Mixed QCD-EW Higgs production & Higgs precision studies

Vittorio Del Duca ETH Zürich & U. Zürich & INFN

Brookhaven 25 March 2022

Flowing into the future: Particle Jets in Quantum Field Theory and Phenomenology

Higgs production at LHC

In proton collisions, the Higgs boson is produced mostly via gluon fusion The gluons do not couple directly to the Higgs boson For matter, the coupling is mediated by a heavy quark loop The largest contribution comes from the top loop The production mode is (roughly) proportional to the top Yukawa coupling yt

QCD NLO corrections (for any heavy quark mass)

Djouadi Graudenz Spira Zerwas 1991-1995

Η



QCD NLO corrections are about 100% larger than leading order

QCD NNLO corrections are known for the top-quark loop only

Czakon Harlander Klappert Niggetiedt 2021



real radiation



K. Ellis Hinchliffe Soldate van der Bij 1988

virtual corrections



Djouadi Graudenz Spira Zerwas 1993 Anastasiou Beerli Bucherer Daleo Kunszt 2006 Aglietti Bonciani Degrassi Vicini 2006 Anastasiou Deutschmann Schweitzer 2020

in terms of Harmonic Polylogarithms (HPL)

Polylogarithms

Euler 1768 Spence 1809

$$\begin{split} H(a,\vec{w};z) &= \int_0^z \mathrm{d}t\,f(a;t)\,H(\vec{w};t) \qquad \qquad f(-1;t) = \frac{1}{1+t}\,, \quad f(0;t) = \frac{1}{t}\,, \quad f(1;t) = \frac{1}{1-t} \\ \text{with} \qquad \{a,\vec{w}\} \in \{-1,0,1\} \qquad \qquad \qquad \text{Remiddi Vermaseren 1999} \end{split}$$

classical polylogarithms are multiple polylogarithms with specific roots (0 and constant a)

$$G(\vec{0}_n; x) = \frac{1}{n!} \ln^n x \qquad G(\vec{a}_n; x) = \frac{1}{n!} \ln^n \left(1 - \frac{x}{a} \right) \qquad G(\vec{0}_{n-1}, a; x) = -\operatorname{Li}_n \left(\frac{x}{a} \right)$$

when the root equals +1,-1,0 multiple polylogarithms become HPLs

Multiple polylogarithms

$$G(a, \vec{w}; z) = \int_0^z \frac{\mathrm{d}t}{t-a} G(\vec{w}; t), \qquad G(a; z) = \ln\left(1 - \frac{z}{a}\right)$$

$$a, \vec{w} \in \mathbb{C}$$

Goncharov 1998-2001

For *a* constant Poincaré Kummer Lappo-Danilevsky 1935

Multiple polylogarithms (MPL) form a shuffle algebra

$$G_{\omega_1}(z)G_{\omega_2}(z) = \sum_{\omega} G_{\omega}(z)$$
 with ω the shuffle of ω_1 and ω_2

example

$$G(a;z) G(b;z) = \int_0^z \frac{dt_1}{t_1 - a} \int_0^z \frac{dt_2}{t_2 - b}$$

= $\int_0^z \frac{dt_1}{t_1 - a} \int_0^{t_1} \frac{dt_2}{t_2 - b} + \int_0^z \frac{dt_2}{t_2 - a} \int_0^{t_2} \frac{dt_1}{t_1 - b}$
= $G(a, b; z) + G(b, a; z)$

 $\lim_{z \to 0} G($

$$d(a_1,\ldots,a_n;z)=0$$
 unless $\vec{a}=\vec{0}$

$$\frac{\partial}{\partial z}G(a_1,\ldots,a_k;z) = \frac{1}{z-a_1}G(a_2,\ldots,a_k;z)$$

MPLs can be represented as nested harmonic sums

$$\sum_{n_1=1}^{\infty} \frac{u_1^{n_1}}{n_1^{m_1}} \sum_{n_2=1}^{n_1-1} \dots \sum_{n_k=1}^{n_{k-1}-1} \frac{u_k^{n_k}}{n_k^{m_k}} = (-1)^k G\left(\underbrace{0,\dots,0}_{m_1-1}, \frac{1}{u_1},\dots,\underbrace{0,\dots,0}_{m_k-1}, \frac{1}{u_1\dots u_k}; 1\right)$$

Higgs production in HEFT

 $m_{\rm H} << 2 m_t$



all amplitudes are reduced by one loop

... but, beware of quark mass effects

σ^{LO}_{EFT}	15.05 pb	σ_{EFT}^{NLO}	34.66 pb
$R_{LO}\sigma^{LO}_{EFT}$	16.00 pb	$R_{LO}\sigma_{EFT}^{NLO}$	$36.84~\rm pb$
$\sigma^{LO}_{ex;t}$	16.00 pb	$\sigma^{NLO}_{ex;t}$	36.60 pb
$\sigma^{LO'}_{ex:t+b}$	14.94 pb	$\sigma^{NLO}_{ex:t+b}$	34.96 pb
$\sigma^{LO}_{ex;t+b+c}$	$14.83~\rm{pb}$	$\sigma^{NLO}_{ex;t+b+c}$	$34.77~\rm{pb}$

Anastasiou Duhr Dulat Furlan Gehrmann Herzog Lazopoulos Mistlberger 2016

$$R_{LO} = \frac{\sigma_{ex:t}^{LO}}{\sigma_{EFT}^{LO}} = 1.063$$

rescaled EFT (rEFT) does a good job (< 1%) in approximating the exact (only top) NLO σ but misses the *t-b* interference

Higgs production

QCD corrections have been computed at N³LO in HEFT

Anastasiou Duhr Dulat Herzog Mistlberger 2015 Mistlberger 2018 (in terms of MPLs and elliptic integrals)



including quark-mass effects and QCD-EW interference the cross section is

 $\sigma = 48.58 \,\mathrm{pb}_{-3.27 \,\mathrm{pb} \,(-6.72\%)}^{+2.22 \,\mathrm{pb} \,(+4.56\%)} \,(\mathrm{theory}) \pm 1.56 \,\mathrm{pb} \,(3.20\%) \,(\mathrm{PDF} + \alpha_s)$

The breakdown of the cross section

 $\begin{array}{rll} 48.58\,\mathrm{pb} = & 16.00\,\mathrm{pb} & (+32.9\%) & (\mathrm{LO},\,\mathrm{rEFT}) \\ & & + 20.84\,\mathrm{pb} & (+42.9\%) & (\mathrm{NLO},\,\mathrm{rEFT}) \\ & & - & 2.05\,\mathrm{pb} & (-4.2\%) & ((t,b,c),\,\mathrm{exact}\,\,\mathrm{NLO}) \\ & & + & 9.56\,\mathrm{pb} & (+19.7\%) & (\mathrm{NNLO},\,\mathrm{rEFT}) \\ & & + & 0.34\,\mathrm{pb} & (+0.2\%) & (\mathrm{NNLO},\,1/m_t) \\ & & + & 2.40\,\mathrm{pb} & (+4.9\%) & (\mathrm{EW},\,\mathrm{QCD}\text{-EW}) \\ & & + & 1.49\,\mathrm{pb} & (+3.1\%) & (\mathrm{N}^3\mathrm{LO},\,\mathrm{rEFT}) \end{array}$

Anastasiou Duhr Dulat Furlan Gehrmann Herzog Lazopoulos Mistlberger 2016 Handbook 4 of LHC Higgs Cross Sections 2016 Higgs production

Handbook 4 of LHC Higgs Cross Sections 2016

 6 sources of uncertainties due to: higher orders truncation of the threshold expansion PDFs
 NLO corrections to QCD-EW interference quark mass effects (2: top mass and top-b interference) at NNLO

δ (scale)	δ (trunc)	δ (PDF-TH)	$\delta(EW)$	$\delta(t,b,c)$	$\delta(1/m_t)$
+0.10 pb -1.15 pb	±0.18 pb	±0.56 pb	±0.49 pb	±0.40 pb	±0.49 pb
$^{+0.21\%}_{-2.37\%}$	20.37%	$\pm 1.16\%$	±1%	$\pm 0.83\%$	±1%

 δ (trunc) = 0.11 pb Mistlberger 2018

 $\delta(1/m_t) = -0.26\%$ Czakon Harlander Klappert Niggetiedt 2021

QCD NNLO corrections

General Top-quark mass corrections are known at NNLO

Czakon Harlander Klappert Niggetiedt 2021

channel	$\sigma^{ ext{NNLO}}_{ ext{HEFT}} ext{ [pb]} \ \mathcal{O}(lpha_s^2) + \mathcal{O}(lpha_s^3) + \mathcal{O}(lpha_s^4)$	$egin{array}{l} (\sigma^{ m NNLO}_{ m exact} \ {\cal O}(lpha^3_s) \end{array}$	$-\sigma^{ m NNLO}_{ m HEFT})[{ m pb}] \ {\cal O}(lpha_s^4)$	$(\sigma_{ m exact}^{ m NNLO}/\sigma_{ m HEFT}^{ m NNLO}-1)~[\%]$		
$\sqrt{s} = 8 { m TeV}$						
gg	7.39 + 8.58 + 3.88	+0.0353	$+0.0879\pm0.0005$	+0.62		
qg	0.55 + 0.26	-0.1397	-0.0021 ± 0.0005	-18		
qq	0.01 + 0.04	+0.0171	-0.0191 ± 0.0002	-4		
total	7.39 + 9.15 + 4.18	-0.0873	$+0.0667\pm0.0007$	-0.10		
$\sqrt{s} = 13 \mathrm{TeV}$						
gg	16.30 + 19.64 + 8.76	+0.0345	$+0.2431\pm 0.0020$	+0.62		
qg	1.49 + 0.84	-0.3696	-0.0115 ± 0.0010	-16		
qq	0.02 + 0.10	+0.0322	-0.0501 ± 0.0006	-15		
total	16.30 + 21.15 + 9.79	-0.3029	$+0.1815\pm 0.0023$	-0.26		

- HEFT not so good for qg and qq channels
- for top-quark mass only the on-shell scheme

QCD NNLO corrections

Higgs + 4-parton amplitudes at one loop

VDD Kilgore Oleari Schmidt Zeppenfeld 2001 Budge Campbell De Laurentis K. Ellis Seth 2020



Higgs + 3-parton amplitudes at two loops



top loop: Jones Kerner Luisoni 2018 Czakon Harlander Klappert Niggetiedt 2021

arbitrary heavy quark masses (only Master Int):

Bonciani VDD Frellesvig Henn Moriello V. Smirnov 2016 all above + Hidding Maestri Salvatori 2019

multi-scale problem with complicated analytic structure elliptic iterated integrals appear

G

 $gg \rightarrow Higgs$ amplitudes at three loops



one scale: one & two top loops one top loop + light-quark loop

two scales: one top loop + b-quark loop



virtual corrections



Harlander Prausa Usovitsch 2019

(one top & one light quark, in terms of HPLs)

Czakon Niggetiedt 2020

(one & two top)

Anastasiou Deutschmann Schweitzer 2020

real-virtual corrections





Bonciani VDD Frellesvig Henn Moriello V. Smirne	ov 2016
all above + Hidding Maestri Salvatori 2019	(arbitrary m _Q)

double-real radiation



VDD Kilgore Oleari Schmidt Zeppenfeld 2001 Budge Campbell De Laurentis K. Ellis Seth 2020

Higgs+3-parton Master Integrals at two loops

4 scales, s, t, m_H , $m_t \rightarrow 3$ external parameters 6 seven-propagator integral families

> Bonciani VDD Frellesvig Henn Moriello V. Smirnov 2016 (A, B, C, D) Bonciani VDD Frellesvig Henn Hidding Maestri Moriello Salvatori V. Smirnov 2019 (F) Frellesvig Hidding Maestri Moriello Salvatori 2019 (G)



Family F: 73 MIs (65 in the polylogarithmic sector, 8 in the elliptic sector)

alphabet: 69 independent letters, with 12 independent square roots

solved through generalised power series expansion Moriello 2019 of the differential equations, defining the parameter *n*-ples along a contour

73 Master Integrals



Elliptic iterated integrals



2-loop sunrise graph



Sabry 1962: ...;Broadhurst 1989; ...; Bloch Vanhove 2013; ... Brödel Duhr Dulat Penante Tancredi 2017-2019



2-loop 3-pt functions

electroweak form factor



Aglietti Bonciani Grassi Remiddi 2007



t-tbar

von Manteuffel Tancredi 2017



2-loop 4-pt function for Higgs + 1 jet



Bonciani VDD Frellesvig Henn Moriello V. Smirnov 2016



$$G(a, \vec{w}; z) = \int_0^z \frac{\mathrm{d}t}{t - a} G(\vec{w}; t), \qquad G(a; z) = \ln\left(1 - \frac{z}{a}\right) \qquad a, \vec{w} \in \mathbb{C}$$



iterated integrals on a torus ...

$$\tilde{\Gamma}\left(\begin{array}{c}n_1\dots n_k\\z_1\dots z_k\end{array};z,\tau\right) = \int_0^z dt\,g^{(n_1)}(t-z_1,\tau)\,\tilde{\Gamma}\left(\begin{array}{c}n_2\dots n_k\\z_2\dots z_k\end{array};t,\tau\right)$$

kernels $g^{(n)}$ have at most simple poles at $z = m + n\tau$



Brown Levin 2011

... are elliptic multiple polylogarithms (eMPL)

$$E_3\left(\begin{array}{cc}n_1\dots n_k\\z_1\dots z_k\end{array};z,\vec{a}\right) = \int_0^z dt\,\varphi_{n_1}(z_1,t,\vec{a})\,E_3\left(\begin{array}{cc}n_2\dots n_k\\z_2\dots z_k\end{array};t,\vec{a}\right) \qquad n_i\in\mathbb{Z}, \quad z_i\in\mathbb{C} \quad a_i\in\mathbb{R}$$
with $\vec{a} = (a_1,a_2,a_3)$ are the zeroes of the elliptic curve $y^2 = (x-a_1)(x-a_2)(x-a_3)$
and $E_3\left(;z,\vec{a}\right) = 1$

2-loop sunrise can be written in terms of eMPLs

Brödel Duhr Dulat Penante Tancredi 2017

Differential Equations

G

Differential Equation method to solve the MIs

 $\partial_i f(x_n;\varepsilon) = A_i(x_n;\varepsilon) f(x_n;\varepsilon)$

f: N-vector of MIs, A_i : NxN matrix, i=1,...,n external parameters

but in some cases E-independent form

 $\partial_i f(x_n;\varepsilon) = \varepsilon A_i(x_n) f(x_n;\varepsilon)$

Henn 2013

solution in terms of iterated integrals



Take two points $(a_1, ..., a_n)$ and $(b_1, ..., b_n)$ in the *n*-dim parameter space, and parametrise the contour $\gamma(t)$ that connects the two points

 $\gamma(t): t \to \{x_1(t), \dots, x_n(t)\}$ $\vec{x}(0) = \vec{a}, \quad \vec{x}(1) = \vec{b}$

and write the differential equation with respect to t. Then find a solution about a point τ by series expanding the coefficient matrix A and then iteratively integrating it. The procedure works in general, for canonical or elliptic sectors **QCD-EW** interference

The Higgs boson may (indirectly) couple to gluons also via the gauge coupling i.e. through a double (electroweak boson + quark) loop



Aglietti Bonciani Degrassi Vicini 2004 (light fermion loop) Degrassi Maltoni 2004 Actis Passarino Sturm Uccirati 2008 (heavy fermion loop)

(in terms of MPLs) (numerically

... elliptic integrals appear)

 $O(\alpha_s^2 \alpha^2)$

the top loop yields a 2% correction to the 5 light fermion loops

- gg-initiated QCD NLO corrections (light fermion loop) computed in various approximations:
 - $-m_{w,z} \rightarrow \infty$ limit
 - soft approximation
 - $-m_{w,z} \rightarrow 0$ limit

Bonetti Melnikov Tancredi 2018

Anastasiou Boughezal Petriello 2009

Anastasiou VDD Furlan Mistlberger Moriello Schweitzer Specchia 2018

and found to be about 5% wrt NLO (HEFT) cross section

D-EW interference





Factorization ansatz $G_{EW\,ij}^{(\ell)} = G_{O\,ij}^{(\ell)}$

Actis Passarino Sturm Uccirati 2008

QCD-EW interference



QCD-EW interference

gg-initiated QCD NLO corrections (light fermion loop): $O(\alpha_s^3\alpha^2)$

Bonetti Melnikov Tancredi 2016



Becchetti Bonciani VDD Hirschi Moriello Schweitzer 2020

IR local subtraction schemes

MadGraph MC@NLO

Frixione Kunszt Signer 1995 Frederix Frixione Maltoni Stelzer 2009

COLORFUL

VDD Somogyi Trocsanyi 2006 Somogyi 2009 VDD Deutschmann Lionetti 2019

Becchetti Bonciani Casconi VDD Moriello 2018 Bonetti Panzer V. Smirnov Tancredi 2020 Becchetti Moriello Schweitzer 2021

thus our NLO result 4.8% wrt gg-initiated NLO HEFT

QCD-EW Higgs+3-parton master integrals at two loops

4 scales, s, t, m_H , $m_V \rightarrow 3$ external parameters

7 seven-propagator integral families

48 MIs (planar), 61 MIs (non-planar)

alphabet: square roots are present, but an MPL representation is possible

Becchetti Bonciani Casconi VDD Moriello 2018 (planar MIs) Becchetti Moriello Schweitzer 2021 (non-planar MIs)



solved through generalised power series expansion Moriello 2019



less than 2% of NLO mixed QCD-EW corrections to $gg \longrightarrow H$

Hirschi Lionetti Schweitzer 2019



qg-initiated QCD-EW interference

 $O(\alpha_s^2 \alpha^2)$



Hirschi Lionetti Schweitzer 2019



Becchetti Bonciani Casconi VDD Moriello 2018 (planar MIs)

Higgs *p*^T distribution at LHC



leading order

K. Ellis Hinchliffe Soldate van der Bij 1988

- high-p_T tail of the Higgs p_T distribution is sensitive to the structure of the loop-mediated Higgs-gluon coupling New Physics particles circulating in the loop would modify it
- QCD NLO corrections known for the top-quark only (on-shell scheme)

Jones Kerner Luisoni 2018 Chen Huss Jones Kerner Lang Lindert Zhang 2021

- Full (=t+b) QCD NLO corrections are not known
- $P = HEFT m_H << 2m_t$ and $p_T << m_t$

QCD corrections are known at NNLO in HEFT, and yield a 15% increase wrt NLO

Boughezal Caola Melnikov Petriello Schulze 2015 Boughezal Focke Giele Liu Petriello 2015 Chen Cruz-Martinez Gehrmann Glover Jaquier 2016

Higgs p_T distribution at **NLO**

virtual corrections



real corrections

Jones Kerner Luisoni 2018 (top) Czakon Harlander Klappert Niggetiedt 2021

Bonciani VDD Frellesvig Henn Moriello V. Smirnov 2016 all above + Hidding Maestri Salvatori 2019 (arbitrary m_Q)

VDD Kilgore Oleari Schmidt Zeppenfeld 2001 Budge Campbell De Laurentis K. Ellis Seth 2020



Higgs pT distribution at LHC

QCD (top) NLO corrections have been computed numerically



Jones Kerner Luisoni 2018 Chen Huss Jones Kerner Lang Lindert Zhang 2021

No *t-b* interference On-shell mass renormalisation

QCD NLO corrections to *t*-*b* interference, using top loop in HEFT and *b*-quark loop in small m_b limit



Lindert Melnikov Tancredi Wever 2017

Higgs *p*^T distribution due to **QCD-EW** interference



Becchetti Bonciani VDD Hirschi Moriello Schweitzer 2020

gg-initiated QCD-EW p_T spectrum harder than HEFT