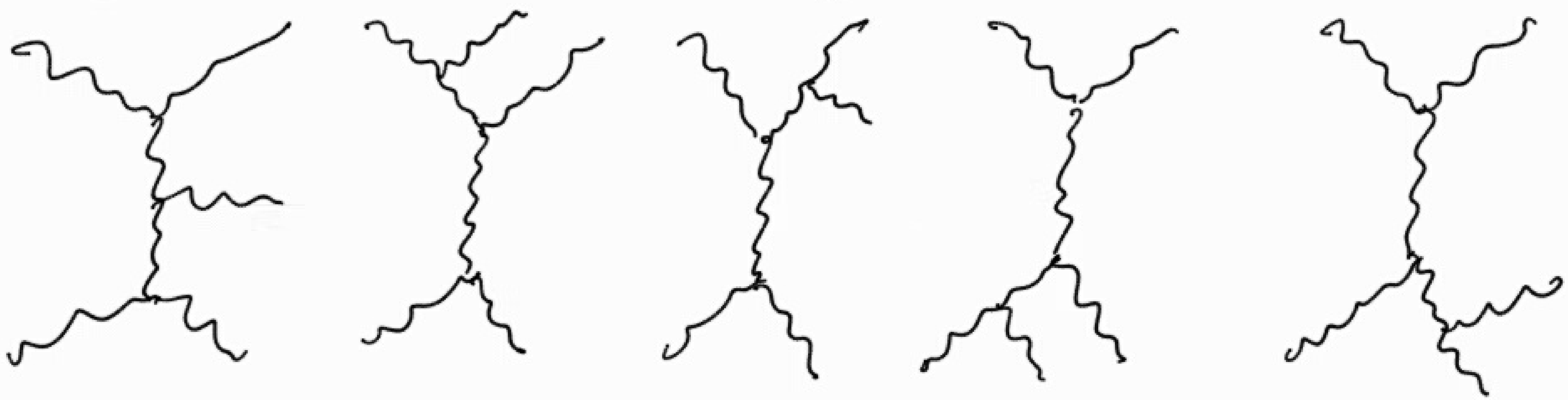
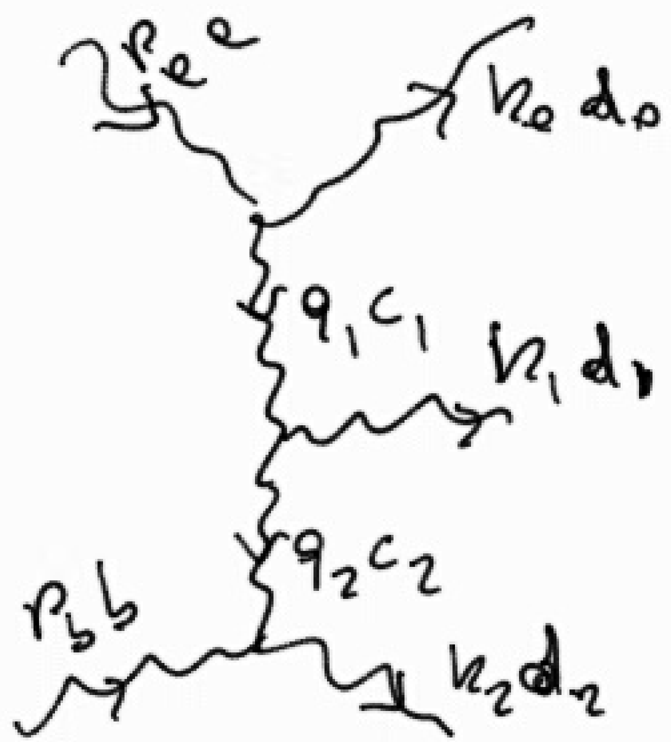
Ex. 3 <u>3-parton</u> <u>production in multi Regge kinematics</u>

As before, we work in a physical gauge and consider only diagrams with gluon exchange in the t channel.



We shall compute explicitly the first 3 diagrams

For the gluon propagator in the t channel, as done in Ex. 2 we use the decomposition $g^{\mu\nu} = 2\,\frac{p_a^{\mu} p_b^{\nu} + p_a^{\nu} p_b^{\mu}}{\hat{s}} - \delta_{\perp}^{\mu\nu}$



The gluon emission along the ladder is:

$$iM(t)^{a b d_0 d_1 d_2}_{\mu_a \mu_b \mu_0 \mu_1 \mu_2}$$

$$= \left(g f^{a d_0 c_1} g_{\mu_a \mu_0}\right) \hat{s}(-i) \frac{2}{\hat{s}\hat{t}_1} p_a^{\nu_1}$$

$$\cdot g f^{c_1 c_2 d_1}\left[g^{\nu_1 \nu_2}(q_1+q_2)^{\mu_1} + g^{\mu_1 \nu_2}(k_1-q_2)^{\nu_1} - (k_1+q_1)^{\nu_2} g^{\nu_1 \mu_1}\right]$$

$$\cdot (-i) p_b^{\nu_2} \frac{2}{\hat{s}\hat{t}_2} \hat{s}\left(g f^{b d_2 c_2} g_{\mu_b \mu_2}\right)$$

we define the central-emission vertex as

$$\hat{C}^{\mu_1}(q_1, q_2) = \frac{2}{\hat{s}} p_a^{\nu_1}\left[g^{\nu_1 \nu_2}(q_1+q_2)^{\mu_1} + g^{\mu_1 \nu_2}(k_1-q_2)^{\nu_1} - (k_1+q_1)^{\nu_2} g^{\nu_1 \mu_1}\right] p_b^{\nu_2}$$

We write the first term, $(q_1+q_2)^{\mu_1}$, of $\hat{C}_{\mu_1}$

through the usual decomposition

$$(q_1+q_2)_\nu g^{\mu_1\nu} = (q_1+q_2)_\nu \left[2\, \frac{p_a^{\mu_1} p_b^{\nu} + p_a^{\nu} p_b^{\mu_1}}{\hat{s}} - \delta_\perp^{\mu_1\nu} \right]$$

To evaluate it, we write q_1+q_2 in light-cone coords:

$$q_1 = p_a - k_0 \qquad \text{so} \qquad q_1 = \left(k_1^+ + k_2^+ , -k_0^- ; -\vec{k}_{0\perp} \right)$$

$$q_2 = q_1 - k_1 \qquad\qquad q_2 = \left(k_2^+ , -k_0^- - k_1^- ; -\vec{k}_{0\perp} - \vec{k}_{1\perp} \right)$$

$$2 q_1 \cdot p_b = \left(k_1^+ + k_2^+ \right) p_b^- \qquad\qquad 2 q_2 \cdot p_b = k_2^+ p_b^-$$

in MRK : $\quad 2(q_1+q_2)\cdot p_b \simeq k_1^+ p_b^- = -\hat{s}_{b1}$

$$2 q_1 \cdot p_a = - p_a^+ k_0^-$$

$$2 q_2 \cdot p_a = - p_a^+ (k_0^- + k_1^-)$$

in MRK : $2(q_1+q_2)\cdot p_a \simeq - p_a^+ k_1^- = \hat{s}_{a1}$

so $(q_1+q_2)^\nu g^{\mu_1 \nu} = \frac{\hat{s}_{a1}}{\hat{s}} p_b^{\mu_1} - \frac{\hat{s}_{b1}}{\hat{s}} p_a^{\mu_1} - (\vec{q}_{1_\perp} + \vec{q}_{2_\perp})^{\mu_1}$

Promoting $\vec{q}_\perp^{\,\mu}$ to a 4-vector: $q_\perp^\mu \equiv (0, 0; \vec{q}_\perp)$

we write

with $q_\perp \cdot q_\perp = -|q_\perp|^2$

$$(q_1+q_2)^{\mu_1} = (q_{1_\perp} + q_{2_\perp})^{\mu_1} + \frac{\hat{s}_{a1}}{\hat{s}} p_b^{\mu_1} - \frac{\hat{s}_{b1}}{\hat{s}} p_a^{\mu_1}$$

The 2nd term of $\hat{C}^{\mu_1}$ contains $(k_1 - q_2)\cdot p_a$

but $k_1 - q_2 = (k_1^+ - k_2^+, k_0^- + 2k_1^-; k_{0\perp} + 2k_{1\perp})$

So $(k_1 - q_2)\cdot p_a = \frac{1}{2}\left(k_0^- + 2k_1^-\right) p_a^+ \simeq -\hat{s}_{a1}$

The 3^{rd} term of $\hat{C}^{\mu_1}$ conteins $(k_1 + q_1)\cdot p_b$

but $k_1 + q_1 = \left(2k_1^+ + k_2^+ , -k_0^- + k_1^- ; -k_{0_\perp} + k_{1_\perp}\right)$

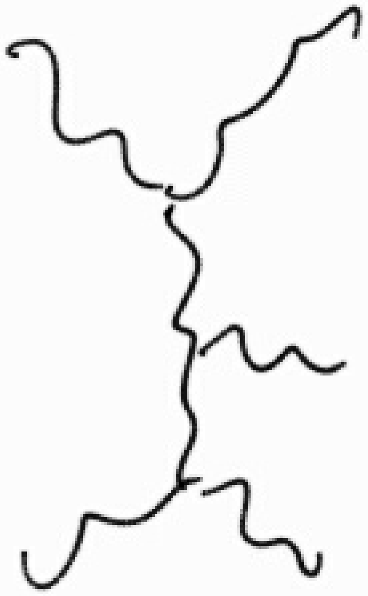
so $(k_1 + q_1)\cdot p_b = \frac{1}{2}\left(2k_1^+ + k_2^+\right) p_b^- \simeq -\hat{s}_{b1}$

So the central-emission vertex is

$$\hat{C}^{\mu_1}(q_1, q_2) = (q_{1_\perp} + q_{2_\perp})^{\mu_1} + \frac{s_{a1}}{s} p_b^{\mu_1} - \frac{s_{b1}}{s} p_a^{\mu_1} + \frac{2}{s}(-s_{a1}) p_b^{\mu_1} - \frac{2}{s}(-s_{b1}) p_a^{\mu_1}$$

$$= (q_{1_\perp} + q_{2_\perp})^{\mu_1} - \frac{s_{a1}}{s} p_b^{\mu_1} + \frac{s_{b1}}{s} p_a^{\mu_1}$$

Then the diagram for the gluon emission along the ladder is



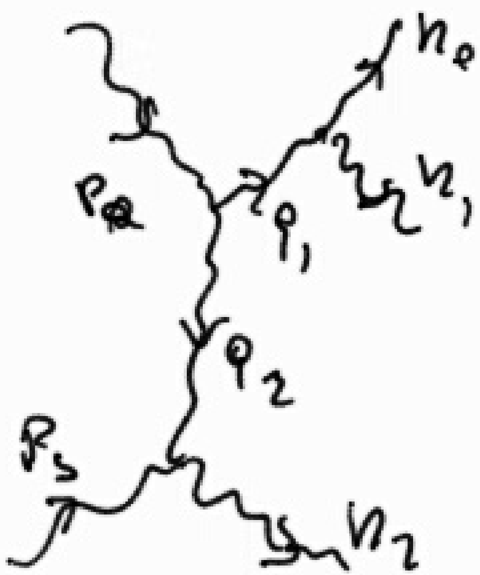
$$iM(t)^{abd_0d_1d_2}_{\mu_a\mu_b\mu_0\mu_1\mu_2} = 2i\hat{s}\left(igf^{ad_0c_1}\, g_{\mu_a\mu_0}\right)\frac{1}{t_1}$$

$$\cdot\left[igf^{c_1d_1c_2}\,\hat{C}^{\mu_1}(q_1,q_2)\right]\frac{1}{t_2}$$

$$\cdot\left(igf^{bd_2c_2}\, g_{\mu_b\mu_2}\right)$$

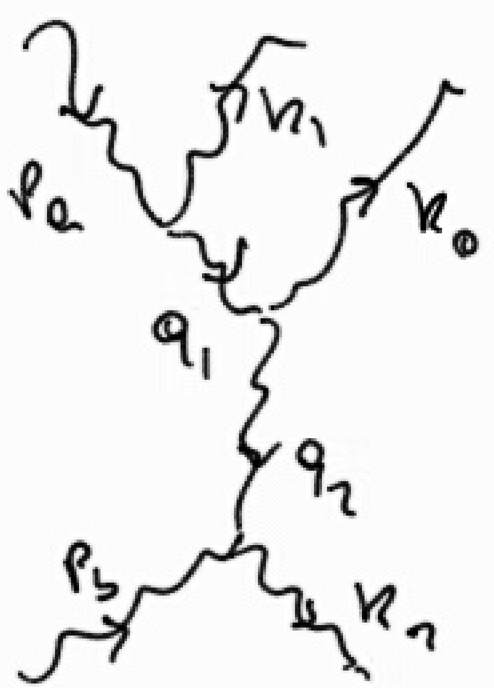
$$t_i = q_i^2 = -q_{\perp_i}^2$$

The emissions from the upper line are:



$$iM(fu) = 2\hat{s}\, f^{c_1d_0d_1} f^{ac_1c_2} f^{bd_2c_2}\, g^3$$

$$\cdot\ \frac{2p_a^{\mu_1}}{\hat{s}_{a1}\,\hat{t}_2}\, g^{\mu_0\mu_a}\, g^{\mu_b\mu_2}$$



$$iM(iu) = -2\hat{s}\, g^3 f^{a c_1 d_1} f^{c_1 d_0 c_2} f^{b d_2 c_2}$$

$$\frac{2 p_a^{\mu_1}}{\hat{s}_{a1}\hat{t}_2}\, g^{\mu_0 \mu_a} g^{\mu_b \mu_2}$$

we can combine $M(iu)$ and $M(fu)$ using Jacobi identity

$$f^{a c_1 c_2} f^{d_0 d_1 c_1} + f^{d_0 c_1 c_2} f^{d_1 a c_1} + f^{d_1 c_1 c_2} f^{a d_0 c_1} = 0$$

so $$i(M(iu)+M(fu)) = i\,2\hat{s}\left(ig f^{a d_0 c_1} g^{\mu_a \mu_0}\right)\frac{1}{t_1}$$

$$\left(ig f^{c_1 d_1 c_2} p_a^{\mu_1} \frac{2\hat{t}_1}{\hat{s}_{a1}}\right)\frac{1}{t_2}$$

$$\left(ig f^{b d_2 c_2} g^{\mu_b \mu_2}\right)$$

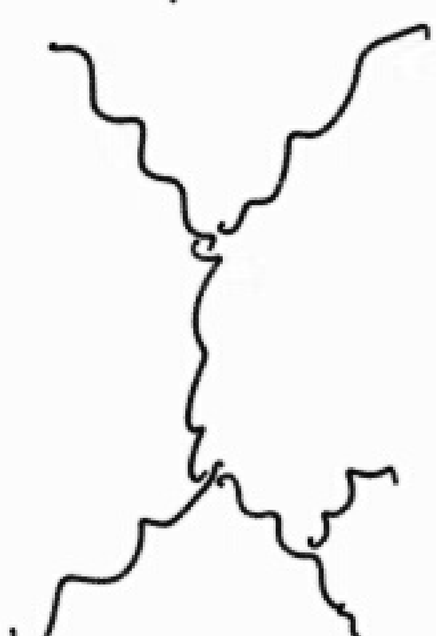
Likewise, we can compute also the emissions from the lower lines (please check it)

$$i\left(\mathcal{M}(id) + \mathcal{M}(fd)\right) = i\, 2\hat{s} \left(i g f^{a d_a c_1} g_{\mu_a \mu_a}\right) \frac{1}{\hat{t}_1}$$

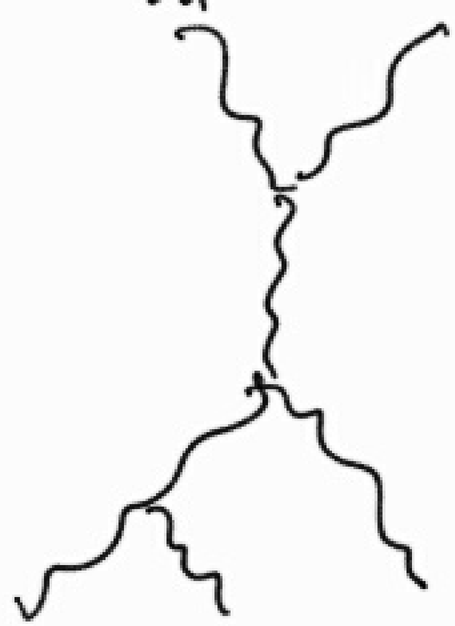
$$\left(i g f^{c_1 d_1 c_2} \left(-\frac{2\hat{t}_2}{\hat{s}_{b1}}\right) p_b^{\mu_1}\right) \frac{1}{\hat{t}_2}$$

$$\left(i g f^{b d_2 c_2} g^{\mu_b \mu_2}\right)$$

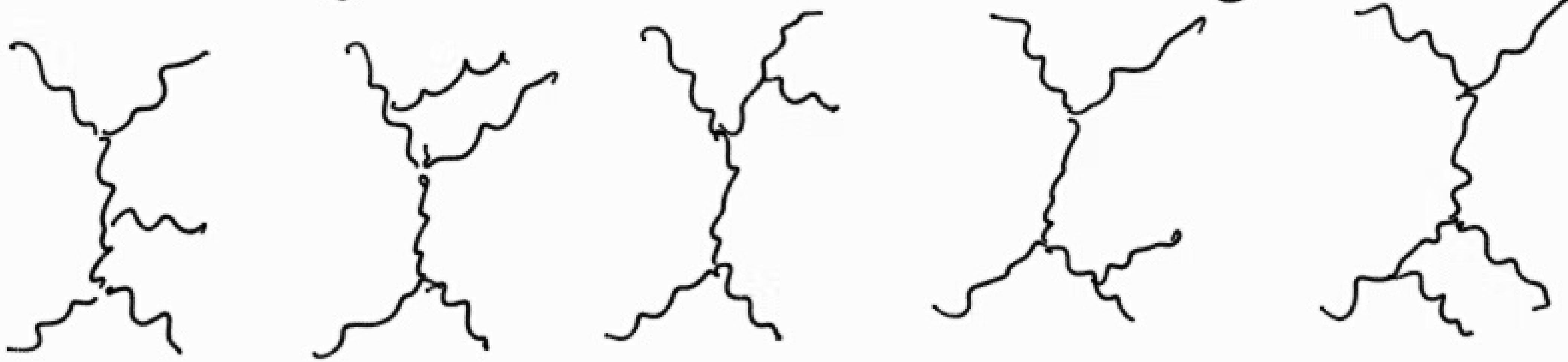
f d



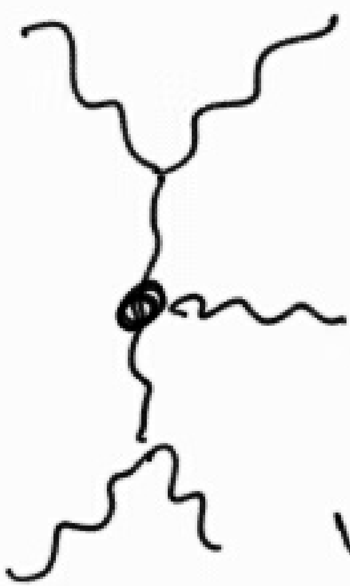
i d



Since the colour structure is the same, we can combine the 5 diagrams with gluon exchange in T channel



into an effective amplitude:



$$iM^{a b d_0 d_1 d_2}_{\mu_a \mu_b \mu_0 \mu_1 \mu_2} = 2\hat{s}\left(i g f^{a d_0 c_1} g^{\mu_a \mu_0}\right)\frac{1}{\hat{t}_1}\left(i g f^{c_1 d_1 c_2} C^{\mu_1}(q_1, q_2)\right)\frac{1}{\hat{t}_2}\left(i g f^{b d_2 c_2} g^{\mu_b \mu_2}\right)$$

with

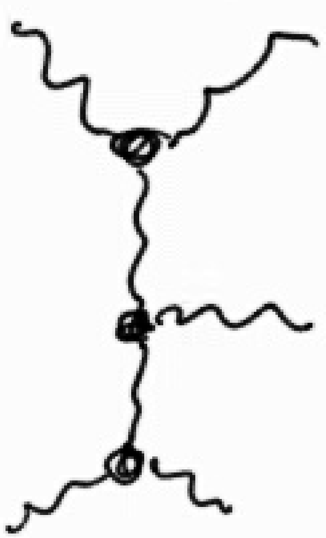
$$C^{\mu_1}(q_1, q_2) = (q_1 + q_2)_\perp^{\mu_1} + \left(\frac{\hat{s}_{b1}}{\hat{s}} + 2\frac{\hat{t}_1}{\hat{s}_{a1}}\right) p_a^{\mu_1} - \left(\frac{\hat{s}_{a1}}{\hat{s}} + \frac{2\hat{t}_2}{\hat{s}_{b1}}\right) p_b^{\mu_1}$$

where the central emission vertex $C^\mu(q_1, q_2)$ is an effective non-local vertex.

It is gauge invariant, in fact it satisfies the Ward identity

$$C^\mu(q_1, q_2)\, k_1^\mu = 0 \qquad \text{(check it)}$$

We may include s and u channel contributions through the effective vertex $\Gamma^{\mu_a \mu_{a'}}$ introduced in Ex. 2 and write the full amplitude in MRK as



$$iM^{a b d_0 d_1 d_2}_{\mu_a \mu_b \mu_0 \mu_1 \mu_2} = 2\hat{s}\left(i g f^{a d_0 c_1} \Gamma^{\mu_a \mu_0}\right) \frac{1}{\hat{t}_1}$$

$$\left(i g f^{c_1 d_1 c_2} C^{\mu_1}(q_1, q_2)\right) \frac{1}{\hat{t}_2}$$

$$\left(i g f^{b d_2 c_2} \Gamma^{\mu_b \mu_2}\right)$$

We square the amplitude and sum over helicities and colours.

Using its gauge invariance, the sum over helicities of the central emission vertex is

$$C^{\mu}(q_1, q_2)\, C^{\nu}(q_1, q_2)^{*} \sum_{\lambda} \epsilon_{\lambda}^{\mu}(k_1)\, \epsilon_{\lambda}^{\nu *}(k_1)$$

$$= - C^{\mu}(q_1, q_2)\, C^{\nu}(q_1, q_2)^{*}\, g^{\mu\nu}$$

$$= 4\, \frac{q_{1_\perp}^2\, q_{2_\perp}^2}{k_{1_\perp}^2} \qquad \text{(check it)}$$

where we used the kinematic constraint $\hat{s}_{a1} \hat{s}_{b1} = k_{1_\perp}^2 \hat{s}$

(Note that the square of the vertex is much simpler than the vertex...)

Thus, the square of the amplitude, summed (averaged) over final (initial) helicities and colours is

$$\overline{\sum_{hel,col}} |M_{gg\to ggg}|^2 = \frac{16\, g^6 C_A^3}{N_c^2-1} \frac{\hat{s}^2}{k_{0_\perp}^2 k_{1_\perp}^2 k_{2_\perp}^2}$$