Superconformal symmetry and higher-derivative Lagrangians

Antoine Van Proeyen

KU Leuven

Frascati,

Breaking of supersymmetry and Ultraviolet Divergences in extended Supergravity

March 25, 2013

Interest in higher-derivative terms

- appear as α' terms in effective action of string theory
- corrections to black hole entropy
- higher order to AdS/CFT correspondence
- counterterms for UV divergences of quantum loops

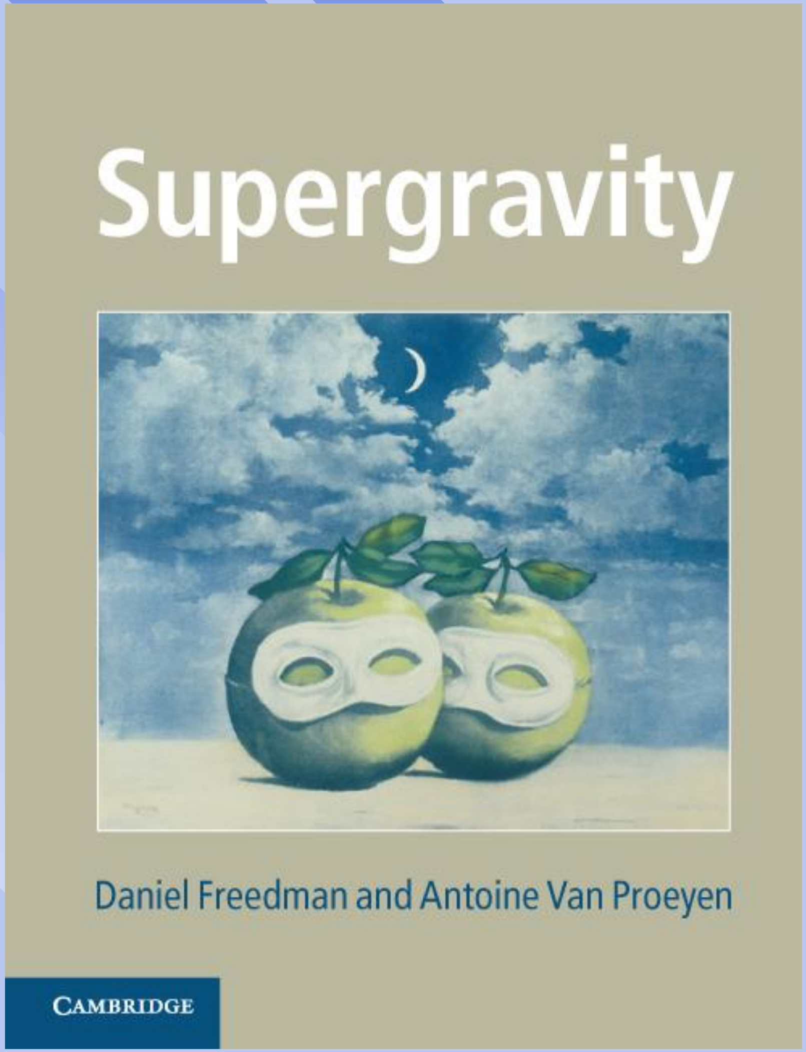
Plan

1. What we know about general sugra/susy theories
2. The superconformal method
   (and in which SUGRAs can we use it)
3. Higher derivative sugra actions and sugra loop results
4. Dirac-Born-Infeld– Volkov-Akulov
   and deformation of supersymmetry
   (example of an all order higher-derivative susy action)
5. Conclusions

http://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&cad=rja&ved=0CDYQtwIwAA&url=http%3A%2F%2Fwww.youtube.com%2Fwatch%3Fv%3DNJDGOJZvwDU&ei=25BNUa2-A4aHPbiEgfAM&usg=AFQjCNGOkkquod6FFe1CGlOxXTJagTaXhw&sig2=DZ2lr-9ohYUJZYk88gR_6A&bvm=bv.44158598,d.Z

1. What we know about general sugra/susy theories

- There are many good books that explain the basics, but ...



‘Ordinary’ susy/sugra

- Bosonic terms in the action have at most two spacetime derivatives, and fermionic terms at most one.
- E.g. *D=4:* fields of spin 2, 1, 0, 3/2, 1/2

$$e_\mu^a\,,A_\mu^I\,,\varphi^u\,,\psi_\mu\,,\lambda^A$$

$$\begin{aligned} e^{-1}\mathcal{L} = & \tfrac{1}{2}R + \\ & +\tfrac{1}{4}(\mathrm{Im}\,\mathcal{N}_{IJ})\mathcal{F}^I_{\mu\nu}\mathcal{F}^{\mu\nu J} \\ & -\tfrac{i}{8}(\mathrm{Re}\,\mathcal{N}_{IJ})\epsilon^{\mu\nu\rho\sigma}\mathcal{F}^I_{\mu\nu}\mathcal{F}^J_{\rho\sigma} \\ & -\tfrac{1}{2}g_{uv}D_\mu\varphi^u\,D^\mu\varphi^v - V \\ & \Big\{-\bar\psi_{\mu i}\gamma^{\mu\nu\rho}D_\nu\psi_\rho{}^i \\ & -\tfrac{1}{2}g_A{}^B\bar\lambda^A \not{D}\lambda_B + \mathrm{h.c.}\Big\} + \ldots \end{aligned}$$

Possibilities for susy depend on the properties of irreducible spinors in each dimension

- Dependent on signature.
  Here: Minkowski
- M: Majorana
  MW: Majorana-Weyl
  S: Symplectic
  SW: Symplectic-Weyl

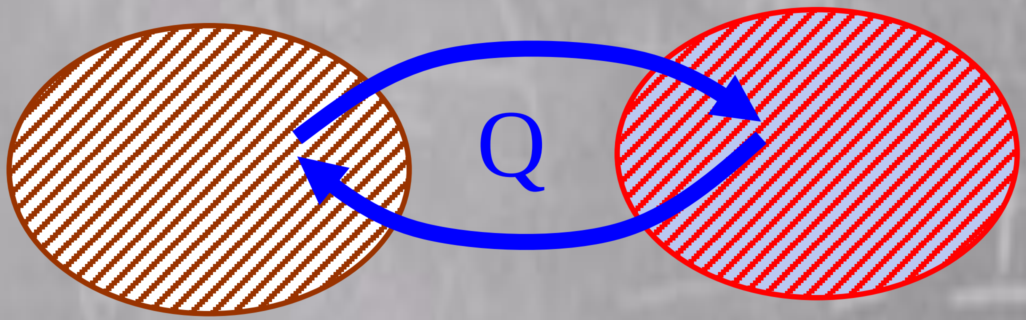
Dim	Spinor	min.# comp
2	MW	1
3	M	2
4	M	4
5	S	8
6	SW	8
7	S	16
8	M	16
9	M	16
10	MW	16
11	M	32

Maximal susy / sugra

- From representations in 4 dimensions:
  maximal N=8 → 32 susys for supergravity
  maximal N=4 → 16 susys for supersymmetry
- based on particle states and susy operator
  - transforming a boson state in a fermion state,
  - and squaring to translations

Particle representations of $\mathcal{N}$ – extended supersymmetry

- # bosonic d.o.f. = # fermionic d.o.f., based on {Q,Q}=P (invertible)



$$\sum_{s,h} \langle p^\mu, s, h| \left(Q_{i\alpha} Q^{\dagger j\alpha} + Q^{\dagger j\alpha} Q_{i\alpha} \right) \mathrm{e}^{-2\pi \mathrm{i} J_3} |p^\mu, s, h\rangle$$
$$= \delta_i^j \sum_{s,h} \langle p^\mu, s, h| P^0 \mathrm{e}^{-2\pi \mathrm{i} J_3} |p^\mu, s, h\rangle$$

Maximal susy / sugra

- From representations in 4 dimensions:
  maximal N=8 → 32 susys for supergravity
  maximal N=4 → 16 susys for supersymmetry
- based on particle states and susy operator
  - transforming a boson state in a fermion state,
  - and squaring to translations
- maximal spin 2 for gravity theories → $N \leq 8$ or 32 susys
  maximal spin 1 without gravity → $N \leq 4$ or 16 susys
- any higher dimensional theory can be dimensionally reduced on tori to D=4.
  This keeps the same number of susy generators

Strathdee,1987

The map: dimensions and # of supersymmetries

D	susy	32		24	20	16		12	8	4
11	M	M								
10	MW	IIA	IIB			I				
9	M	N=2				N=1				
8	M	N=2				N=1				
7	S	N=4				N=2				
6	SW	(2,2)		(2,1)		(1,1)	(2,0)		(1,0)	
5	S	N=8		N=6		N=4			N=2	
4	M	N=8		N=6	N=5	N=4		N=3	N=2	N=1
		SUGRA				SUGRA/SUSY		SUGRA	SUGRA/SUSY	

vector multiplets

tensor multiplet

vector multiplets + multiplets up to spin 1/2

Basic supergravities and deformations

- Basic supergravities:
  have only gauged supersymmetry and general coordinate transformations (and U(1)'s of vector fields).
  - No potential for the scalars.
  - Only Minkowski vacua.
- In any entry of the table there are 'deformations':
  without changing the kinetic terms of the fields, the couplings are changed.
  - Many deformations are 'gauged supergravities':
    gauging of a YM group, introducing a potential.
  - Produced by fluxes on branes
  - There are also other deformations
    (e.g. massive deformations, superpotential)

Embedding tensor formalism

- The gauge group is a subgroup of the isometry group G, defined by an embedding tensor. $\left(\partial_\mu - A_\mu{}^M \Theta_M{}^\alpha \delta_\alpha\right)\phi$

all the rigid symmetries

determines which symmetries are gauged, and how:
e.g. also the coupling constants.
There are several constraints on the tensor.

- Structure to get a complete picture of supergravities with at most two spacetime derivatives in Lagrangian
(although: to get all the explicit solutions of constraints still needs more work)

Cordaro, Frè, Gualtieri,Termonia and Trigiante, 9804056

Nicolai, Samtleben, 0010076

de Wit, Samtleben and Trigiante, 0507289

Higher-derivative actions: no systematic knowledge

- Various constructions of higher derivative terms
  - e.g. susy Dirac-Born-Infeld: Cecotti, Ferrara, 1987; Tseytlin; Bagger, Galperin; Roček; Kuzenko, Theisen; Ivanov, Krivonos, Ketov; Bellucci,
- but no systematic construction,
  or classification of what are the possibilities;
  (certainly not in supergravity)

Constructions of actions

Possible constructions:

- order by order Noether transformations: the only possibility for the maximal theories (Q>16)
- superspace:
  - very useful for rigid N=1: shows structure of multiplets.
  - very difficult for supergravity. Needs many fields and many gauge transformations
- (super)group manifold:
  - Optimal use of the symmetries using constraints on the curvatures
- superconformal tensor calculus:
  - keeps the structure of multiplets as in superspace but avoids its immense number of unphysical degrees of freedom
  - extra symmetry gives insight in the structure

2. The superconformal method

- Superconformal symmetry is the maximal extension of spacetime symmetries according to Coleman-Mandula theorem
- Here: *not* about Weyl supergravity: ~~$\int \mathrm{d}^4x \left[R^2_{\mu\nu\rho\sigma} - 2R^2_{\mu\nu} + \frac{1}{3}R^2\right]$~~

  $\int \mathrm{d}^4x \frac{1}{2\kappa^2} R$

- Tool for construction of actions
  - allows to use multiplet calculus similar to superspace
  - makes hidden symmetries explicit

Gravity as a conformal gauge theory

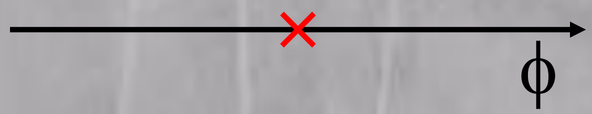
The strategy

- scalar field (compensator)

conformal gravity: $\mathcal{L} = -\frac{1}{2}\sqrt{g}\phi\Box^C\phi = -\frac{1}{2}\sqrt{g}\phi\Box\phi + \frac{1}{12}\sqrt{g}R\phi^2$

dilatational gauge fixing $\phi = \sqrt{6}/\kappa$ → $\mathcal{L} = \frac{1}{2\kappa^2}\sqrt{g}R$

- First action is conformal invariant,
- gauge-fixed one is Poincaré invariant.
- Scalar field had scale transformation $\delta\phi(x) = \lambda_D(x)\phi(x)$



Schematic: Conformal construction of gravity

conformal scalar action
(contains Weyl fields)

local conformal symmetry

Gauge fix
dilatations and
special conformal transformations

Poincaré gravity action

local ⊡ symmetry

Superconformal construction
The idea of superconformal methods

- Difference susy- sugra: the concept of multiplets is clear in susy, they are mixed in supergravity
- Superfields are an easy conceptual tool for rigid susy

- (Super)gravity can be obtained by starting with (super)conformal symmetry and gauge fixing.
- With matter:
  Before gauge fixing: everything looks like in rigid supersymmetry + covariantizations

Superconformal algebra

- In general $\begin{pmatrix} \text{conformal algebra} & Q, S \\ Q, S & R\text{-symmetry} \end{pmatrix}$

- according to dilatational weight: (e.g. N=1)

$$\begin{array}{rcl} 1 & : & P_\mu \\ \frac{1}{2} & : & Q \\ 0 & : & D, \quad M_{ab}, \quad \mathrm{U}(1) \\ -\frac{1}{2} & : & S \\ -1 & : & K_\mu \end{array}$$

$$[D, Q] = \tfrac{1}{2} Q, \qquad [D, S] = -\tfrac{1}{2} S$$

$$\left\{Q_\alpha, Q^\beta\right\} = -\tfrac{1}{2}(\gamma^a)_\alpha{}^\beta P_a\,, \qquad \left\{S_\alpha, S^\beta\right\} = -\tfrac{1}{2}(\gamma^a)_\alpha{}^\beta K_a\,,$$

$$\left\{Q_\alpha, S^\beta\right\} = -\tfrac{1}{2}\delta_\alpha{}^\beta D - \tfrac{1}{4}(\gamma^{ab})_\alpha{}^\beta M_{ab} + \tfrac{1}{2}\mathrm{i}(\gamma_*)_\alpha{}^\beta\, \mathrm{U}(1)$$

The strategy : superconformal construction of N=1 supergravity

chiral multiplet + Weyl multiplet
superconformal action

↓ Gauge fix dilatations,
special conformal transformations,
local R-symmetry and
special supersymmetry

Poincaré supergravity action

Superconformal construction of N=4 supergravity

De Roo, 1985

Weyl multiplet + 6 gauge compensating multiplets (on-shell)

superconformal action

gauge-fixing

Weyl symmetry, local SU(4), local U(1) ,
S-supersymmetry and K-conformal boosts

pure N=4 Cremmer-Scherk-Ferrara supergravity

$$\frac{1}{4}R - \frac{1}{8}\frac{\partial\tau\partial\bar{\tau}}{(\mathrm{Im}\tau)^2} + \frac{i}{4}\tau F^{+I}_{\mu\nu}\delta_{IJ}F^{+J\mu\nu} + h.c.$$

On-shell and off-shell multiplets

- Action should be invariant
- Algebra can be closed only modulo field equations
- Problem: No flexibility in field equations
- Examples:
  - hypermultiplets N=2;
  - N=4 gauge multiplets (are compensator multiplets);

In which sugras can we use superconformal methods ?

- We should have a superconformal algebra
- We should have compensating multiplets

Nahm, 1978

Superconformal groups

conformal algebra is so(D,2)

D	supergroup	conf	R	ferm.
3	$\mathrm{OSp}(N\|4)$	$\mathrm{SO}(3,2)=\mathrm{Sp}(4)$	$\mathrm{SO}(N)$	$4N$
4	$\mathrm{SU}(2,2\|N)$	$\mathrm{SO}(4,2)=\mathrm{SU}(2,2)$	$\mathrm{U}(N)$ for $N\neq 4$	$8N$
			$\mathrm{SU}(4)$ for $N=4$	
5	$F^2(4)$	$\mathrm{SO}(5,2)$	$\mathrm{SU}(2)$	16
6	$\mathrm{OSp}(8^*\|2N)$	$\mathrm{SO}(6,2)=\mathrm{SO}^*(8)$	$\mathrm{USp}(2N)$	$16N$

covering group — always compact

Other superalgebras have been considered, where
the conformal algebra is not a factor, but a subalgebra of the bosonic part symmetry e.g. SO(11,2) $\subset$ Sp(64) $\subset$ OSp(1| 64)
But not sucessfully applied for constructing actions

JW van Holten, AVP, 1982
D'Auria, S. Ferrara, M. Lledò, V. Varadarajan, 2000

The map: dimensions and # of supersymmetries

D	susy	32		24	20	16		12	8	4
11	M	M								
10	MW	IIA	IIB			I				
9	M	N=2				N=1				
8	M	N=2				N=1				
7	S	N=4				N=2				
6	SW	(2,2)		(2,1)		(1,1)	(2,0)		(1,0)	
5	S	N=8		N=6		N=4			N=2	
4	M	N=8		N=6	N=5	N=4		N=3	N=2	N=1
		SUGRA				SUGRA/SUSY		SUGRA	SUGRA/SUSY	

have SC algebra

can be used for SC methods

vector multiplets

tensor multiplet

vector multiplets + multiplets up to spin 1/2

3. Higher derivative supergravity actions and supergravity loop results

The Null Results

Miracle #1 2007 N= 8, D=4 is UV finite up to 3-loops

Bern, Carrasco, Dixon, Johansson, Kosower, Roiban

Miracle #2 2009 N= 8, D=5 is UV finite up to 4-loops

Bern, Carrasco, Dixon, Johansson, Roiban

Miracle #3 2012 N= 4, D=4 is UV finite up to 3-loops

Miracle #4 2012 N= 4, D=5 is UV finite up to 2-loops

Bern, Davies, Dennen, Huang : 3-loop D=4 computation in pure supergravity

September: 2-loop D=5 UV finite

- If there are divergences:
  supersymmetric counterterms should exist
  (or supersymmetry anomalies)
- We do not know enough to be sure whether invariants do exist.

Higher derivative sugra actions

- examples with superconformal tensor calculus
  - S. Cecotti and S. Ferrara, `Supersymmetric Born-Infeld actions', 1986
  - N=2 constructions:
    B. de Wit, S. Katmadas, M. van Zalk, arXiv:1010.2150 ;
    W. Chemissany, S. Ferrara, R. Kallosh, C. S. Shahbazi, 1208.4801
  - Higher derivative extension in D=6 (1,0)
    E. Bergshoeff, F. Coomans, E. Sezgin, AVP 1203.2975)
- other methods
  starting with S. Deser, J.H. Kay and K.S. Stelle, 1977, ...
  more recent: G. Bossard, P. Howe, K. Stelle, P. Vanhove; M. Koehn, J-L Lehners, B. Ovrut ... (using superspace)

N=2 D=4 construction

- Based on tensor calculus as in superspace:
  - chiral multiplets: $S = \{X, \Omega_i, \ldots, C\}$
  - also Weyl multiplet $W^2 = \{T^-_{ab} T^{ab\,-}, \ldots\}$
  - kinetic multiplet: $\mathbb{T}(\bar{S}) = \{\bar{C}, \ldots\}$

arbitrary power is still chiral

- made superconformal invariant
  - restriction on possible actions (homogeneity)
- Everything off-shell
- many possibilities, e.g. invariants contributing to entropy and central charges of black holes

de Wit, Katmadas, van Zalk, 2011

N=2 higher derivative terms with auxiliary fields

- The term quartic in the auxiliary field from the Weyl multiplet is a partner of the term quartic in the Weyl curvature.

$$\lambda(C_{....})^4 \qquad \lambda(\partial\mathcal{T})^4$$

- Deformed EOM for the Weyl multiplet auxiliary

$$T^+_{ab} = \frac{2}{X}F^+_{ab} + \lambda(\partial^4 T^3)^+_{ab} + \ldots$$

- Solve recursively: infinite number of higher derivative terms with higher and higher powers of the graviphoton, N=2 supergravity vector

$$\mathcal{T}^{def} = \mathcal{F} + \lambda\,[\partial^4\mathcal{F}^3] + \lambda^2[\partial^4\mathcal{F}^2][\partial^4\mathcal{F}^3] + \cdots$$

- The action with auxiliary field eliminated: Born-Infeld with higher derivatives

$$S^{def} = -\frac{1}{4}\mathcal{F}^2 + \lambda([\partial\mathcal{F}])^4 + \lambda^2[\partial^8\mathcal{F}^6] + \cdots$$

Chemissany, Ferrara, Kallosh, Shahbazi, 1208.4801

Also transformation laws deform

EXACT

$$\delta\psi_\mu^i = D_\mu\epsilon^i - \frac{1}{16}\gamma^{ab}T_{ab}^{-}\varepsilon^{ij}\gamma_\mu\epsilon_j - \gamma_\mu\eta^i$$

Deformation of the supergravity local N=2 supersymmetry after S-supersymmetry gauge-fixing and expanding near the lowest order solution for auxiliary fields

Order by order

$$\phi_{aux} = \phi_{aux}^0 + \Delta\phi_{aux} \qquad \Delta\phi_{aux} = \sum_{n=1} \lambda^n \phi_{aux}^{(n)},$$

The deformation of the gravitino supersymmetry due to higher derivative term is

$$\Delta\psi_\mu = -4\lambda[\partial^4\mathcal{F}^3]_\mu{}^\nu\gamma_\nu\,\epsilon^i + \ldots$$

Are all valid counterterms (broken) superconformal actions ?

N=0 : Locally conformal R^4 ; gauge fixed: is 3-loop counterterm

$$\int d^4x\,\sqrt{-g}\,\phi^{-4}C_{\alpha\beta\gamma\delta}C_{\dot\alpha\dot\beta\dot\gamma\dot\delta}C^{\alpha\beta\gamma\delta}C^{\dot\alpha\dot\beta\dot\gamma\dot\delta}$$

N=2 superconformal R^4

chiral kinetic action with inverse powers of the compensator superfield S

$$\int d^4\theta\,\frac{W^2}{S^2}\,\mathbb{T}\left(\frac{\overline{W^2}}{\overline{S^2}}\right)$$

N=4 superconformal R^4

???

N=4 has no tensor calculus

- Since compensating multiplets cannot be multiplied, ...
  we cannot make constructions as those for N=2.
- Algebra only valid on shell:
  modified actions imply modified field equations:

  $\Rightarrow$ transformations (or superfields) have to be deformed.

How for N=4 ?

- No tensor calculus; no auxiliary fields
- How to establish the existence/non-existence of the consistent order by order deformation of N=4 on shell superspace ?
- Conjecture: if it does not exist: explanation of finiteness (if Bern et al do not find N=4, D=4 is divergent at higher loops)
- Until invariant counterterms are constructed (conformal?) we have no reason to expect UV divergences

Two points of view

1. **Legitimate counterterms are not available yet**
2. **Legitimate counterterms are not available, period**

???

S. Ferrara, R. Kallosh, AVP, 1209.0418

N=4 conjecture

If the UV finiteness will persist in higher loops, one would like to view this as an opportunity to test some new ideas about gravity.
E.g. : is superconformal symmetry more fundamental ?

Repeat: Classical N=4 is obtained from gauge fixing a superconformal invariant action:
The mass M_{Pl} appears in the gauge-fixing procedure

Bergshoeff, de Roo, de Wit, van Holten and AVP, 1981; de Roo, 1984

Analogy:

- Mass parameters M_W and M_Z of the massive vector mesons are not present in the gauge invariant action of the standard model.
- Show up when the gauge symmetry is spontaneously broken.
- In **unitary gauge** they give an impression of being fundamental.
- In **renormalizable gauge** (where UV properties analyzed) : absent

S. Ferrara, R. Kallosh, AVP, 1209.0418

The non-existence of (broken) superconformal-invariant counterterms and anomalies in N=4, D=4 could explain 'miraculous' vanishing results.

- **simplest** possible explanation of the 3-loop finiteness and predicts perturbative UV finiteness in higher loops
- the same conjecture applies to higher derivative superconformal invariants and to a consistent superconformal anomaly
- the conjecture is **economical**, sparing in the use of resources: either the local N=4 superconformal symmetry is a good symmetry, or it is not.
- **Falsifiable** by N=4 L=4 computations (which are already underway)

If the conjecture survives these computations (if UV finite) :
hint that the models with superconformal symmetry serve as a basis for constructing a consistent quantum theory where M_{Pl} appears in the process of **gauge-fixing** superconformal symmetry.

Also **falsifiable** by our own calculations:
if we find a way to construct
(non-perturbative) superconformal invariants

The non-existence of (broke **anomalies in N=4, D=4 coul**

“We are trying to prove ourselves wrong as quickly as possible, because only in that way can we find progress.” (Feynman)

- **simplest** possible explanati
finiteness in higher loops
- the same conjecture applies to higher derivative superconformal invariants and to a consistent superconformal anomaly
- the conjecture is **economical**, sparing in the use of resources: either the local N=4 superconformal symmetry is a good symmetry, or it is not.
- **Falsifiable** by N=4 L=4 computations
(which are already underway)

If the conjecture survives these computations (if UV finite) :
hint that the models with superconformal symmetry serve as a basis for constructing a consistent quantum theory where M_{Pl} appears in the process of **gauge-fixing** superconformal symmetry.

Also **falsifiable** by our own calculations:
if we find a way to construct
(non-perturbative) superconformal invariants

4. Dirac-Born-Infeld– Volkov-Akulov and deformation of supersymmetry

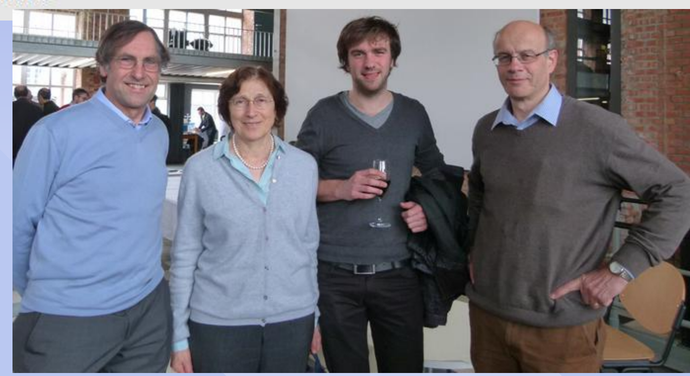
Preview

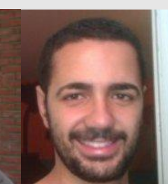
Dirac-Born-Infeld-Volkov-Akulov and Deformation of Supersymmetry

Eric Bergshoeff, Frederik Coomans, Renata Kallosh, C.S. Shahbazi and Antoine Van Proeyen

supersym … action and the supersymmetry transformations of the d=10 and d=4 Maxwell supermultiplets so that at each order of the deformation the theory has 16 Maxwell multiplet deformed
supersym…es as well as 16 Volkov-Akulov type non-linear supersymmetries. The result agrees with the expansion in the string tension of the explicit action of the Dirac-Born-Infeld model and its
supersym…es, extracted from D9 and D3 superbranes, respectively. The half-maximal Dirac-Born-Infeld models with 8 Maxwell supermultiplet deformed supersymmetries and 8 Volkov-Akulov type
…es are described by a new class of d=6 vector branes related to chiral (2,0) supergravity, which we denote as 'Vp-branes'. We use a space-filling V5 superbrane for the d=6 model and a V3
…r the d=4 half-maximal Dirac-Born-Infeld (DBI) models. In this way we present a completion to all orders of the deformation of the Maxwell supermultiplets with maximal 16+16
superbran…ies in d=10 and 4, and half-maximal 8+8 supersymmetries in d=6 and 4.
supersym

a super new paper (arXiv:1303.5662)





on the search of deformations of N=4 theories,
we find all-order invariant actions in rigid susy with extra supersymmetries (Volkov-Akulov (VA) – type)

Down-up approach: start deformations

$$S = \int \mathrm{d}^D x \, \{ -\tfrac{1}{4}(F_{\mu\nu})^2 + \bar{\lambda}\partial\!\!\!/\lambda \}$$

gauge field (D-2) on-shell dof;
fermion = #spinor comp / 2

Dim	Spinor	min.# comp
2	MW	1
3	M	2
4	M	4
5	S	8
6	SW	8
7	S	16
8	M	16
9	M	16
10	MW	16
11	M	32

The map: dimensions and # of supersymmetries

D	susy	32		24	20	16		12	8	4
11	M	M								
10	MW	IIA	IIB			I				
9	M	N=2				N=1				
8	M	N=2				N=1				
7	S	N=4				N=2				
6	SW	(2,2)		(2,1)		(1,1)	(2,0)		(1,0)	
5	S	N=8		N=6		N=4			N=2	
4	M	N=8		N=6	N=5	N=4		N=3	N=2	N=1
		SUGRA				SUGRA/SUSY		SUGRA	SUGRA/SUSY	

vector multiplets

tensor multiplet

vector multiplets + multiplets up to spin 1/2

Down-up approach: start deformations

D=10: MW;
D=6 SW;
D=4 M;
D=3 M;
D=2 MW

$$S = \int \mathrm{d}^D x \, \{ -\tfrac{1}{4}(F_{\mu\nu})^2 + \bar{\lambda}\partial\!\!\!/\lambda \}$$

gauge field (D-2) on-shell dof;
fermion = #spinor comp / 2

$$\delta_\epsilon A_\mu = \bar{\epsilon}\Gamma_\mu\lambda \,, \qquad \delta_\epsilon\lambda = \frac{1}{4}\Gamma^{\mu\nu}F_{\mu\nu}\epsilon$$

extra (trivial) fermionic shift symmetry

$$\delta_\eta A_\mu = 0 \,, \qquad \delta_\eta\lambda = -\frac{1}{2\alpha}\eta$$

normalization for later use

Bottom-up deformation

$$
\begin{aligned}
S \;=\; & \int \mathrm{d}^D x \,\{ -\tfrac{1}{4} F^2 + \bar\lambda \partial\!\!\!/ \lambda \} - 2\alpha c_4 F^{\mu\nu} \bar\lambda \Gamma_\mu \partial_\nu \lambda \\
& + \tfrac{1}{8}\alpha^2 [\, \mathrm{Tr}\, F^4 - \tfrac{1}{4} \left(F^2\right)^2 + 4(1+4c_4^2)(F^2)^{\mu\nu} \bar\lambda \Gamma_\mu \partial_\nu \lambda \\
& + (1-4c_4^2) F_\mu{}^\lambda (\partial_\lambda F_{\nu\rho}) \bar\lambda \Gamma^{\mu\nu\rho} \lambda + \tfrac{1}{2}(c_1 + 8c_4^2) F^2 \bar\lambda \partial\!\!\!/ \lambda \\
& - \tfrac{1}{2} c_2 F_{\mu\nu} (\partial_\lambda F^\lambda{}_\rho) \bar\lambda \Gamma^{\mu\nu\rho} \lambda - \tfrac{1}{2}(c_3 + 4c_4^2) F_{\mu\nu} F_{\rho\sigma} \bar\lambda \Gamma^{\mu\nu\rho\sigma} \partial\!\!\!/ \lambda] \\
& + \mathcal{O}(\alpha^2 \lambda^4) + \mathcal{O}(\alpha^3)
\end{aligned}
$$

free coefficients c_i , but these are related to field redefinitions

$$
\begin{aligned}
A_\mu(0) &= A_\mu - \tfrac{1}{16}\alpha^2 c_2 F^{\nu\rho} \bar\lambda \Gamma_{\mu\nu\rho} \lambda \,, \\
\lambda(0) &= \lambda + \tfrac{1}{2}\alpha c_4 F_{\mu\nu} \Gamma^{\mu\nu} \lambda + \tfrac{1}{32}\alpha^2 c_1 F^2 \lambda - \tfrac{1}{32}\alpha^2 c_3 F_{\mu\nu} F_{\rho\sigma} \Gamma^{\mu\nu\rho\sigma} \lambda \,,
\end{aligned}
$$

$\Rightarrow$answer unique;

also transformation rules deformed.

As well ϵ as η parameter transformations can be defined

continues from E. Bergshoeff, M. Rakowski and E. Sezgin, 1987

bottom-up deformation

e.g.

$$\begin{aligned}
\delta_\eta A^\mu &= \tfrac{\alpha}{4}\bar\eta F^{\nu\mu}\Gamma_\nu\lambda + \tfrac{\alpha}{8}\bar\eta\Gamma^{\mu\nu\rho}F_{\nu\rho}\lambda - \tfrac{1}{16}\alpha c_2 F_{\nu\rho}\bar\eta\Gamma^{\mu\nu\rho}\lambda + \mathcal{O}(\alpha\eta\lambda^3) + \mathcal{O}(\alpha^2)\,,\\
\delta_\eta\lambda &= -\tfrac{1}{2\alpha}\eta + \alpha\left[-\tfrac{1}{32}F^2 - \tfrac{1}{64}\Gamma^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\right]\eta\\
&\quad + \tfrac{1}{4}c_4 F_{\mu\nu}(c)\Gamma^{\mu\nu}\left[\eta - \tfrac{1}{2}\alpha c_4 F_{\rho\sigma}(c)\Gamma^{\rho\sigma}\eta\right]\\
&\quad + \tfrac{1}{64}\alpha c_1 F^2\eta - \tfrac{1}{64}\alpha c_3 F_{\mu\nu}F_{\rho\sigma}\Gamma^{\mu\nu\rho\sigma}\eta + \mathcal{O}(\alpha\eta\lambda^2) + \mathcal{O}(\alpha^2)
\end{aligned}$$

already complicated; but only use of

-Majorana flip relations $\bar\lambda_1\Gamma^\mu\lambda_2 = -\bar\lambda_2\Gamma^\mu\lambda_1$

-cyclic (Fierz) identity

$$\Gamma_\mu\lambda_1\bar\lambda_2\Gamma^\mu\lambda_3 + \Gamma_\mu\lambda_2\bar\lambda_3\Gamma^\mu\lambda_1 + \Gamma_\mu\lambda_3\bar\lambda_1\Gamma^\mu\lambda_2 = 0\,.$$

which are valid in D=10,6,4,3,2

looks hopeless to continue to all orders

Dp-brane action

Start with κ–symmetric Dp brane action

$$S_{\rm DBI}+S_{\rm WZ} = -\frac{1}{\alpha^2}\int \mathrm{d}^{p+1}\sigma\,\sqrt{-\det(G_{\mu\nu}+\alpha\mathcal{F}_{\mu\nu})}+\frac{1}{\alpha^2}\int \Omega_{p+1}$$

$$G_{\mu\nu} = \eta_{mn}\Pi_\mu^m\Pi_\nu^n\ , \qquad \Pi_\mu^m = \partial_\mu X^m - \bar\theta\Gamma^m\partial_\mu\theta$$

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} - 2\alpha^{-1}\bar\theta\sigma_3\Gamma_m\partial_{[\mu}\theta\left(\partial_{\nu]}X^m - \tfrac{1}{2}\bar\theta\Gamma^m\partial_{\nu]}\theta\right)$$

Dp brane: IIB theory m=0,..., 9 and μ=0,..., $p=2n+1$

space-time coördinates X^m; θ is doublet of MW spinors;

$F_{\mu\nu}$ Abelian field strength

Symmetries:

rigid susy doublet ϵ^1; ϵ^2

local κ symmetry doublet (effectively only half (reducible symmetry)

world volume gct

$$\delta_\kappa\theta = (1+\Gamma)\kappa$$

Dp-brane action

Start with κ–symmetric Dp brane action

$$S_{\rm DBI}+S_{\rm WZ} = -\frac{1}{\alpha^2}\int \mathrm{d}^{p+1}\sigma\,\sqrt{-\det(G_{\mu\nu}+\alpha\mathcal{F}_{\mu\nu})}+\frac{1}{\alpha^2}\int \Omega_{p+1}$$

$$G_{\mu\nu} = \eta_{mn}\Pi_\mu^m\Pi_\nu^n\ , \qquad \Pi_\mu^m = \partial_\mu X^m - \bar\theta\Gamma^m\partial_\mu\theta$$

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} - 2\alpha^{-1}\bar\theta\sigma_3\Gamma_m\partial_{[\mu}\theta\left(\partial_{\nu]}X^m - \tfrac{1}{2}\bar\theta\Gamma^m\partial_{\nu]}\theta\right)$$

Dp brane: IIB theory m=0,..., 9 and μ=0,..., $p=2n+1$

space-time coördinates X^m; θ is doublet of MW spinors;

$F_{\mu\nu}$ Abeli

Same applies for D=6 (2,0) (also called iib): m=0,...,5

brane interpretation: see talk Eric Bergshoeff

Symmetries:

rigid susy do

local κ symm)

Also D=4 N=2, m=0,...,3 (BH solutions)

world volume get

$\delta_\kappa\theta = (1+\Gamma)\kappa$

Gauge fixing

$$X^m = \{\delta^{m'}_\mu \sigma^\mu, \phi^I\}\,, \quad m' = 0, 1, \ldots, p\,, \quad I = 1, \ldots, 9-p$$
$$\theta = (\theta^1 = 0,\ \theta^2 \equiv \alpha\lambda)$$

worldvolume gct $\xi^{m'}$ and κ symmetry gauge-fixed

to stay in the gauge ('decomposition laws'): parameters become function of parameters of other symmetries

$\Rightarrow$ the two deformed ϵ^1 and ϵ^2 supersymmetries preserved

suitable combinations are called ϵ and ζ

Complete DBI-VA model for the p=9 case (no scalars ϕ^{I})

$$S = -\frac{1}{\alpha^2}\int \mathrm{d}^{10}x \left\{ \sqrt{-\det(G_{\mu\nu} + \alpha\mathcal{F}_{\mu\nu})} - 1 \right\}$$

$$G_{\mu\nu} = \eta_{mn}\Pi_\mu^m\Pi_\nu^n \,, \qquad \Pi_\mu^m = \delta_\mu^m - \alpha^2\bar\lambda\Gamma^m\partial_\mu\lambda \,,$$

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} + 2\alpha\bar\lambda\Gamma_{[\nu}\partial_{\mu]}\lambda \,, \qquad \mu = 0, 1, ..., 9 \,, \qquad m = 0, 1, ..., 9 \,,$$

16 ϵ transformations, deformation of the Maxwell supermultiplet supersymmetries

$$\begin{aligned}
\delta_\epsilon\lambda &= -\tfrac{1}{2\alpha}\left(\mathbb{1} - \beta\right)\epsilon - \tfrac{1}{2}\alpha\partial_\mu\lambda\bar\lambda\Gamma^\mu\left(\mathbb{1} + \beta\right)\epsilon \,, \\
\delta_\epsilon A_\mu &= -\tfrac{1}{2}\bar\lambda\Gamma_\mu(\mathbb{1} + \beta)\epsilon + \tfrac{1}{2}\alpha^2\bar\lambda\Gamma_m(\tfrac{1}{3}\mathbb{1} + \beta)\epsilon\bar\lambda\Gamma^m\partial_\mu\lambda - \tfrac{1}{2}\alpha\bar\lambda\Gamma^\rho\left(\mathbb{1} + \beta\right)\epsilon F_{\rho\mu}
\end{aligned}$$

16 VA-type ζ transformations

$$\begin{aligned}
\delta_\zeta\lambda &= \alpha^{-1}\zeta + \alpha\partial_\mu\lambda\bar\lambda\Gamma^\mu\zeta \,, \\
\delta_\zeta A_\mu &= \bar\lambda\Gamma_\mu\zeta + \alpha\bar\lambda\Gamma^\rho\zeta F_{\rho\mu} - \tfrac{1}{3}\alpha^2\bar\lambda\Gamma_m\zeta\bar\lambda\Gamma^m\partial_\mu\lambda
\end{aligned}$$

$$\beta = \left[\det\left(\delta_\mu{}^\nu + \alpha\mathcal{F}_{\mu\rho}G^{\rho\nu}\right)\right]^{-1/2} \sum_{k=0}^{5} \frac{\alpha^k}{2^k k!}\widehat{\Gamma}^{\mu_1\nu_1\cdots\mu_k\nu_k}\mathcal{F}_{\mu_1\nu_1}\cdots\mathcal{F}_{\mu_k\nu_k} = 1 + \mathcal{O}(\alpha)$$

are not the regular susys

Complete DBI-VA model for the p=9 case (no scalars ϕ^I)

$$S = -\frac{1}{\alpha^2}\int \mathrm{d}^{10}x \left\{\sqrt{-\det(G_{\mu\nu}+\alpha\mathcal{F}_{\mu\nu})} - 1\right\}$$

$$G_{\mu\nu} = \eta_{mn}\Pi_\mu^m\Pi_\nu^n \,, \qquad \Pi_\mu^m = \delta_\mu^m - \alpha^2\bar\lambda\Gamma^m\partial_\mu\lambda \,,$$
$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} + 2\alpha\bar\lambda\Gamma_{[\nu}\partial_{\mu]}\lambda \,, \qquad \mu = 0,1,...,9 \,, \qquad m = 0,1,...,9 \,,$$

16 ϵ transformations, deformation of the Maxwell supermultiplet supersymmetries

$$\begin{aligned}\delta_\epsilon\lambda &= -\tfrac{1}{2\alpha}(\mathbb{1}-\beta)\,\epsilon - \tfrac{1}{2}\alpha\partial_\mu\lambda\bar\lambda\Gamma^\mu(\mathbb{1}+\beta)\,\epsilon \,,\\ \delta_\epsilon A_\mu &= -\tfrac{1}{2}\bar\lambda\Gamma_\mu(\mathbb{1}+\beta)\epsilon + \tfrac{1}{2}\alpha^2\bar\lambda\Gamma_m(\tfrac{1}{3}\mathbb{1}+\beta)\epsilon\bar\lambda\Gamma^m\partial_\mu\lambda - \tfrac{1}{2}\alpha\bar\lambda\Gamma^\rho(\mathbb{1}+\beta)\,\epsilon F_{\rho\mu}\end{aligned}$$

16 VA-type ζ transformations

$$\begin{aligned}\delta_\zeta\lambda &= \alpha^{-1}\zeta + \alpha\partial_\mu\lambda\bar\lambda\Gamma^\mu\zeta \,,\\ \delta_\zeta A_\mu &= \bar\lambda\Gamma_\mu\zeta + \alpha\bar\lambda\Gamma^\rho\zeta F_{\rho\mu} - \tfrac{1}{3}\alpha^2\bar\lambda\Gamma_m\zeta\bar\lambda\Gamma^m\partial_\mu\lambda\end{aligned}$$

Note: VA susys: do not transform fermion to boson states; are not the regular susys

Comparing bottom-up with top-down

$$
\begin{aligned}
S \;=\; & \int \mathrm{d}^D x \, \{ -\tfrac{1}{4}F^2 + \bar\lambda \partial\!\!\!/ \lambda \} - 2\alpha c_4 F^{\mu\nu}\bar\lambda\Gamma_\mu\partial_\nu\lambda \\
& + \tfrac{1}{8}\alpha^2 [\, \mathrm{Tr}\, F^4 - \tfrac{1}{4}\left(F^2\right)^2 + 4(1+4c_4^2)(F^2)^{\mu\nu}\bar\lambda\Gamma_\mu\partial_\nu\lambda \\
& + (1-4c_4^2)F_\mu{}^\lambda(\partial_\lambda F_{\nu\rho})\bar\lambda\Gamma^{\mu\nu\rho}\lambda + \tfrac{1}{2}(c_1+8c_4^2)F^2\bar\lambda\partial\!\!\!/\lambda \\
& - \tfrac{1}{2}c_2 F_{\mu\nu}(\partial_\lambda F^\lambda{}_\rho)\bar\lambda\Gamma^{\mu\nu\rho}\lambda - \tfrac{1}{2}(c_3+4c_4^2)F_{\mu\nu}F_{\rho\sigma}\bar\lambda\Gamma^{\mu\nu\rho\sigma}\partial\!\!\!/\lambda] + \mathcal{O}(\alpha^2\lambda^4) + \mathcal{O}(\alpha^3)
\end{aligned}
$$

- Expanding the all-order result, one re-obtains indeed the result that was obtained in the bottom-up calculation to order α^2 using a particular field definition (choice of coefficients c_i : such that no ∂ F terms: $c_1=2$, $c_2=0$, $c_3=-1$, $c_4=-1/2$).
- For the transformation laws: agree modulo a ‘zilch symmetry’: (on-line trivial symmetry)
  (ζ is linear combination of ϵ and η)
- **This proves that our all-order result is indeed the full deformation that we were looking for !**

The map: dimensions and # of supersymmetries

D	susy	32	24	20	16	12	8	4
11	M	M						
10	MW	IIA, IIB			D9 I			
9	M	N=2			N=1			
8	M	N=2			D7 =1			
7	S	N=4			N=2			
6	SW	(2,2)	(2,1)		D5 (1,1), (2,0)		(1,0)	
5	S	N=8	N=6		N=4		N=2	
4	M	N=8	N=6	N=5	D3 N=4	N=3	N=2	N=1
		SUGRA			SUGRA/SUSY	SUGRA	SUGRA/SUSY	

vector multiplets

tensor multiplet

vector multiplets + multiplets up to spin 1/2

Complete DBI-VA model for the p=3 case

$$S = -\frac{1}{\alpha^2}\int \mathrm{d}^4x \left\{\sqrt{-\det(G_{\mu\nu}+\alpha\mathcal{F}_{\mu\nu})} - 1\right\}, \quad \mu = 0,1,2,3$$

$$\begin{aligned}
G_{\mu\nu} &= \eta_{mn}\Pi^m_\mu\Pi^n_\nu = \eta_{m'n'}\Pi^{m'}_\mu\Pi^{n'}_\nu + \delta_{IJ}\Pi^I_\mu\Pi^J_\nu\,, \qquad m' = 0,1,2,3\,, \quad I = 1,\dots,6 \\
\Pi^{m'}_\mu &= \delta^{m'}_\mu - \alpha^2\bar\lambda\Gamma^{m'}\partial_\mu\lambda\,, \qquad \Pi^I_\mu = \partial_\mu\phi^I - \alpha^2\bar\lambda\Gamma^I\partial_\mu\lambda \\
\mathcal{F}_{\mu\nu} &\equiv F_{\mu\nu} - 2\alpha\bar\lambda\Gamma_{[\mu}\partial_{\nu]}\lambda - 2\alpha\bar\lambda\Gamma_I\partial_{[\mu}\lambda\partial_{\nu]}\phi^I
\end{aligned}$$

16 ϵ and 16 ζ symmetries and shift symmetry of scalars

Can be compared with N = 4 , D=4

N=4, D=4

$$S_{\mathsf{Maxw}} = \int \mathrm{d}^4x\left(-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + 2\bar{\psi}_i \partial\!\!\!/ \psi^i - \tfrac{1}{8}\partial_\mu\varphi_{ij}\partial^\mu\varphi^{ij}\right)$$

one vector; 4 Majorana (or Weyl) spinors and 6 scalars)

recognized as α=0 part of all order action and transformations when

$$\alpha\varphi_{ij} = \phi_a\beta^a_{ij} - \mathrm{i}\phi_{a+3}\alpha^a_{ij}\,, \qquad a = 1,2,3$$

Gliozzi, Scherk, Olive 4× 4 matrices

rewriting the D=10 Majorana-Weyl fermion as $\lambda = \begin{pmatrix}\psi^i \\ \psi_i\end{pmatrix}$

$$\Gamma^\mu = \gamma^\mu \otimes \mathbb{1}_8\,, \qquad \Gamma^a = \gamma_* \otimes \begin{pmatrix} 0 & \beta^a \\ -\beta^a & 0 \end{pmatrix}, \qquad \Gamma^{a+3} = \gamma_* \otimes \begin{pmatrix} 0 & \mathrm{i}\alpha^a \\ \mathrm{i}\alpha^a & 0 \end{pmatrix},$$

$$C_{10} = C_4 \otimes \begin{pmatrix} 0 & \mathbb{1}_4 \\ \mathbb{1}_4 & 0 \end{pmatrix}, \qquad \Gamma_* = \gamma_* \otimes \begin{pmatrix} \mathbb{1}_4 & 0 \\ 0 & -\mathbb{1}_4 \end{pmatrix}$$

All-order deformations of N=4 ,D=4

- Since the action is invariant at all orders under 16+16 supersymmetries, this is the full result !
- The action has both type of supersymmetries: ordinary SUSY and VA-type
- The 10-dimensional formulation is much simpler.

$$S = -\frac{1}{\alpha^2}\int \mathrm{d}^4x \left\{\sqrt{-\det(G_{\mu\nu}+\alpha\mathcal{F}_{\mu\nu})}-1\right\} , \qquad \mu = 0,1,2,3$$

$$\begin{aligned}
G_{\mu\nu} &= \eta_{mn}\Pi_\mu^m\Pi_\nu^n = \eta_{m'n'}\Pi_\mu^{m'}\Pi_\nu^{n'} + \delta_{IJ}\Pi_\mu^I\Pi_\nu^J , \qquad m' = 0,1,2,3 \\
\Pi_\mu^{m'} &= \delta_\mu^{m'} - \alpha^2\bar\lambda\Gamma^{m'}\partial_\mu\lambda , \qquad \Pi_\mu^I = \partial_\mu\phi^I - \alpha^2\bar\lambda\Gamma^I\partial_\mu\lambda , \qquad I = 1,\dots,6 \\
\mathcal{F}_{\mu\nu} &\equiv F_{\mu\nu} - 2\alpha\bar\lambda\Gamma_{[\mu}\partial_{\nu]}\lambda - 2\alpha\bar\lambda\Gamma_I\partial_{[\mu}\lambda\partial_{\nu]}\phi^I
\end{aligned}$$

World volume theory in AdS background

$$S_{cl} = S_{\mathrm{DBI}} + S_{WZ}$$

$$S_{\mathrm{DBI}} = -\int \mathrm{d}^{p+1}\sigma \sqrt{-\det\left(g^{\mathrm{ind}}_{\mu\nu} + F_{\mu\nu}\right)}$$

$$g^{\mathrm{ind}}_{\mu\nu} = \partial_\mu X^M \partial_\nu X^N G_{MN}$$

↓ solution metric

rigid symmetries inherited from solution:

- AdS superisometries (incl. susys)
- isometries of sphere

local

- GCT in *(p+1)*-dimensional worldvolume
- κ symmetry

P. Claus, R. Kallosh and AVP, hep-th/9711161 and 9812066
P. Claus, R. Kallosh, J. Kumar, P. Townsend and AVP, hep-th/9711161

Gauge fixing of GCT on brane and of κ - symmetry

- After gauge fixing: remaining symmetries (from rigid super-AdS and gauge-fixed GCT and κ) appear as conformal rigid symmetries on the brane.
- fermionic ones are ϵ and η.
  (similar to ϵ and ζ in this new work).
- Hope to get all-order result with conformal symmetry in the cases where these superalgebras exist (as for D3)

V-branes: DBI-VA actions with 8+8 supersymmetries

- Our results apply also when we start with D=6, and then can take either p=5 or p=3: these are related to branes called V5 and V3 (see talk Eric Bergshoeff)
- Same formulas as for D=10 lead to actions in 6 and 4 dimensions with 8+8 supersymmetries

The map: dimensions and # of supersymmetries

D	susy	32	24	20	16	12	8	4
11	M	M						
10	MW	IIA IIB			D9 I			
9	M	N=2			N=1			
8	M	N=2			D7 =1			
7	S	N=4			N=2			
6	SW	(2,2)	(2,1)		D5 (1,1) (2,0)		V5 (1,0)	
5	S	N=8	N=6		N=4		N=2	
4	M	N=8	N=6	N=5	D3 N=4	N=3	V3 N=2	N=1
		SUGRA			SUGRA/SUSY	SUGRA	SUGRA/SUSY	

vector multiplets

tensor multiplet

vector multiplets + multiplets up to spin 1/2

5. Conclusions 1

- Superconformal symmetry has been used as a tool for constructing classical actions of supergravity.
  - also **higher-derivative terms** can be constructed
- Quantum calculations show that there are unknown relevant properties of supergravity theories.
- Can (broken) superconformal symmetry be such an extra quantum symmetry?
- The non-existence of (broken) superconformal-invariant counterterms and anomalies in N=4, D=4 could in that case explain ‘miraculous’ vanishing results.

Conclusions 2

- We do not have a systematic knowledge of higher-derivative supergravity action
- Perturbative approach: construct actions and transformation laws order by order in α .
- Starting from Dp brane actions in D=10 we can construct DBI-VA actions with 16+16 susys (*p*=9,7,5,3,...). For *p*=3 this is the deformation of N=4, D=4 with higher order derivatives
- Same can be done from D=6: DBI-VA actions (V-branes) with 8+8 susys, e.g. containing the deformation of D=4, N=2.
- Hope that this can lead also to supergravity actions using the superconformal methods.