The S matrix
and
the high energy limit

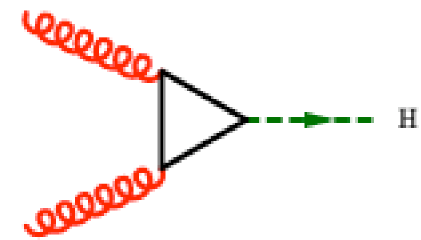
Vittorio Del Duca
ETH Zürich & INFN LNF

Max Planck Institute 21 June 2016

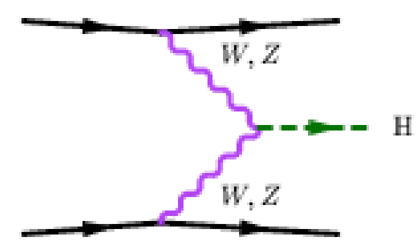
Higgs production modes at LHC

In proton collisions, the Higgs boson is produced mostly via

- gluon fusion $gg \to H$
  - largest rate for all M_H
  - proportional to the top Yukawa coupling y_t

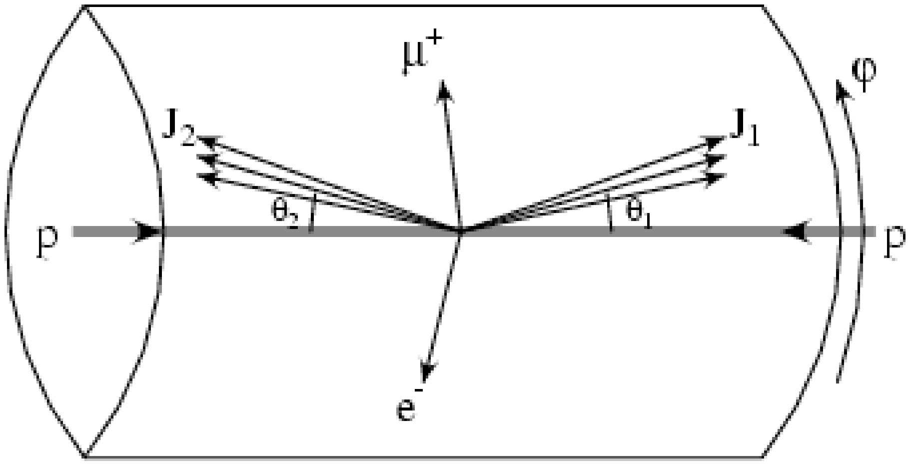


- vector-boson fusion (VBF) $qq \to qqH$
  - second largest rate (mostly *u d* initial state)
  - proportional to the VVH coupling

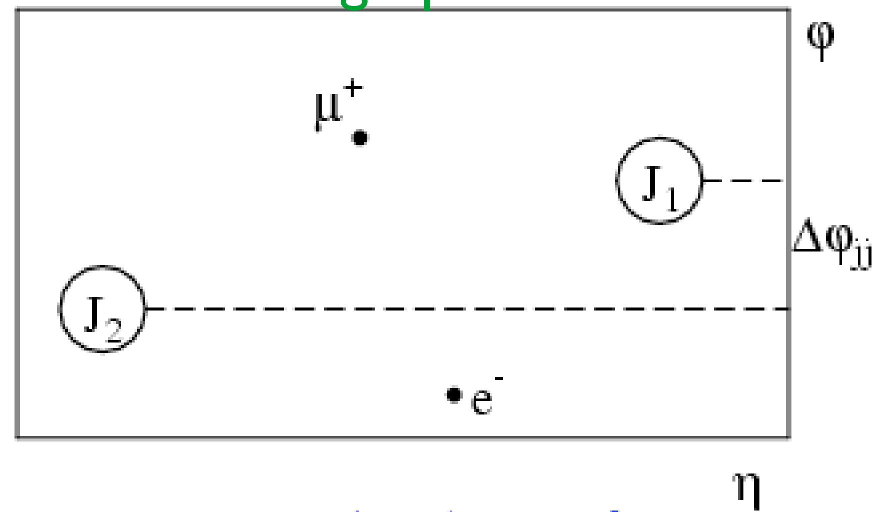


Vector boson fusion $qq \to qqH$

A VBF event



Lego plot

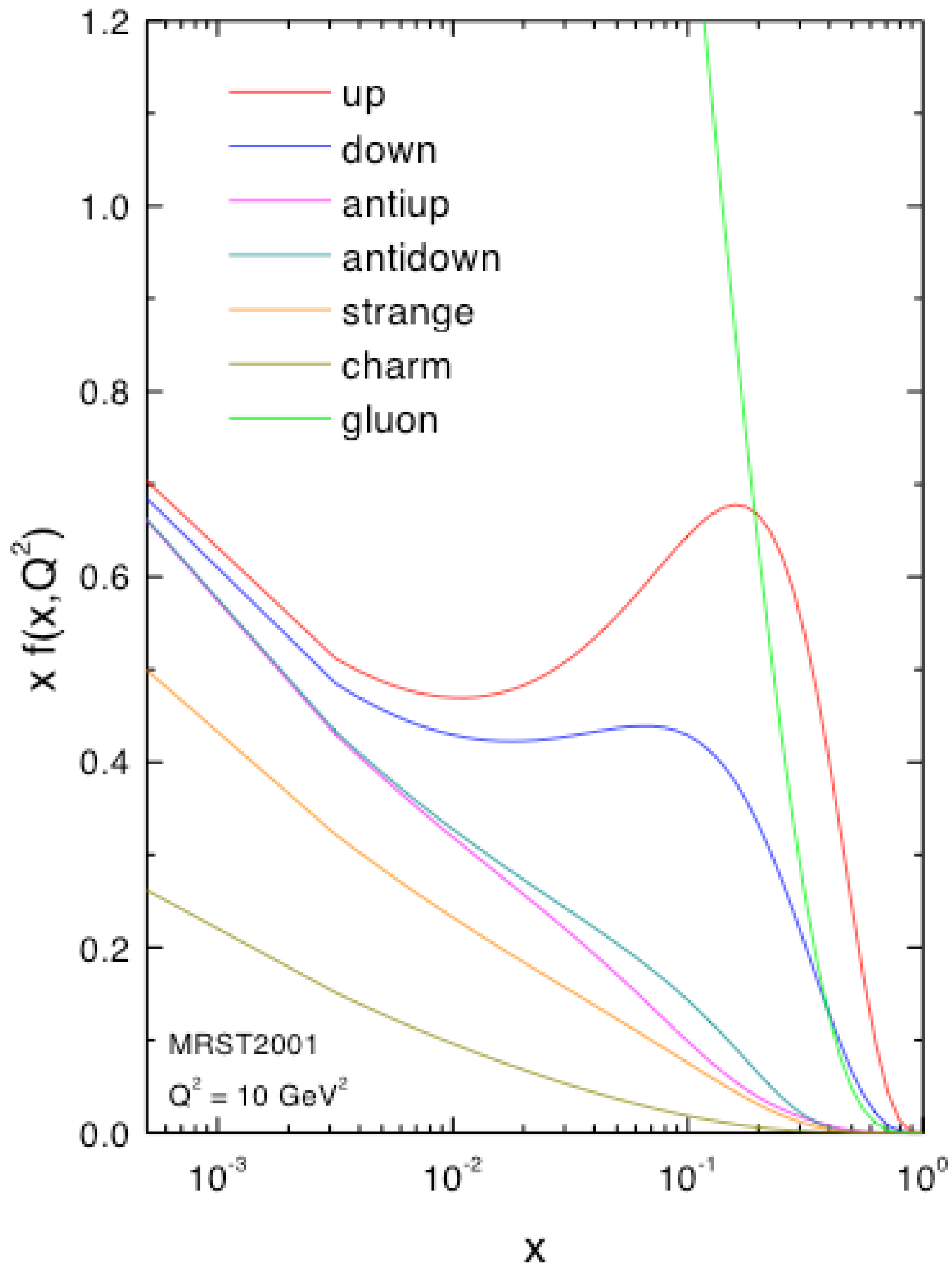


$$\eta = \frac{1}{2} \ln \frac{1+\cos\theta}{1-\cos\theta}$$

VBF features

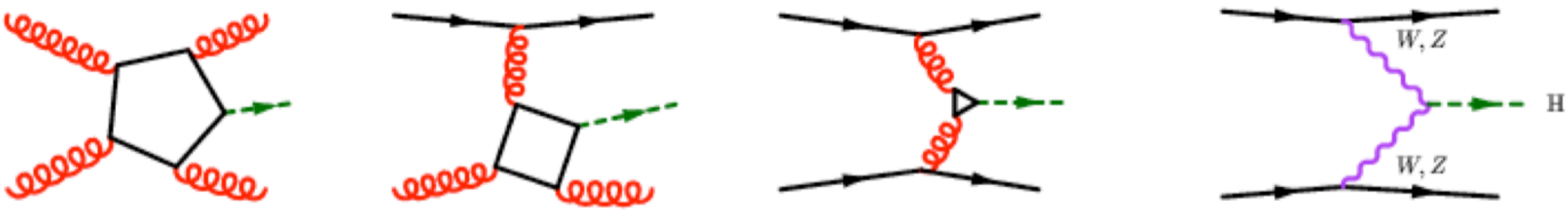
- energetic jets in the forward and backward directions
- sparse gluon radiation in the central-rapidity region, due to colourless W/Z exchange → *rapidity gaps*
- Higgs decay products between the tagging jets
- α_s corrections are known up to N^3LO for inclusive Higgs production (up to NNLO for Higgs + 2-jet production) in the DIS approximation, i.e. no gluon radiation exchanged between the two quark lines. They are ≤ 1% (about 5-10%)

Parton distribution functions (PDF)



Higgs + 2 jet production

gluon fusion *vs.* VBF



inclusive cuts

$$p_{jT} > 20\,\text{GeV} \quad |y_j| < 5 \quad R_{jj} > 0.6$$

VBF cuts

$$|y_{j_1} - y_{j_2}| > 4.2 \quad y_{j_1} \cdot y_{j_2} < 0 \quad \sqrt{s_{j_1 j_2}} > 600\text{GeV}$$

- from inclusive to VBF cuts:
  H+2jets production from gluon fusion goes from 9-10 pb to 0.3-0.4 pb
  H+2jets production from VBF goes from about 3 pb to about 1.2 pb
  Kilgore Oleari Schmidt Zeppenfeld VDD 2001

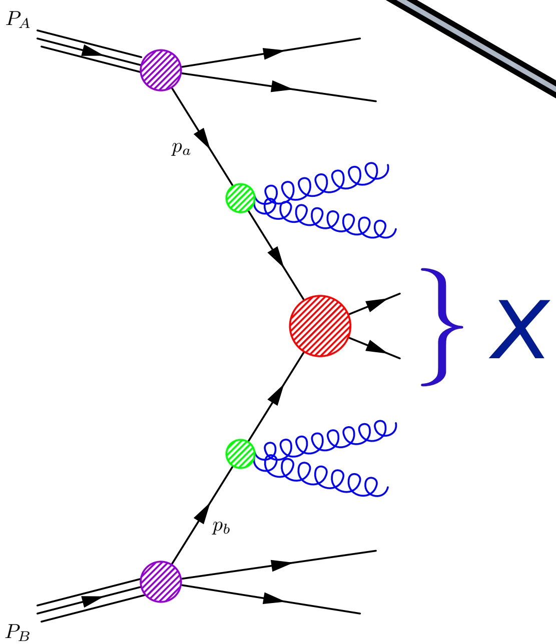
- VBF cuts enhance VBF wrt gluon fusion by a factor 10

Factorisation

computed in pQCD

extracted from data
evolved through DGLAP

is the separation between
the short- and the long-range interactions



$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_{a/A}(x_1,\mu_F^2)\, f_{b/B}(x_2,\mu_F^2)$$
$$\times\ \hat{\sigma}_{ab\to X}\left(x_1, x_2, \{p_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_F^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}\right)$$

$$X = W, Z, H, Q\bar{Q}, \text{high-}E_T\text{jets}, \ldots$$

$\hat{\sigma}$ is known as a fixed-order expansion in α_S

$$\hat{\sigma}(\alpha_S,\mu_R,\mu_F) = (\alpha_S(\mu_R))^n \left[\hat{\sigma}^{(0)} + \left(\frac{\alpha_S}{2\pi}\right)\hat{\sigma}^{(1)}(\mu_R,\mu_F) + \left(\frac{\alpha_S}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_R,\mu_F) + \ldots\right]$$

$\hat{\sigma}^{(0)}$ = LO $\quad$ $\hat{\sigma}^{(1)}$ = NLO $\quad$ $\hat{\sigma}^{(2)}$ = NNLO $\quad$ $\hat{\sigma}^{(3)}$ = N^{3}LO

LO: maximal dependence on scales. Poor convergenge of expansion in α_S
NLO: (usually) good estimate of x-sect
NNLO: good estimate of uncertainty

Factorisation-breaking contributions

- underlying event: we expect that the soft gluon radiation exchanged between the protons fills up the rapidity gap present in the hard interaction, although occasionally the gap might survive
Bjorken's rapidity gap survival probability Bjorken 1992

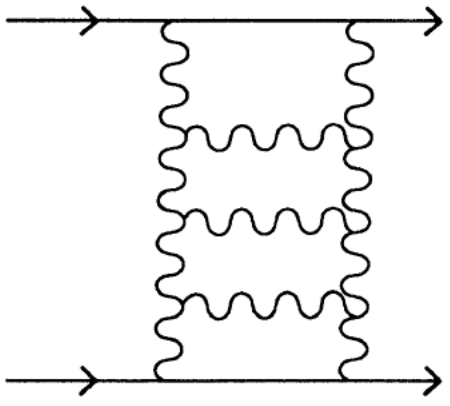
In hadron-hadron collisions, production of Higgs bosons and other color-singlet systems can occur via fusion of electroweak bosons, occasionally leaving a “rapidity gap” in the underlying-event structure. This observation, due to Dokshitzer, Khoze, and Troyan, is studied to see whether it serves as a signature for detection of the Higgs bosons, etc. We find it is a very strong signature at subprocess c.m. energies in excess of a few TeV. The most serious problem with this strategy is the estimation of the fraction of events containing the rapidity gap; most of the time the gap is filled by soft interactions of spectator degrees of freedom. We also study this question and estimate this “survival probability of the rapidity gap” to be of order 5%, with an uncertainty of a factor 3

abstract of PRD47 (1993) 101

Rapidity gaps

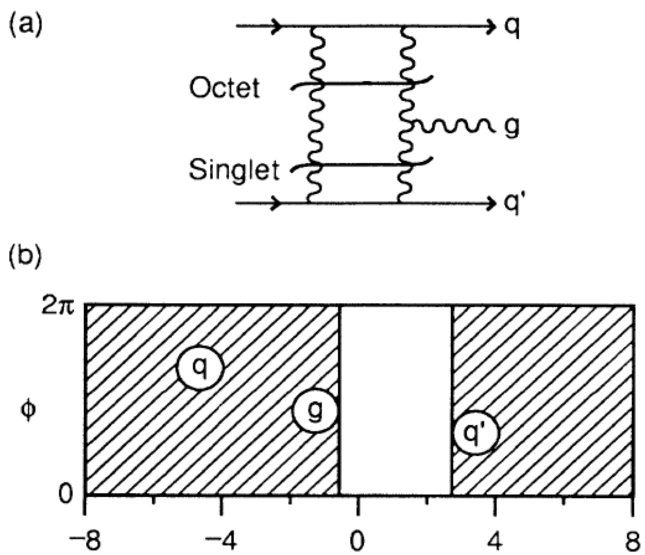
- a rapidity gap is the near-absence of secondary hadrons in a given rapidity interval

- what makes a rapidity gap?
  - an electroweak process, with exchange of bosons in the *t* channel
  - a strong interaction process, with exchange of two gluons in the *t* channel in a singlet configuration (*pomeron*)

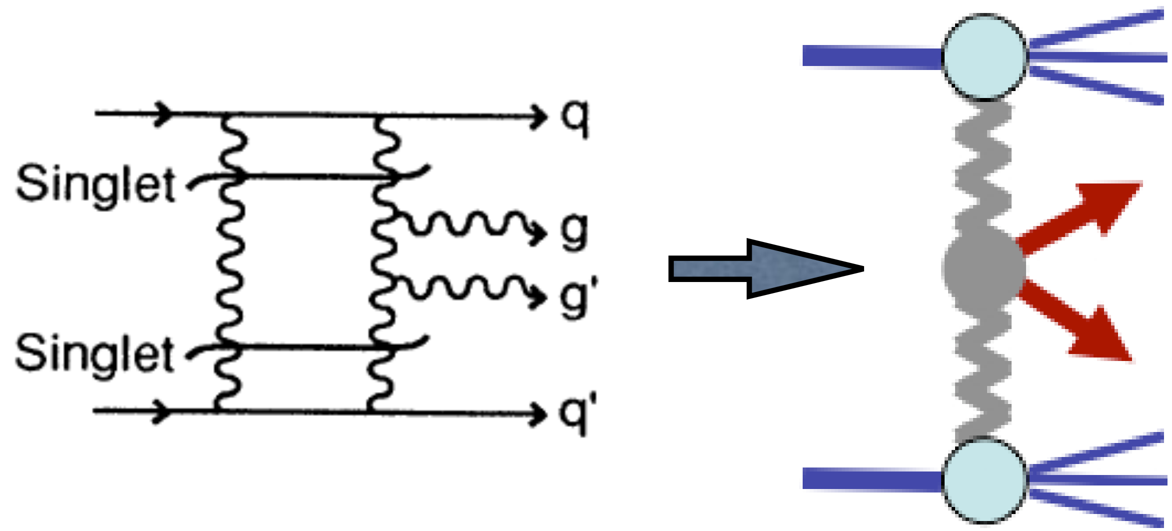


2-gluon ladder

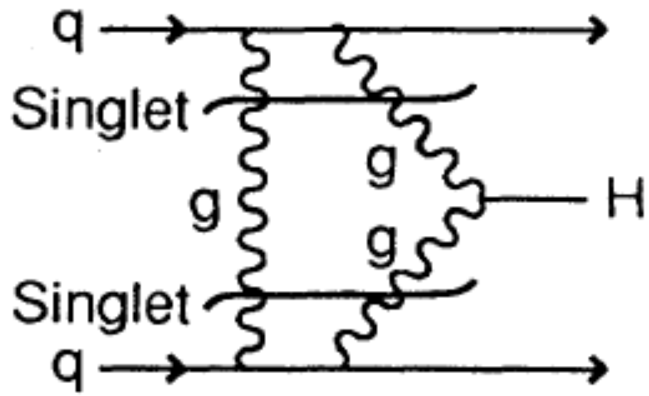
Examples of rapidity gaps



singlet-octet ladder



double pomeron exchange



all drawings taken from PRD47 (1993) 101 from PRD47 (1993) 101

… There are many difficult issues involved. They include the following.

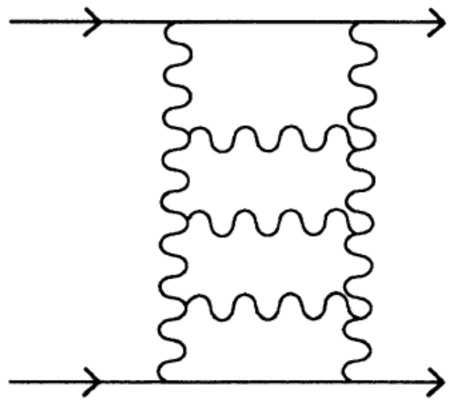
1. How big must the rapidity gaps be in order that multiplicity fluctuations do not mimic their effect?

2. How big are strong-interaction (Pomeron-exchange) backgrounds and how do they scale with energy and p_T ?

3. What fraction of a given electroweak-boson exchange process, as defined at the parton level, really leads to a final state containing the rapidity gap? Most of the time spectator interactions will fill in the gap present at the naive level considered above. We estimate … that the survival probability of the rapidity gap is of order 5%, but there are serious theoretical issues here which need further exploration …

- through a simple leading order (octet exchange) evaluation, Bjorken estimates the probability of a rapidity gap from strong interactions to be about 0.1-1%

In the above considerations, we have uncritically assumed that higher orders in α_s, do not significantly change this result. This is naive, and a proper estimate should include at the least the ladders of exchanged gluons … This is more properly described by the BFKL evolution equation … Qualitatively, the result of these additional contributions is an increase in strength of the qq interaction at very large s, as well as an increase in the relative importance of the color-singlet-exchange contribution … Mueller and Navelet …



Pile-up events

- At the LHC, there is one more player in the game. Every time two tightly packed bunches of protons cross, they generate up to 25-30 primary collisions between protons: *pile-up events*
- In such an environment, it might be very difficult to observe a rapidity gap
- we may re-define the rapidity gap as the absence of low-E_T jets (*minijets*) in the rapidity interval

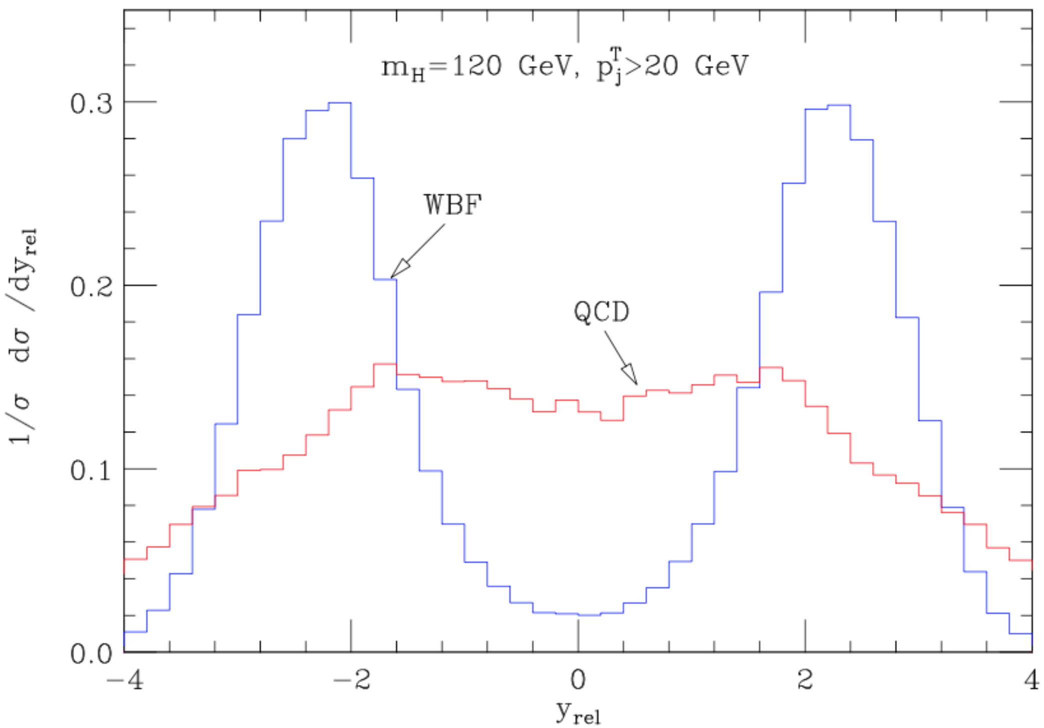
Inter-jet radiation

- we expect that gluon radiation in the t channel be very different for octet *vs.* singlet exchange: gluon radiation from an octet will fill the gap, while a singlet should have no radiation within the gap
- Can the gluon radiation be much softer than the tagging forward jets, but still be hard enough to be modelled through a pQCD computation?
- Firstly, this is an experimental question:
  what is the minimum E_T for which a jet can still be called a jet (and not be a fluctuation from the underlying event)?
- suppose we have established an $E_{T,min}$ of, say, 10-20 GeV.
  About that value, the jets are hard, but not so hard (minijets)

- How do we model an inter-jet radiation of minijets?

- One way is through a fixed-order computation of 3 or more jets, where two jets are the tagging forward jets, while the additional jets model the inter-jet radiation

Frizzo Maltoni VDD 2004



$$y_{\rm rel} = y_{j_3} - \frac{y_{j_1} + y_{j_2}}{2}$$

with inclusive cuts

$$p_{jT} > 20\,\text{GeV} \quad |y_j| < 5 \quad R_{jj} > 0.6$$

plus VBF cuts

$$|y_{j_1} - y_{j_2}| > 4.2 \quad y_{j_1} \cdot y_{j_2} < 0$$

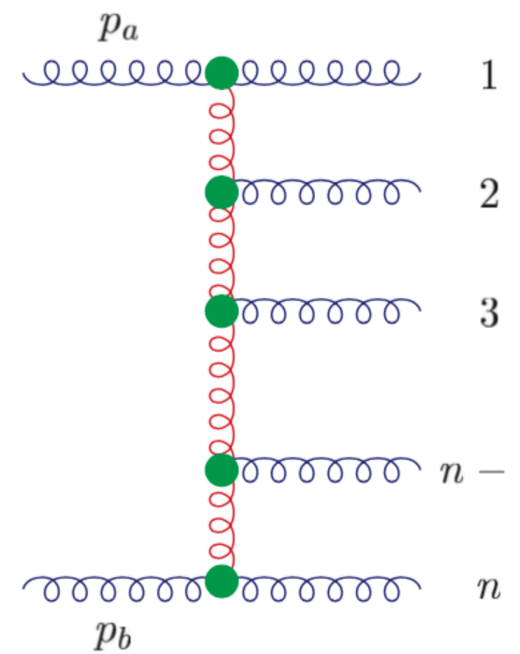
$$\sqrt{s_{j_1 j_2}} > 600\,\text{GeV}$$

- But in order to model two-gluon exchange in a singlet configuration, we need a computation which is at least $O(\alpha_s^2)$ wrt to the leading order, i.e. at least NNLO, and possibly $O(\alpha_s^3)$ or higher, in order to analyse if/how the singlet evolves wrt the octet

- as of now, we have:
  - NNLO computations of $2 \to 2$ processes
  - N^3LO computations of $2 \to 1$ processes

Balitski Fadin Kuraev Lipatov

- In perturbative QCD, in the Regge limit *s* » |*t*|, any scattering process is dominated by gluon exchange in the *t* channel



- BFKL is a resummation of multiple gluon radiation out of the gluon exchanged in the *t* channel

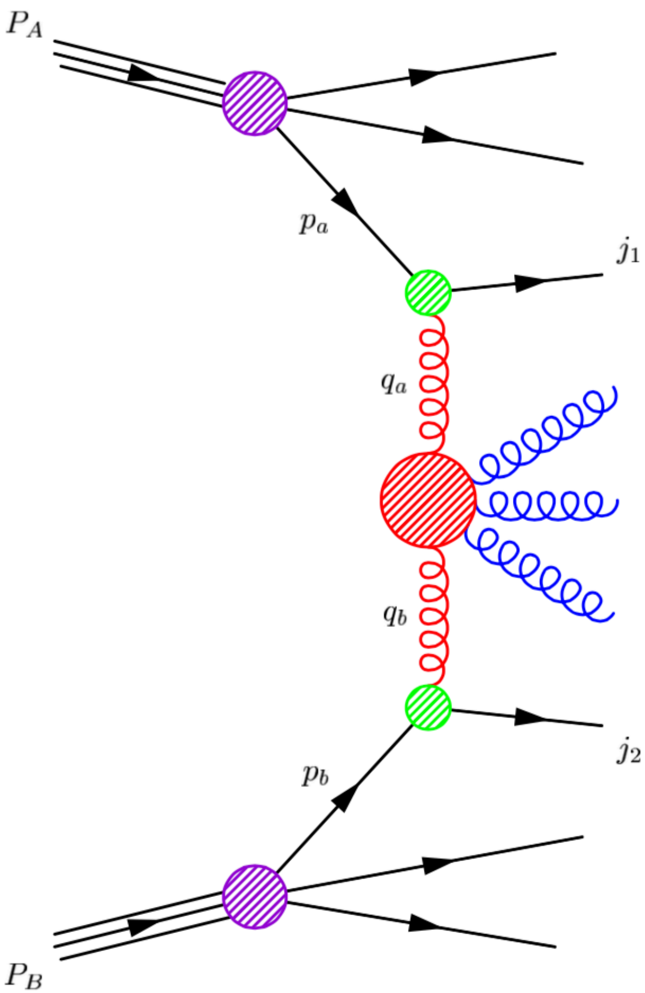
- the resummation yields an integral equation for the evolution of the gluon propagator in 2-dim transverse momentum space

- the Leading Logarithmic (BFKL 1976-77) and Next-to-Leading Logarithmic (Fadin-Lipatov 1998) contributions in log(s/|t|) of the radiative corrections to the gluon propagator in the *t* channel are resummed to all orders in α_s

- the BFKL equation is obtained in the limit of strong rapidity order of the emitted gluons - *multi-Regge kinematics* (MRK)

Mueller-Navelet jets

Mueller Navelet 1987



Dijet production cross section with two tagging jets in the forward and backward directions

$p_a = x_a P_A \quad p_b = x_b P_B$ incoming parton momenta

S: hadron centre-of-mass energy

$s = x_a x_b S$: parton centre-of-mass energy

E_{Tj}: jet transverse energies

$$\Delta y = |y_{j_1} - y_{j_2}| \simeq \log \frac{s}{E_{Tj_1} E_{Tj_2}}$$

is the rapidity interval between the tagging jets

gluon radiation is considered in MRK and resummed through the LL BFKL equation

Mueller-Navelet evaluated the inclusive dijet cross section up to 5 loops

- Solution of the LL BFKL equation is analytic

- The kernel of the LL BFKL equation is also implemented numerically in a Monte Carlo - High Energy Jets (HEJ)

  Andersen Smillie 2009

- However, the LLA is unsatisfactory (α_s is fixed, not running)

- It is known that NLL corrections to the BFKL equation are large, but the solution of the NLL BFKL equation is rather complicated → not used in practice in realistic simulations

One of the most remarkable discoveries in elementary particle physics has been that of the existence of the complex plane …

incipit of *The analytic S-matrix*
Eden Landshoff Olive Polkinghorne 1966

N=4 Super Yang Mills

- maximal supersymmetric theory (without gravity)
  conformally invariant, β fn. = 0
  - spin 1 gluon
    4 spin 1/2 gluinos
    6 spin 0 real scalars
- 't Hooft limit: $N_c \to \infty$ with $\lambda = g^2 N_c$ fixed
  - only planar diagrams
- AdS/CFT duality Maldacena 97
  - large-λ limit of 4dim CFT $\leftrightarrow$ weakly-coupled string theory

    (aka weak-strong duality)

N=4 Super Yang Mills

- amplitudes in N=4 SYM are much simpler than in Standard Model processes
- use N=4 SYM as a computational lab:
  - to learn techniques and tools to be used in Standard Model calculations
  - to learn about the bases of special functions which may occur in the scattering processes

N=4 Super Yang Mills

- In the last years, a huge progress has been made in understanding the analytic structure of the S-matrix of N=4 SYM
- Besides the ordinary conformal symmetry, in the planar limit the S-matrix exhibits a dual conformal symmetry
  Drummond Henn Smirnov Sokatchev 2006
- Accordingly, the analytic structure of the scattering amplitudes is highly constraint
- 4- and 5-point amplitudes are fixed to all loops by the symmetries in terms of the one-loop amplitudes and the cusp anomalous dimension
  Bern Dixon Smirnov 2005; Drummond Henn Korchemsky Sokatchev 2007
- Beyond 5 points, the amplitudes are given in terms of a remainder function R. The symmetries only fix the variables of R (some conformally invariant cross ratios) but not the analytic dependence of R on them

$\mathcal{N}$=4 Super Yang Mills

- The progress in understanding the analytic structure of the *S*-matrix in planar *N*=4 SYM is also due to an improved understanding of the mathematical structures underlying the scattering amplitudes

- *n*-point amplitudes are expected to be iterated integrals on the space of configurations of points in 3-dim projective space $\mathrm{Conf}_n(\mathrm{CP}^3)$, with singularities given by a certain cluster algebra

  Golden Paulos Spradlin Volovich 2014

- The simplest case of iterated integrals are the iterated integrals over rational functions, i.e. the multiple polylogarithms

  Goncharov 2001

$$G(a,\vec{w};z) = \int_0^z \frac{\mathrm{d}t}{t-a} G(\vec{w};t)\,, \qquad G(a;z) = \ln\left(1-\frac{z}{a}\right)$$

- It is thought that maximally helicity violating (MHV) and next-to-MHV (NMHV) amplitudes can be expressed in terms of multiple polylogarithms of uniform transcendental weight

  Arkani-Hamed et al. *Scattering Amplitudes and the Positive Grassmannian* 2012

N=4 Super Yang Mills

- 6-pt MHV and NMHV amplitudes are known analytically up to five loops

  Duhr Smirnov VDD 2009
  Goncharov Spradlin Vergu Volovich 2010
  Dixon Drummond Henn 2011
  Dixon Drummond von Hippel Pennington 2013
  Dixon Drummond Duhr Pennington 2014
  Dixon Drummond Henn 2011
  Dixon von Hippel 2014
  Dixon von Hippel McLeod 2015

- 7-pt MHV amplitudes are known analytically at two loops

  Golden Spradlin 2014

- No analytic result is known beyond seven-pt
  The cluster algebra which defines the multiple polylogarithms
  is infinite starting from eight points

MRK in *N*=4 Super Yang Mills

- In the Euclidean region (where all Mandelstam invariants are negative), amplitudes in MRK factorise completely in terms of building blocks which are expressed in terms of Regge poles and can be determined to all orders through the 4-pt and 5-pt amplitudes.
  Thus the remainder functions *R* vanish at all points — Duhr Glover VDD 08

- After analytic continuation to some regions of the Minkowski space, the amplitudes develops cuts which are described by a dispersion relation for octet exchange, which is similar to the singlet BFKL equation in QCD
  Bartels Lipatov Sabio-Vera 08

- Accordingly, 6-pt amplitudes have been thoroughly examined, both at weak and at strong coupling

- In particular, 6-pt amplitudes can be expressed in terms of single-valued harmonic polylogarithms — Dixon Duhr Pennington 2012

Regge factorisation of the *n*-pt amplitude

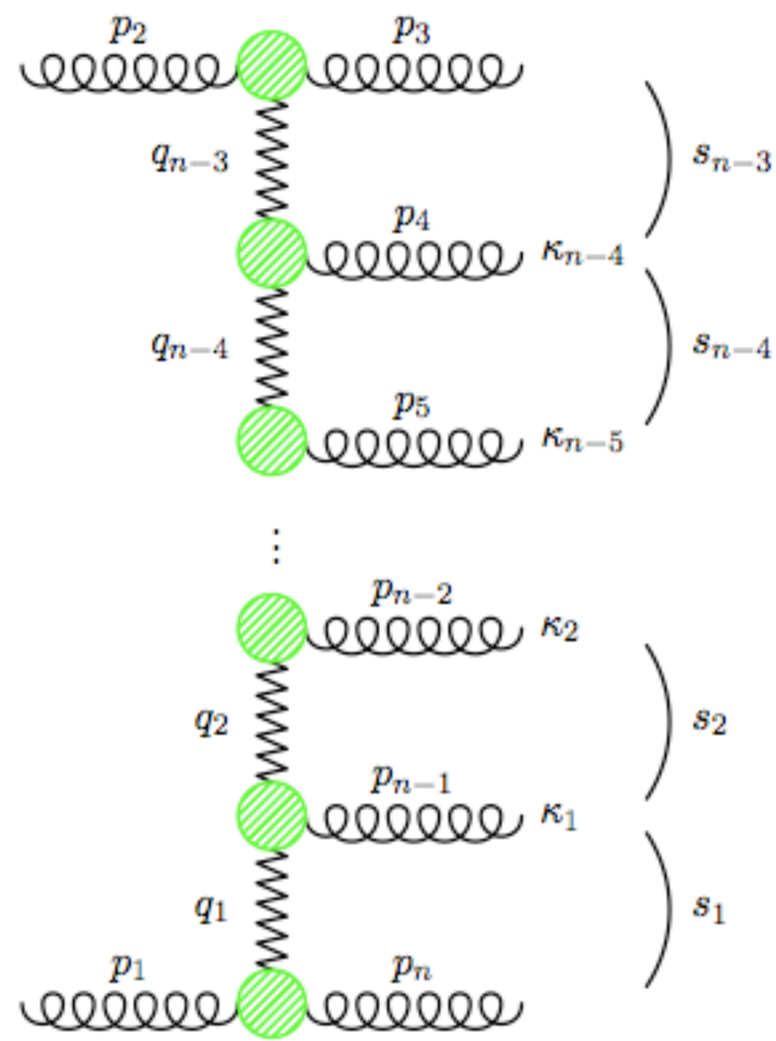
$$m_n(1,2,\ldots,n) = s\,[g\,C(p_2,p_3)]\;\frac{1}{t_{n-3}}\left(\frac{-s_{n-3}}{\tau}\right)^{\alpha(t_{n-3})}[g\,V(q_{n-3},q_{n-4},\kappa_{n-4})]$$

$$\cdots\times\;\frac{1}{t_2}\left(\frac{-s_2}{\tau}\right)^{\alpha(t_2)}[g\,V(q_2,q_1,\kappa_1)]\;\frac{1}{t_1}\left(\frac{-s_1}{\tau}\right)^{\alpha(t_1)}[g\,C(p_1,p_n)]$$

n-pt amplitude in the multi-Regge limit

$$y_3 \gg y_4 \gg \cdots \gg y_n; \qquad |p_{3\perp}| \simeq |p_{4\perp}|\ldots \simeq |p_{n\perp}|$$

$$s \gg s_1, s_2, \ldots, s_{n-3} \gg -t_1, -t_2 \ldots, -t_{n-3}$$



the *l*-loop *n*-pt amplitude can be assembled using the *l*-loop trajectories, vertices and coefficient functions, determined through the *l*-loop 4-pt and 5-pt amplitudes

⇒ in Euclidean space,
no violation of the BDS ansatz can be found in the multi-Regge limit

Duhr Glover VDD 08

Single-valued polylogarithms

- Single-valued functions are real analytic functions on the complex plane

- Because the discontinuities of the classical polylogarithms are known

$$\Delta \mathrm{Li}_n(z) = 2\pi i \frac{\log^{n-1} z}{(n-1)!}$$

one can build combinations of classical polylogarithms
such that all branch cuts cancel on the punctured plane C/{0,1}
(Riemann sphere with punctures)

- An example is the Bloch-Wigner dilogarithm

$$D_2(z) = \mathrm{Im}[\mathrm{Li}_2(z)] + \arg(1-z)\log|z|$$

Single-valued harmonic polylogarithms

- Harmonic polylogarithms (HPLs) are multiple polylogarithms with the root = +1,-1,0

$$H(a,\vec{w};z) = \int_0^z \mathrm{d}t\, f(a;t)\, H(\vec{w};t)$$

$$f(-1;t) = \frac{1}{1+t}\,, \quad f(0;t) = \frac{1}{t}\,, \quad f(1;t) = \frac{1}{1-t} \qquad \{a,\vec{w}\} \in \{-1,0,1\}$$

- Single-valued HPLs (SVHPL) are combinations of HPLs with no branch cuts. Some SVHPLs

weight 1: $\log|z|^2 \qquad \frac{1}{2}\log|z|^2 - \log|1+z|^2$

weight 2: $\mathrm{Li}_2(z) - \mathrm{Li}_2(\bar{z}) + \frac{1}{2}\log|z|^2(\log(1-z) - \log(1-\bar{z}))$

Mueller-Navelet jets and SVHPLs

- The singlet LL BFKL ladder in QCD, and thus the dijet cross section a la Mueller-Navelet, can be expressed in terms of SVHPLs

  Dixon Duhr Pennington VDD 2013

- MN evaluated analytically the inclusive dijet cross section up to 5 loops. We evaluated it analytically up to 13 loops

- Also, we could evaluate analytically the dijet cross section differential wrt the jet transverse energies or the azimuthal angle between the jets (up to 6 loops)

MRK in N=4 Super Yang Mills

- Beyond 6 points, only 2-loop MHV amplitudes were known in MRK

- In MRK and at LLA, the 2-loop n-pt remainder function $R_n^{(2)}$ can be written as a sum of 2-loop 6-pt remainder functions $R_6^{(2)}$

  Prygarin Spradlin Vergu Volovich; Bartels Kormilitzin Lipatov Prygarin 2011

- In MRK and at LLA, we can write all MHV amplitudes at L loops in terms of amplitudes with up to (L+4) points

  Drummond Druc Duhr Dulat Marzucca Papathanasiou Verbeek VDD, to appear

- We show explicitly that the MHV amplitudes at:
  - 2 loops are determined by the 2-loop 6-pt amplitudes
  - 3 loops are determined by the 6- and 7-pt amplitudes through 3 loops
  - 4 loops are determined by the 6-, 7-, and 8-pt amplitudes through 4 loops
  - 5 loops are determined by the 6-, 7-, 8- and 9-pt amplitudes through 5 loops

- we can also write all non-MHV amplitudes up 8 points and 4 loops

MRK in $\mathcal{N}$=4 Super Yang Mills

- the *n*-pt scattering amplitudes in MRK at LLA are single-valued iterated integrals on the moduli space $M_{0,n-2}$ of Riemann spheres with *n* marked points

  Drummond Druc Duhr Dulat Marzucca Papathanasiou Verbeek VDD, to appear

- in fact, single-valued iterated integrals on $M_{0,4}$ are SVHPLs

- the single-valuedness is not related to the conformal symmetry; rather to the fact that the essential degrees of freedom are the 2-dim transverse momenta which have no branch cuts