The issue of the cut in the multi-Regge kinematics of the 6-pt amplitude in $N=4$ SYM, due to the exchange of 2 Reggeised gluons in the octet channel, was described and solved by Bartels, Lipatov and Sabio-Vera in 2008 (BLS)

The Hamiltonian for the octet exchange of 2 Reggeised gluons is

$$H = h^{T(8)}_{12} + \bar{h}^{T(8)}_{12} \qquad w/ \quad h^{T(8)}_{12} = \ln(p_1 p_2) + p_1 \ln(\rho_{12}) \frac{1}{p_1} + p_2 \ln(\rho_{12}) \frac{1}{p_2} - 2\psi(1)$$

which is the transposed of the singlet Hamiltonian of large-N_c QCD (the kernel of the BFKL eq. for singlet exchange has no IR divergences, while the kernel of the dispersion eq. for octet exchange has IR divergences, but they can be isolated so as to deal w/ a reduced singlet-like kernel with no IR divergences).

The transposed Ham. $h^{T(8)}_{12}$ is related to $h^{(0)}$ by a similarity transf.

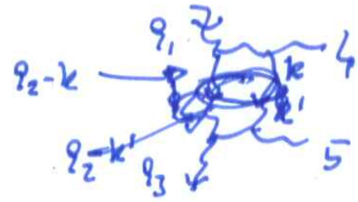
$$h^{(0)}_{12} = \frac{1}{p_1 p_2} h^{T(8)}_{12} p_1 p_2$$

then the hermiticity of H is : $\quad H^{+} = |p_1|^2 |p_2|^2 \; H \; \frac{1}{|p_1|^2 |p_2|^2}$

~~which is compatible with the normalization~~

DLS expressed the discontinuity of the 6-pt amplitude as

$$\frac{1}{u} \mathrm{Im}\, M_{2\to 4} = s_{45}^{\omega(t_2)} \int \frac{d\omega}{2\pi i} \left(\frac{s_{45}}{\mu^2}\right)^{\omega} f_2(\omega)$$



w/
$$f_2(\omega) \propto \int d^2k\, d^2k'\, \Phi_1(k, q_2, q_1)\, G_\omega(k, k', q_2)\, \Phi_2(k', q_2, q_3)$$

where Φ_1, Φ_2 are functions (called impact factors) describing the emission on the ends of the BFKL-like ladder, and G_ω is the Green's fn. accounting for the BFKL-like ladder (which features the cut),

$$G_\omega(k, k'; q) = \frac{1}{2\pi^2} \frac{|q|^2}{|k|^2 |q-k|^2} \sum_n \int d\nu\, \frac{f^*_{\nu n}(k', q)\, f_{\nu n}(k, q)}{\omega - \omega_{\nu, n}}$$

w/ $\omega_{\nu,n} = -\frac{\alpha_s N_c}{4\pi} E_{m,\bar{m}}$ the eigenvalue

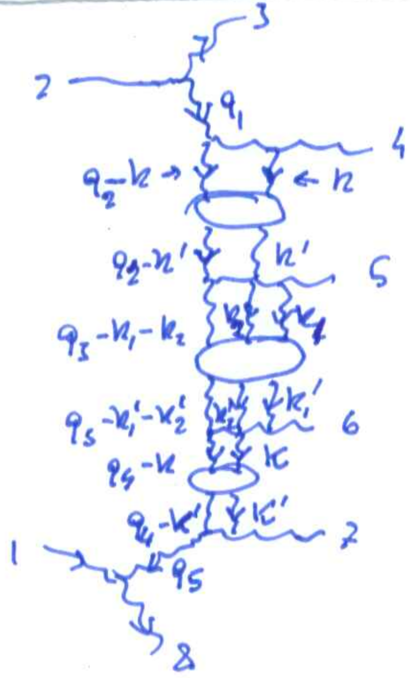
$$E_{m,\bar{m}} = \epsilon_m + \epsilon_{\bar{m}} \qquad \epsilon_m = \psi\left(m - \tfrac{1}{2}\right) + \psi\left(\tfrac{1}{2} - m\right) - 2\psi(1)$$

and as usual
$$m = \frac{n+1}{2} + i\nu$$
$$\bar{m} = \frac{1-n}{2} + i\nu$$

the eigenfunctions are: $f_{\nu,n}(q, k) = \left(\frac{k}{q-k}\right)^{m-1/2} \left(\frac{k^*}{q^*-k^*}\right)^{\bar{m}-\frac{1}{2}}$

the impact factors (discussed in Duhr's lectures) are given by

$$\chi \propto \int d^2k\, \Phi_1(k, q_2, q_1)$$



The double discontinuity of the 8-pt ampl.

$$\mathrm{Disc}\left(\mathrm{Disc}\, M_{2\to 8}\right) \propto s_{45}^{\omega(t_2)}\, s_{56}^{\omega(t_3)}\, s_{67}^{\omega(t_4)}$$

$$\cdot \int \frac{d\omega_2}{(2\pi i)} \frac{d\omega_3}{2\pi i} \frac{d\omega_4}{2\pi i} \left(\frac{s_{45}}{\mu^2}\right)^{\omega_2} \left(\frac{s_{56}}{\mu^2}\right)^{\omega_3} \left(\frac{s_{67}}{\mu^2}\right)^{\omega_4} \cdot$$

$$\cdot\, f_3(\omega_2, \omega_3, \omega_4)$$

shorthand $\omega_i = \omega(t_i)$

in the region

$$s_{12} > 0 \qquad s_{34}, s_{78}, s_{45}, s_{67} < 0$$

$$s_{4567} > 0 \quad , \quad s_{56} > 0$$

would be characterised by a 3-Reggeised gluon cut in the partial wave fn. f_3,

$$f_3(\omega_2, \omega_3, \omega_4) \propto \int d^2k\, d^2k'\, d^2k_1\, d^2k_2\, d^2k_1'\, d^2k_2'\, d^2K\, d^2K'$$

$$\Phi(k, q_2, q_1)\, G_{\omega_2}(k, k', q_2)\, \Phi^{(3)}(k', k_1, k_2, q_3, q_2)$$

$$G^{(3)}_{\omega_3}(k_1, k_2, k_1', k_2', q_3)\, \Phi^{(3)}(K, k_1', k_2', q_4, q_3)$$

$$G_{\omega_4}(K, K', q_4)\, \Phi(K', q_5, q_4)$$

for the impact factors,

$$\int d^2k\, \Phi(k, q_2, q_1) \to \chi$$

$$\int d^2k'\, d^2k_1\, d^2k_2\, \Phi^{(3)}(k', k_1, k_2, q_3, q_2) \to \chi^{(3)}$$

A ladder of N reggeised gluons with only nearest-neighbour interactions in the octet t-channel exchange of a $(2+2N)$-pt amplitude in planar $\mathcal{N}=4$ SYM in the high-energy limit will be characterised by the Hamiltonian

$$H = h^{T(8)} + \bar{h}^{T(8)} \qquad w/ \quad h^{T(8)} = \ln \frac{p_1 p_N}{q^2} + \sum_{k=1}^{N-1} h^{T(8)}_{k,k+1} \qquad q = \sum_{k=1}^{N} p_k$$

$$\text{and} \quad h^{T(8)}_{k,k+1} = \ln(p_k p_{k+1}) + p_k \ln(\rho_{k,k+1}) \frac{1}{p_k} + p_{k+1} \ln(\rho_{k,k+1}) \frac{1}{p_{k+1}} - 2\psi(1)$$

Apart from the transposition of the nearest-neighbour Hamiltonian $h^{T(8)}_{k,k+1}$, $h^{T(8)}$ differs from the singlet $h^{(0)}$ of large-N_c QCD by the b.c.

$$h^{(0)}_{N,1} \to \ln \frac{p_1 p_N}{q^2}$$

(there are IR divergences in Regge trajectories of particle 1 and n)

The hermiticity $H^\dagger = \prod_{i=1}^{N} |p_i|^2 \; H \left(\prod_{i=1}^{N} |p_i|^2 \right)^{-1}$

implies that the wavefn's are normalised as

$$\|\psi\|^2 = \int \prod_{k=1}^{N-1} d^2 p_k \; \psi^* \left(\prod_{j=1}^{N} |p_j|^2 \right)^{-1} \psi$$

One can then use the duality transformation:

$$P_1 = x_0 - x_1 \qquad P_i = x_{i-1} - x_i \qquad q = \sum_{i=1}^{N} P_i = x_0 - x_N \qquad \rho_{i,i+1} = i\frac{\partial}{\partial x_i} = i\partial_i$$

then, after various algebra manipulations, one can write the holomorphic Hamilt. as,

$$h^{T(8)} = -\ln x_{0,N}^2 + \ln(x_{0,1}^2\partial_1) + \ln(x_{N-1,N}^2\partial_{N-1}) - 2\psi(1) + \sum_{k=1}^{N-2} h_{k,k+1}$$

$$\text{w/ } h_{k,k+1} = \ln(x_{k,k+1}^2\partial_k) + \ln(x_{k,k+1}^2\partial_{k+1}) - \ln x_{k,k+1}^2$$

For $N=2$, one gets (there's no $h_{k,k+1}$ to consider)

$$h^{T(8)} = -\ln x_{0,2}^2 + \ln(x_{01}^2\partial_1) + \ln(x_{12}^2\partial_1) - 2\psi(1)$$

which is also a form of h obtained by BLS (2008)

Using the invariance $\qquad x_k \to \frac{ax_k+b}{cx_k+d}$

one can set $x_0 = 0 \qquad x_N \to \infty$

then h simplifies to

$$\boxed{h^{T(8)} = \ln(x_1^2\partial_1) + \ln(\partial_{N-1}) - 2\psi(1) + \sum_{k=1}^{N-2} h_{k,k+1}}$$

to go back to initial variables, use $\qquad x_k \to \frac{x_k - x_0}{x_k - x_N} = \frac{\sum_{i=1}^{k} P_i}{q - \sum_{i=1}^{k} P_i}$

for $N=2 \Rightarrow x_1 \to \frac{P}{q-P}$

For $N=3$, the arguments of the wave fn. are

$$x_1 \to \frac{p_1}{q-p_1} \qquad x_2 \to \frac{p_1+p_2}{q-(p_1+p_2)}$$

the holomorphic Hamiltonian is

$$h^{T(3)} = \ln(x_1^2\partial_1) + \ln(\partial_2) - 2\psi(1) + \ln(x_{1,2}^2\partial_1) + \ln(x_{1,2}^2\partial_2) - \ln x_{12}^2$$

— o —

Back to N reggeised gluons:

consider the Lax operator for the n^{th} gluon, constructed on the generators of the conformal group

$$L_{n,a}(u) = u\, \mathbb{1}_n \otimes \mathbb{1}_a + i M_n^a \otimes \sigma^a$$

$$= \begin{pmatrix} u + i x_n \partial_n & i\partial_n \\ -i x_n^2 \partial_n & u - i x_n \partial_n \end{pmatrix}$$

$$M_n^+ = -x_k^2 \partial_k$$

$$M_n^3 = x_k \partial_k$$

$$M_n^- = \partial_k$$

and the monodromy matrix

$$T_a(u) = \begin{pmatrix} A(u) & B(u) \\ C(u) & D(u) \end{pmatrix} = L_{1,a}(u) \cdots L_{N,a}(u) \qquad \text{on } N-1 \text{ pairs}$$

and the pair hamiltonian

$$h_{n,n+1} = \ln(x_{n,n+1}^2 \partial_n) + \ln(x_{n,n+1}^2 \partial_{n+1}) - \ln x_{n,n+1}^2 .$$

Lipatov (2009) uses the conf. symmetry of the pair hamiltonian

$$[h_{n,n+1}, M^a_{n,n+1}] = 0 \qquad \text{w/ } M^a_{n,n+1} = M_n^a + M_{n+1}^a$$

and the commutation rel :

$$\left[h_{n,n+1}, \left[M^2_{n,n+1}, M^a_n - M^a_{n+1}\right]\right] = 4\left(M^a_n - M^a_{n+1}\right)$$

which can be proven using the fact that $h_{n,n+1}$ is diagonal

$$h_{n,n+1}\,|m_{n,n+1}\rangle = \left(\psi\left(m_{n,n+1}-\tfrac{1}{2}\right)+\psi\left(\tfrac{1}{2}-m_{n,n+1}\right)-2\psi(1)\right)|m_{n,n+1}\rangle$$

in the conformal weight rep :

$$M^2_{n,n+1}\,|m_{n,n+1}\rangle = m_{n,n+1}\left(m_{n,n+1}-1\right)|m_{n,n+1}\rangle$$

to show that : $\left[L_n(u)L_{n+1}(u), h_{n,n+1}\right] = -i\left(L_n(u) - L_{n+1}(u)\right)$

which implies that :

$$\left[T_0(u), \sum_{n=1}^{N-2} h_{n,n+1}\right] = iL_2L_3\cdots L_{N-1} - iL_1L_2\cdots L_{N-2}$$

then ~~one~~ can ~~write~~ check that for the T_{22} entry :

$$\left[D(u), \ln(x_1^2\partial_1)+\ln(\partial_{N-1})\right] = (0,1)\left[T(u), \ln(x_1^2\partial_1)+\ln(\partial_{N-1})\right]\begin{pmatrix}0\\1\end{pmatrix}$$

$$= -i(0,1)\left(L_2L_3\cdots L_{N-1} - L_1L_2\cdots L_{N-2}\right]\begin{pmatrix}0\\1\end{pmatrix}$$

~~and~~ because $h^{T(8)} = \ln(x_1^2\partial_1) + \ln(\partial_{N-1}) - 2\psi(1) + \sum_{n=1}^{N-2} h_{n,n+1}$

the relations above imply :

$\left[D(u), h^{T(8)}\right] = 0$, so $D(u)$ in an IEM in involution w/ $h^{T(8)}$

using the explicit 2×2 matrix repr. of the Lax operator,

one can ~~explicit~~ display $D(u)$ as a power series in u,

$$D(u) = (0,1)\big(L_1(u)\,L_2(u)\,\cdots\,L_{N-1}(u)\big)\begin{pmatrix}0\\1\end{pmatrix}$$

$$= \sum_{k=0}^{N-1} q_k\, u^{N-1-k} \qquad \text{w/} \quad q_0 = 1$$

$$q_1 = -i \sum_{k=1}^{N-1} x_k \partial_k$$

$$\vdots$$

$$q_k = \sum_{0<r_1<r_2<\cdots r_k<N} x_{r_1} \prod_{s=1}^{k-1} x_{r_s, r_{s+1}} \prod_{t=1}^{k} i\partial_{r_t}$$

in particular, $q_{N-1} = -i^{N-1} x_1 \prod_{s=1}^{N-2} x_{s,s+1} \prod_{t=1}^{N-1} \partial_t$

For $N=2$, there is the only IEM:

$q_1 = -i x_1 \partial_1 = -i M_1^3$ which is the z-comp. of the Casimir

the wavefu. norm. $\|\psi\|^2 = \int \frac{d^2 x_1}{x_1} |\psi|^2$

w/ eigenfu's $\psi^{(2)}_{m,\bar m} = x_1^{m-\frac12} (x_1^*)^{\bar m - \frac12}$ which we already have seen

For $N=3$, the T_{22} entry of the Lax operator is

$$D_3(u) = u^2 - iu\left(x_1\partial_1 + x_2\partial_2\right) + x_1 x_{1,2}\partial_1\partial_2 \qquad \text{(check it!)}$$

The IEM are:

$$q_1 = -i\left(x_1\partial_1 + x_2\partial_2\right) \quad \text{which is the } z\text{-component of the Casimir}$$

$$q_2 = x_1 x_{1,2}\partial_1\partial_2$$

the norm. condition on the wavefn. is

$$\|\psi\|^2 \equiv \int \frac{d^2x_1\, d^2x_2}{|x_1|^2 |x_{1,2}|^2} |\psi|^2$$

For the eigenvalue eq. $q_2\psi = x_1 x_{1,2}\partial_1\partial_2\psi = \lambda\psi$

we look for a solution of the type $\psi_m^{(3)} = x_2^{m-\frac{1}{2}} f\left(\frac{x_2}{x_1}\right)$

this leads to the diff. eq. (show it!)

$$\left[x(1-x)\frac{\partial^2}{\partial x^2} + \left(m+\frac{1}{2}\right)(1-x)\frac{\partial}{\partial x} + \lambda\right] f(x) = 0 \quad \text{w/} \quad x = \frac{x_2}{x_1}$$

Proof: do the transf. of variables from the set x_1, x_2 to the set $\frac{x_2}{x_1}, x_2$

then
$$\frac{\partial}{\partial x_1} = \frac{\partial x}{\partial x_1}\frac{\partial}{\partial x} = -\frac{x_2}{x_1^2}\frac{\partial}{\partial x} = -\frac{x^2}{x_2}\frac{\partial}{\partial x}$$

$$\frac{\partial}{\partial x_2} = \frac{\partial x}{\partial x_2}\frac{\partial}{\partial x} + \frac{\partial}{\partial x_2} = \frac{1}{x_1}\frac{\partial}{\partial x} + \frac{\partial}{\partial x_2} = \frac{x}{x_2}\frac{\partial}{\partial x} + \frac{\partial}{\partial x_2}$$

so
$$x_1(x_1 - x_2)\frac{\partial}{\partial x_1}\frac{\partial}{\partial x_2} = \frac{x_2}{x}\left(\frac{x_2}{x} - x_2\right)\left(-\frac{x^2}{x_2}\frac{\partial}{\partial x}\right)\left(\frac{x}{x_2}\frac{\partial}{\partial x} + \frac{\partial}{\partial x_2}\right)$$

$$= -x_2(1-x)\left(\frac{1}{x_2}\frac{\partial}{\partial x} + \frac{x}{x_2}\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial x}\frac{\partial}{\partial x_2}\right)$$

$$= -(1-x)\left(\frac{\partial}{\partial x} + x\frac{\partial^2}{\partial x^2} + x_2\frac{\partial}{\partial x}\frac{\partial}{\partial x_2}\right)$$

For $N=3$, the monodromy matrix is

$$T(u) = L_1(u)\,L_2(u) = \begin{pmatrix} u_1 + i x_1\partial_1 & i\partial_1 \\ -i x_1^2\partial_1 & u - i x_1\partial_1 \end{pmatrix} \begin{pmatrix} u + i x_2\partial_2 & i\partial_2 \\ -i x_2^2\partial_2 & u - i x_2\partial_2 \end{pmatrix}$$

the T_{22} entry is

$$\begin{aligned} T_{22} \equiv \Delta(u) &= -i x_1^2\partial_1(i\partial_2) + (u - i x_1\partial_1)(u - i x_2\partial_2) \\ &= x_1^2\partial_1\partial_2 + u^2 - iu(x_1\partial_1 + x_2\partial_2) - x_1 x_2\partial_1\partial_2 \\ &= u^2 - iu(x_1\partial_1 + x_2\partial_2) + x_1 x_{1,2}\partial_1\partial_2 \end{aligned}$$

so the eigenvalue eq. becomes

$$\left[(1-x)\left(\frac{\partial}{\partial x} + x\frac{\partial^2}{\partial x^2} + x_2 \frac{\partial}{\partial x_2}\frac{\partial}{\partial x}\right) + \lambda\right]\psi_m^{(3)} = 0$$

since we try $\psi_m^{(3)} = x_2^{m-\frac{1}{2}} f\left(\frac{x_2}{x_1}\right)$ then $x_2 \frac{\partial}{\partial x_2} x_2^{m-\frac{1}{2}} = \left(m-\frac{1}{2}\right) x_2^{m-\frac{1}{2}}$

and the eigenvalue eq. is reduced to:

$$\left[x(1-x)\frac{\partial^2}{\partial x^2} + \left(m+\frac{1}{2}\right)(1-x)\frac{\partial}{\partial x} + \lambda\right] f(x) = 0$$ ✓

which is a hypergeom. eq. of type:

$$x(1-x)\frac{d^2}{dx^2}F + \left[C(a+b+1)x\right]\frac{dF}{dx} - abF = 0$$

w/ $F = {}_2F_1(a,b;c;x) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n}$ w/ $(a)_n = a(a+1)\cdots(a+n-1)$

the solutions are:

$$f_1(x) = {}_2F_1(a_1, a_2; 1+a_1+a_2; x)$$

$$f_2(x) = x^{a_1+a_2}\, {}_2F_1(-a_2, -a_1; 1-a_1-a_2; x)$$

w/ $\begin{cases} a_1 + a_2 = m - \frac{1}{2} \\ a_1 a_2 = -\lambda \end{cases}$

After requiring SV-ness at $x = 0, 1, \infty$, Lipatov can write the eigenfunctions for 3 reggeized gluons a 2-fold integral

$$\psi_{m,\bar{m}}^{(3)} \propto x_2^{m-\frac{1}{2}} (x_2^*)^{\bar{m}-\frac{1}{2}} \int \frac{d^2y}{|y|^2}\, y^{-a_2} (y^*)^{-\bar{a}_2} \left(\frac{y-1}{y-x}\right)^{a_1} \left(\frac{y^*-1}{y^*-x^*}\right)^{\bar{a}_1} \qquad x = \frac{x_2}{x_1}$$

Alternatively, Lipatov follows Sklyanin's approach, i.e. consider the $T_{12}(u) \equiv B(u)$ entry of the monodromy matrix

$$B(u) = +\sum_{k=0}^{N-2} b_k u^{N-2-k} \qquad w/\ b_0 = P_{N-1} = \sum_{i=1}^{N-1} P_i = i\sum_{k=1}^{N-1} \partial_i$$

the operators B w/ different spectral parameters commute

$$[B(u), B(v)] = 0$$

Then one can write $B(u) = P_{N-1} \prod_{k=1}^{N-2} (u - \hat{b}_k)$ in terms of the zeroes $\hat{b}_k$

and therefore $[\hat{b}_r, \hat{b}_s] = 0$ and $[\hat{b}_r, P_{N-1}] = 0$

In Sklyanin's approach, the wave-fn. is written as the product of the ground state, defined through $C(u)\Psi_0 = 0$ w/ $\Psi_0 = \prod_{k=1}^{N-1} z_k^{-2}$, and of the Baxter fn's, defined through the eigenvalues of the transfer matrix.

~~For N=3~~ Furthermore, in Sklyanin's approach, one works w/ the transposed entries of the monodromy matrix:

$$T^T(u) \equiv L_1^T(u) L_2^T(u) \cdots L_{N-1}^T(u) \qquad w/\ L_n^T(u) = \begin{pmatrix} u + i\partial_n x_n & i\partial_n \\ -i\partial_n x_n^2 & u - i\partial_n x_n \end{pmatrix}$$

For $N=3$, $$T^T(u) = L_1^T L_2^T = \begin{pmatrix} u + i\partial_1 x_1 & i\partial_1 \\ -i\partial_1 x_1^2 & u - i\partial_1 x_1 \end{pmatrix} \begin{pmatrix} u + i\partial_2 x_2 & i\partial_2 \\ -i\partial_2 x_2^2 & u - i\partial_2 x_2 \end{pmatrix}$$

so $$B(u) = (u + i\partial_1 x_1) i\partial_2 + i\partial_1 (u - i\partial_2 x_2)$$

$$= iu(\partial_1 + \partial_2) - \partial_1\partial_2(x_1 - x_2)$$

$$= \quad '' \quad -\partial_1\partial_2 x_{1,2}$$

Then $B(u) = P_{N-1} \prod_{k=1}^{N-2} (u - \hat{b}_k)$

for $N=3$ implies that $\quad iu(\partial_1 + \partial_2) - \partial_1\partial_2 x_{1,2} = i(\partial_1 + \partial_2)(u - \hat{b}_1)$

whose formal sol. is $\quad \hat{b}_1 = -i \dfrac{\partial_1 \partial_2}{\partial_1 + \partial_2} x_{1,2}$

The operator B is diagonalised easily in the momentum representation, for which $i\partial_k f = p_k f$, so we replace $\quad i\dfrac{\partial}{\partial x_k} \to p_k \quad x_k \to -i\dfrac{\partial}{\partial p_k}$

and the eq. for B becomes the eigenvalue eq.:

$$\left[u(p_1 + p_2) - i p_1 p_2 \left(\frac{\partial}{\partial p_1} - \frac{\partial}{\partial p_2}\right)\right] f = (p_1 + p_2)(u - b_1) f$$

and thus $\quad i p_1 p_2 \left(\dfrac{\partial}{\partial p_1} - \dfrac{\partial}{\partial p_2}\right) f = P b_1 f \qquad w/ \quad P = p_1 + p_2$

Because the norm. condition ~~fixes~~ requires that the wavefu.

$\Psi \sim (p_1 + p_2)^{-(a_1 + a_2)}$, then we look for a solution of type

$f = \chi(p_1 + p_2) \tilde{f}\left(\dfrac{p_2}{p_1}\right)$

so we transform from the variables p_1, p_2 to $\quad P = p_1 + p_2 \quad y = \dfrac{p_2}{p_1}$

Inverting, $\quad p_1 = \dfrac{P}{1+y} \quad p_2 = \dfrac{y}{1+y} P$

$$\frac{\partial}{\partial p_1} = \frac{\partial P}{\partial p_1}\frac{\partial}{\partial P} + \frac{\partial y}{\partial p_1}\frac{\partial}{\partial y} = \frac{\partial}{\partial P} - \frac{p_2}{p_1^2}\frac{\partial}{\partial y} = \frac{\partial}{\partial P} - \frac{y(1+y)}{P}\frac{\partial}{\partial y}$$

$$\frac{\partial}{\partial p_2} = \frac{\partial P}{\partial p_2}\frac{\partial}{\partial P} + \frac{\partial y}{\partial p_2}\frac{\partial}{\partial y} = \frac{\partial}{\partial P} + \frac{1}{p_1}\frac{\partial}{\partial y} = \frac{\partial}{\partial P} + \frac{1+y}{P}\frac{\partial}{\partial y}$$

then the eq. for B : $i p_1 p_2 \left(\frac{\partial}{\partial p_1} - \frac{\partial}{\partial p_2} \right) f = (p_1+p_2) b_1 f$

becomes $\quad i \frac{y}{\cancel{(1+y)^2}} \mathcal{P}^{\cancel{2}} \left(- \frac{\cancel{(1+y)^2}}{\cancel{\mathcal{P}}} \frac{\partial}{\partial y} \right) f = \mathcal{P} b_1 f$

which is reduced to $\quad -iy \frac{\partial}{\partial y} f = b_1 f$

then if we look for a solution of type $f = \chi(\mathcal{P}) \bar{f}(y)$

$\chi(\mathcal{P})$ is arbitrary, and $\quad y \frac{d\bar{f}}{dy} = i b_1 \bar{f} \Rightarrow \bar{f}(y) = y^{ib_1} = \left(\frac{p_2}{p_1} \right)^{ib_1}$

so the sol. is $f(p_1, p_2) = \chi(p_1+p_2) \left(\frac{p_2}{p_1} \right)^{ib_1}$ w χ arbitrary

Let us look then at the T_{22} entry of the transposed monodromy matrix

$$D(u) = u^2 - iu(\partial_1 x_1 + \partial_2 x_2) + \partial_1 \partial_2 x_1 x_{1,2}$$

and let's write the eigenvalue eq. $\partial_1 \partial_2 x_1 x_{1,2} \psi = \lambda \psi$

in momentum repr : $p_1 p_2 \frac{\partial}{\partial p_1} \left(\frac{\partial}{\partial p_1} - \frac{\partial}{\partial p_2} \right) \psi = \lambda \psi$

we'll look for a solution in the form : $\psi = (p_1+p_2)^{-(\ell_1+\ell_2)} \eta(y) \quad y = \frac{p_2}{p_1}$

let's transform again from the variables p_1, p_2 to $\mathcal{P}, y$ w/ $\mathcal{P} = p_1 + p_2$

then we get:

$$\frac{y}{(1+y)^2} \mathcal{P}^2 \left(\frac{\partial}{\partial \mathcal{P}} - \frac{y(1+y)}{\mathcal{P}} \frac{\partial}{\partial y} \right) \left(- \frac{(1+y)^2}{\mathcal{P}} \frac{\partial}{\partial y} \right) \psi = \lambda \psi$$

$$\frac{y}{(1+y)^2} \mathcal{P}^2 \left(\frac{(1+y)^2}{\mathcal{P}^2} \frac{\partial}{\partial y} - \frac{(1+y)^2}{\mathcal{P}} \frac{\partial}{\partial \mathcal{P}} \frac{\partial}{\partial y} + \frac{2y(1+y)^2}{\mathcal{P}^2} \frac{\partial}{\partial y} + \frac{y(1+y)^3}{\mathcal{P}^2} \frac{\partial^2}{\partial y^2} \right) \psi = \lambda \psi$$

which reduces to, using $\lambda = -a_1 a_2$,

$$\left(y^2(1+y)\frac{\partial^2}{\partial y^2} - y\rho\frac{\partial}{\partial \rho}\frac{\partial}{\partial y} + (1+2y)y\frac{\partial}{\partial y} + a_1 a_2 \right)\psi = 0$$

since we look for a solution of type: $\psi = \rho^{-(a_1+a_2)}\eta(y)$

we get the eigenvalue eq. for η: $\rho\frac{\partial}{\partial\rho}\psi = -(a_1+a_2)\psi$

$$\left(y^2(1+y)\frac{\partial^2}{\partial y^2} + (1+a_1+a_2+2y)y\frac{\partial}{\partial y} + a_1 a_2 \right)\eta(y) = 0$$

again, this is a hypergeom. eq. with solutions:

$$\eta_1(y) = \frac{1}{y}\,{}_2F_1\left(1-a_1, 1-a_2; 2; -\frac{1}{y}\right)$$ which can be transformed to ${}_2F_1(-y)$

$$\eta_2(y) = -\frac{\Gamma(-a_1)\Gamma(a_2)}{\Gamma(1+a_2-a_1)}\, y^{-a_1}\,{}_2F_1(-a_1, 1-a_1; 1+a_2-a_1; -y)$$

Again, using the sv-ness of the ~~wave~~ fn. at 0, 1, ∞ and an integral repr. of the hypergeom. fn., Lipatov writes the total wavefn. as

$$\psi(\vec{\rho}_1, \vec{\rho}_2) = (\rho_1+\rho_2)^{-(a_1+a_2)}(\rho_1^*+\rho_2^*)^{-(\bar{a}_1+\bar{a}_2)}\phi(\vec{y})$$

w/ $$\phi(\vec{y}) = \int d^2t\left(\frac{1}{ty}+1\right)^{a_1}\left(\frac{1}{t^*y^*}+1\right)^{\bar{a}_1}(1-t)^{a_2-1}(1-t^*)^{\bar{a}_2-1}$$

Introducing a Fourier-Mellin transform of $\phi(\vec{y})$

$$\phi(u,\bar{u}) = \int\frac{d^2y}{|y|^2}\, y^{-iu}(y^*)^{-i\bar{u}}\phi(\vec{y})$$ w/ $-iu = i\nu_u + \frac{N_u}{2}$

$-i\bar{u} = i\nu_u - \frac{N_u}{2}$

where $\phi(u,\bar{u})$ is a product of ratios of Γ fu's:

$\Psi(\vec{\rho}_1,\vec{\rho}_2)$ is then written by transforming back

$$\Psi(\vec{\rho}_1,\vec{\rho}_2) = (\rho_1+\rho_2)^{-(a_1+a_2)}(\rho_1^*+\rho_2^*)^{-(\bar{a}_1+\bar{a}_2)}\int_{-\infty}^{\infty} d\nu_\mu \sum_{N_\mu=-\infty}^{\infty} \phi(u,\bar{u}) \left(\frac{\rho_2}{\rho_1}\right)^{iu} \left(\frac{\rho_2^*}{\rho_1^*}\right)^{i\bar{u}}$$

i.e. the wavefu of 3 reggeized gluons through a Fourier-Mellin Transform