Ex. 5 Virtual corrections in MRK

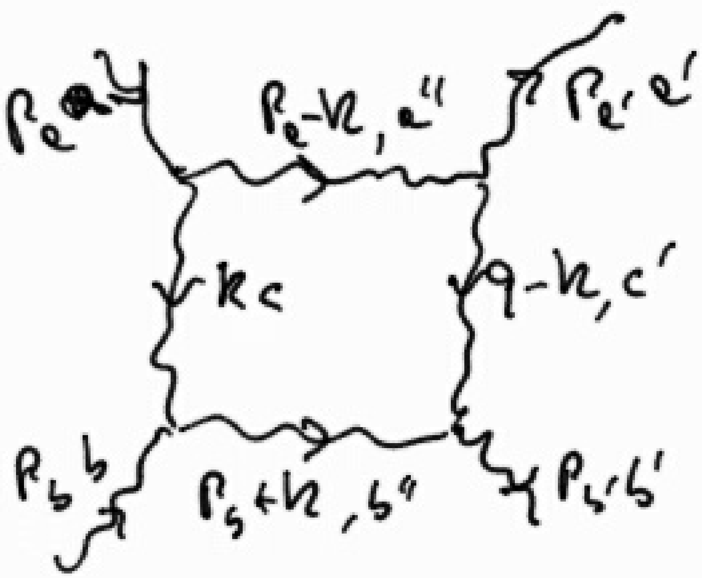
We consider now the one-loop corrections to $gg \to gg$ scattering in MRK.

The leading contribution is given by the t-channel exchange of two gluons

The momentum transfer $q^2 = \hat{t}$ is parametrized by the vector $q \simeq (0, 0; \vec{q}_2)$, since only the transverse degrees of freedom are relevant.

For the gluon propagators in the $\hat{t}$ channel we use the decomposition
$$g^{\mu\nu} = 2\,\frac{p_a^\mu p_b^\nu + p_a^\nu p_b^\mu}{\hat{s}} - \delta_2^{\mu\nu}$$

Then, for the diagram, the leading contribution is



$$iM^{aa'bb'}_{\mu_a \mu_{a'} \mu_b \mu_{b'}} \simeq 2\hat{s}^3 g^4 f^{aa''c} f^{a''a'c'} f^{bb''c} f^{b''b'c'} g_{\mu_a \mu_{a'}} g_{\mu_b \mu_{b'}} \cdot I$$

with
$$I = \int \frac{d\alpha\, d\beta\, d^2k_\perp}{(2\pi)^4} \frac{1}{\alpha\beta\hat{s} - k_2^2 + i\epsilon} \frac{1}{-(1-\alpha)\beta\hat{s} - k_2^2 + i\epsilon} \cdot \frac{1}{\alpha(1+\beta)\hat{s} - k_2^2 + i\epsilon} \frac{1}{\alpha\beta\hat{s} - (q-k)_2^2 + i\epsilon}$$

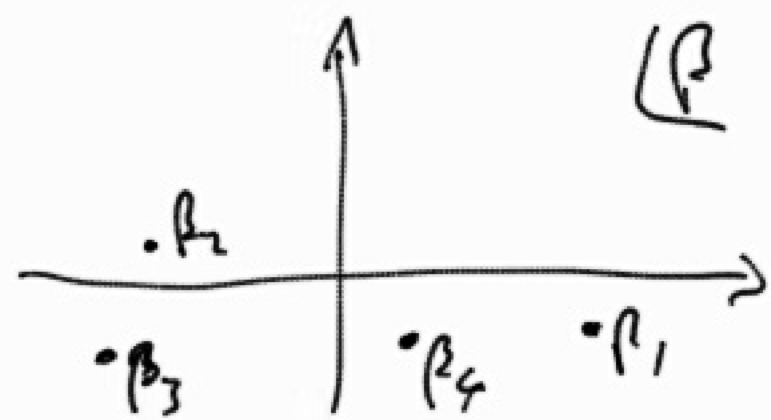
where for the momentum k we use the Sudakov decomposition

$$k^\mu = \alpha p_a^\mu + \beta p_b^\mu + k_\perp^\mu \qquad 0 < \alpha, \beta < 1$$

$$d^4 k = \frac{\hat{s}}{2} d\alpha\, d\beta\, d^2 k_\perp \qquad k^2 = \alpha\beta\hat{s} - k_\perp^2$$

In the β plane, the poles are

$$\beta_1 = \frac{k_\perp^2 - i\epsilon}{\alpha\hat{s}} \qquad \beta_2 = \frac{-k_\perp^2 + i\epsilon}{(1-\alpha)\hat{s}} \qquad \beta_3 = \frac{-\alpha\hat{s} + k_\perp^2 - i\epsilon}{\alpha\hat{s}} \qquad \beta_4 = \frac{(q-k)_\perp^2 - i\epsilon}{\alpha\hat{s}}$$



since all the poles but β_2 are in the lower half-plane, it's convenient to perform the integral in the upper half-plane. Then we take $\alpha \ll 1$

We obtain $$I = -\frac{i}{\hat{s}}\int \frac{d\alpha\, d^2k_2}{(2\pi)^3}\,\frac{1}{\alpha\hat{s} - k_2^2}\,\frac{1}{k_2^2}\,\frac{1}{(q-k)_2^2}$$

We look at the IR region $k_2^2 < -t$,

The integral over α is logarithmic over the range $-\frac{\hat{t}}{\hat{s}} < \alpha < 1$

Then we get $$I \simeq -\frac{i}{2\pi}\,\frac{1}{\hat{s}^2}\,\ln\frac{\hat{s}}{-\hat{t}}\int\frac{d^2k}{(2\pi)^2}\,\frac{1}{k_2^2}\,\frac{1}{(q-k)_2^2}$$

So the diagram  can be written as

$$iM^{aa'bb'}_{\mu_a\mu_{a'}\mu_b\mu_{b'}} = -i\,\frac{4g^2}{N_c}\,\frac{\hat{s}}{t}\,\ln\frac{\hat{s}}{-\hat{t}}\,\alpha(\hat{t})\, f^{aa''c} f^{a''a'c'} f^{bb''c} f^{b''b'c'}\, g_{\mu_a\mu_{a'}}\, g_{\mu_b\mu_{b'}} \quad *)$$

with $$\alpha(\hat{t}) = \alpha_s N_c\, \hat{t}\int\frac{d^2k_2}{(2\pi)^2}\,\frac{1}{k_2^2\,(q-k)_2^2}$$

the gluon Regge trajectory

Evaluating the integral over k_2 with an IR cutoff μ we obtain that $\alpha(\hat{t}) \simeq -\frac{\alpha_s N_c}{2\pi} \ln \frac{q_\perp^2}{\mu^2}$

showing that the diagram is doubly logarithmic divergent.

We may regulate the integral of the gluon Regge trajectory in dimensional regularisation in $d = 4-2\epsilon$ dim

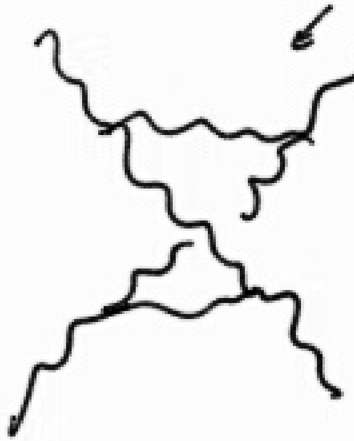
Then $$\alpha(t) = \alpha_s N_c \hat{t} \int \frac{d^{2-2\epsilon} k_\perp}{(2\pi)^2} \frac{1}{k_\perp^2 (q-k)_\perp^2}$$

$$= 8\pi \alpha_s N_c c_\Gamma \frac{1}{\epsilon} \left(\frac{\mu^2}{-\hat{t}}\right)^\epsilon$$

with $$c_\Gamma = \frac{1}{(4\pi)^{2-\epsilon}} \frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}$$

Thus, in DR the Regge trajectory is exhibited through an IR simple pole.

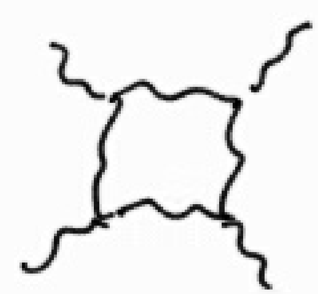
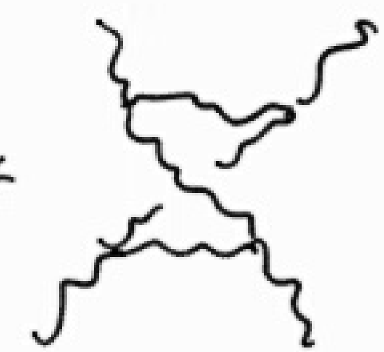
The diagram is obtained by crossing the channels $\hat{s}$ and $\hat{u}$



in the previous diagram (eq. *)

Using momentum conservation $\hat{u} = -\hat{s} - \hat{t}$

$$\ln \frac{\hat{u}}{-t} \simeq \ln \frac{-\hat{s}}{-\hat{t}} = \ln \frac{\hat{s}}{-t} + i\pi$$

we can write the amplitude for  +  as

$$iM^{aa'bb'}_{\mu_a\mu_{a'}\mu_b\mu_{b'}} \simeq -i\,\frac{4g^2}{N_c}\,\frac{\hat{s}}{\hat{t}}\,\alpha(\hat{t})\, g_{\mu_a\mu_{a'}}\, g_{\mu_b\mu_{b'}}\, f^{aa''c} f^{a''a'c'}$$

$$\cdot\left(\ln\frac{\hat{s}}{-\hat{t}}\, f^{bb''c} f^{b''b'c'} - \left(\ln\frac{\hat{s}}{-\hat{t}} + i\pi\right) f^{bb''c'} f^{b''b'c}\right)$$

Performing the contractions over the repeated colour indices,

$$iM^{aa'bb'}_{\mu_a\mu_{a'}\mu_b\mu_{b'}} \simeq -i\,\frac{4g^2}{N_c}\,\frac{\hat{s}}{\hat{t}}\,\alpha(\hat{t})\, g_{\mu_a\mu_{a'}}\, g_{\mu_b\mu_{b'}}$$

$$\left(\frac{N_c}{4}\, f^{ada'} f^{bdb'}\left(2\ln\frac{\hat{s}}{-\hat{t}} + i\pi\right) - \frac{N_c^2}{N_c^2-1}\,\delta^{aa'}\delta^{bb'}\, i\pi\right)$$

we obtain a leading logarithmic term with the same colour structure as at tree level,

$$iM^{aa'bb'}_{\mu_a\mu_{a'}\mu_b\mu_{b'}} = -i\,\frac{2\hat{s}}{\hat{t}}\, g^2\, g_{\mu_a\mu_{a'}}\, g_{\mu_b\mu_{b'}}\, f^{ada'} f^{bdb'}\, \alpha(\hat{t}) \ln\frac{\hat{s}}{-\hat{t}}$$

In fact, to leading log accuracy, we can write that the real part of the one-loop amplitude is

$$\mathrm{Re}\, M_4^{1\text{-loop}} = \alpha(t) \ln \frac{\hat{s}}{-\hat{t}} \cdot M_4^{\text{tree}}$$

In order to analise the colour structure, we decompose the amplitude in terms of the SU(3) representations occurring in the product $\underline{8} \otimes \underline{8}$ of the two gluons exchanged in the $\hat{t}$ channel

$$M^{aa'bb'}_{\mu_a\mu_{a'};\mu_b\mu_{b'}} = g^{\mu_a\mu_{a'}} g^{\mu_b\mu_{b'}} \sum_{T} P^{aa'}_{bb'}(T)\, A^{T}(\hat{s},\hat{t})$$

with $A^{T}(\hat{s},\hat{t})$ colourless amplitudes

and $P^{aa'}_{bb'}(T)$ colour projectors

$$P^{aa'}_{bb'}(T)\, P^{bb'}_{cc'}(T') = P^{aa'}_{cc'}\, \delta_{TT'}$$

Now $\underline{8} \otimes \underline{8} = (\underline{8} \otimes \underline{8})_S + (\underline{8} \otimes \underline{8})_A$

with $(\underline{8} \otimes \underline{8})_S = \underline{1} + \underline{8}_S + \underline{27}$

$(\underline{8} \otimes \underline{8})_A = \underline{8}_A + \underline{10} + \overline{\underline{10}}$

Because the projectors parity under $\hat{s} \leftrightarrow \hat{u}$ crossing is

$$P^{aa'}_{b'b}(T) = (-)^T P^{aa'}_{bb'}(T)$$

with
$$(-)^T = \begin{cases} -1 & \text{for } (\underline{8} \otimes \underline{8})_A \\ +1 & \text{for } (\underline{8} \otimes \underline{8})_S \end{cases}$$

the $\hat{s} \leftrightarrow \hat{u}$ crossed term, which has a relative negative sign, must belong to $(\underline{8} \otimes \underline{8})_A$.

Using the explicit projectors

$$P^{aa'}_{bb'}(\underline{8}_A) = \frac{1}{N_c} f^{acа'} f^{bcb'}$$

$$P^{aa'}_{bb'}(\underline{10} + \underline{\overline{10}}) = \frac{1}{2}\left(\delta_{ab}\delta_{a'b'} - \delta_{ab'}\delta_{a'b}\right) - \frac{1}{N_c} f^{aca'} f^{bcb'}$$

it is immediate to see that the leading logarithmic term belongs to the antisymmetric octet.

In fact, the contraction with $P^{aa'}_{bb'}(\underline{10} + \underline{\overline{10}})$ vanishes.

We note that we neglected subleading one-loop corrections, like the self-energy and the vertex corrections, which determine the running of the coupling. Thus, in a leading log treatment, α_s must be considered as fixed.

$$A^{\underline{8}_a}(s,t) = -2g^2 N_c \frac{\hat{s}}{t}\, \alpha(\hat{t}) \ln\left(\frac{\hat{s}}{-\hat{t}}\right)$$

The one-loop computation above is not gauge invariant, although correct in the physical gauge where we performed it. The sub-leading contributions have not been correctly taken into account either.

However, the behaviour of the one-loop 4-gluon amplitude in the high-energy limit can easily be extracted from $M_4^{1\text{-loop}}$ in general kinematics at fixed helicities.

The colour decomposition of the one-loop n-gluon amplitude, is usually given in terms of colour matrices T of the fundamental repr. A more convenient form is in terms of structure const.

(Dixon, Meltoni, VDD '99)

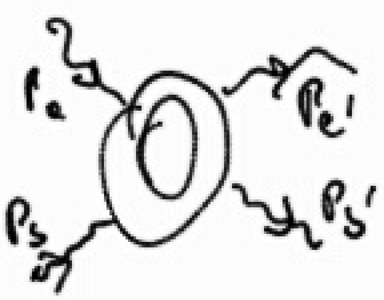
$$\mathcal{M}_4^{(1)} = g^n \sum_{\sigma \in S_{n-1}/\mathbb{R}} \Big[\mathrm{tr}\left(F^{a\sigma_1} \cdots F^{a\sigma_n}\right) A_{n;1}^{(1)}(\sigma_1, \ldots, \sigma_n) + 2 n_f \, \mathrm{tr}\left(T^{a\sigma_1} \cdots T^{a\sigma_n}\right) A_{n;1}^{(1/2)}(\sigma_1, \ldots, \sigma_n) \Big]$$

with $S_{n-1} = S_n / \mathbb{Z}_n$ the non-cyclic permutations

and $\mathbb{R}(1,2,\ldots,n) = (n,\ldots,2,1)$ the reflection

and $(F^a)_{bc} = i f^{bac}$

For a one-loop 4-gluon amplitude in the high-energy limit, $\hat{s} \gg |\hat{t}|$, one obtains (unrenormalised)



(Schmidt, VDD '97)

$$\mathrm{Re}\, M_4^{(1)}(p_a^-, p_{a'}^+, p_{b'}^+, p_b^-) = M_4^{\mathrm{Tree}}(p_a^-, p_{a'}^+, p_{b'}^+, p_b^-)\, g^2 c_\Gamma \cdot \left\{ \left(\frac{\mu^2}{-t}\right)^{\epsilon} \left[N_c \left(-\frac{4}{\epsilon^2} - \frac{11}{3\epsilon} + \frac{2}{\epsilon} \ln \frac{\hat{s}}{-\hat{t}} - \frac{67}{9} + \pi^2 \right) + n_f \left(\frac{2}{3\epsilon} + \frac{10}{9} \right) \right] - \frac{\beta_0}{\epsilon} \right\}$$

To LL accuracy, this yields

$$\mathrm{Re}\, M_4^{(1)} = M_4^{\mathrm{tree}} \cdot \quad c_\Gamma\, \alpha(\hat{t}) \ln \frac{\hat{s}}{-\hat{t}}$$

in agreement with what we found through the direct calculation. However, the expression above allows us also to obtain the NLO corrections to the helicity-conserving vertices (called "impact factors")