Towards jet cross sections at NNLO through subtraction

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Precision QCD

Precise determination of

- strong coupling constant α_s
- parton distributions
- LHC parton luminosity

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Precise determination of

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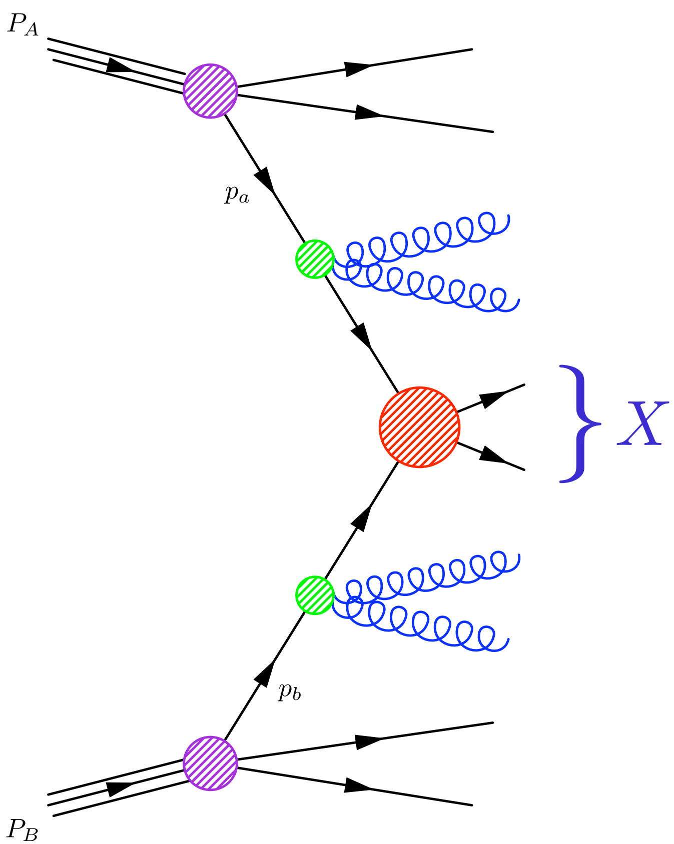
- parton distributions
- LHC parton luminosity

Precise prediction for

- Higgs production
- new physics processes
- their backgrounds

Cross sections at high Q^2



separate the short- and the long-range interactions through factorisation

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_{a/A}(x_1,\mu_F^2)\, f_{b/B}(x_2,\mu_F^2)$$
$$\times\ \hat{\sigma}_{ab\to X}\left(x_1, x_2, \{p_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_F^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}\right)$$

$$X = W, Z, H, Q\bar{Q}, \text{high-}E_T\text{jets}, \ldots$$

$\hat{\sigma}$ is known as a fixed-order expansion in α_S

$$\hat{\sigma} = C\alpha_S^n(1 + c_1\alpha_S + c_2\alpha_S^2 + \ldots)$$

$c_1 =$ NLO $\qquad c_2 =$ NNLO

or as an all-order resummation

$$\hat{\sigma} = C\alpha_S^n[1 + (c_{11}L + c_{10})\alpha_S + (c_{22}L^2 + c_{21}L + c_{20})\alpha_S^2 + \ldots]$$

where $L = \ln(M/q_T), \ln(1-x), \ln(1/x), \ln(1-T), \ldots$

$c_{11}, c_{22} =$ LL $\qquad c_{10}, c_{21} =$ NLL $\qquad c_{20} =$ NNLL

NLO features

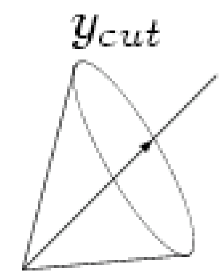
- Jet structure: final-state collinear radiation
- PDF evolution: initial-state collinear radiation
- Opening of new channels
- Reduced sensitivity to fictitious input scales: μ_R, μ_F
  - predictive normalisation of observables
    - first step toward precision measurements
    - accurate estimate of signal and background for Higgs and new physics
- Matching with parton-shower MC's:

  MC@NLO   POWHEG

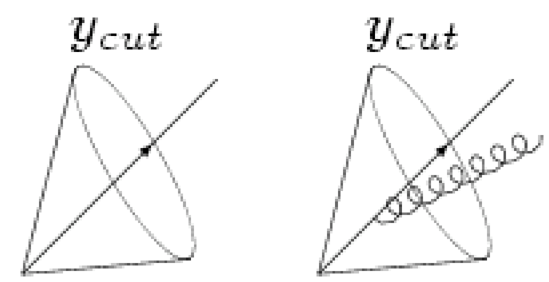
Jet structure

the jet non-trivial structure shows up first to NLO

leading order



NLO



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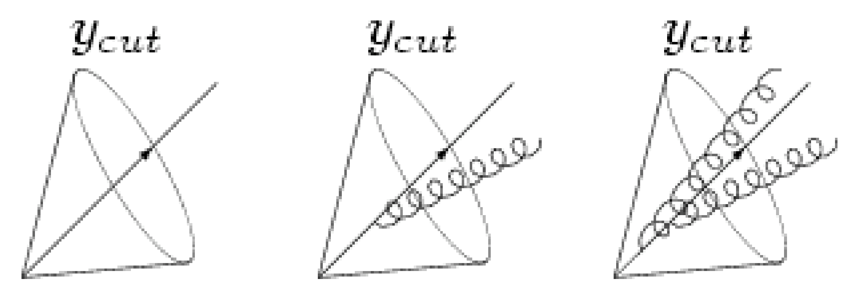
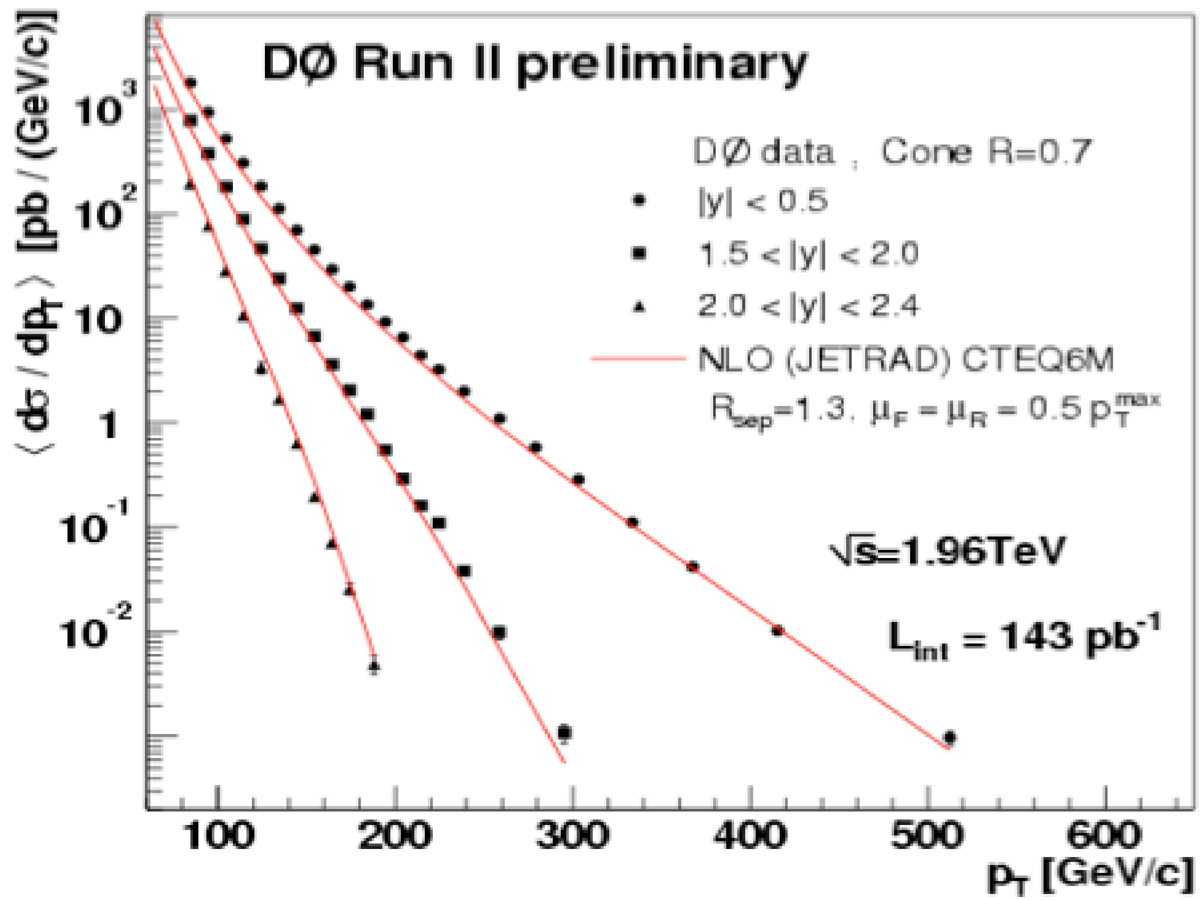


Is NLO enough to describe data ?

Inclusive jet p_T cross section at Tevatron



good agreement between NLO and data over several orders of magnitude

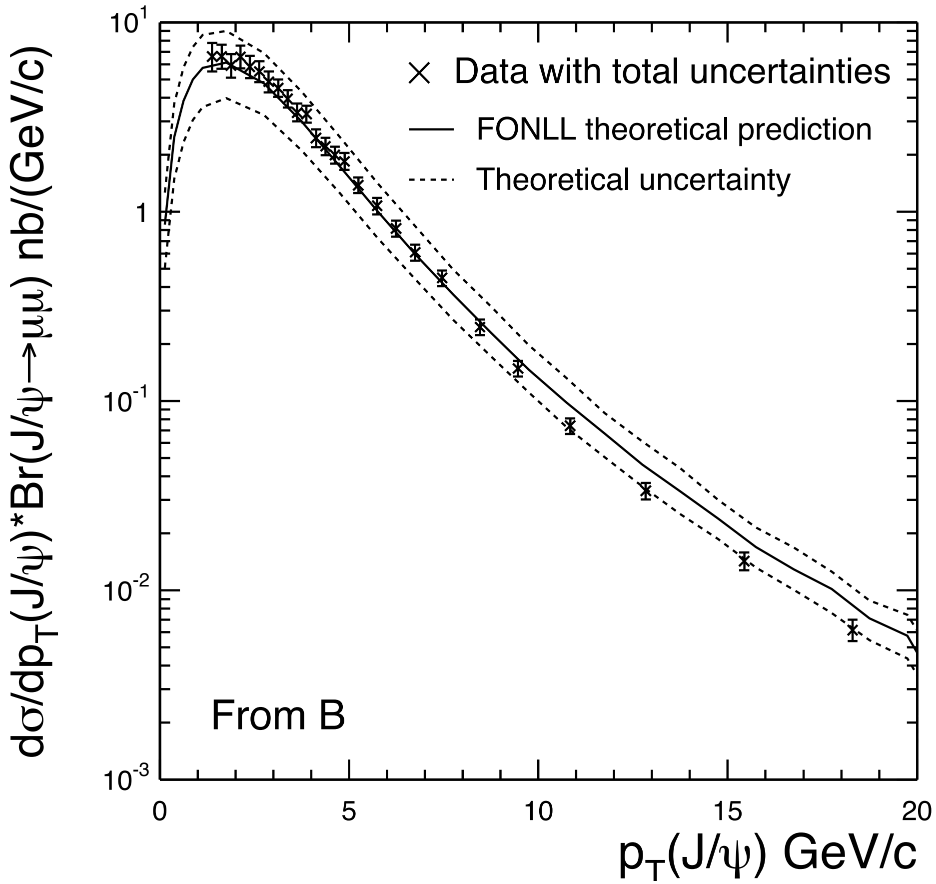
constrains the gluon distribution at high x

Is NLO enough to describe data ?

b cross section in $p\bar{p}$ collisions at 1.96 TeV

$$d\sigma(p\bar{p} \to H_b X, H_b \to J/\psi\ X)/dp_T(J/\psi)$$

CDF hep-ex/0412071



total x-sect is

$$19.4 \pm 0.3(stat)^{+2.1}_{-1.9}(syst)\ \text{nb}$$

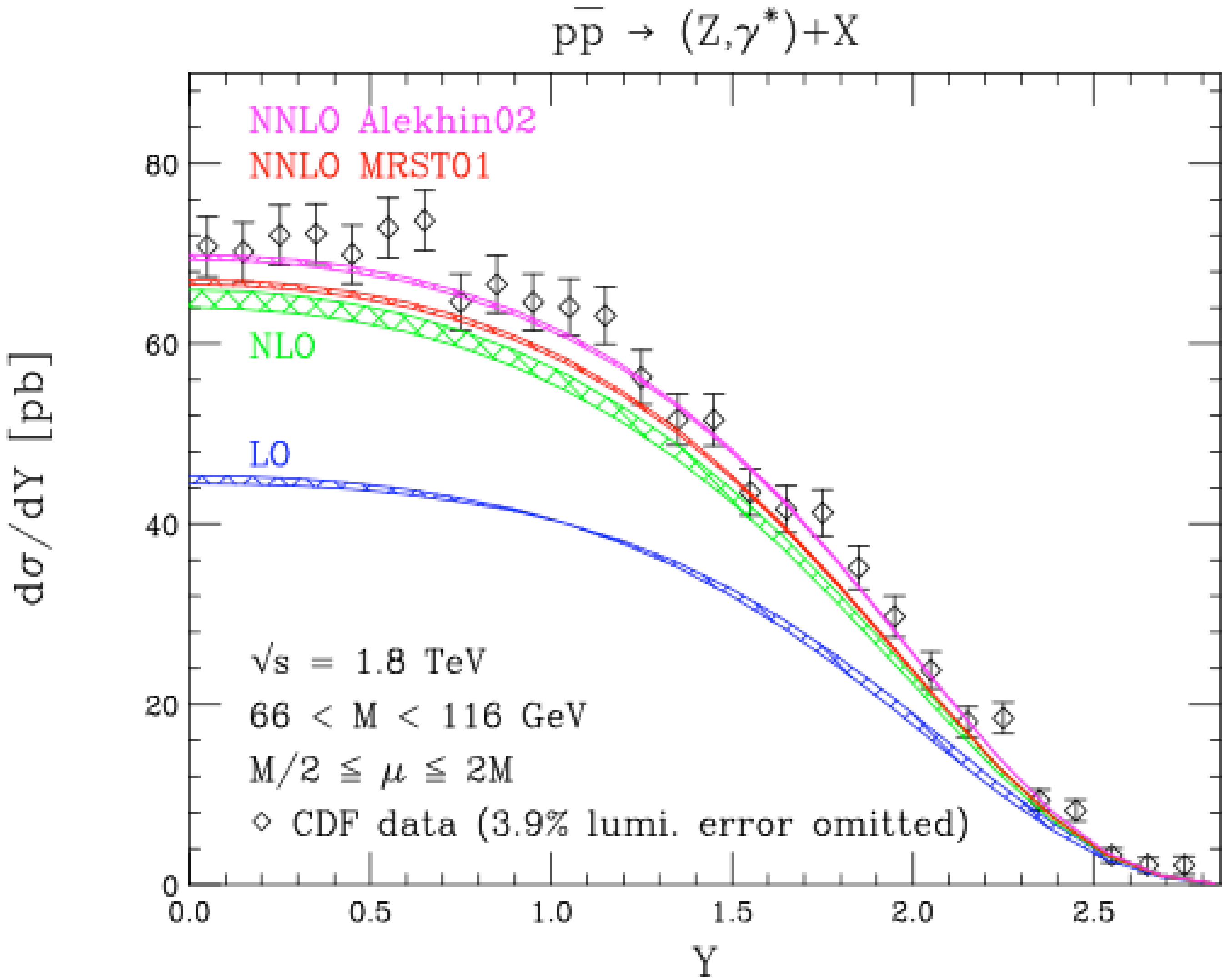
FONLL = NLO + NLL

Cacciari, Frixione, Mangano, Nason, Ridolfi 2003

good agreement with data (with use of updated FF's by Cacciari & Nason)

Is NLO enough to describe data ?

di-lepton rapidity distribution for (Z, γ^*) production vs. Tevatron Run I data



LO and NLO curves are for the MRST PDF set

no spin correlations

C. Anastasiou L. Dixon K. Melnikov F. Petriello 2003

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Drell-Yan W acceptances at LHC with leptonic decay of the W

Cuts A $\longrightarrow |\eta^{(e)}| < 2.5,\ p_T^{(e)} > 20$ GeV, $p_T^{(\nu)} > 20$ GeV

Cuts B $\longrightarrow |\eta^{(e)}| < 2.5,\ p_T^{(e)} > 40$ GeV, $p_T^{(\nu)} > 20$ GeV

	LO		LO+HW	NLO		MC@NLO
Cuts A	0.5249	$\xrightarrow{-7.7\%}$	0.4843	0.4771	$\xrightarrow{+1.5\%}$	0.4845
	↓5.4%			↓7.0%		↓6.3%
Cuts A, no spin	0.5535			0.5104		0.5151
Cuts B	0.0585	$\xrightarrow{+208\%}$	0.1218	0.1292	$\xrightarrow{+2.9\%}$	0.1329
	↓29%			↓16%		↓18%
Cuts B, no spin	0.0752			0.1504		0.1570

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- $|\mathrm{MC@NLO} - \mathrm{NLO}| = \mathcal{O}(2\%)$ S. Frixione M.L. Mangano 2004
- NNLO useless without spin correlations
- Precisely evaluated Drell-Yan W, Z cross sections could be used as ``standard candles'' to measure the parton luminosity at LHC

Drell-Yan W acceptances at LHC with leptonic decay of the W

$p_\perp^{e,\min}$ (GeV)	A(NLO)	A(NNLO)
20	0.487,0.488,0.489	0.497,0.492,0.491
30	0.379,0.378,0.378	0.379,0.376,0.377
40	0.127,0.125,0.122	0.161,0.155,0.152
50	0.0312,0.0295,0.0277	0.0427,0.0397,0.0387

$\mu = m_W/2, m_W, 2m_W$

K. Melnikov, F. Petriello 2006

Drell-Yan *W* acceptances at LHC with leptonic decay of the *W*

$p_\perp^{e,\min}$ (GeV)	A(NLO)	A(NNLO)
20	0.487,0.488,0.489	0.497,0.492,0.491
30	0.379,0.378,0.378	0.379,0.376,0.377
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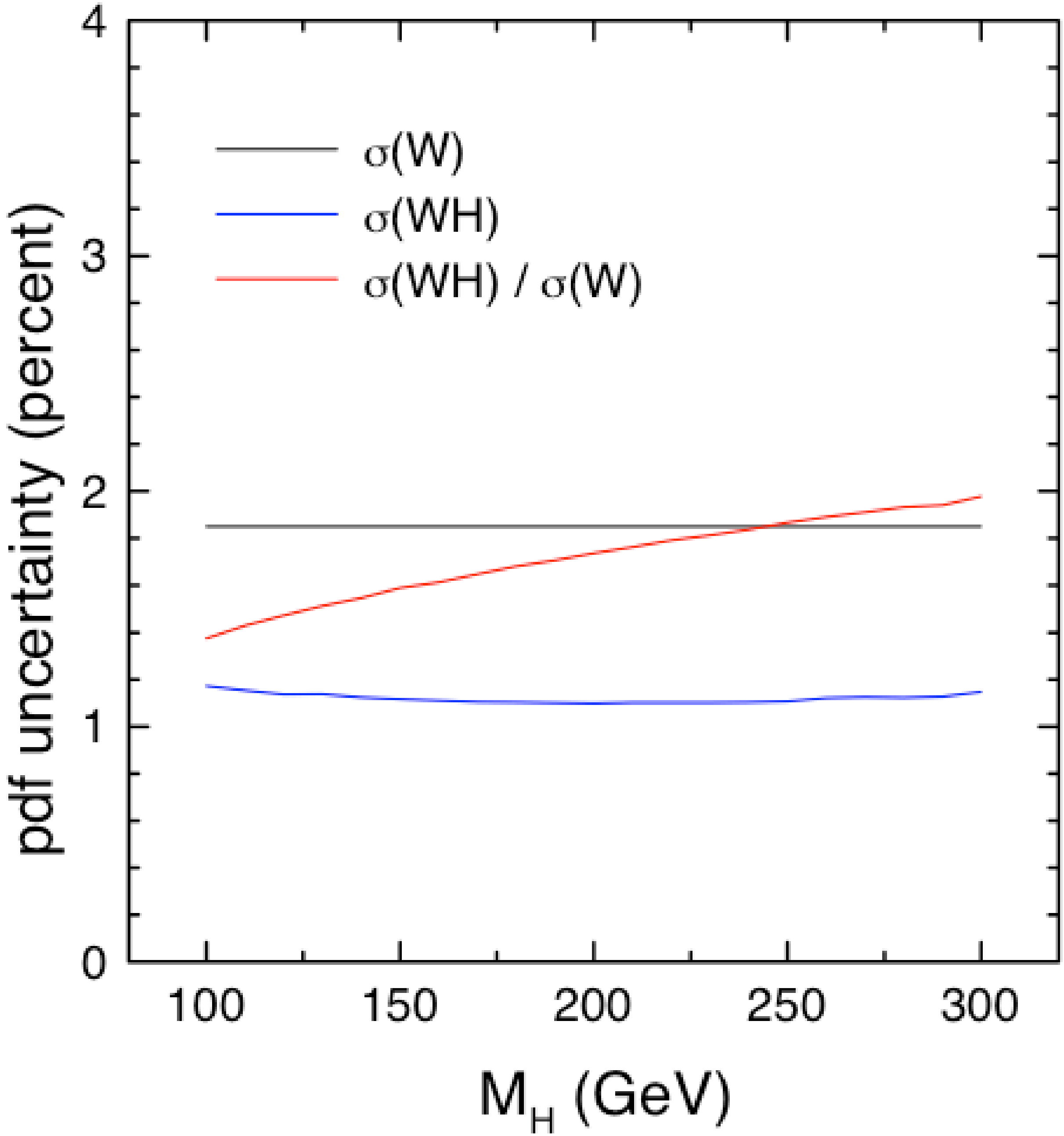
$\mu = m_W/2, m_W, 2m_W$

- At LO, $p_{e,\perp} \leq m_W/2$
- NNLO corrections are large for $p_{e,\perp}$ = 40, 50 GeV
  but are at the percent level for $p_{e,\perp}$ = 20, 30 GeV

K. Melnikov, F. Petriello 2006

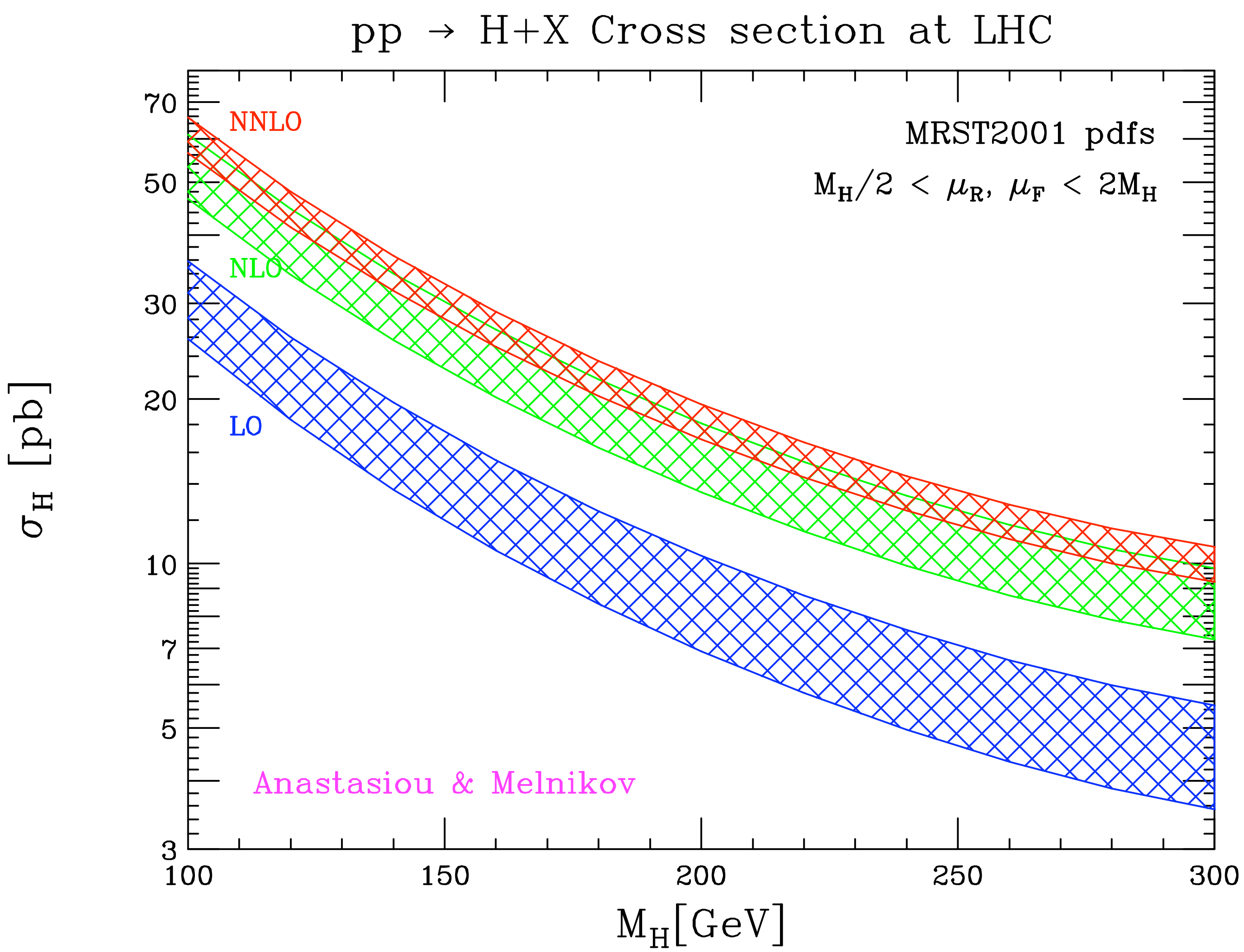
PDF uncertainty on W, WH cross sections at LHC



- MRST2001E
- use $\sigma(W), \sigma(Z)$ as ``standard candles'', i.e. to calibrate other cross sections, e.g. $\sigma(WH)$
- $\sigma(WH)$ more precisely predicted because it samples quark PDF's at higher x than $\sigma(W)$

Is NLO enough to describe data ?

Total cross section for inclusive Higgs production at LHC



contour bands are

lower

$\mu_R = 2M_H \quad \mu_F = M_H/2$

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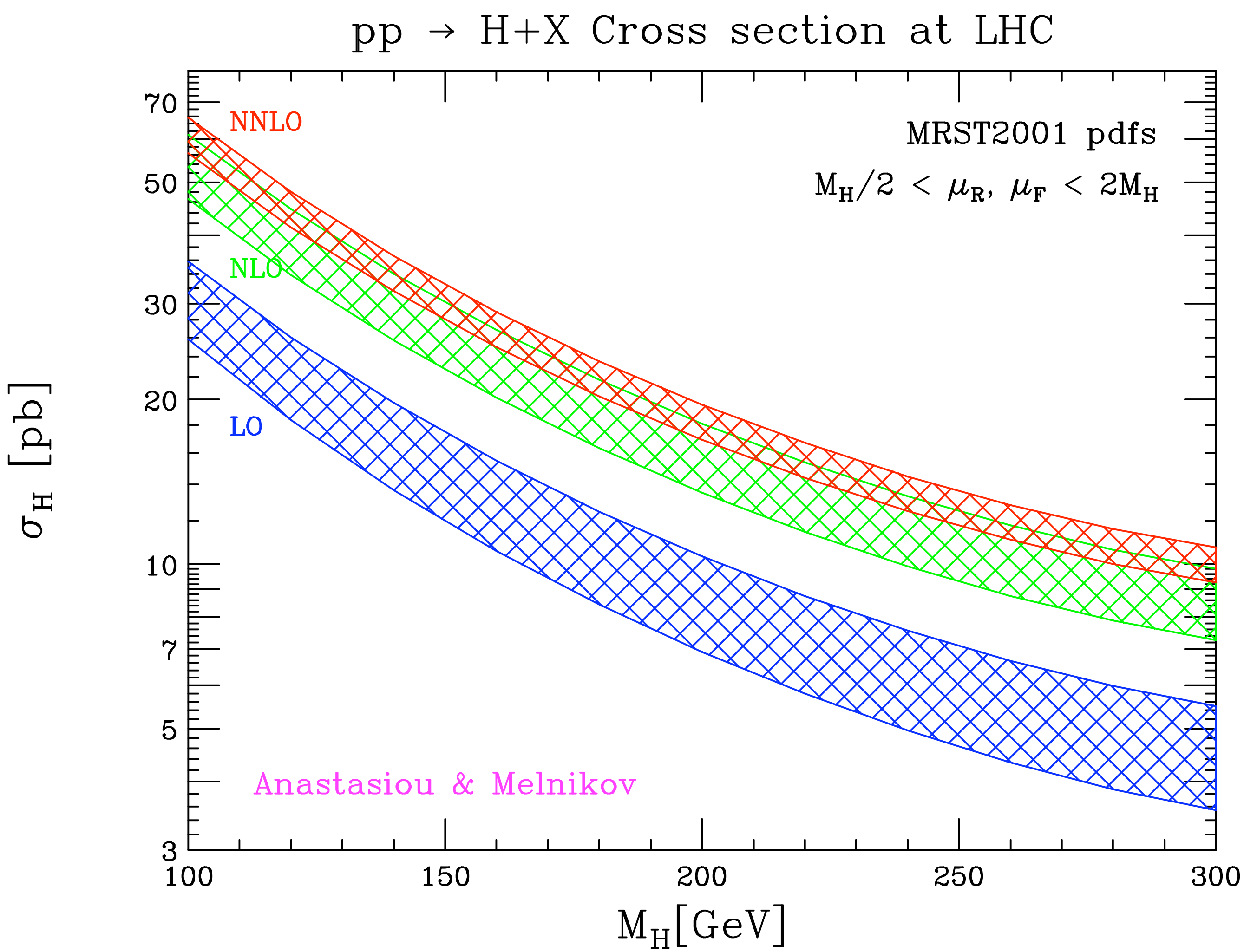

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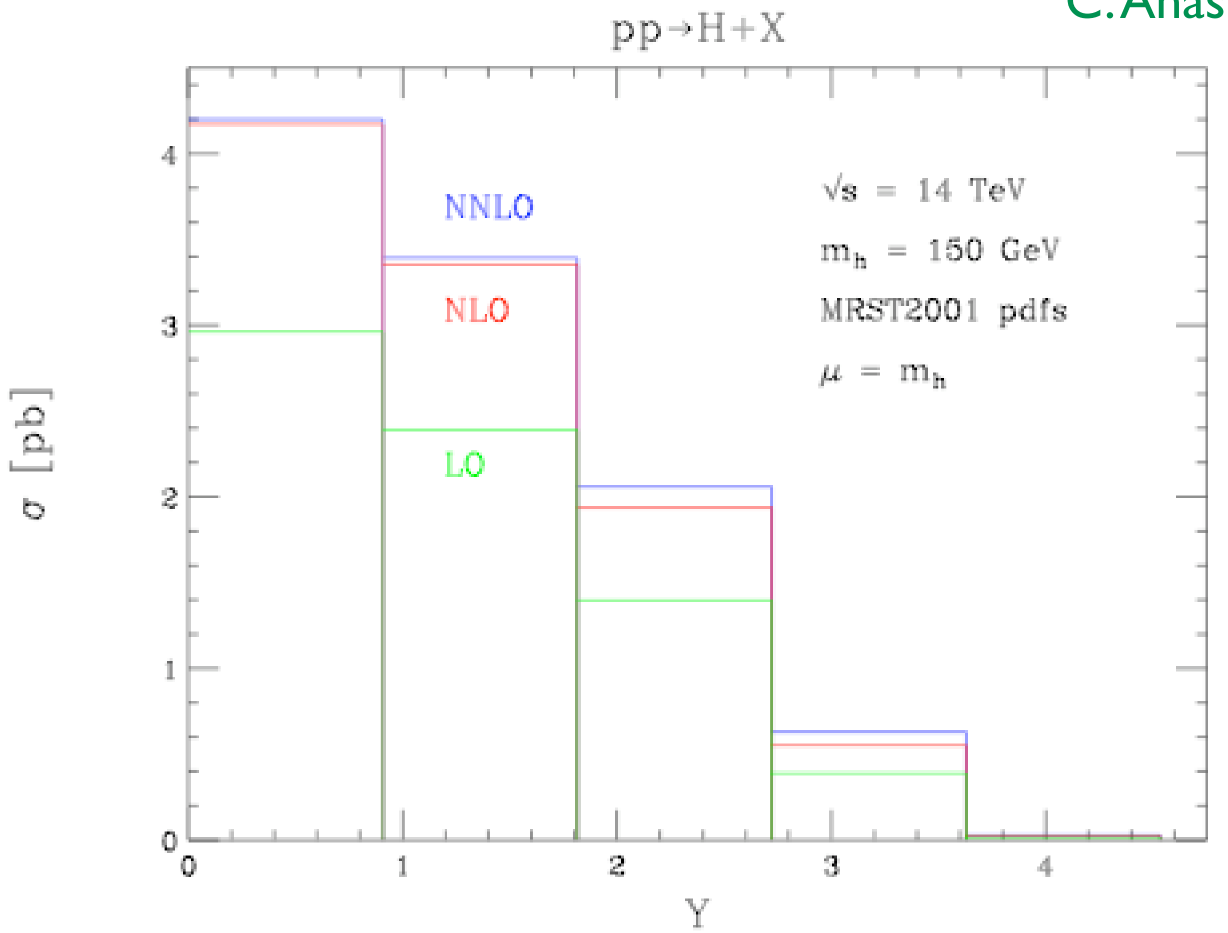
NNLO prediction stabilises the perturbative series

Higgs production at LHC

a fully differential cross section:
bin-integrated rapidity distribution, with a jet veto

C. Anastasiou K. Melnikov F. Petriello 2004



jet veto: require

$R = 0.4$

$|\mathbf{p}_T^j| < p_T^{veto} = 40\ \mathrm{GeV}$

for 2 partons

$R_{12}^2 = (\eta_1 - \eta_2)^2 + (\phi_1 - \phi_2)^2$

if $R_{12} > R$

$|\mathbf{p}_T^1|, |\mathbf{p}_T^2| < p_T^{veto}$

if $R_{12} < R$

$|\mathbf{p}_T^1 + \mathbf{p}_T^2| < p_T^{veto}$

- $M_H = 150\ \mathrm{GeV}$ (jet veto relevant in the $H \to W^+W^-$ decay channel)

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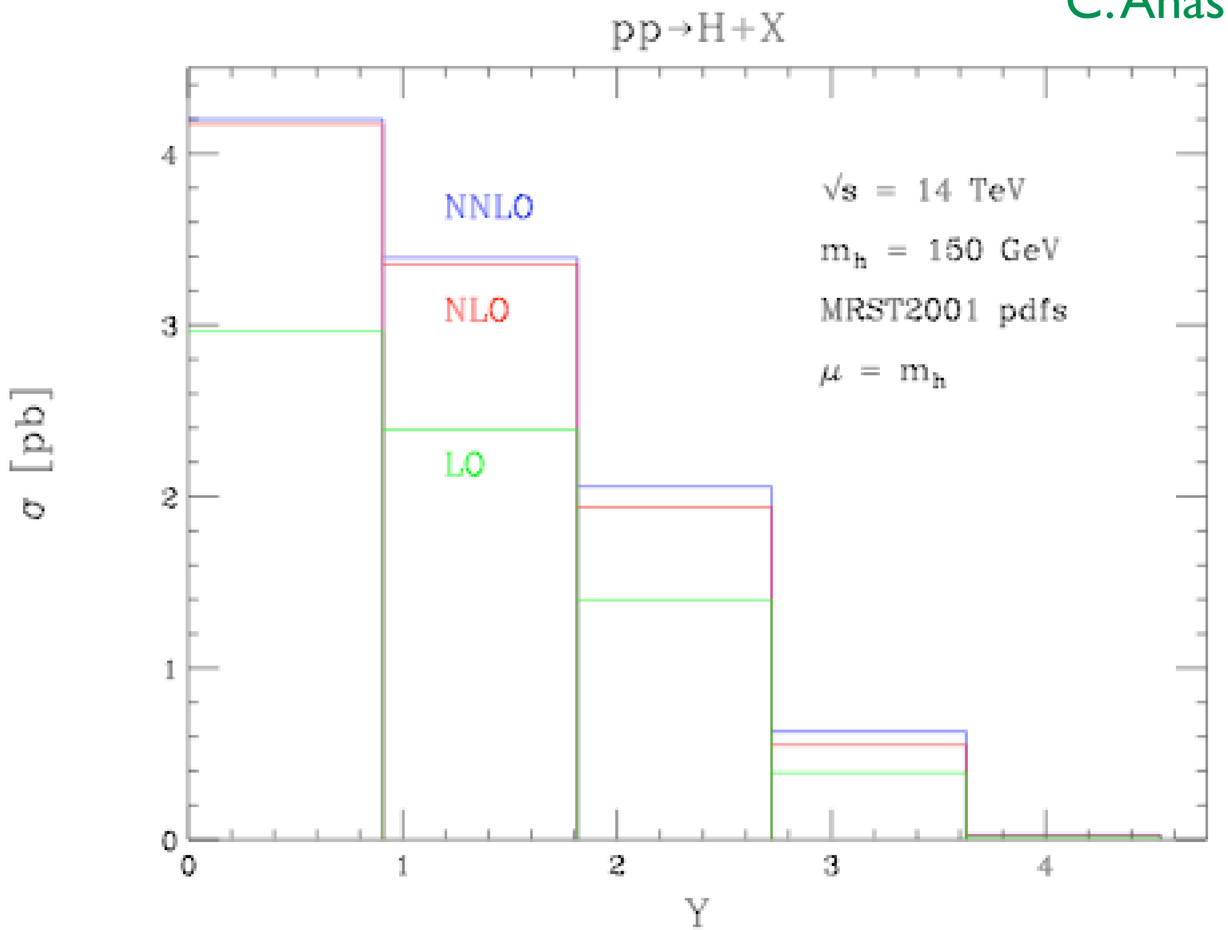

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- $M_H = 150 \text{ GeV}$ (jet veto relevant in the $H \to W^+W^-$ decay channel)
- K factor is much smaller for the vetoed x-sect than for the inclusive one: average $|\mathbf{p}_T^j|$ increases from NLO to NNLO: less x-sect passes the veto

World average of $\alpha_S(M_Z)$

$$\alpha_S(M_Z) = 0.1189 \pm 0.0010$$

S. Bethke hep-ex/0606035

Process	Q [GeV]	$\alpha_s(M_{Z^0})$	excl. mean $\alpha_s(M_{Z^0})$	std. dev.
DIS [Bj-SR]	1.58	$0.121\ ^{+\ 0.005}_{-\ 0.009}$	0.1189 ± 0.0008	0.3
τ-decays	1.78	0.1215 ± 0.0012	0.1176 ± 0.0018	1.8
DIS [ν; xF_3]	2.8 - 11	$0.119\ ^{+\ 0.007}_{-\ 0.006}$	0.1189 ± 0.0008	0.0
DIS [e/μ; F_2]	2 - 15	0.1166 ± 0.0022	0.1192 ± 0.0008	1.1
DIS [e-p $\rightarrow$ jets]	6 - 100	0.1186 ± 0.0051	0.1190 ± 0.0008	0.1
Υ decays	4.75	0.118 ± 0.006	0.1190 ± 0.0008	0.2
$\mathrm{Q\overline{Q}}$ states	7.5	0.1170 ± 0.0012	0.1200 ± 0.0014	1.6
e^+e^- [$\Gamma(Z \rightarrow had)$]	91.2	$0.1226^{+\ 0.0058}_{-\ 0.0038}$	0.1189 ± 0.0008	0.9
e^+e^- 4-jet rate	91.2	0.1176 ± 0.0022	0.1191 ± 0.0008	0.6
e^+e^- [jets & shps]	189	0.121 ± 0.005	0.1188 ± 0.0008	0.4

Rightmost 2 columns give the exclusive mean value of $\alpha_S(M_Z)$ calculated without that measurement, and the number of std. dev. between this measurement and the respective excl. mean

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- NLO corrections are large:
  Higgs production from gluon fusion in hadron collisions
- NLO uncertainty bands are too large to test theory vs. data: b production in hadron collisions
- NLO is effectively leading order:
  energy distributions in jet cones

NNLO state of the art

- Drell-Yan W, Z production
  - total cross section — Hamberg, van Neerven, Matsuura 1990; Harlander, Kilgore 2002
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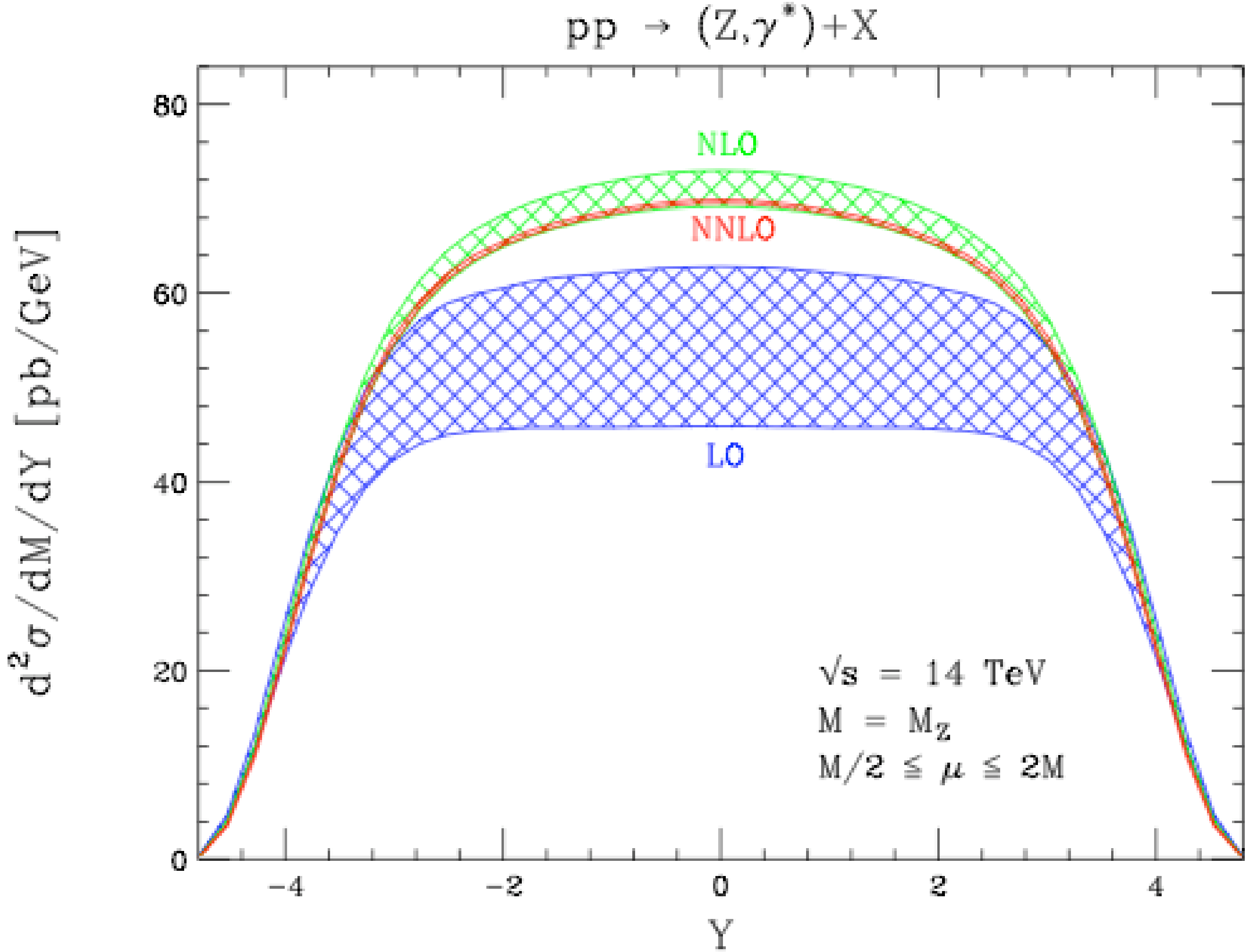
- $e^+e^- \to 3$ jets
  - almost complete — De Ridder, Gehrmann, Glover 2004-6

NNLO Drell-Yan Z production at LHC



Rapidity distribution for an on-shell Z boson

C. Anastasiou L. Dixon K. Melnikov F. Petriello 2003

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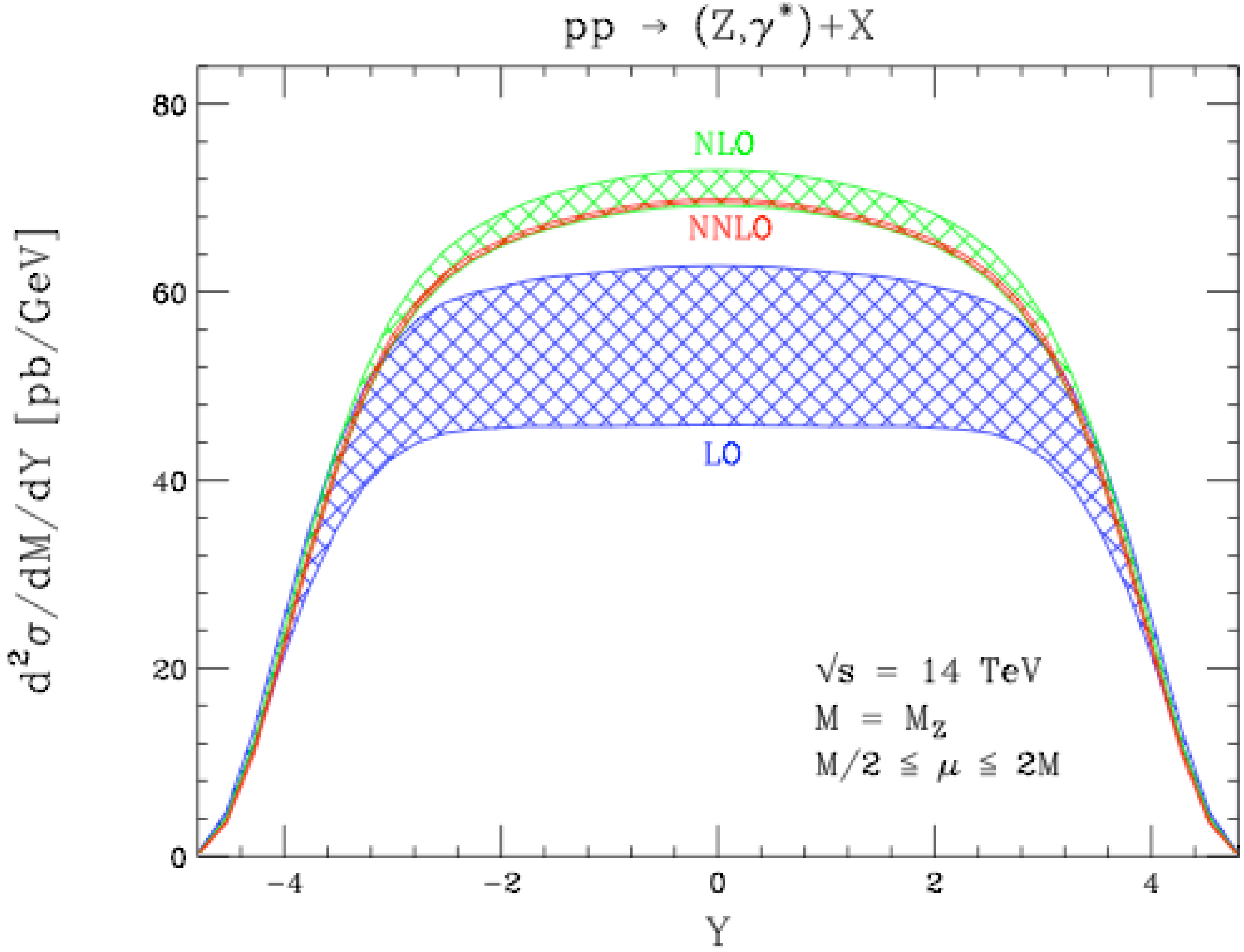

Rapidity distribution for an on-shell Z boson

- $30\%(15\%)$ NLO increase wrt to LO at central Y's (at large Y's) NNLO decreases NLO by $1-2\%$

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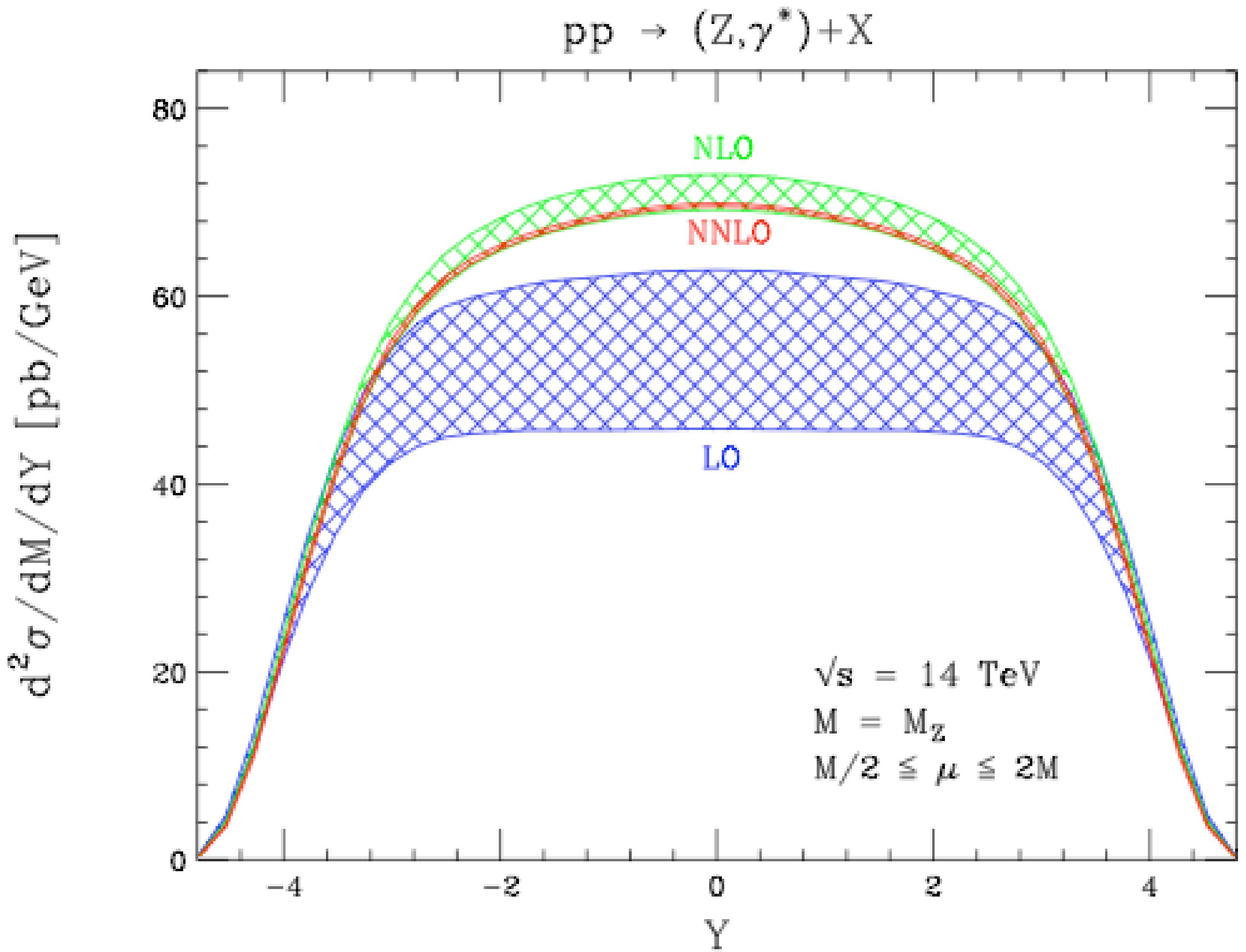
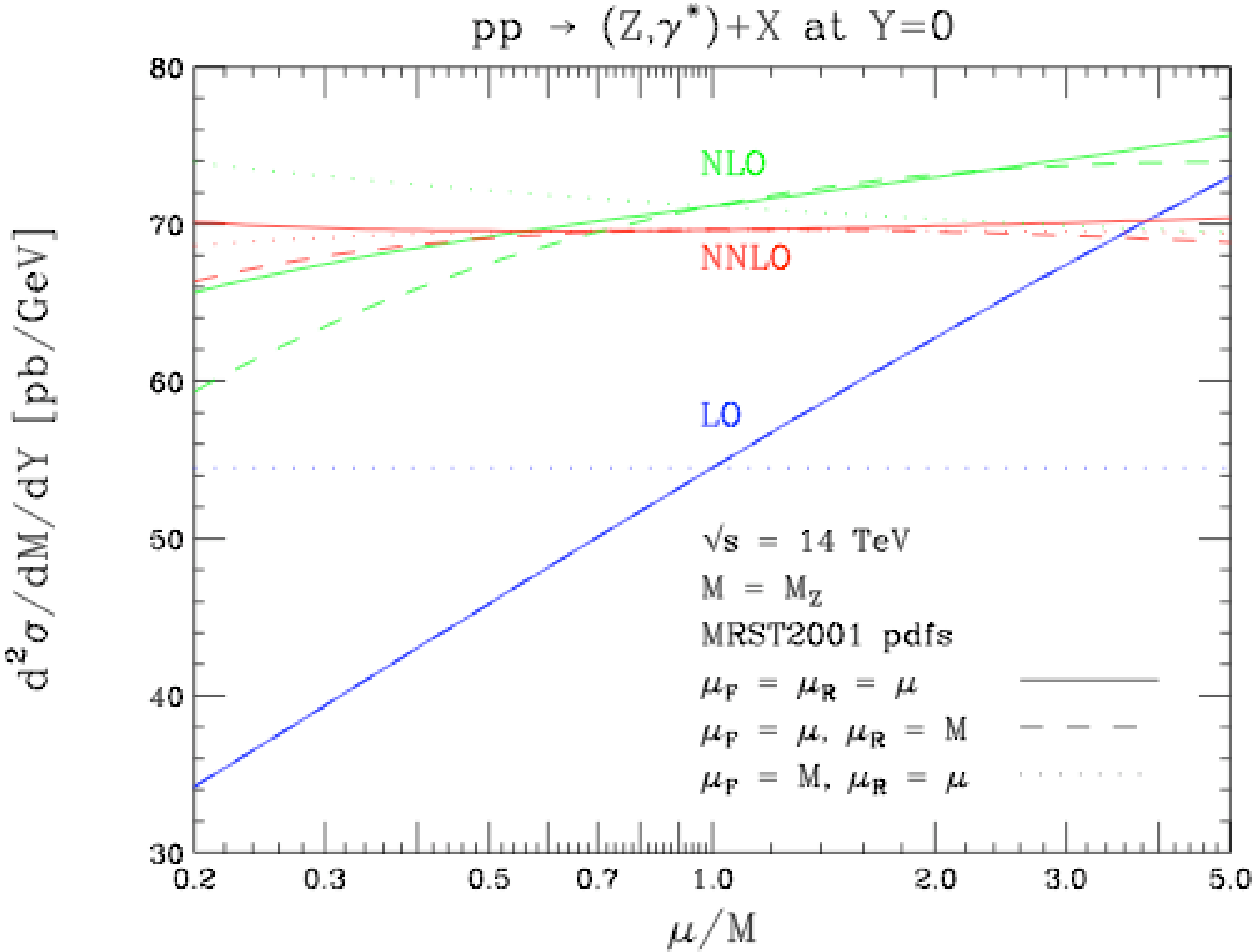


Rapidity distribution for an on-shell Z boson

- $30\%(15\%)$ NLO increase wrt to LO at central Y's (at large Y's)
  NNLO decreases NLO by $1-2\%$
- scale variation: $\approx 30\%$ at LO; $\approx 6\%$ at NLO; less than 1% at NNLO

C. Anastasiou L. Dixon K. Melnikov F. Petriello 2003

Scale variations in Drell-Yan Z production



- solid: vary μ_R and μ_F together
- dashed: vary μ_F only
- dotted: vary μ_R only

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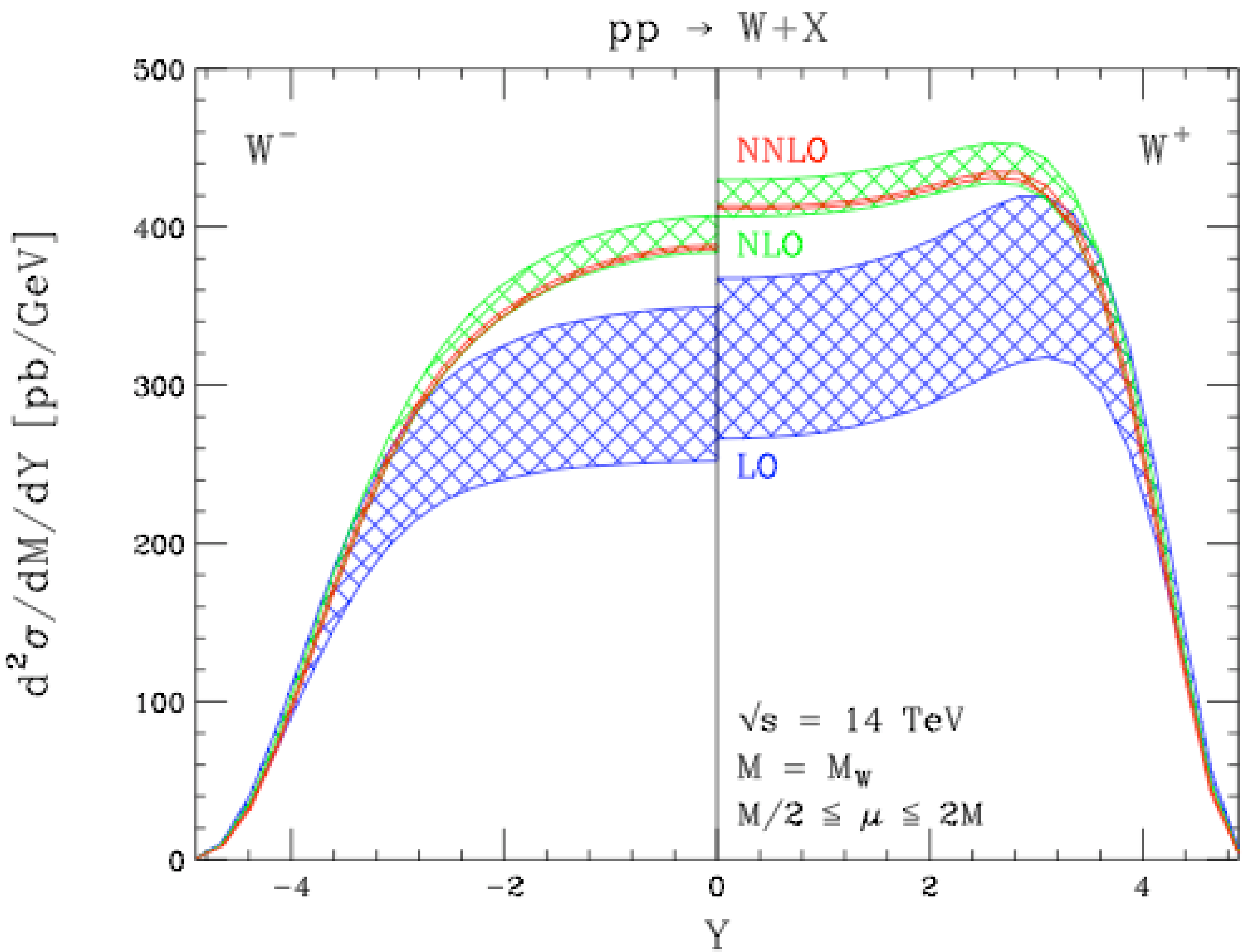
Drell-Yan W production at LHC



Rapidity distribution for an on-shell W^- boson (left) W^+ boson (right)

- distributions are symmetric in Y

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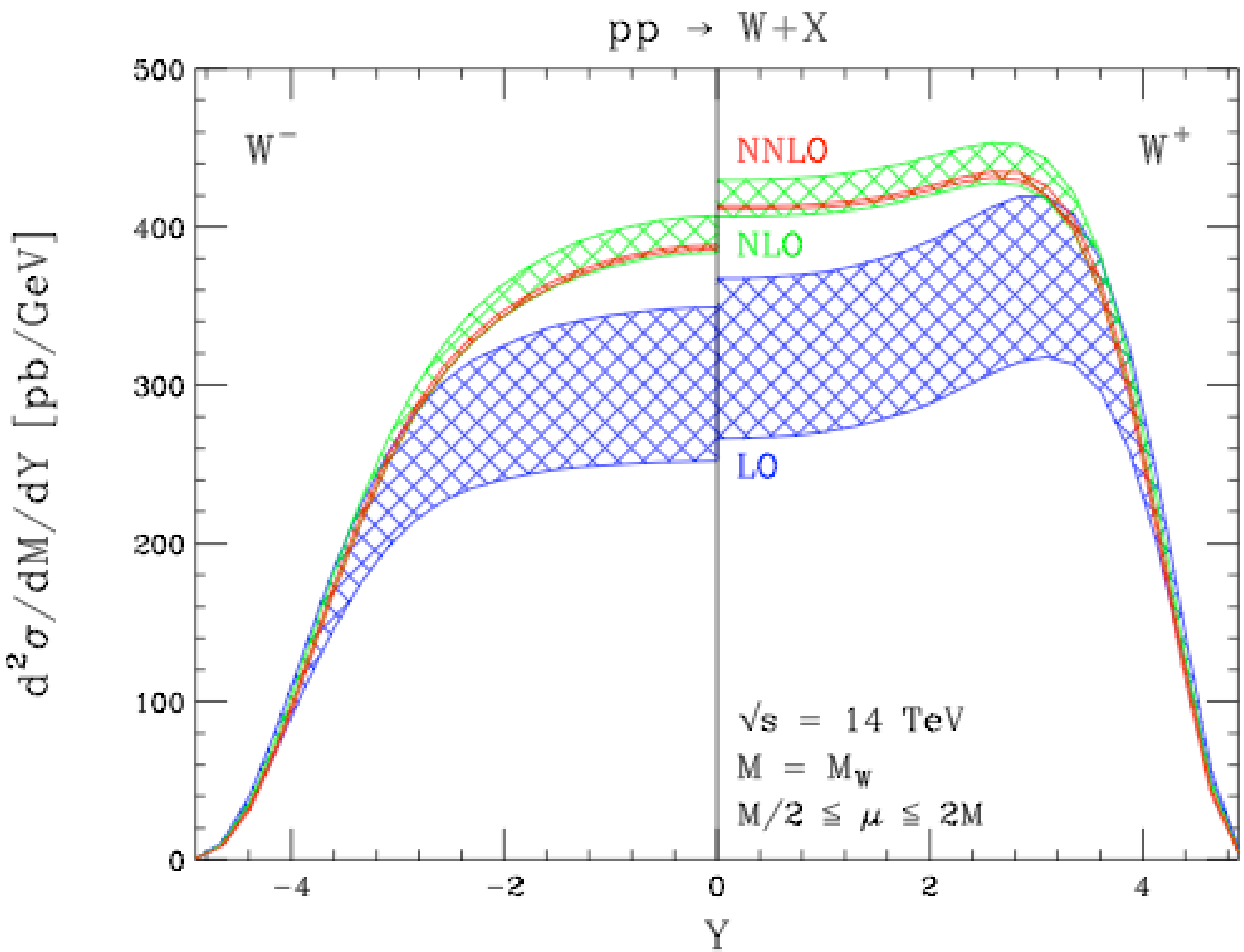

Rapidity distribution for an on-shell W^- boson (left) W^+ boson (right)

- distributions are symmetric in Y
- NNLO scale variations are $1\%(3\%)$ at central (large) Y

C. Anastasiou L. Dixon K. Melnikov F. Petriello 2003

NNLO cross sections

- Analytic integration

Hamberg, van Neerven, Matsuura 1990
Anastasiou Dixon Melnikov Petriello 2003

⇑ first method

⇒ flexible enough to include a limited class of acceptance cuts by modelling cuts as ``propagators''

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- Sector decomposition

  Denner Roth 1996; Binoth Heinrich 2000
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  ⇧ flexible enough to include any acceptance cuts

  ⇧ cancellation of divergences is performed numerically

  ⇨ can it handle many final-state partons ?

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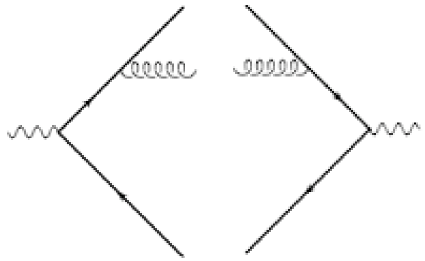
- **Sector decomposition** — Denner Roth 1996; Binoth Heinrich 2000; Anastasiou, Melnikov, Petriello 2004
  - ↑ flexible enough to include any acceptance cuts
  - ↑ cancellation of divergences is performed numerically
  - ⇒ can it handle many final-state partons ?
- **Subtraction**
  - ↑ process independent
  - ⇒ cancellation of divergences is analytic can it be automatised ?

NLO assembly kit

$e^+e^- \rightarrow 3$ jets

leading order $|\mathcal{M}_n^{tree}|^2$



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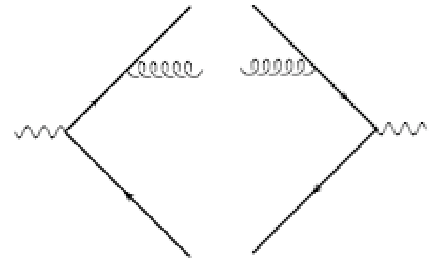
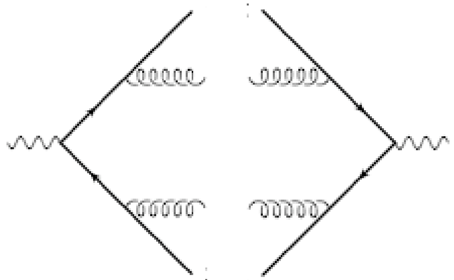
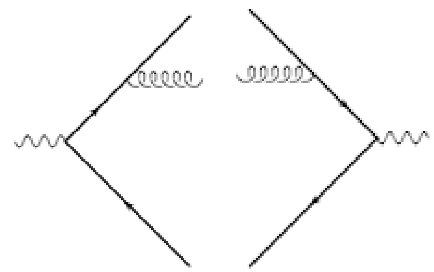


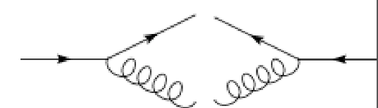
NLO real



IR



$\otimes$



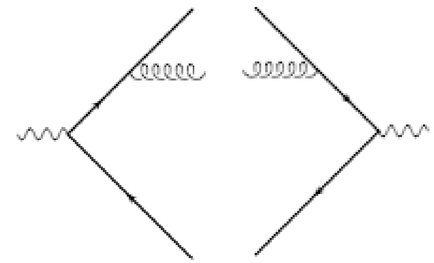
$$|\mathcal{M}_{n+1}^{\text{tree}}|^2 \quad \to \quad |\mathcal{M}_n^{\text{tree}}|^2 \quad \times \quad \int dPS |P_{\text{split}}|^2$$

$$= \quad -\left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon}\right)$$

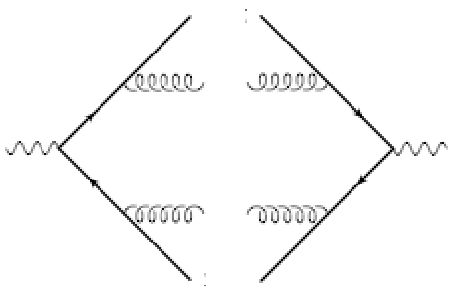
NLO assembly kit

$e^+e^- \to 3$ jets

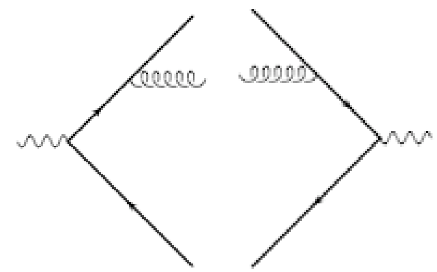
leading order $|\mathcal{M}_n^{tree}|^2$



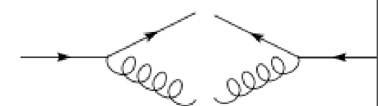
NLO real



IR $\Rightarrow$



$\otimes$



$$|\mathcal{M}_{n+1}^{\text{tree}}|^2 \quad \to \quad |\mathcal{M}_n^{\text{tree}}|^2 \times \int dPS|P_{\text{split}}|^2$$

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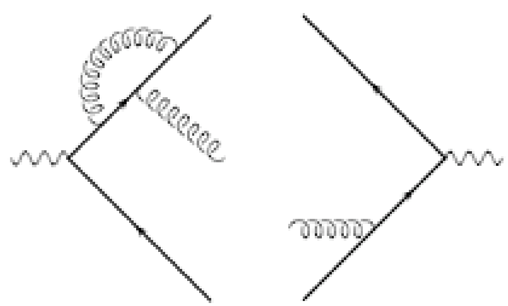
NLO virtual



$d = 4 - 2\epsilon$

$$\int d^dl \; 2(\mathcal{M}_n^{loop})^* \mathcal{M}_n^{tree} \quad = \quad \left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon}\right) |\mathcal{M}_n^{tree}|^2 + fin.$$

NLO production rates

$$\sigma = \sigma^{\rm LO} + \sigma^{\rm NLO} = \int_m d\sigma_m^B \; J_m + \sigma^{\rm NLO}$$

$$\sigma^{\rm NLO} = \int_{m+1} d\sigma_{m+1}^{\rm R} J_{m+1} + \int_m d\sigma_m^{\rm V} J_m$$

the 2 terms on the rhs are divergent in d=4

NLO production rates

Process-independent procedure devised in the 90's

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- slicing — Giele Glover & Kosower
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  - dipole — Catani & Seymour
  - antenna — Kosower; Campbell Cullen & Glover

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the 2 terms on the rhs are divergent in d=4

use universal IR structure to subtract divergences

$$\sigma^{\mathrm{NLO}} = \int_{m+1} \left[d\sigma_{m+1}^{\mathrm{R}} J_{m+1} - d\sigma_{m+1}^{\mathrm{R,A}} J_m \right] + \int_m \left[d\sigma_m^{\mathrm{V}} + \int_1 d\sigma_{m+1}^{\mathrm{R,A}} \right] J_m$$

the 2 terms on the rhs are finite in d=4

Observable (jet) functions

J_m vanishes when one parton becomes soft or collinear to another one

$$J_m(p_1, ..., p_m) \to 0\,, \quad \text{if} \quad p_i \cdot p_j \to 0$$

$d\sigma_m^{\mathrm{B}}$ is integrable over 1-parton IR phase space

J_{m+1} vanishes when two partons become simultaneously soft and/or collinear

$$J_{m+1}(p_1, ..., p_{m+1}) \to 0\,, \quad \text{if} \quad p_i \cdot p_j \text{ and } p_k \cdot p_l \to 0 \quad (i \neq k)$$

R and V are integrable over 2-parton IR phase space

observables are IR safe

$$J_{n+1}(p_1, .., p_j = \lambda q, .., p_{n+1}) \to J_n(p_1, ..., p_{n+1}) \quad \text{if} \quad \lambda \to 0$$

$$J_{n+1}(p_1, .., p_i, .., p_j, .., p_{n+1}) \to J_n(p_1, .., p, .., p_{n+1}) \quad \text{if} \quad p_i \to zp\,,\; p_j \to (1-z)p$$

for all $n \geq m$

NLO IR limits

NLO IR limits

collinear operator

$$\mathrm{C}_{ir}|\mathcal{M}^{(0)}_{m+2}(p_i,p_r,\ldots)|^2 \propto \frac{1}{s_{ir}}\,\langle\mathcal{M}_{m+1}(0)(p_{ir},\ldots)|\hat{P}^{(0)}_{f_i f_r}|\mathcal{M}_{m+1}(0)(p_{ir},\ldots)\rangle$$

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soft operator

$$\mathrm{S}_r|\mathcal{M}^{(0)}_{m+2}(p_r,\ldots)|^2 \propto \frac{s_{ik}}{s_{ir}s_{rk}}\langle\mathcal{M}_{m+1}(0)(\ldots)|T_i\cdot T_k|\mathcal{M}_{m+1}(0)(\ldots)\rangle$$

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counterterm

$$\sum_r\left(\sum_{i\neq r}\frac{1}{2}\mathrm{C}_{ir}+\mathrm{S}_r\right)|\mathcal{M}^{(0)}_{m+2}(p_i,p_r,\ldots)|^2$$

performs double subtraction in overlapping regions

NLO overlapping divergences

$\mathrm{C}_{ir}\mathrm{S}_r$ can be used to cancel double subtraction

$$\mathrm{C}_{ir}\left(\mathrm{S}_r - \mathrm{C}_{ir}\mathrm{S}_r\right)|\mathcal{M}_{m+2}^{(0)}|^2 = 0$$

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the NLO counterterm

$$\mathrm{A}_1|\mathcal{M}^{(0)}_{m+2}|^2 = \sum_r \left[\sum_{i\neq r}\frac{1}{2}\mathrm{C}_{ir} + \left(\mathrm{S}_r - \sum_{i\neq r}\mathrm{C}_{ir}\mathrm{S}_r\right)\right]|\mathcal{M}^{(0)}_{m+2}(p_i,p_r,\ldots)|^2$$

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- has the same singular behaviour as SME, and is free of double subtractions

$$\mathrm{C}_{ir}\left(1 - \mathrm{A}_1\right)|\mathcal{M}_{m+1}^{(0)}|^2 = 0 \qquad \mathrm{S}_r\left(1 - \mathrm{A}_1\right)|\mathcal{M}_{m+1}^{(0)}|^2 = 0$$

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- contains spurious singularities when parton $s \neq r$ becomes unresolved, but they are screened by J_m

Collinear mapping

$$\tilde{p}^\mu_{ir} = \frac{1}{1-\alpha_{ir}}\left(p^\mu_i + p^\mu_r - \alpha_{ir} Q^\mu\right), \qquad \tilde{p}^\mu_n = \frac{1}{1-\alpha_{ir}} p^\mu_n, \qquad n \neq i, r$$

$$\alpha_{ir} = \frac{1}{2}\left[y_{(ir)Q} - \sqrt{y^2_{(ir)Q} - 4y_{ir}}\right] \qquad y_{ir} = \frac{2p_i \cdot p_r}{Q^2}$$

momentum is conserved $\quad \tilde{p}^\mu_{ir} + \sum_n \tilde{p}^\mu_n = p^\mu_i + p^\mu_r + \sum_n p^\mu_n$

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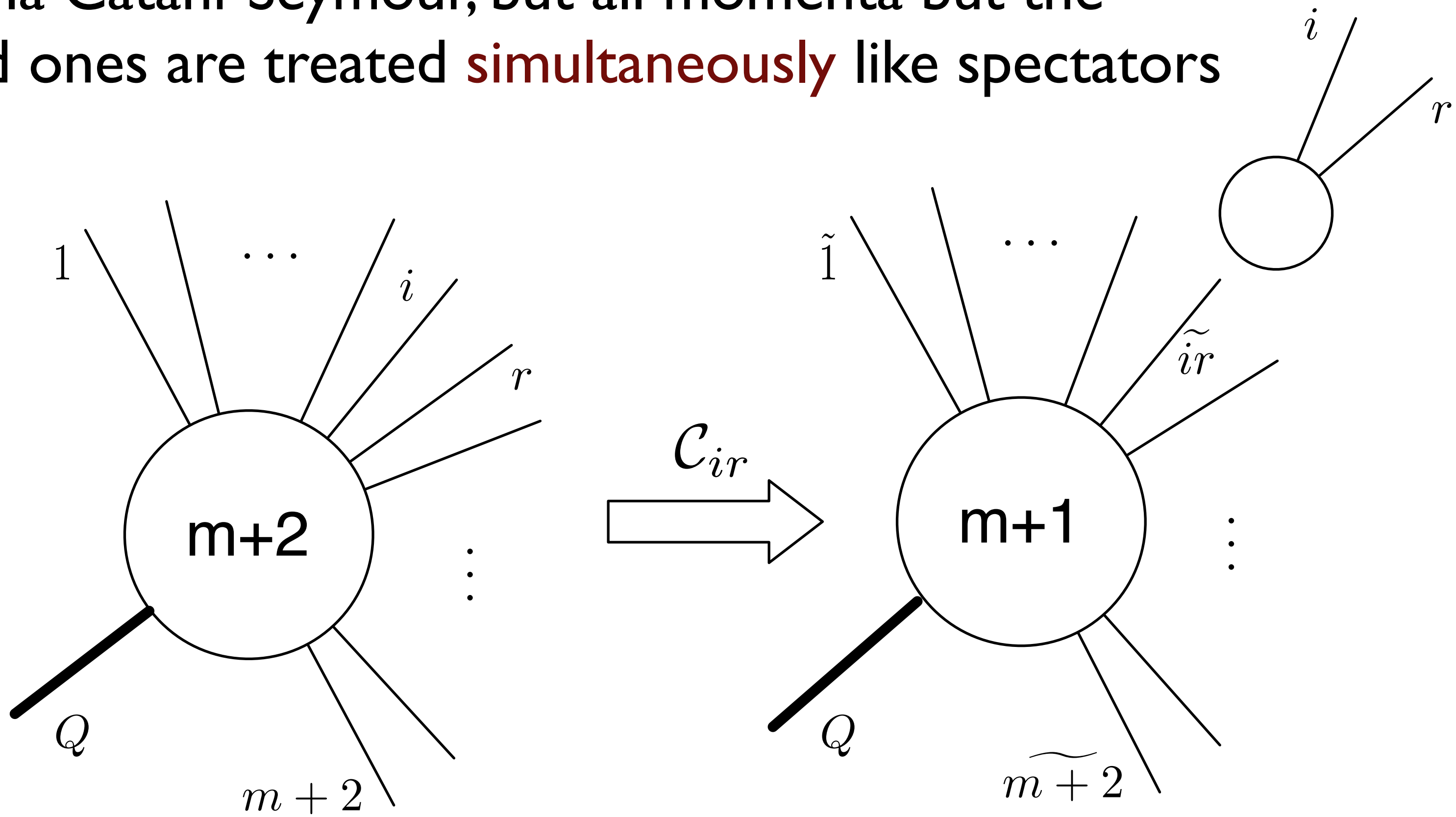
momentum is conserved $\tilde{p}^{\mu}_{ir} + \sum_n \tilde{p}^{\mu}_{n} = p^{\mu}_{i} + p^{\mu}_{r} + \sum_n p^{\mu}_{n}$

mapping à la Catani-Seymour, but all momenta but the unresolved ones are treated simultaneously like spectators



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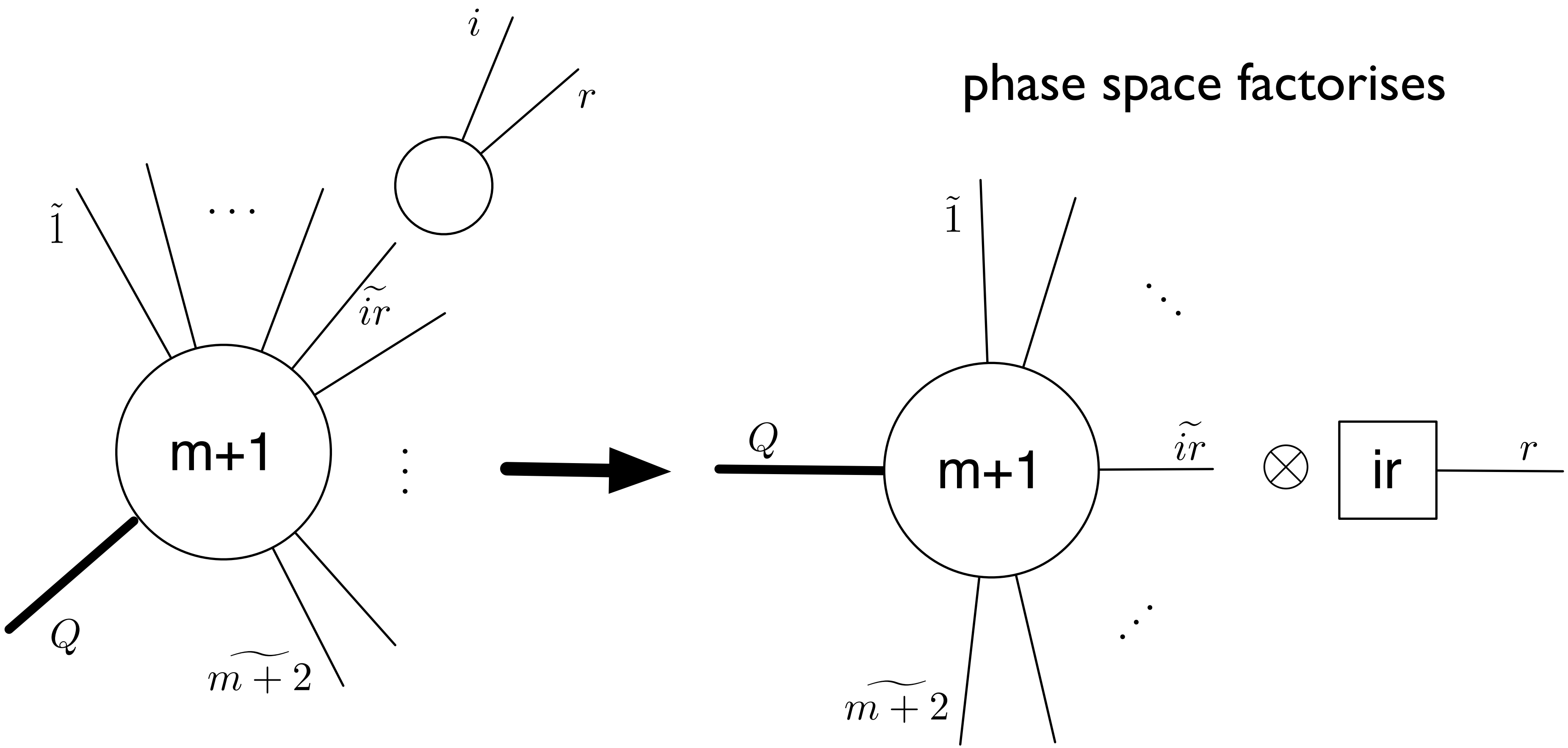
phase space factorises



Soft mapping

$$\tilde{p}_n^\mu = \Lambda_\nu^\mu[Q, (Q - p_r)/\lambda_r](p_n^\nu/\lambda_r)\,, \qquad n \neq r$$

$$\lambda_r = \sqrt{1 - y_{rQ}}$$

$$\Lambda_\nu^\mu[K, \widetilde{K}\,] = g_\nu^\mu - \frac{2(K + \widetilde{K}\,)^\mu (K + \widetilde{K}\,)_\nu}{(K + \widetilde{K}\,)^2} + \frac{2K^\mu \widetilde{K}_\nu}{K^2}$$

Lorentz transformation that preserves total momentum

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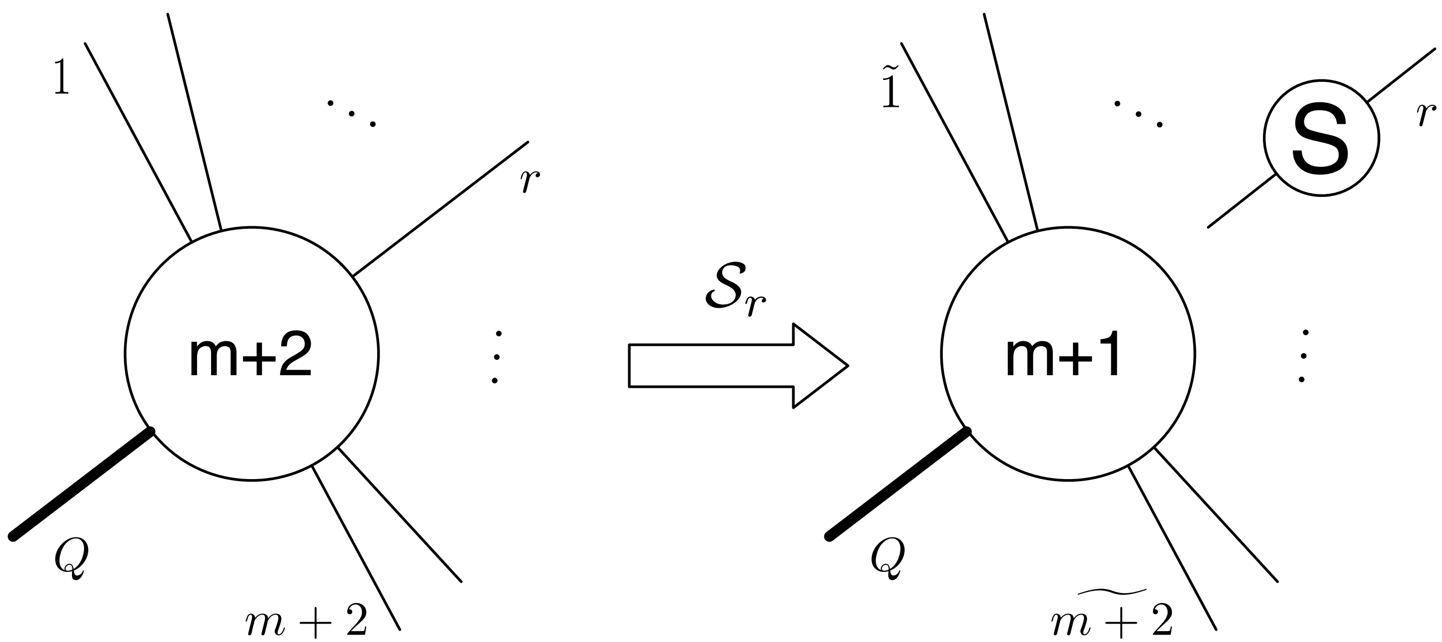

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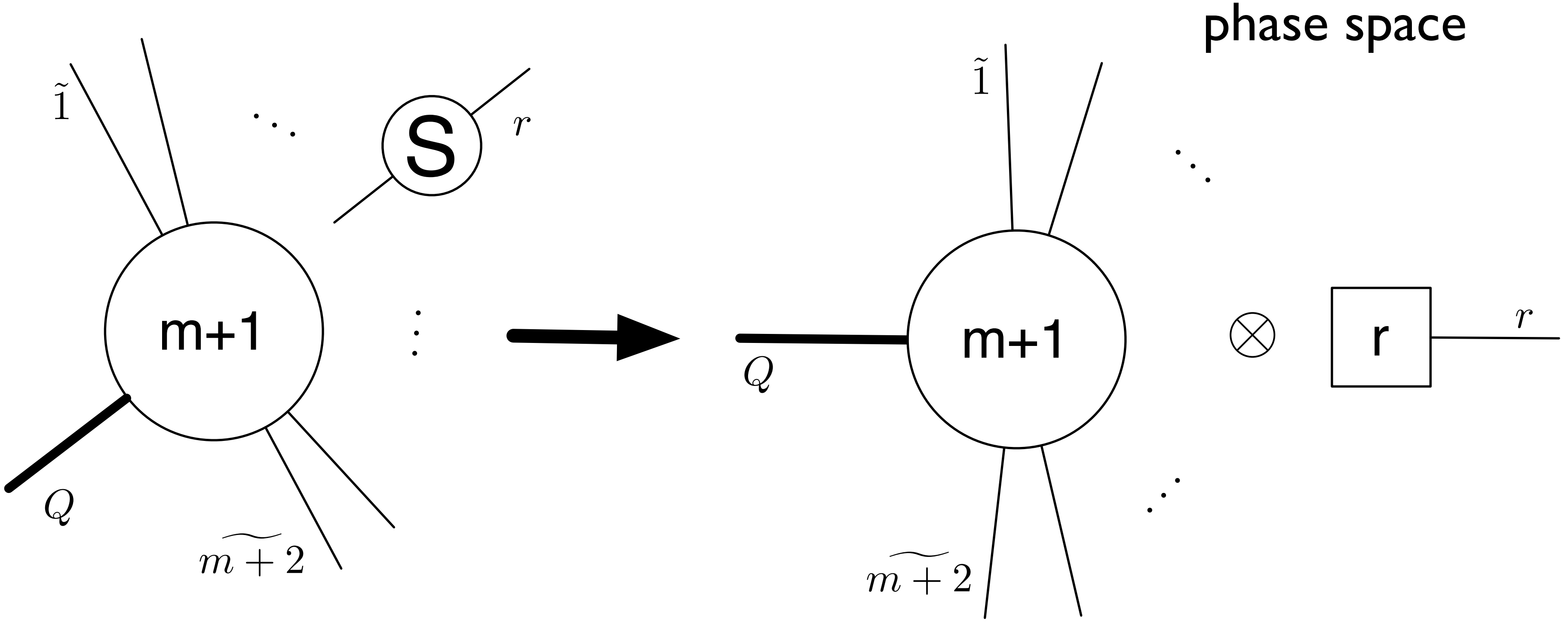

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$$\mathrm{d}\sigma_{m+2}^{\mathrm{R},\mathrm{A}_1} = \mathrm{d}\phi_{m+1}\,[\mathrm{d}p_1]\,\boldsymbol{\mathcal{A}}_1|\mathcal{M}_{m+2}^{(0)}|^2$$

$$\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{R},\mathrm{A}_1} = \mathrm{d}\phi_{m+1}|\mathcal{M}_{m+1}^{(0)}|^2 \otimes \boldsymbol{I}(m+1,\varepsilon)$$

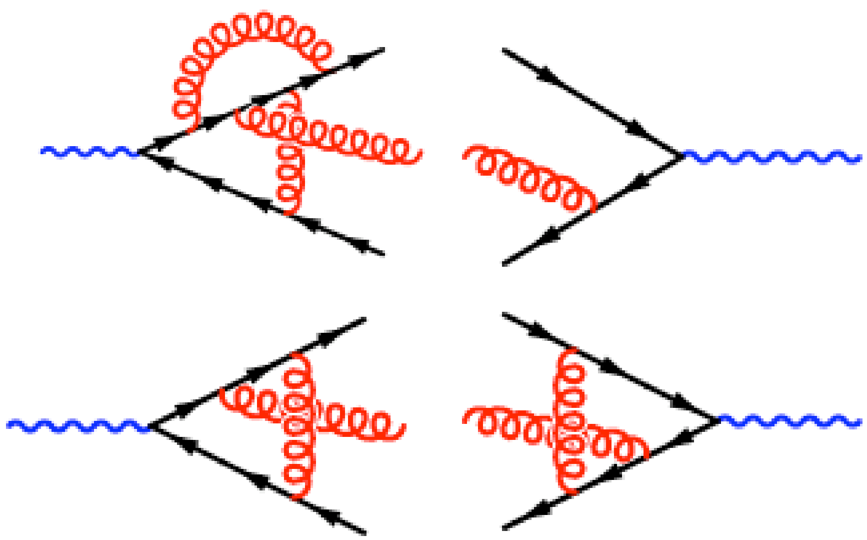
NNLO assembly kit

$e^+e^- \rightarrow 3 \text{ jets}$

NNLO assembly kit

$e^+e^- \to 3$ jets

double virtual



NNLO assembly kit

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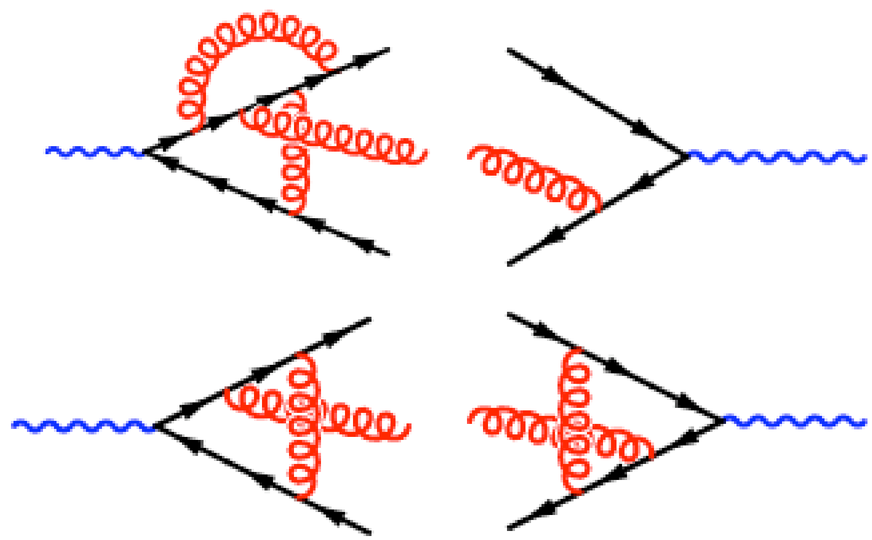
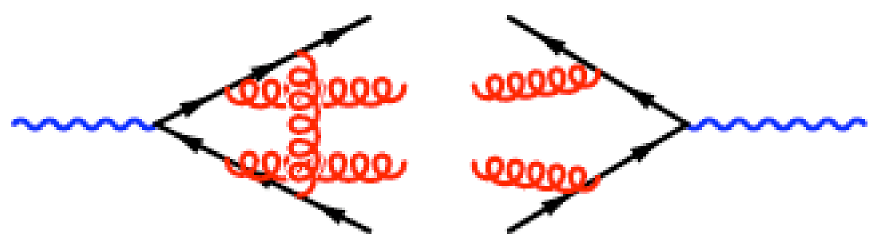


real-virtual



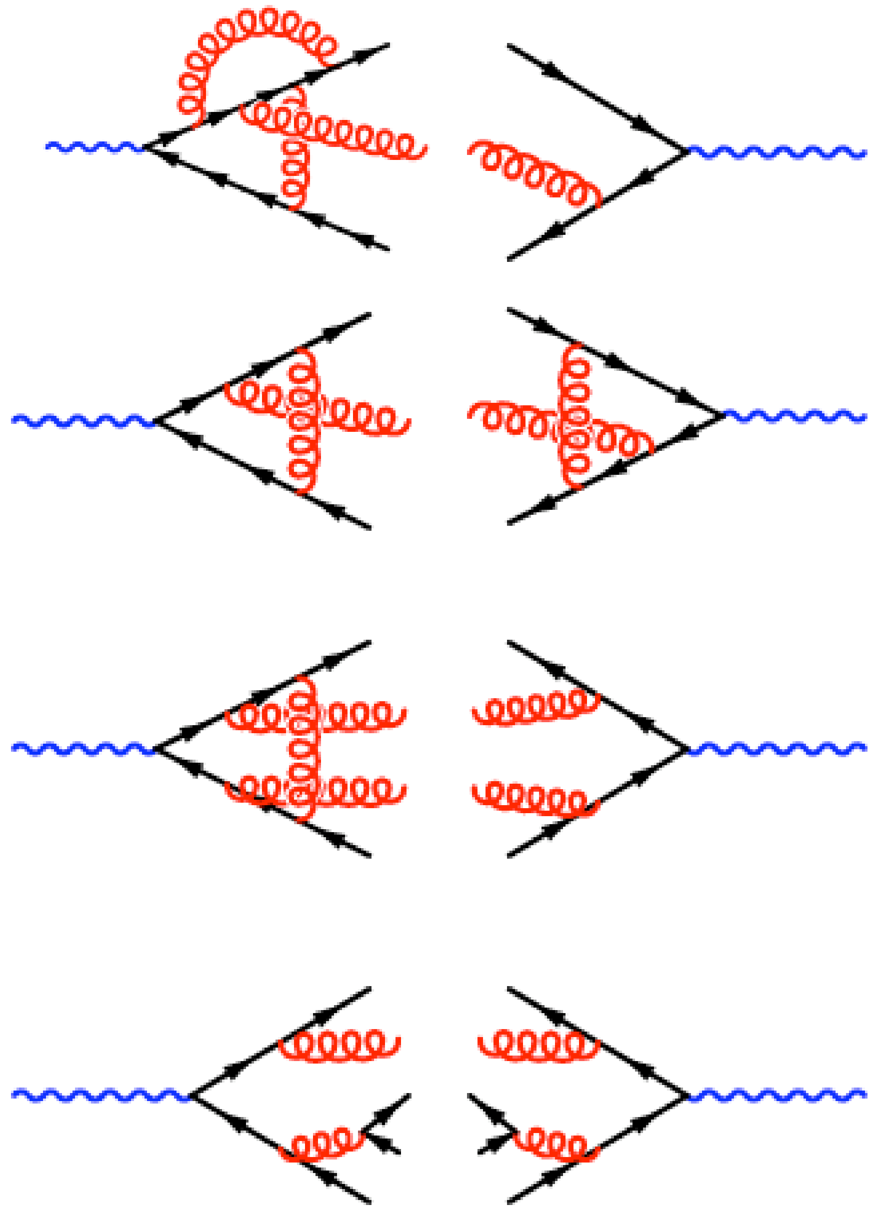
NNLO assembly kit

$e^+e^- \to 3$ jets

double virtual

real-virtual

double real



Two-loop matrix elements

Two-loop matrix elements

- two-jet production $qq' \to qq',\ q\bar{q} \to q\bar{q},\ q\bar{q} \to gg,\ gg \to gg$

  C. Anastasiou N. Glover C. Oleari M. Tejeda-Yeomans 2000-01

  Z. Bern A. De Freitas L. Dixon 2002

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- photon-pair production $q\bar{q} \to \gamma\gamma, \; gg \to \gamma\gamma$

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  L. Garland T. Gehrmann N. Glover A. Koukoutsakis E. Remiddi 2002

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  C. Anastasiou N. Glover M. Tejeda-Yeomans 2002
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  L. Garland T. Gehrmann N. Glover A. Koukoutsakis E. Remiddi 2002

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  T. Gehrmann E. Remiddi 2002

Two-loop matrix elements

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  C. Anastasiou N. Glover M. Tejeda-Yeomans 2002
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  R. Hamberg W. van Neerven T. Matsuura 1991

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  R. Hamberg W. van Neerven T. Matsuura 1991

- Higgs production $gg \to H$ (in the $m_t \to \infty$ limit)

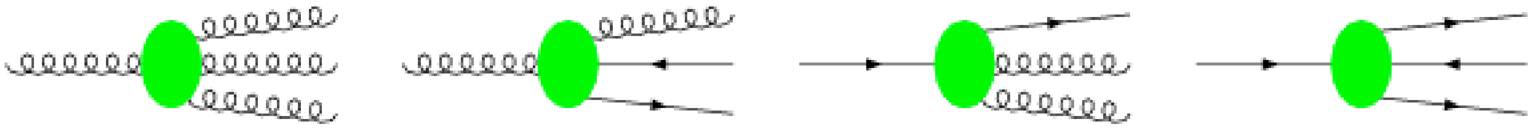
  R. Harlander 2001

Collinear and soft currents

- universal IR structure → process-independent procedure

Collinear and soft currents

- universal IR structure ⇒ process-independent procedure
- universal collinear and soft currents
  - 3-parton tree splitting functions



J. Campbell N. Glover 1997; S. Catani M. Grazzini 1998; A. Frizzo F. Maltoni VDD 1999; D. Kosower 2002

Collinear and soft currents

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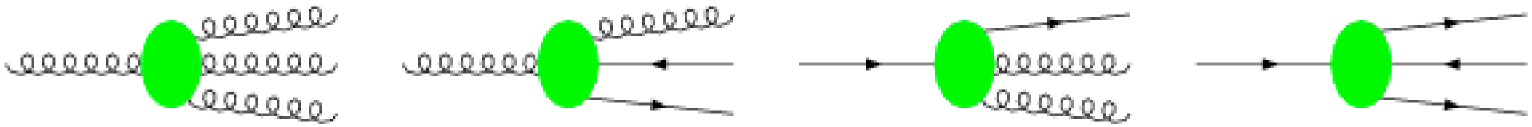
- universal collinear and soft currents
  - 3-parton tree splitting functions



J. Campbell N. Glover 1997; S. Catani M. Grazzini 1998; A. Frizzo F. Maltoni VDD 1999; D. Kosower 2002

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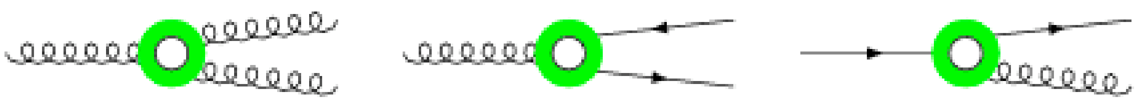
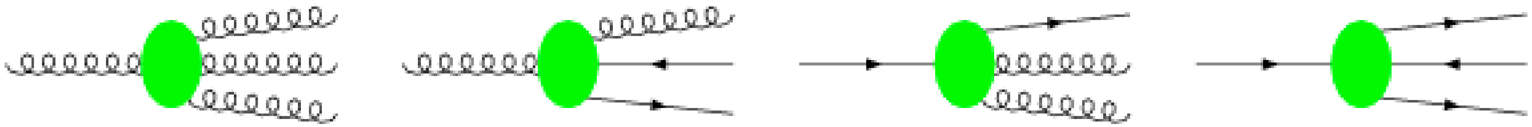


Z. Bern L. Dixon D. Dunbar D. Kosower 1994; Z. Bern W. Kilgore C. Schmidt VDD 1998-99; D. Kosower P. Uwer 1999; S. Catani M. Grazzini 1999; D. Kosower 2003

Collinear and soft currents

- universal IR structure ⟹ process-independent procedure
- universal collinear and soft currents
  - 3-parton tree splitting functions



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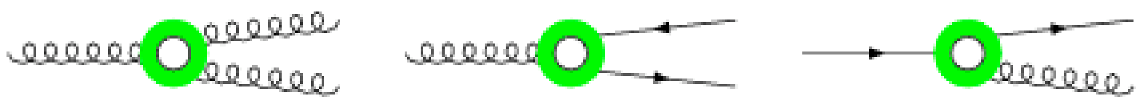

Z. Bern L. Dixon D. Dunbar D. Kosower 1994; Z. Bern W. Kilgore C. Schmidt VDD 1998-99; D. Kosower P. Uwer 1999; S. Catani M. Grazzini 1999; D. Kosower 2003

- universal subtraction counterterms
  - several ideas and works in progress

D. Kosower; S. Weinzierl; A. De Ridder, T. Gehrmann, G. Heinrich 2003
S. Frixione M. Grazzini 2004; G. Somogyi Z. Trocsanyi VDD 2005

  - but devised only for $e^+e^- \to 3$ jets

A. De Ridder, T. Gehrmann, N. Glover 2005; G. Somogyi Z. Trocsanyi VDD 2006

NNLO subtraction

$$\sigma^{\rm NNLO} = \int_{m+2} d\sigma^{\rm RR}_{m+2} J_{m+2} + \int_{m+1} d\sigma^{\rm RV}_{m+1} J_{m+1} + \int_{m} d\sigma^{\rm VV}_{m} J_{m}$$

the 3 terms on the rhs are divergent in d=4

use universal IR structure to subtract divergences

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use universal IR structure to subtract divergences

$$\sigma^{\rm NNLO} = \int_{m+2} \left[d\sigma^{\rm RR}_{m+2} J_{m+2} - d\sigma^{\rm RR,A_2}_{m+2} J_{m} \right]$$

takes care of doubly-unresolved regions,
but still divergent in singly-unresolved ones

NNLO subtraction

$$\sigma^{\text{NNLO}} = \int_{m+2} d\sigma^{\text{RR}}_{m+2} J_{m+2} + \int_{m+1} d\sigma^{\text{RV}}_{m+1} J_{m+1} + \int_m d\sigma^{\text{VV}}_m J_m$$

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$$+ \int_{m+1} \left[d\sigma^{\text{RV}}_{m+1} J_{m+1} - d\sigma^{\text{RV,A}_1}_{m+1} J_m \right]$$

still contains $1/\epsilon$ poles in regions away from 1-parton IR regions

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$$\sigma^{\rm NNLO} = \int_{m+2} d\sigma^{\rm RR}_{m+2} J_{m+2} + \int_{m+1} d\sigma^{\rm RV}_{m+1} J_{m+1} + \int_m d\sigma^{\rm VV}_m J_m$$

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$$+ \int_{m+1} \left[d\sigma^{\rm RV}_{m+1} J_{m+1} - d\sigma^{\rm RV,A_1}_{m+1} J_m \right]$$

still contains $1/\epsilon$ poles in regions away from 1-parton IR regions

$$+ \int_m \left[d\sigma^{\rm VV}_m + \int_2 d\sigma^{\rm RR,A_2}_{m+2} + \int_1 d\sigma^{\rm RV,A_1}_{m+1} \right] J_m$$

2-step procedure

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- construct subtraction terms that regularise the singularities of the SME in all unresolved parts of the phase space, avoiding multiple subtractions

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2-step procedure

- construct subtraction terms that regularise the singularities of the SME in all unresolved parts of the phase space, avoiding multiple subtractions

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- perform momentum mappings, such that the phase space factorises exactly over the unresolved momenta and such that it respects the structure of the cancellations among the subtraction terms

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A_2 counterterm

- construct the 2-unresolved-parton counterterm using the IR currents

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$$
\begin{aligned}
\mathbf{A}_2|\mathcal{M}_{m+2}^{(0)}|^2 = \sum_r \sum_{s\neq r} \Bigg\{ \sum_{i\neq r,s} \Bigg[& \frac{1}{6}\mathbf{C}_{irs} + \sum_{j\neq i,r,s} \frac{1}{8}\mathbf{C}_{ir;js} + \frac{1}{2}\mathbf{S}_{rs} \\
& + \frac{1}{2}\Bigg(\mathbf{CS}_{ir;s} - \mathbf{C}_{irs}\mathbf{CS}_{ir;s} - \sum_{j\neq i,r,s} \mathbf{C}_{ir;js}\mathbf{CS}_{ir;s} \Bigg) \Bigg] \\
& - \sum_{i\neq r,s} \Bigg[\mathbf{CS}_{ir;s}\mathbf{S}_{rs} + \mathbf{C}_{irs} \left(\frac{1}{2}\mathbf{S}_{rs} - \mathbf{CS}_{ir;s}\mathbf{S}_{rs} \right) \\
& + \sum_{j\neq i,r,s} \mathbf{C}_{ir;js} \left(\frac{1}{2}\mathbf{S}_{rs} - \mathbf{CS}_{ir;s}\mathbf{S}_{rs} \right) \Bigg] \Bigg\} |\mathcal{M}_{m+2}^{(0)}|^2
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A_2 counterterm

- construct the 2-unresolved-parton counterterm using the IR currents

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$$+\frac{1}{2}\Bigg(\mathbf{CS}_{ir;s} - \mathbf{C}_{irs}\mathbf{CS}_{ir;s} - \sum_{j\neq i,r,s} \mathbf{C}_{ir;js}\mathbf{CS}_{ir;s} \Bigg) \Bigg]$$

$$- \sum_{i\neq r,s} \Bigg[\mathbf{CS}_{ir;s}\mathbf{S}_{rs} + \mathbf{C}_{irs}\Bigg(\frac{1}{2}\mathbf{S}_{rs} - \mathbf{CS}_{ir;s}\mathbf{S}_{rs} \Bigg)$$

$$+ \sum_{j\neq i,r,s} \mathbf{C}_{ir;js}\Bigg(\frac{1}{2}\mathbf{S}_{rs} - \mathbf{CS}_{ir;s}\mathbf{S}_{rs} \Bigg) \Bigg] \Bigg\} |\mathcal{M}^{(0)}_{m+2}|^2$$

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performing double and triple subtractions in overlapping regions

$$\mathrm{C}_{irs}\,(1-\mathrm{A}_2)\,|\mathcal{M}^{(0)}_{m+2}|^2 = 0 \qquad \mathrm{S}_{rs}\,(1-\mathrm{A}_2)\,|\mathcal{M}^{(0)}_{m+2}|^2 = 0$$

$$\mathrm{C}_{ir;js}\,(1-\mathrm{A}_2)\,|\mathcal{M}^{(0)}_{m+2}|^2 = 0 \qquad \mathrm{CS}_{ir;s}\,(1-\mathrm{A}_2)\,|\mathcal{M}^{(0)}_{m+2}|^2 = 0$$

Triple-collinear mapping

$$\tilde{p}^{\mu}_{irs} = \frac{1}{1-\alpha_{irs}}\left(p^{\mu}_i + p^{\mu}_r + p^{\mu}_s - \alpha_{irs} Q^{\mu}\right), \qquad \tilde{p}^{\mu}_n = \frac{1}{1-\alpha_{irs}} p^{\mu}_n, \qquad n \neq i, r, s$$

$$\alpha_{irs} = \frac{1}{2}\left[y_{(irs)Q} - \sqrt{y^2_{(irs)Q} - 4 y_{irs}}\,\right]$$

momentum conservation $\tilde{p}^{\mu}_{irs} + \sum_n \tilde{p}^{\mu}_n = p^{\mu}_i + p^{\mu}_r + p^{\mu}_s + \sum_n p^{\mu}_n$

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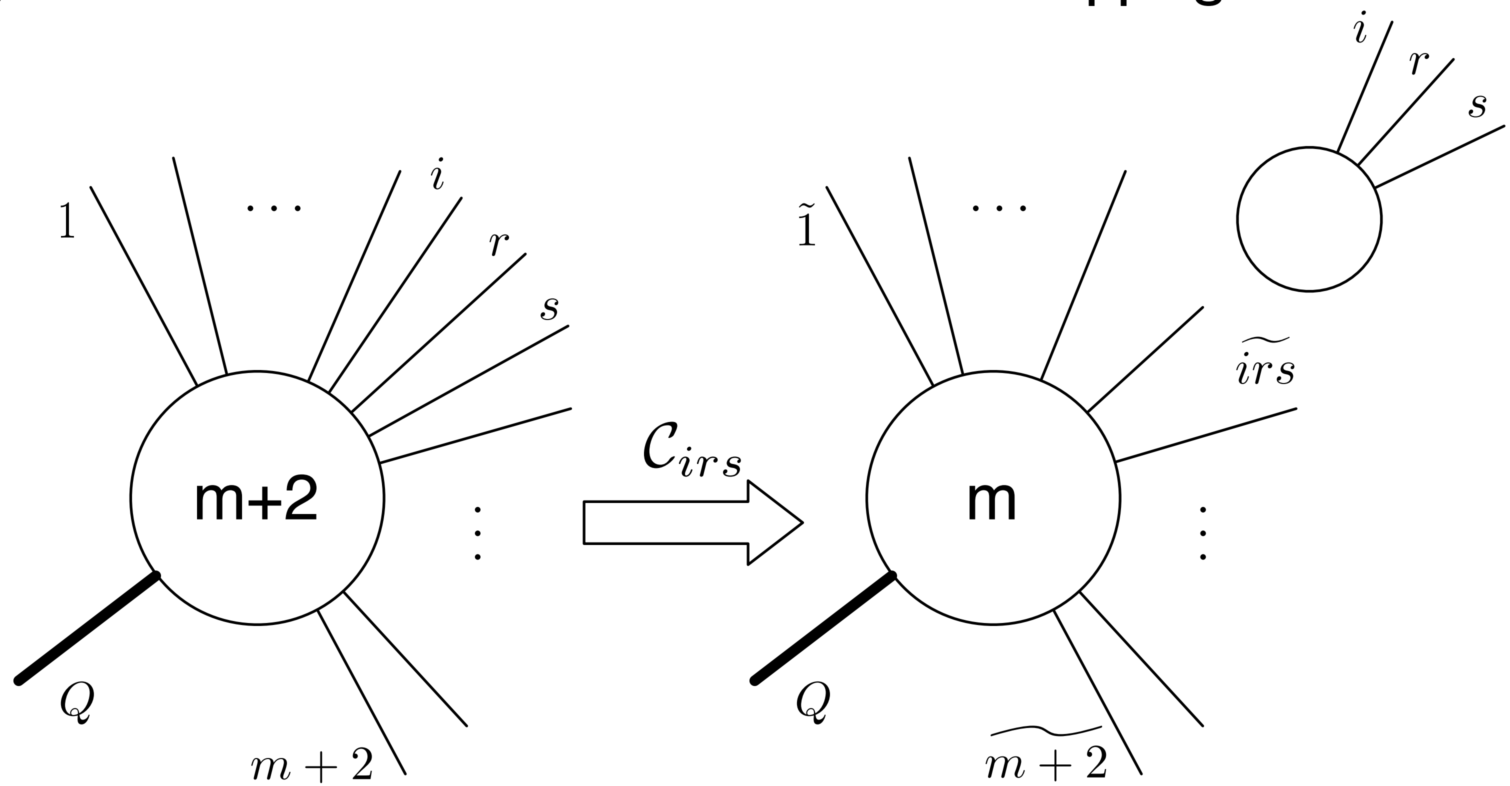
straightforward extension of NLO collinear mapping



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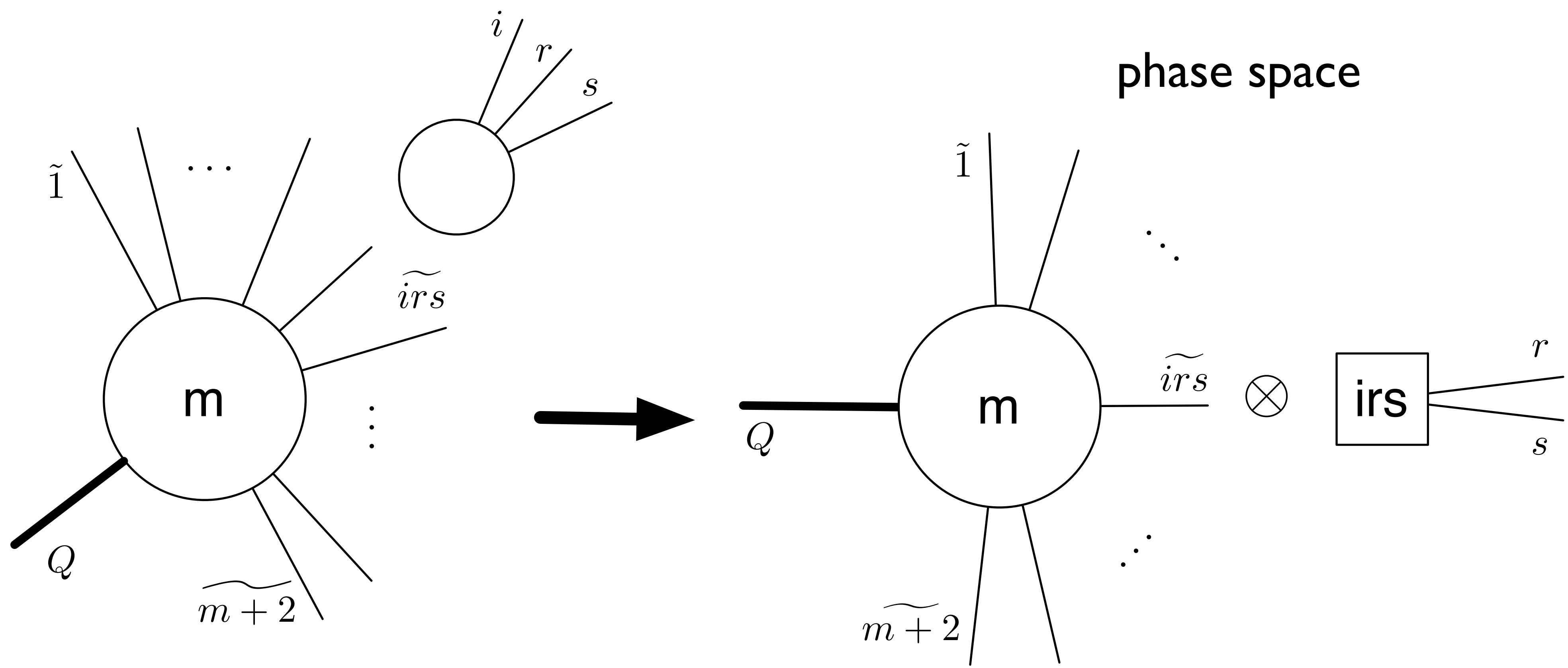
phase space



Double-soft mapping

$$\tilde{p}_n^\mu = \Lambda_\nu^\mu[Q, (Q - p_r - p_s)/\lambda_{rs}](p_n^\nu/\lambda_{rs})\,, \qquad n \neq r, s$$

$$\lambda_{rs} = \sqrt{1 - \left(y_{(rs)Q} - y_{rs}\right)}$$

straightforward extension of NLO soft mapping

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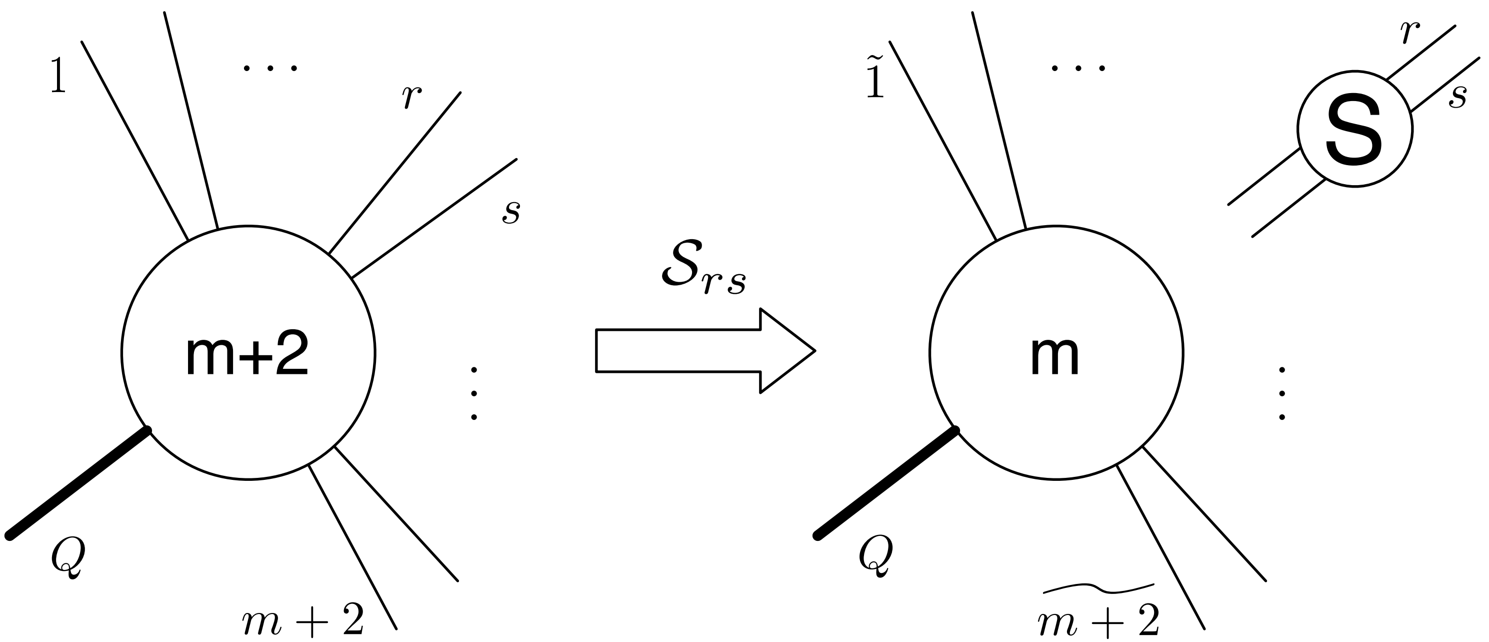

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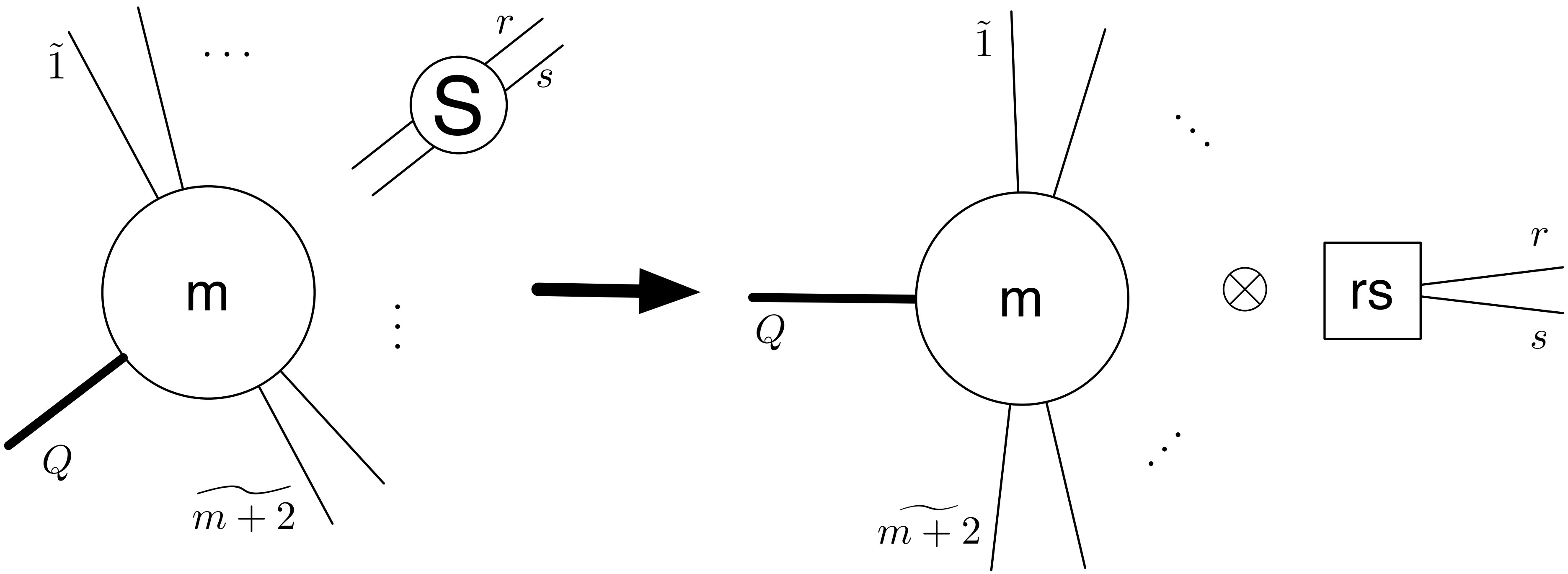

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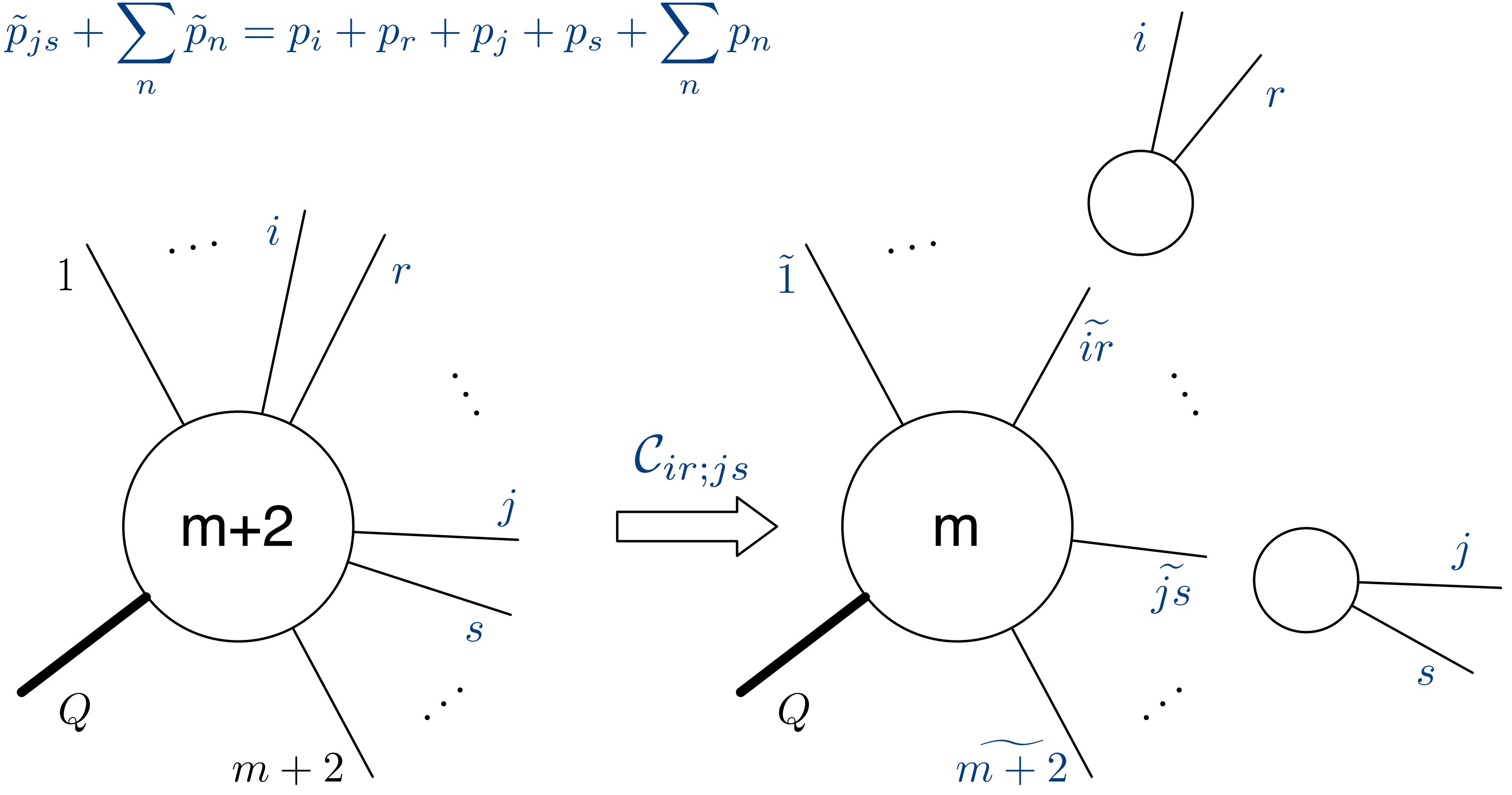
$$\tilde{p}_{ir} + \tilde{p}_{js} + \sum_n \tilde{p}_n = p_i + p_r + p_j + p_s + \sum_n p_n$$



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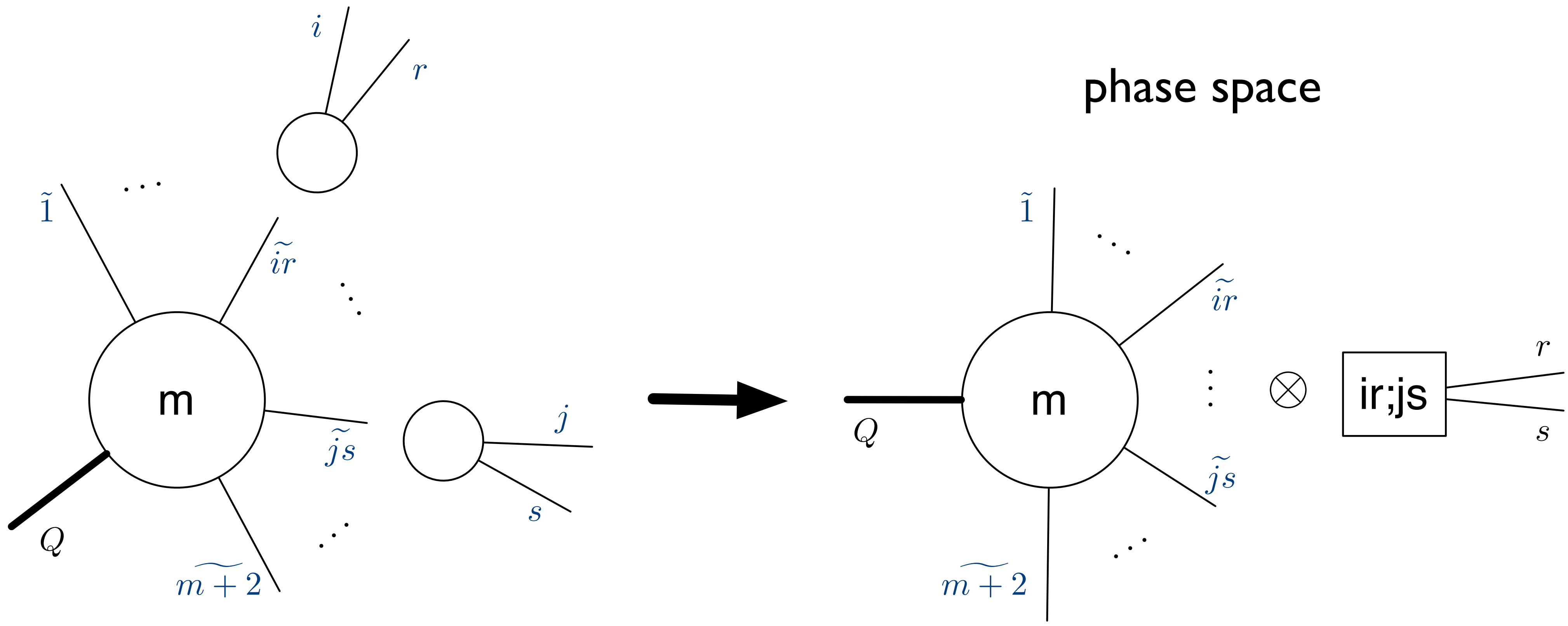

Soft-collinear mapping

composition of a collinear and a soft mapping

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in this case, the order of the mappings is irrelevant

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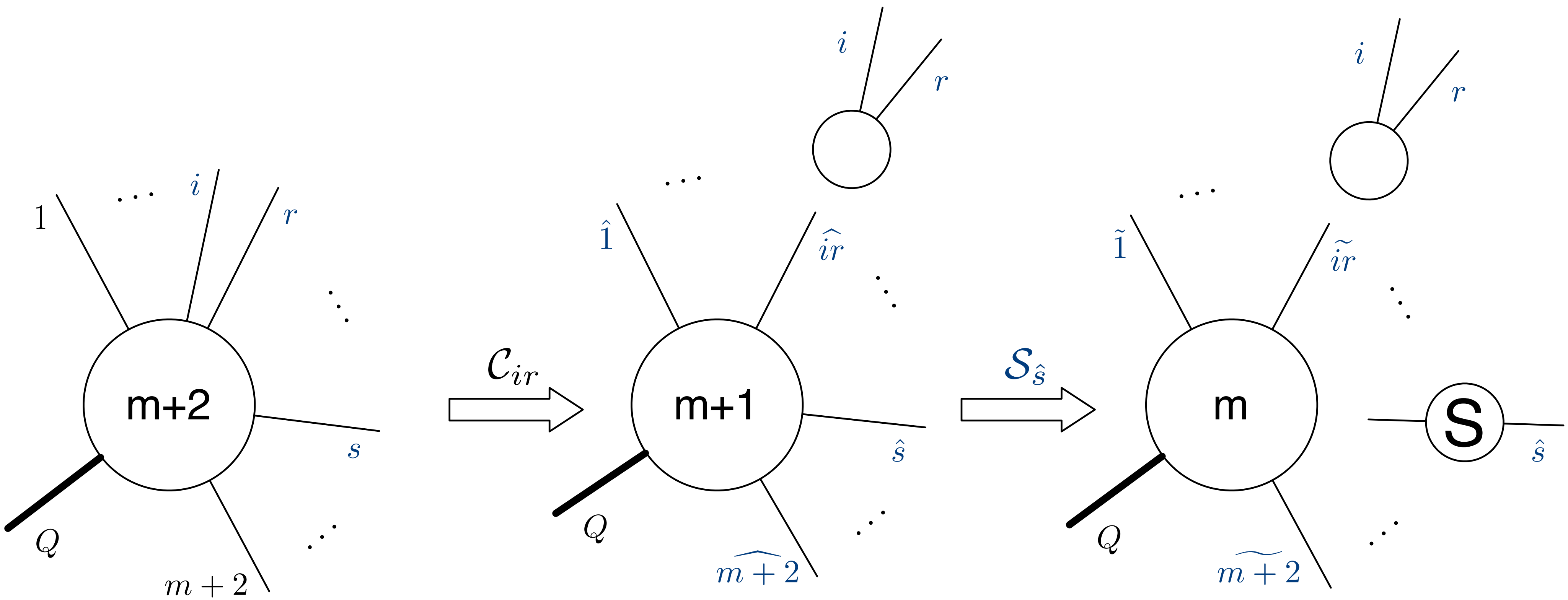

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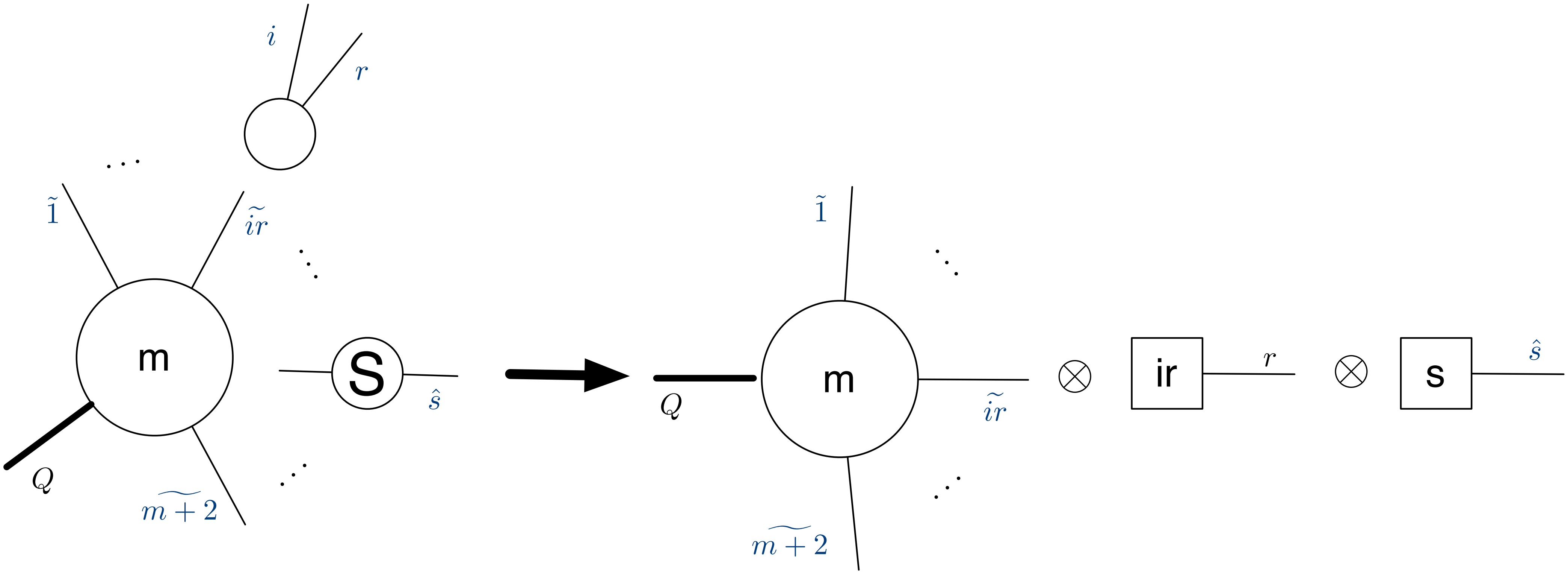

RR counterterm

needs a NLO-type subtraction
between the m+2- and the m+1-parton contributions

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$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{NNLO}}_{\{m+2\}} + \sigma^{\mathrm{NNLO}}_{\{m+1\}} + \sigma^{\mathrm{NNLO}}_{\{m\}}$$

$$\sigma^{\mathrm{NNLO}}_{\{m+2\}} = \int_{m+2} \Big[\mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} J_m - \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} J_{m+1} + \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} J_m \Big]_{d=4}$$

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A_1 takes care of the singly-unresolved regions and A_{12} of the over-subtracting

RR counterterm

needs a NLO-type subtraction
between the m+2- and the m+1-parton contributions

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{NNLO}}_{\{m+2\}} + \sigma^{\mathrm{NNLO}}_{\{m+1\}} + \sigma^{\mathrm{NNLO}}_{\{m\}}$$

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must be finite in the doubly-unresolved regions ⇒

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$$\sigma^{\rm NNLO} = \sigma^{\rm NNLO}_{\{m+2\}} + \sigma^{\rm NNLO}_{\{m+1\}} + \sigma^{\rm NNLO}_{\{m\}}$$

$$\sigma^{\rm NNLO}_{\{m+2\}} = \int_{m+2} \Big[{\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m$$

must be finite in
the doubly-unresolved regions

$$- {\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} + {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \Big]_{d=4}$$

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A_1 takes care of the singly-unresolved regions and A_{12} of the over-subtracting

RR counterterm = $A_2 + A_1 - A_{12}$

$$ {\rm d}\sigma^{\rm RR,A_2}_{m+2} = {\rm d}\phi_m \, [{\rm d}p_2] \, \boldsymbol{\mathcal{A}}_2 |\mathcal{M}^{(0)}_{m+2}|^2$$

$$ {\rm d}\sigma^{\rm RR,A_1}_{m+2} = {\rm d}\phi_{m+1} \, [{\rm d}p_1] \, \boldsymbol{\mathcal{A}}_1 |\mathcal{M}^{(0)}_{m+2}|^2$$

$$ {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} = {\rm d}\phi_m \, [{\rm d}p_1] \, [{\rm d}p_1] \, \boldsymbol{\mathcal{A}}_{12} |\mathcal{M}^{(0)}_{m+2}|^2$$

need to construct A_{12} such that all overlapping regions in
1-parton and 2-parton IR phase space regions are counted only once

$$\mathbf{C}_{ir}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{C}_{ir}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{S}_{r}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{S}_{r}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{C}_{irs}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{C}_{irs}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{C}_{ir;js}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{C}_{ir;js}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{CS}_{ir;s}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{CS}_{ir;s}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{S}_{rs}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{S}_{rs}|\mathcal{M}^{(0)}_{m+2}|^2$$

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$$\mathbf{S}_{r}(\mathbf{A}_1+\mathbf{A}_2-\mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2=\mathbf{S}_{r}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{C}_{irs}(\mathbf{A}_1+\mathbf{A}_2-\mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2=\mathbf{C}_{irs}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{C}_{ir;js}(\mathbf{A}_1+\mathbf{A}_2-\mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2=\mathbf{C}_{ir;js}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{CS}_{ir;s}(\mathbf{A}_1+\mathbf{A}_2-\mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2=\mathbf{CS}_{ir;s}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{S}_{rs}(\mathbf{A}_1+\mathbf{A}_2-\mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2=\mathbf{S}_{rs}|\mathcal{M}^{(0)}_{m+2}|^2$$

the definition of A_{12} is rather simple

$$\mathbf{A}_{12}|\mathcal{M}^{(0)}_{m+2}|^2\equiv\mathbf{A}_1\mathbf{A}_2|\mathcal{M}^{(0)}_{m+2}|^2$$

need to construct A_{12} such that all overlapping regions in 1-parton and 2-parton IR phase space regions are counted only once

$$\mathbf{C}_{ir}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{C}_{ir}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{S}_{r}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{S}_{r}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{C}_{irs}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{C}_{irs}|\mathcal{M}^{(0)}_{m+2}|^2$$

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the definition of A_{12} is rather simple

$$\mathbf{A}_{12}|\mathcal{M}^{(0)}_{m+2}|^2 \equiv \mathbf{A}_1\mathbf{A}_2|\mathcal{M}^{(0)}_{m+2}|^2$$

but showing that it has the right properties is non trivial, and requires considering iterated singly-unresolved limits and strongly-ordered doubly-unresolved limits

Iterated counterterms

$$\boldsymbol{\mathcal{A}}_{12}|\mathcal{M}^{(0)}_{m+2}(\{p\})|^2 = \sum_t \Bigg[\sum_{k\neq t} \frac{1}{2} \mathcal{C}_{kt} \boldsymbol{\mathcal{A}}_2 |\mathcal{M}^{(0)}_{m+2}(\{p\})|^2 + \left(\mathcal{S}_t \boldsymbol{\mathcal{A}}_2 |\mathcal{M}^{(0)}_{m+2}(\{p\})|^2 - \sum_{k\neq t} \mathcal{C}_{kt} \mathcal{S}_t \boldsymbol{\mathcal{A}}_2 |\mathcal{M}^{(0)}_{m+2}(\{p\})|^2 \right) \Bigg]$$

Iterated counterterms

$$\begin{aligned}\boldsymbol{\mathcal{A}}_{12}|\mathcal{M}^{(0)}_{m+2}(\{p\})|^2 &= \sum_t \Bigg[\sum_{k\neq t}\frac{1}{2}\,\mathcal{C}_{kt}\boldsymbol{\mathcal{A}}_2|\mathcal{M}^{(0)}_{m+2}(\{p\})|^2 \\ &\quad + \left(\mathcal{S}_t\boldsymbol{\mathcal{A}}_2|\mathcal{M}^{(0)}_{m+2}(\{p\})|^2 - \sum_{k\neq t}\mathcal{C}_{kt}\mathcal{S}_t\boldsymbol{\mathcal{A}}_2|\mathcal{M}^{(0)}_{m+2}(\{p\})|^2\right)\Bigg]\end{aligned}$$

where

$$\begin{aligned}\mathcal{C}_{kt}\boldsymbol{\mathcal{A}}_2 &= \sum_{r\neq k,t}\Bigg[\mathcal{C}_{kt}\mathcal{C}_{ktr} + \mathcal{C}_{kt}\mathcal{CS}_{kt;r} - \mathcal{C}_{kt}\mathcal{C}_{ktr}\mathcal{CS}_{kt;r} - \mathcal{C}_{kt}\mathcal{C}_{rkt}\mathcal{S}_{kt} \\ &\quad + \sum_{i\neq r,k,t}\left(\frac{1}{2}\,\mathcal{C}_{kt}\mathcal{C}_{ir;kt} - \mathcal{C}_{kt}\mathcal{C}_{ir;kt}\mathcal{CS}_{kt;r}\right)\Bigg] + \mathcal{C}_{kt}\mathcal{S}_{kt}\end{aligned}$$

and likewise for $\mathcal{S}_t\boldsymbol{\mathcal{A}}_2\,,\ \mathcal{C}_{kt}\mathcal{S}_t\boldsymbol{\mathcal{A}}_2$

Iterated counterterms

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- the momentum mapping for each of the iterated counterterms is built out of a composition of either the NLO collinear or the NLO soft mappings, or of both

- the treatment of colour in iterated singly-unresolved limits differs for spin-correlated SME from that of colour-correlated SME

  → no soft factorization formulae for simultaneously colour-correlated and spin-correlated SME.
  This was a no-go in the direction of generalised dipole-type counterterms

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{NNLO}}_{\{m+2\}} + \sigma^{\mathrm{NNLO}}_{\{m+1\}} + \sigma^{\mathrm{NNLO}}_{\{m\}}$$

$$\sigma^{\mathrm{NNLO}}_{\{m+2\}} = \int_{m+2} \Big[\mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} J_m - \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} J_{m+1} + \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} J_m \Big]_{d=4}$$

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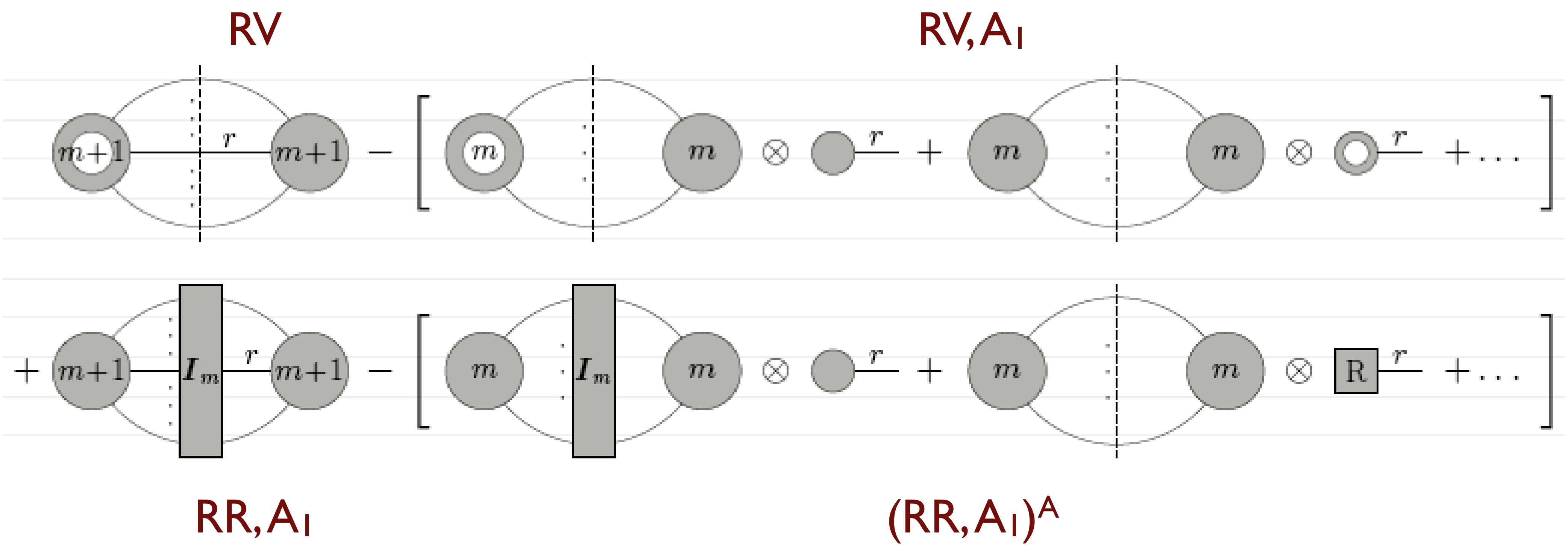
$$\sigma^{\mathrm{NNLO}}_{\{m+1\}} = \int_{m+1} \bigg\{ \bigg[\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \bigg] J_{m+1} - \bigg[\mathrm{d}\sigma^{\mathrm{RV,A}_1}_{m+1} + \bigg(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \bigg)^{\mathrm{A}_1} \bigg] J_m \bigg\}_{\varepsilon=0}$$

G. Somogyi Z. Trocsanyi 2006

RV counterterm

$$\sigma^{\rm NNLO}_{\{m+1\}} = \int_{m+1} \left\{ \left[{\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right] J_{m+1} - \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}_{\varepsilon=0}$$

G. Somogyi Z. Trocsanyi 2006



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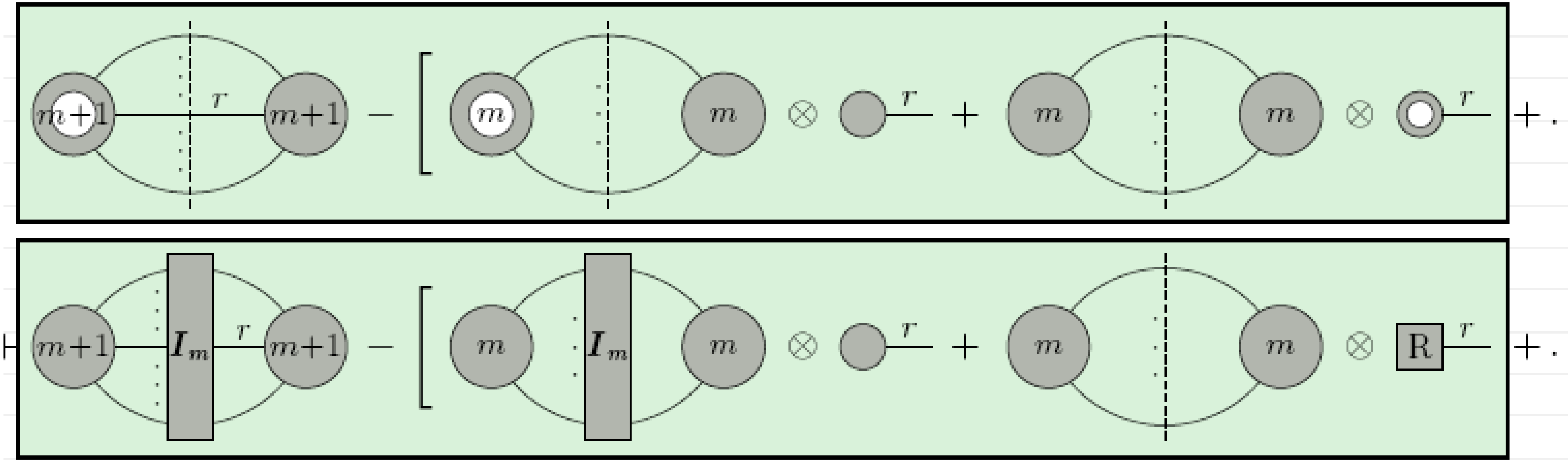
RV kinematic singularities RV, A_1



RR, A_1 kinematic singularities $(RR, A_1)^A$

RV counterterm

$$\sigma_{\{m+1\}}^{\mathrm{NNLO}} = \int_{m+1} \left\{ \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV}} + \int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right] J_{m+1} - \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \right] J_m \right\}_{\varepsilon=0}$$

G. Somogyi Z. Trocsanyi 2006

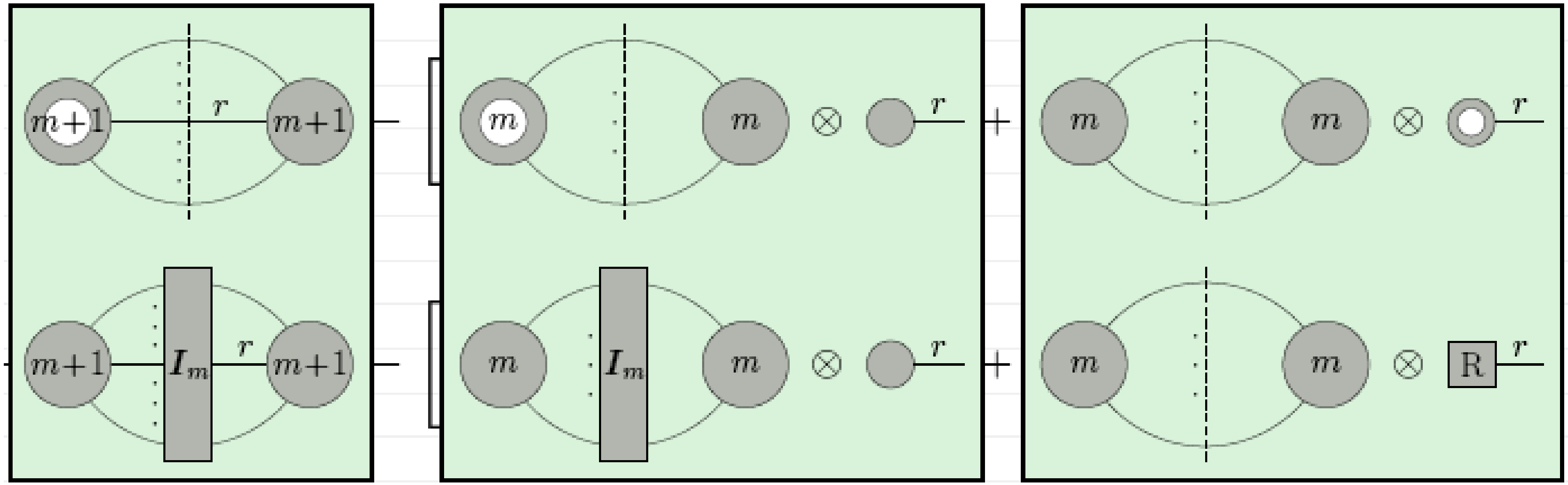
RV counterterm

$$\sigma^{\mathrm{NNLO}}_{\{m+1\}} = \int_{m+1} \left\{ \left[\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right] J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] J_m \right\}_{\varepsilon=0}$$

G. Somogyi Z. Trocsanyi 2006

RV

RV, A1



RR, A1

(RR, A1)^A

ϵ poles

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RV counterterm

$$\sigma_{\{m+1\}}^{\mathrm{NNLO}} = \int_{m+1} \left\{ \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV}} + \int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} \right] J_{m+1} - \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} \right)^{\mathrm{A}_1} \right] J_m \right\}_{\varepsilon=0}$$

G. Somogyi Z. Trocsanyi 2006

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G. Somogyi Z. Trocsanyi 2006

$$\int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} = \mathrm{d}\phi_{m+1} |\mathcal{M}^{(0)}_{m+1}|^2 \otimes \boldsymbol{I}(m,\varepsilon)$$

$$\mathrm{d}\sigma^{\mathrm{RV,A}_1}_{m+1} = \mathrm{d}\phi_m [\mathrm{d}p_1]\, \boldsymbol{\mathcal{A}}_1 2\,\mathrm{Re}\langle \mathcal{M}^{(0)}_{m+1} || \mathcal{M}^{(1)}_{m+1} \rangle$$

$$\left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right)^{\mathrm{A}_1} = \mathrm{d}\phi_m [\mathrm{d}p_1] \boldsymbol{\mathcal{A}}_1 \Big(|\mathcal{M}^{(0)}_{m+1}|^2 \otimes \boldsymbol{I}(m,\varepsilon) \Big)$$

NNLO counterterms

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{NNLO}}_{\{m+2\}} + \sigma^{\mathrm{NNLO}}_{\{m+1\}} + \sigma^{\mathrm{NNLO}}_{\{m\}}$$

$$\sigma^{\mathrm{NNLO}}_{\{m+2\}} = \int_{m+2} \Big[\mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} J_m - \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} J_{m+1} + \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} J_m \Big]_{d=4}$$

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remainder is finite by KLN theorem

$$\sigma^{\rm NNLO}_{\{m\}} = \int_m \Big\{ {\rm d}\sigma^{\rm VV}_m + \int_2 \Big[{\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \Big] + \int_1 \Big[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \Big(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big)^{\rm A_1} \Big] \Big\}_{\varepsilon=0} J_m$$

Thrust

$$T = \mathrm{Max}\frac{\sum_i |\mathbf{p_i} \cdot \mathbf{n}|}{\sum_i |\mathbf{p_i}|}$$

sum over all final-state particles *i*

n unit vector, varied to maximise *T*

T = 1 for aligned particles

T = 1/2 for isotropic distribution of particles

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sum over all final-state particles *i*

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T = 1 for aligned particles

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C parameter

$$C = 3(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1)$$

where λ_α are eigenvalues of $\Theta^{\alpha\beta} = \frac{\sum_i p_i^\alpha p_i^\beta/|\mathbf{p_i}|}{\sum_j |\mathbf{p_j}|}$ $\quad \alpha,\beta = 1,2,3$

For massless particles

$$C = 3 - \frac{3}{2}\sum_{i,j}\frac{(p_i\cdot p_j)^2}{(p_i\cdot Q)(p_j\cdot Q)} \qquad Q = \sum_i p_i^\mu$$

C = 1 for an isotropic & acoplanar distribution of (at least 4) particles

C = 0 for aligned particles

for 3 particles, $C \leq 3/4$

3-jet event shape variables

$$\langle O^n \rangle \equiv \int \mathrm{d}O \, O^n \, \frac{1}{\sigma_0} \frac{\mathrm{d}\sigma}{\mathrm{d}O} = \left(\frac{\alpha_\mathrm{s}(Q)}{2\pi} \right) A_O^{(n)} + \left(\frac{\alpha_\mathrm{s}(Q)}{2\pi} \right)^2 B_O^{(n)} + \left(\frac{\alpha_\mathrm{s}(Q)}{2\pi} \right)^3 C_O^{(n)}$$

n = moment

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$$C_O^{(n)} = C_{O;5}^{(n)} + C_{O;4}^{(n)} + C_{O;3}^{(n)}$$

is NNLO contribution

RR RV VV

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$$C_O^{(n)} = C_{O;5}^{(n)} + C_{O;4}^{(n)} + C_{O;3}^{(n)}$$ is NNLO contribution

RR RV VV

- $C_{O;5}^{(n)}$ and $C_{O;4}^{(n)}$ have been computed and shown to be finite for $e^+e^- \to q\bar{q}ggg$ and $e^+e^- \to q\bar{q}gg$ Gabor Somogyi 2006

$O = C$ or $O = 1 - T$; $n = 1, 2, 3$

3-jet event shape variables

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n = moment

$$C_O^{(n)} = C_{O;5}^{(n)} + C_{O;4}^{(n)} + C_{O;3}^{(n)}$$ is NNLO contribution

RR RV VV

- $C_{O;5}^{(n)}$ and $C_{O;4}^{(n)}$ have been computed and shown to be finite for $e^+e^- \to q\bar{q}ggg$ and $e^+e^- \to q\bar{q}gg$ Gabor Somogyi 2006
$O = C$ or $O = 1 - T$; $n = 1, 2, 3$

- Perfect agreement with NLO results for $B_O^{(n)}$

from Somogyi's talk at HP^2 Zurich 06

- Prediction for moments of event shapes – RR contribution

n	$C^{(n)}_{\tau;5}$	$C^{(n)}_{C;5}$
1	$-(9.27 \pm 0.34) \cdot 10^1$	$-(3.44 \pm 0.14) \cdot 10^2$
2	-3.07 ± 0.43	$-(1.42 \pm 0.03) \cdot 10^2$
3	2.01 ± 0.12	6.29 ± 1.87

- Technical details
  - No. of MC points used: $n = 40 \times 2.5 \cdot 10^5$ (VEGAS)
  - χ^2/d.o.f. as reported by VEGAS: $\chi^2/\text{d.o.f.} = 0.79$
  - No. of subtractions: 535 at 139 different PS points for each event [compare with 12 subtractions at 12 different PS points for $e^+e^- \to$ 4 jets at NLO needed in this scheme ($q\bar{q}gg$ subprocess)]
  - Speed of code on an AMD Athlon 1.3 GHz machine with 256 MB RAM: $2.5 \cdot 10^5$ pts. $\approx$ 2.5 h

- Prediction for moments of event shapes – RV contribution

n	$C^{(n)}_{\tau;4}$	$C^{(n)}_{C;4}$
1	$(1.23 \pm 0.01) \cdot 10^3$	$(4.33 \pm 0.05) \cdot 10^3$
2	$(2.55 \pm 0.02) \cdot 10^2$	$(3.25 \pm 0.02) \cdot 10^3$
3	$(4.79 \pm 0.03) \cdot 10^1$	$(1.80 \pm 0.01) \cdot 10^3$

- Technical details
  - No. of MC points used: $n = 20 \times 2.5 \cdot 10^5$ (VEGAS)
  - χ^2/d.o.f. as reported by VEGAS: $\chi^2/\text{d.o.f.} = 1.24$
  - No. of subtractions: 15 at 7 different PS points for each event
  - Speed of code on an AMD Athlon 1.3 GHz machine with 256 MB RAM: $2.5 \cdot 10^5$ pts. $\approx$ 7 h

Conclusions

- we devised a NNLO subtraction scheme for $e^+e^- \longrightarrow n$ jets
- the calculation is organised into 3 contributions, RR, RV, VV, each of which supposed to be finite in *d*=4 dimensions
- For $e^+e^- \longrightarrow 3$ jets the RR and RV pieces are shown to be finite
- The VV piece still needs be done (but must be finite in *d*=4 dimensions, because of the KLN theorem)