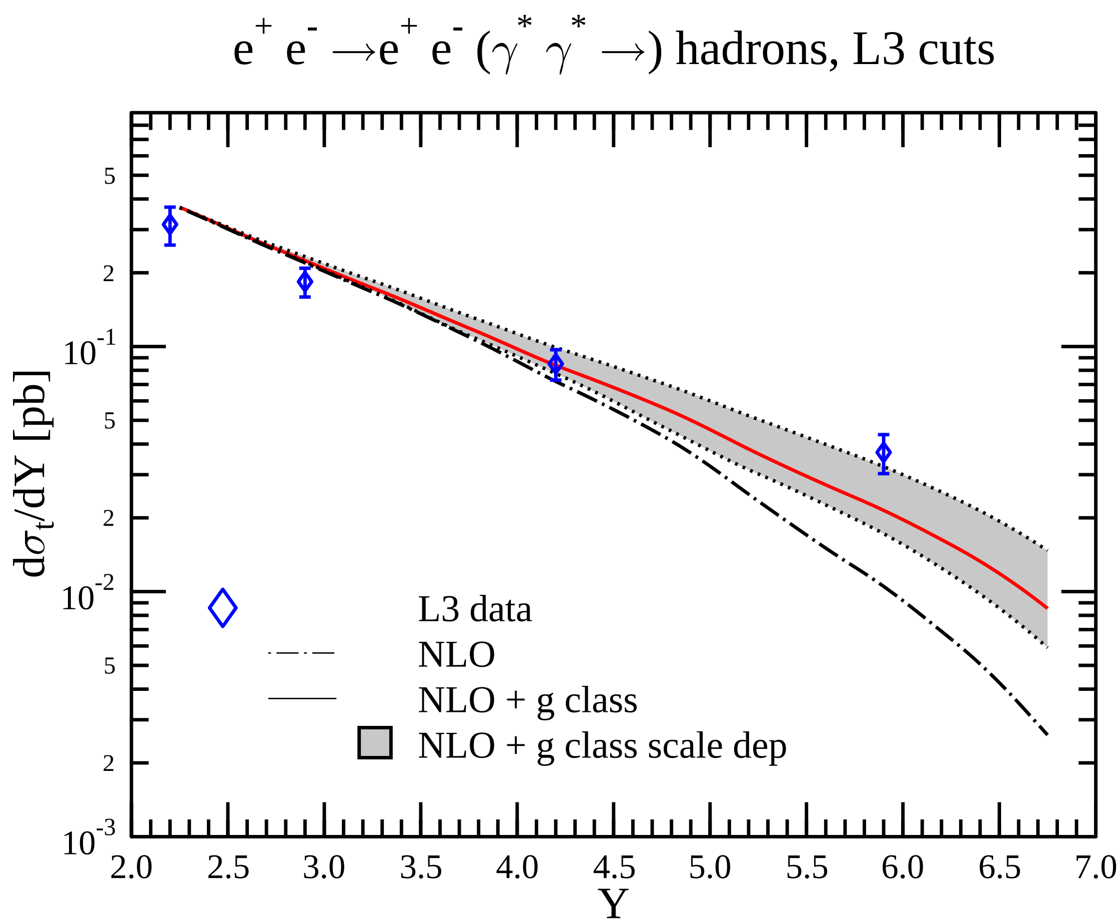
L3 CUTS $E_i \geq 40$ GeV $\quad 30 \leq \theta_i \leq 66$ mrad $\quad W \geq 5$ GeV

L3 Collaboration 2001

- ✱ shaded band: scale variation $\frac{\mu_0}{2} < \mu_R < 2\mu_0$
- ✱ error bars on data: added statistical and systematic errors in quadrature
- ✱ slight excess of NLO over L3 data at small Y (REM: massless limit)
- ✱ NLO & NLO + g class underestimate L3 data at large Y

CAVEAT

❋ however, if as a default scale for μ_R use $\mu_0^2 = \dfrac{Q_1^2 + Q_2^2}{2}$, get



➥ the g class contribution depends sizeably on μ_R scale variations

Conclusions

- NLO theory describes well the data, but for
  - slight excess at small Y, to be reduced by mass dependence
  - a slight deficit at large Y
- 4-parton contributions of the NNLO theory
  - HEL is not accurate at LEP2 energies
  - 4-quark g class must be included exactly
- deficit at large Y
  - NLO $+$ g class reduces discrepancy between theory & L3 data
  - a full NNLO calculation (nowadays unfeasible) would be welcome
  - higher order corrections (BFKL ?) might be relevant