The Ghost Condensate in N=1 Supergravity

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Breaking of Supersymmetry

and Ultraviolet Divergences

in Extended Supergravity

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Motivation

Higher-derivative scalar field theories coupled to gravitation appear in

1) DBI theories
2) ghost-condensate theories of NEC violation
3) Galileon theories of cosmology
4) worldvolume actions of solitonic branes

Can these be extended to N=1 local supersymmetry?

Yes! We have

- given a general formalism for constructing N=1 supersymmetric higher-derivative chiral superfield Lagrangians and coupling them to supergravity
- applied this to DBI, ghost-condensates and Galileons.

Scalar Ghost Condensation

Consider a real scalar field ϕ. Denote the standard kinetic term $X = -\frac{1}{2}(\partial\phi)^2$. A ghost condensate arises from higher-derivative theories of the form

$$\mathcal{L} = \sqrt{-g}P(X)$$

where P(X) is an arbitrary differentiable function of X. In a flat FRW spacetime with $ds^2 = -dt^2 + a(t)^2\eta_{ij}dx^i dx^j$ and assuming $\phi = \phi(t)$, the scalar equation of motion is

$$\frac{d}{dt}\left(a^3 P_{,X}\dot{\phi}\right) = 0$$

Trivial solution is $\phi = \text{constant}$. More interesting is the the solution

$$X(=\frac{1}{2}\dot{\phi}^2) = \text{constant}, \quad P_{,X} = 0$$

Denoting by X_{ext} a constant extremum of P(X), the equation of motion admits the "ghost condensate" solution

$$\phi = ct, \quad c^2 = 2X_{\text{ext}}$$

This vacuum spontaneously breaks Lorentz invariance. It can also lead to violations of the "null energy condition" (NEC). To see this, evaluating the energy and pressure densities ⇒

$$\rho = 2XP_{,X} - P, \quad p = P \quad \Rightarrow \quad \rho + p = 2XP_{,X}$$

The NEC corresponds to the requirement that

$$\rho + p \geq 0$$

Since X>0, $\Rightarrow$ the NEC can be violated if

$$P_{,X} < 0$$

That is, if we are close to an extremum of P(X) then on one side the NEC is violated while on the the other side it is not. Since Einstein's equations $\Rightarrow$

$$\dot{H} = -\frac{1}{2}(\rho + p)$$

it is now possible to obtain a non-singular "bouncing" universe where H increases from negative to positive values.

However, is this NEC violating vacuum "stable"?

Expanding the Lagrangian around the ghost condensate

$$\phi = ct + \delta\phi(x^m)$$

gives to quadratic order

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}\Big((2XP_{,XX} + P_{,X})(\dot{\delta\phi})^2 - P_{,X}\delta\phi^{,i}\delta\phi_{,i}\Big)$$

Note that Lorentz violation $\Rightarrow$ the coefficients of the time- and space-derivative pieces are different. The vacuum will be ghost-free iff

$$2XP_{,XX} + P_{,X} > 0$$

This can be achieved by choosing the condensate to be at a minimum

$$P_{,XX} > 0$$

Note that the theory can remain ghost-free even in the NEC violating region where $P_{,X} < 0$.

However, in the NEC violating region the coefficient $-P_{,X}$ in front of the spatial derivative term has the wrong sign. $\Rightarrow$ The theory suffers from "gradient instabilities"! These can be softened by adding small higher-derivative terms--not of the P(X) type--such as

$$-(\Box\phi)^2$$

These modify the dispersion relation for $\delta\phi$ at high momenta and suppress instabilites for a short--but sufficient--period of time.

Finally, a prototypical choice for P(X) that shows all interesting properties is

$$P(X) = -X + X^2 \quad (\Rightarrow \;\; c = 1)$$

Review of Globally N=1 Supersymmetric Ghost Condensation

Higher-Derivative Superfield Lagrangian:

Consider the chiral superfield

$$\Phi = A + \mathrm{i}\theta\sigma^m\bar{\theta}A_{,m} + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A + \theta\theta F + \sqrt{2}\theta\chi - \frac{\mathrm{i}}{\sqrt{2}}\theta\theta\chi_{,m}\sigma^m\bar{\theta}$$

The ordinary kinetic Lagrangian is

$$\mathcal{L}_{\Phi^\dagger\Phi} = \int \mathrm{d}^4\theta\, \Phi^\dagger\Phi = \Phi^\dagger\Phi\,|_{\theta\theta\bar{\theta}\bar{\theta}} = -\partial A\cdot\partial A^* + F^*F + \frac{\mathrm{i}}{2}(\chi_{,m}\sigma^m\bar{\chi} - \chi\sigma^m\bar{\chi}_{,m})$$

Defining $A = \frac{1}{\sqrt{2}}(\phi + \mathrm{i}\xi)$ the Lagrangian becomes

$$\mathcal{L}_{\Phi^\dagger\Phi} = -\frac{1}{2}(\underset{\substack{\uparrow\\ X}}{\partial}\phi)^2 - \frac{1}{2}(\partial\xi)^2 + F^*F + \frac{\mathrm{i}}{2}(\chi_{,m}\sigma^m\bar{\chi} - \chi\sigma^m\bar{\chi}_{,m})$$

This is the global N=1 supersymmetric generalization of X.

What is the supersymmetric generalization of X^2?

Consider

$$\mathcal{L}_{D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger} = \frac{1}{16} D\Phi D\Phi \bar{D}\Phi^\dagger \bar{D}\Phi^\dagger\Big|_{\theta\theta\bar{\theta}\bar{\theta}}$$

To quadratic order in the spinor component field

$$\begin{aligned}
\mathcal{L}_{D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger} = {} & (\partial A)^2(\partial A^*)^2 - 2F^*F\partial A\cdot\partial A^* + F^{*2}F^2 \\
& -\frac{\mathrm{i}}{2}(\chi\sigma^m\bar{\sigma}^l\sigma^n\bar{\chi}_{,n})A_{,m}A^*_{,l} + \frac{\mathrm{i}}{2}(\chi_{,n}\sigma^n\bar{\sigma}^m\sigma^l\bar{\chi})A_{,m}A^*_{,l} \\
& +\mathrm{i}\chi\sigma^m\bar{\chi}^{,n}A_{,m}A^*_{,n} - \mathrm{i}\chi^{,m}\sigma^n\bar{\chi}A_{,m}A^*_{,n} + \frac{\mathrm{i}}{2}\chi\sigma^m\bar{\chi}(A^*_{,m}\Box A - A_{,m}\Box A^*) \\
& +\frac{1}{2}(F\Box A - \partial F\partial A)\bar{\chi}\bar{\chi} + \frac{1}{2}(F^*\Box A^* - \partial F^*\partial A^*)\chi\chi \\
& +\frac{1}{2}FA_{,m}(\bar{\chi}\bar{\sigma}^m\sigma^n\bar{\chi}_{,n} - \bar{\chi}_{,n}\bar{\sigma}^m\sigma^n\bar{\chi}) + \frac{1}{2}F^*A^*_{,m}(\chi_{,n}\sigma^n\bar{\sigma}^m\chi - \chi\sigma^n\bar{\sigma}^m\chi_{,n}) \\
& +\frac{3\mathrm{i}}{2}F^*F(\chi_{,m}\sigma^m\bar{\chi} - \chi\sigma^m\bar{\chi}_{,m}) + \frac{\mathrm{i}}{2}\chi\sigma^m\bar{\chi}(FF^*_{,m} - F^*F_{,m})
\end{aligned}$$

In terms of ϕ,ξ the pure A part of the Lagrangian is

$$(\partial A)^2(\partial A^*)^2 = \frac{1}{4}(\partial\phi)^4 + \frac{1}{4}(\partial\xi)^4 - \frac{1}{2}(\partial\phi)^2(\partial\xi)^2 + (\partial\phi\cdot\partial\xi)^2$$

X^2

This is the global N=1 supersymmetric generalization of X^2. It is the unique generalization with the properties:

a) When the spinor is set to zero, the only non-vanishing term in $\frac{1}{16} D\Phi D\Phi \bar{D}\Phi^\dagger \bar{D}\Phi^\dagger$ is the top $\theta^2\bar{\theta}^2$ component.

This is very helpful in producing higher-derivative terms that include X^2.

b) When coupled to supergravity, $\frac{1}{16} D\Phi D\Phi \bar{D}\Phi^\dagger \bar{D}\Phi^\dagger$ leads to minimal coupling of ϕ, ξ to gravity.

For example, an alternative generalization of X^2

$$-\frac{1}{16}(\Phi - \Phi^\dagger)^2 \bar{D} D\Phi D\bar{D}\Phi^\dagger \quad \Rightarrow \quad \phi^2(\underline{\partial\xi})^2\mathcal{R}$$

Globally Supersymmetric Ghost Condensate:

Choose the scalar function P(X) to be

$$P(X) = -X + X^2$$

For a pure ghost condensate can take the superpotential

$$W = 0 \quad \Longrightarrow \quad F = 0$$

The associated globally supersymmetric Lagrangian, to quadratic order in the spinor, is

$$\begin{aligned}
\mathcal{L}^{\mathrm{SUSY}} &= \left(-\Phi^\dagger\Phi + \frac{1}{16} D\Phi D\Phi \bar{D}\Phi^\dagger \bar{D}\Phi^\dagger \right)\Big|_{\theta\theta\bar{\theta}\bar{\theta}} \\
&= +\frac{1}{2}(\partial\phi)^2 + \frac{1}{4}(\partial\phi)^4 + \frac{1}{2}(\partial\xi)^2 + \frac{1}{4}(\partial\xi)^4 - \frac{1}{2}(\partial\phi)^2(\partial\xi)^2 + (\partial\phi\cdot\partial\xi)^2 \\
&- \frac{\mathrm{i}}{2}(\chi_{,m}\sigma^m\bar{\chi} - \chi\sigma^m\bar{\chi}_{,m}) - \frac{1}{2}(\partial\phi)^2\frac{\mathrm{i}}{2}(\chi_{,m}\sigma^m\bar{\chi} - \chi\sigma^m\bar{\chi}_{,m}) \\
&- \phi_m\phi_{,n}\frac{\mathrm{i}}{2}(\chi^{,n}\sigma^m\bar{\chi} - \chi\sigma^m\bar{\chi}^{,n})
\end{aligned}$$

The equations of motion admit a ghost condensate vacuum

$$\phi = t, \quad \xi = 0, \quad \chi = 0$$

To assess stability, expand in the small fluctuations

$$\phi = t + \delta\phi(x^m), \quad \xi = \delta\xi(x^m), \quad \chi = \delta\chi(x^m)$$

To quadratic order, the result is

$$\begin{aligned}\mathcal{L}^{\mathrm{SUSY}} &= (\delta\dot{\phi})^2 + 0\cdot\delta\phi^{,i}\delta\phi_{,i} \\ &+ \ 0\cdot(\delta\dot{\xi})^2 + \delta\xi^{,i}\delta\xi_{,i} \\ &+ \ \frac{1}{2}\frac{\mathrm{i}}{2}\left(\delta\chi_{,0}\sigma^0\delta\bar{\chi} - \delta\chi\sigma^0\delta\bar{\chi}_{,0}\right) - \frac{1}{2}\frac{\mathrm{i}}{2}\left(\delta\chi_{,i}\sigma^i\delta\bar{\chi} - \delta\chi\sigma^i\delta\bar{\chi}_{,i}\right)\end{aligned}$$

1) $\delta\phi$ kinetic term: As previously, has a gradient instability in the NEC violating region. ⇒ In the pure boson case, added a $-(\Box\phi)^2$ term. The appropriate SUSY extension is

$$-\frac{1}{2^{11}}\underset{\substack{\uparrow\\ X^2}}{D\Phi D\Phi}\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\left(\{D,\bar{D}\}\{D,\bar{D}\}(\Phi+\Phi^\dagger)\right)^2\Big|_{\theta\theta\bar{\theta}\bar{\theta}} = -(\Box\phi)^2\Big(\tfrac{1}{4}(\partial\phi)^4 + \tfrac{1}{4}(\partial\xi)^4 + (\partial\phi\cdot\partial\xi)^2 - \tfrac{1}{2}(\partial\phi)^2(\partial\xi)^2\Big)$$

Expanding around the ghost condensate using $(\partial\phi)^2 = -1$

$\Rightarrow$
$$\mathcal{L}^{\rm SUSY} = (\dot{\delta\phi})^2 + 0\cdot\delta\phi^{,i}\delta\phi_{,i} - \frac{1}{4}\underline{(\Box\delta\phi)^2} + \dots$$

which softens the gradient instabilities.

2) $\delta\xi$ kinetic term: New to SUSY. Has vanishing time and wrong sign spatial kinetic terms. Cured by adding supersymmetric higher-derivative terms. The appropriate terms are

$$+\frac{8}{16^2}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\Big(\{D,\bar{D}\}(\Phi-\Phi^\dagger)\{D,\bar{D}\}(\Phi^\dagger-\Phi)\Big)\Big|_{\theta\theta\bar{\theta}\bar{\theta}}$$

X^2

$$-\frac{4}{16^3}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\Big(\{D,\bar{D}\}(\Phi+\Phi^\dagger)\{D,\bar{D}\}(\Phi-\Phi^\dagger)\Big)\Big(\{D,\bar{D}\}(\Phi+\Phi^\dagger)\{D,\bar{D}\}(\Phi^\dagger-\Phi)\Big)\Big|_{\theta\theta\bar{\theta}\bar{\theta}}$$

$$= -2(\partial\phi)^4(\partial\xi)^2 - (\partial\phi)^4(\partial\phi\cdot\partial\xi)^2$$

Expanding around the ghost condensate $\Rightarrow$

$$\mathcal{L}^{\rm SUSY} = \dots + (\dot{\delta\xi})^2 - \delta\xi^{,i}\delta\xi_{,i} + \dots$$

which is Lorentz covariant with the correct sign.

3) $\delta\chi$ kinetic term: Ghost free with gradient "instability". Can be cured with the context of supersymmetric Galileons but re-grow a ghost! Won't discuss here.

Michael Koehn's talk

To summarize: The entire supersymmetric ghost consensate Lagrangian is

$$
\begin{aligned}
\mathcal{L}^{\rm SUSY} &= -\Phi^\dagger\Phi\,|_{\theta\theta\bar\theta\bar\theta} + \frac{1}{16}D\Phi D\Phi\bar D\Phi^\dagger\bar D\Phi^\dagger\,|_{\theta\theta\bar\theta\bar\theta} \\
&\quad + D\Phi D\Phi\bar D\Phi^\dagger\bar D\Phi^\dagger\Bigg[-\frac{1}{2^{11}}\Big(\{D,\bar D\}\{D,\bar D\}(\Phi+\Phi^\dagger)\Big)^2 \\
&\qquad\qquad + \frac{1}{2^5}\{D,\bar D\}(\Phi-\Phi^\dagger)\{D,\bar D\}(\Phi^\dagger-\Phi) \\
&\qquad\qquad - \frac{1}{2^{10}}\Big(\{D,\bar D\}(\Phi+\Phi^\dagger)\{D,\bar D\}(\Phi-\Phi^\dagger)\Big)^2\Bigg]\Bigg|_{\theta\theta\bar\theta\bar\theta} \\
&= +\frac{1}{2}(\partial\phi)^2 + \frac{1}{4}(\partial\phi)^4 - \frac{1}{4}(\partial\phi)^4(\Box\phi)^2 \\
&\quad + \frac{1}{2}(\partial\xi)^2 - \frac{1}{2}(\partial\phi)^2(\partial\xi)^2 - 2(\partial\phi)^4(\partial\xi)^2 + (\partial\phi\cdot\partial\xi)^2 - (\partial\phi)^4(\partial\phi\cdot\partial\xi)^2 \\
&\quad + \frac{i}{2}(\chi_{,m}\sigma^m\bar\chi - \chi\sigma^m\bar\chi_{,m})\Big(-1-\frac{1}{2}(\partial\phi)^2\Big) - \phi_m\phi_{,n}\frac{i}{2}(\chi^{,n}\sigma^m\bar\chi - \chi\sigma^m\bar\chi^{,n}) + \ldots
\end{aligned}
$$

The ghost condensate vacuum of this theory breaks N=1 supersymmetry spontaneously in a new form.

Consider the SUSY transformation of the fermion

$$\delta\chi = \mathrm{i}\sqrt{2}\sigma^m\bar{\zeta}\partial_m A + \sqrt{2}\zeta F$$

Usually supersymmetry is broken by a non-vanishing VEV $\langle F\rangle \neq 0$ of the auxiliary field. However, since in the ghost condensate Lagrangian $W=0 \implies F=0$.

Recall that for the ghost condensate $\langle\phi\rangle = ct$ $\Rightarrow$

$$\langle\dot{A}\rangle = \frac{\dot{\phi}}{\sqrt{2}} = \frac{c}{\sqrt{2}}$$

Therefore

$$\delta\chi = \mathrm{i}\sqrt{2}\sigma^m\bar{\zeta}\partial_m A = \mathrm{i}\sigma^0\bar{\zeta}c$$

and the spinor transforms inhomogeneously $\Rightarrow$ SUSY is broken by the time-dependent condensate.

The Ghost Condensate in N=1 Supergravity

In previous work, we showed that a global N=1 supersymmetric Lagrangian of the general form

$$\mathcal{L}^{\text{SUSY}} = K(\Phi,\Phi^\dagger)\mid_{\theta\theta\bar\theta\bar\theta} + \frac{1}{16} D\Phi D\Phi \bar{D}\Phi^\dagger \bar{D}\Phi^\dagger T(\Phi,\Phi^\dagger,\partial_m\Phi,\partial_n\Phi^\dagger)\mid_{\theta\theta\bar\theta\bar\theta}$$

$$+\Big(W(\Phi)\mid_{\theta\theta} + W^\dagger(\Phi^\dagger)\mid_{\bar\theta\bar\theta}\Big) \qquad \text{Michael Koehn's talk}$$

where K is any real function, T is an arbitrary hermitian function (with all derivative indices contracted) and W is a holomorphic superpotential, can consistently be coupled to N=1 supergravity.

Notation: Curved N=1 superspace

$$(x^m, \Theta^\alpha, \bar\Theta_{\dot\alpha}) \quad , \quad \mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha})$$

Gravity supermultiplet

$$(e_m^a, \psi_m, M, b_m)$$

Two superfield expansions we will need are the chiral curvature superfield

$$\begin{aligned} R = &-\frac{1}{6}M - \frac{1}{6}\Theta^\alpha\left(\sigma_{\alpha\dot\alpha}{}^a\bar\sigma^{b\dot\alpha\beta}\psi_{ab\beta} - \mathrm{i}\sigma_{\alpha\dot\alpha}{}^a\bar\psi_a{}^{\dot\alpha}M + \mathrm{i}\psi_{a\alpha}b^a\right) \\ &+ \Theta^\alpha\Theta_\alpha\Big(\frac{1}{12}\mathcal{R} - \frac{1}{6}\mathrm{i}\bar\psi^a{}_{\dot\alpha}\bar\sigma^{b\dot\alpha\beta}\psi_{ab\beta} - \frac{1}{9}MM^* - \frac{1}{18}b^a b_a + \frac{1}{6}\mathrm{i}e_a{}^m\mathcal{D}_m b^a \\ &- \frac{1}{12}\bar\psi_{\dot\alpha}\bar\psi^{\dot\alpha}M + \frac{1}{12}\psi_a{}^\alpha\sigma_{\alpha\dot\alpha}{}^a\bar\psi_c{}^{\dot\alpha}b^c - \frac{1}{48}\varepsilon^{abcd}\left[\bar\psi_{a\dot\alpha}\bar\sigma_b{}^{\dot\alpha\beta}\psi_{cd\beta} + \psi_a{}^\alpha\sigma_{\alpha\dot\alpha b}\bar\psi_{cd}{}^{\dot\alpha}\right]\Big) \end{aligned}$$

and the chiral density superfield

$$2\mathcal{E} = e\Big(1 + \mathrm{i}\Theta^\alpha\sigma_{\alpha\dot\alpha}{}^a\bar\psi_a{}^{\dot\alpha} - \Theta^\alpha\Theta_\alpha\left[M^* + \bar\psi_{a\dot\alpha}\bar\sigma^{ab\dot\alpha}{}_{\dot\beta}\bar\psi_b{}^{\dot\beta}\right]\Big)$$

In terms of these quantities, the supergravity extension of global $\mathcal{L}^{\mathrm{SUSY}}$ is

$$\begin{aligned} \mathcal{L}^{SUGRA} = \int \mathrm{d}^2\Theta 2\mathcal{E}\Big[&\frac{3}{8}(\bar{\mathcal{D}}^2 - 8R)\mathrm{e}^{-K/3} - \frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger T) \\ &+ W(\Phi)\Big] + \mathrm{h.c.} \end{aligned}$$

Since we are interested in the pure ghost condensate, we can take

$$W=0 \implies F=M=0$$

The component expansion of $\mathcal{L}^{SUGRA}$ is then given by

$$\begin{aligned}
\mathcal{L}^{SUGRA} = \Big[& -\frac{3}{32}e(\mathcal{D}^2\bar{\mathcal{D}}^2\mathrm{e}^{-K/3}) + \mathrm{i}\frac{3}{16}e\bar{\psi}_{a\dot{\alpha}}\bar{\sigma}^{a\dot{\alpha}\alpha}(\mathcal{D}_\alpha\bar{\mathcal{D}}^2\mathrm{e}^{-K/3}) - \frac{3}{8}e\bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b(\bar{\mathcal{D}}^2\mathrm{e}^{-K/3}) \\
& +\mathrm{i}\frac{1}{4}e(\bar{\psi}_a\bar{\sigma}^a)^\alpha(\mathcal{D}_\alpha\mathrm{e}^{-K/3}) - \frac{1}{4}e\big(\psi_{ab}\sigma^b\bar{\psi}^a + \mathrm{i}\psi_a b^a\big)^\alpha(\mathcal{D}_\alpha\mathrm{e}^{-K/3}) \\
& +\frac{1}{32}e\mathcal{D}^2\bar{\mathcal{D}}^2(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger T) - \frac{1}{16}e\mathrm{i}(\bar{\psi}_a\bar{\sigma}^a)^\alpha\mathcal{D}_\alpha\bar{\mathcal{D}}^2(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger T) \\
& +\frac{1}{8}e\bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b\bar{\mathcal{D}}^2(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger T)\Big]\Big|^{\text{lowest component}} + \text{h.c.} \\
& +e\Big(-\frac{1}{2}\mathcal{R} + \frac{1}{3}b^a b_a + \frac{1}{4}\varepsilon^{abcd}(\bar{\psi}_a\bar{\sigma}_b\psi_{cd} - \psi_a\sigma_b\bar{\psi}_{cd})\Big)\mathrm{e}^{-K(A,A^*)/3}
\end{aligned}$$

where

$$\psi_{mn}{}^\alpha = \tilde{\mathcal{D}}_m\psi_n{}^\alpha - \tilde{\mathcal{D}}_n\psi_m{}^\alpha, \quad \tilde{\mathcal{D}}_m\psi_n{}^\alpha = \partial_m\psi_n{}^\alpha + \psi_n{}^\beta\omega_{m\beta}{}^\alpha$$

Note that the auxiliary field b_m remains undetermined.

Must evaluate the lowest component of superfield term.

Evaluating the first part of the Lagrangian ⇒

$$
\begin{aligned}
\frac{1}{e}\mathcal{L}^{SUGRA}_{K(\Phi,\Phi^\dagger)} &= \frac{1}{e}\Big[\int \mathrm{d}^2\Theta 2\mathcal{E}\frac{3}{8}(\bar{\mathcal{D}}^2 - 8R)\mathrm{e}^{-K/3}\Big] + \mathrm{h.c.} \\
&= \Big(-\frac{1}{2}\mathcal{R} + \frac{1}{3}b^a b_a + \frac{1}{4}\varepsilon^{abcd}(\bar\psi_a\bar\sigma_b\psi_{cd} - \psi_a\sigma_b\bar\psi_{cd})\Big)\mathrm{e}^{-K(A,A^*)/3} \\
&+ 3|\partial A|^2(\mathrm{e}^{-K/3})_{,AA^*} + \mathrm{i}b^a\big(A_{,a}(\mathrm{e}^{-K/3})_{,A} - A^*{}_{,a}(\mathrm{e}^{-K/3})_{,A^*}\big) \\
&- \mathrm{i}\frac{1}{\sqrt{2}}b^a\big(\psi_a\chi(\mathrm{e}^{-K/3})_{,A} - \bar\psi_a\bar\chi(\mathrm{e}^{-K/3})_{,A^*}\big) \\
&- \sqrt{2}\chi\sigma^{mn}\psi_{mn}(\mathrm{e}^{-K/3})_{,A} - \sqrt{2}\bar\chi\bar\sigma^{mn}\bar\psi_{mn}(\mathrm{e}^{-K/3})_{,A^*} \\
&- \mathrm{i}\frac{3}{2}\psi_a\sigma^{ab}\sigma^c\bar\psi_b A_{,c}(\mathrm{e}^{-K/3})_{,A} - \mathrm{i}\frac{3}{2}\bar\psi_a\bar\sigma^{ab}\bar\sigma^c\psi_b A^*{}_{,c}(\mathrm{e}^{-K/3})_{,A^*} \\
&+ \frac{1}{2}\chi\sigma^a\bar\chi b_a(\mathrm{e}^{-K/3})_{,AA^*} + \mathrm{i}\frac{3}{2}\big(\chi\sigma^a e_a{}^m\mathcal{D}_m\bar\chi + \bar\chi\bar\sigma^a e_a{}^m\mathcal{D}_m\chi\big)(\mathrm{e}^{-K/3})_{,AA^*} \\
&+ \frac{3}{2}\sqrt{2}A^*{}_{,b}\psi_a\sigma^b\bar\sigma^a\chi(\mathrm{e}^{-K/3})_{,AA^*} + \frac{3}{2}\sqrt{2}A_{,b}\bar\psi_a\bar\sigma^b\sigma^a\bar\chi(\mathrm{e}^{-K/3})_{,AA^*} \\
&- \frac{3}{2}(\partial A)^2(\mathrm{e}^{-K/3})_{,AA} - \frac{3}{2}(\partial A^*)^2(\mathrm{e}^{-K/3})_{,A^*A^*} \\
&+ \mathrm{i}\frac{3}{2}\chi\sigma^a\bar\chi\Big(A^*{}_{,a}(\mathrm{e}^{-K/3})_{,AA^*A^*} - A_{,a}(\mathrm{e}^{-K/3})_{,AAA^*}\Big)
\end{aligned}
$$

This is the supergravity extension of the -X scalar term if one takes

$$K(\Phi,\Phi^\dagger) = -\Phi\Phi^\dagger$$

Evaluating the second part of the Lagrangian taking

$$T = \frac{\tau}{16} \quad \Rightarrow$$

$$\frac{1}{e}\mathcal{L}^{SUGRA}_{\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger,\tau} = \frac{1}{e}\Big(-\frac{\tau}{2^7}\int \mathrm{d}^2\Theta 2\mathcal{E}(\bar{\mathcal{D}}^2 - 8R)(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger)\Big) + \text{h.c.}$$
$$= \Big(+ \frac{\tau}{2^9}\mathcal{D}^2\bar{\mathcal{D}}^2(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger) - \frac{\tau}{2^8}\mathrm{i}(\bar{\psi}_a\bar{\sigma}^a)^\alpha\mathcal{D}_\alpha\bar{\mathcal{D}}^2(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger)$$
$$+ \frac{\tau}{2^7}\bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b\bar{\mathcal{D}}^2(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger)\Big)\Big| + \text{h.c.}$$
$$= +\tau(\partial A)^2(\partial A^*)^2 - \frac{1}{2}\sqrt{2}\tau\bar{\psi}_a\bar{\sigma}^a\sigma^c\bar{\chi}A^*{}_{,c}(\partial A)^2 - \frac{1}{2}\sqrt{2}\tau\chi\sigma^c\bar{\sigma}^a\psi_a A_{,c}(\partial A^*)^2$$
$$- \sqrt{2}\tau(\partial A^*)^2 A_{,m}\chi\psi^m - \sqrt{2}\tau(\partial A)^2 A^*{}_{,m}\bar{\psi}^m\bar{\chi}$$
$$-\frac{\mathrm{i}}{2}\tau\chi\sigma^a\bar{\chi}A_{,a}e_b{}^m\mathcal{D}_m A^*{}_{,b} + \frac{5}{6}\tau\chi\sigma^a\bar{\chi}A_{,a}A^*{}_{,b}b^b$$
$$+\frac{\mathrm{i}}{2}\tau\chi\sigma^a\bar{\chi}A^*{}_{,a}e_b{}^m\mathcal{D}_m A_{,b} + \frac{5}{6}\tau\chi\sigma^a\bar{\chi}A^*{}_{,a}A_{,b}b^b$$
$$-\mathrm{i}\tau(\mathcal{D}_m\chi)\sigma^b\bar{\chi}A^{,m}A^*{}_{,b} + \sqrt{2}\tau\bar{\psi}_a\bar{\sigma}^c\sigma^b\bar{\chi}A^{,a}A^*{}_{,b}A_{,c} + \frac{1}{3}\tau\bar{\chi}\bar{\sigma}^b\sigma_c\bar{\sigma}_a\chi b^c A^{,a}A^*{}_{,b}$$
$$+\mathrm{i}\tau\chi\sigma^b(\mathcal{D}_m\bar{\chi})A^{*,m}A_{,b} + \sqrt{2}\tau\chi\sigma^b\bar{\sigma}^c\psi_a A^{*,a}A_{,b}A^*{}_{,c}$$
$$-\frac{\mathrm{i}}{2}\tau\chi\sigma^a\bar{\sigma}^b\sigma^m(\mathcal{D}_m\bar{\chi})A_{,a}A^*{}_{,b} - \frac{1}{12}\tau\chi\sigma^a\bar{\sigma}^b\sigma^c\bar{\chi}b_c A_{,a}A^*{}_{,b}$$
$$+\frac{\mathrm{i}}{2}\tau(\mathcal{D}_m\chi)\sigma^m\bar{\sigma}^b\sigma^a\bar{\chi}A^*{}_{,a}A_{,b} - \frac{1}{12}\tau\chi\sigma^c\bar{\sigma}^b\sigma^a\bar{\chi}b_c A^*{}_{,a}A_{,b}\ .$$

This is the supergravity extension of the X^2 scalar term if one takes

$$\tau = 1$$

The equation of motion for b_m is given by

$$
\begin{aligned}
b_m = &-\frac{3}{2}\mathrm{i}\left(A_{,m}(\mathrm{e}^{-K/3})_{,A} - A^*{}_{,m}(\mathrm{e}^{-K/3})_{,A^*}\right)\mathrm{e}^{K/3} - \frac{3}{4}\chi\sigma_m\bar{\chi}(\mathrm{e}^{-K/3})_{,AA^*}\mathrm{e}^{K/3} \\
&+\frac{3}{4}\sqrt{2}\mathrm{i}\left(\psi_m\chi(\mathrm{e}^{-K/3})_{,A} - \bar{\psi}_m\bar{\chi}(\mathrm{e}^{-K/3})_{,A^*}\right)\mathrm{e}^{K/3} \\
&-\frac{5}{4}\tau\chi\sigma^a\bar{\chi}(A_{,a}A^*{}_{,m} + A^*{}_{,a}A_{,m})\mathrm{e}^{K/3} \\
&+\frac{1}{2}\tau\chi\sigma^a\bar{\sigma}_m\sigma^b\bar{\chi}A_{,a}A^*{}_{,b}\mathrm{e}^{K/3} \\
&+\frac{1}{8}\tau\left(\chi\sigma^a\bar{\sigma}^b\sigma_m\bar{\chi} + \chi\sigma_m\bar{\sigma}^a\sigma^b\bar{\chi}\right)A_{,a}A^*{}_{,b}\mathrm{e}^{K/3}
\end{aligned}
$$

Inserting this back into the Lagrangian, Weyl rescaling as

$$
\begin{aligned}
e_n{}^a &\xrightarrow{\text{WEYL}} \mathrm{e}^{K/6}e_n{}^a \\
\chi &\xrightarrow{\text{WEYL}} \mathrm{e}^{-K/12}\chi \\
\psi_m &\xrightarrow{\text{WEYL}} \mathrm{e}^{K/12}\psi_m
\end{aligned}
$$

and shifting

$$
\psi_m \xrightarrow{\text{SHIFT}} \psi_m + \mathrm{i}\frac{\sqrt{2}}{6}\sigma_m\bar{\chi}K_{,A^*}
$$

$\Rightarrow$ keeping terms with at most two fermions

$$
\begin{aligned}
\frac{1}{e}\mathcal{L}^{SUGRA}_{K(\Phi,\Phi^\dagger),\mathrm{Weyl}} =& \frac{1}{e}\Big[\int \mathrm{d}^2\Theta 2\mathcal{E}\frac{3}{8}(\bar{\mathcal{D}}^2 - 8R)\mathrm{e}^{-K/3}\Big]_{\mathrm{Weyl}} + \mathrm{h.c.} \\
=& -\frac{1}{2}\mathcal{R} - K_{,AA^*}|\partial A|^2 \\
& - \mathrm{i}K_{,AA^*}\bar{\chi}\bar{\sigma}^m\mathcal{D}_m\chi + \varepsilon^{klmn}\bar{\psi}_k\bar{\sigma}_l\tilde{\mathcal{D}}_m\psi_n \\
& - \frac{1}{2}\sqrt{2}K_{,AA^*}A^*{}_{,n}\chi\sigma^m\bar{\sigma}^n\psi_m - \frac{1}{2}\sqrt{2}K_{,AA^*}A_{,n}\bar{\chi}\bar{\sigma}^m\sigma^n\bar{\psi}_m
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{1}{e}\mathcal{L}^{SUGRA}_{\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger,\tau,\mathrm{Weyl}} =& \frac{1}{e}\Big[\int \mathrm{d}^2\Theta 2\mathcal{E}(-\frac{\tau}{2^7})(\bar{\mathcal{D}}^2 - 8R)(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger)\Big]_{\mathrm{Weyl}} + \mathrm{h.c.} \\
=& +\tau(\partial A)^2(\partial A^*)^2 - \frac{1}{2}\sqrt{2}\tau\bar{\psi}_a\bar{\sigma}^a\sigma^c\bar{\chi}A^*{}_{,c}(\partial A)^2 - \frac{1}{2}\sqrt{2}\tau\chi\sigma^c\bar{\sigma}^a\psi_a A_{,c}(\partial A^*)^2 \\
& - \sqrt{2}\tau(\partial A^*)^2 A_{,m}\chi\psi^m - \sqrt{2}\tau(\partial A)^2 A^*{}_{,m}\bar{\psi}^m\bar{\chi} \\
& -\frac{\mathrm{i}}{2}\tau\chi\sigma^a\bar{\chi}A_{,a}e^{bm}(\mathcal{D}_m A^*{}_{,b}) + \frac{\mathrm{i}}{2}\tau\chi\sigma^a\bar{\chi}A^*{}_{,a}e^{bm}(\mathcal{D}_m A_{,b}) \\
& - \frac{\mathrm{i}}{6}\tau\chi\sigma^a\bar{\chi}A_{,a}A^*{}_{,b}K^{,b} + \frac{\mathrm{i}}{6}\tau\chi\sigma^a\bar{\chi}A^*{}_{,a}A_{,b}K^{,b} \\
& -\mathrm{i}\tau(\mathcal{D}_m\chi)\sigma_n\bar{\chi}A^{,m}A^{*,n} + \sqrt{2}\tau\bar{\psi}_a\bar{\sigma}^c\sigma^b\bar{\chi}A^{,a}A^*{}_{,b}A_{,c} \\
& + \frac{\mathrm{i}}{12}\tau\chi\sigma^a\bar{\chi}A_{,b}A^*{}_{,a}K^{,b} + \frac{\mathrm{i}}{6}\tau\chi\sigma^{cb}\sigma^a\bar{\chi}A_{,c}A^*{}_{,a}K_{,b} \\
& +\mathrm{i}\tau\chi\sigma^b(\mathcal{D}_m\bar{\chi})A^{*,m}A_{,b} + \sqrt{2}\tau\chi\sigma^b\bar{\sigma}^c\psi_a A^{*,a}A_{,b}A^*{}_{,c} \\
& - \frac{\mathrm{i}}{12}\tau\chi\sigma^a\bar{\chi}A^*{}_{,b}A_{,a}K^{,b} - \frac{\mathrm{i}}{6}\tau\chi\sigma^a\bar{\sigma}^{bc}\bar{\chi}A^*{}_{,c}A_{,a}K_{,b} \\
& -\frac{\mathrm{i}}{2}\tau\chi\sigma^p\bar{\sigma}^q\sigma^m(\mathcal{D}_m\bar{\chi})A_{,p}A^*{}_{,q} + \frac{\mathrm{i}}{2}\tau(\mathcal{D}_m\chi)\sigma^m\bar{\sigma}^p\sigma^q\bar{\chi}A_{,p}A^*{}_{,q} \\
& + \frac{\mathrm{i}}{6}\tau\chi\sigma^c\bar{\sigma}^b\sigma^a\bar{\chi}K_{,a}A^*{}_{,b}A_{,c} - \frac{\mathrm{i}}{6}\tau\chi\sigma^a\bar{\sigma}^b\sigma^c\bar{\chi}K_{,a}A_{,b}A^*{}_{,c} \\
& - \frac{7}{4}\mathrm{i}\tau\chi\sigma^a\bar{\chi}\big(A^*{}_{,a}(\partial A)^2(\mathrm{e}^{-K/3})_{,A} - A_{,a}(\partial A^*)^2(\mathrm{e}^{-K/3})_{,A^*}\big)\mathrm{e}^{K/3} \\
& - \frac{3}{2}\mathrm{i}\tau\chi\sigma^a\bar{\chi}\big(A_{,a}(\mathrm{e}^{-K/3})_{,A} - A^*{}_{,a}(\mathrm{e}^{-K/3})_{,A^*}\big)|\partial A|^2\mathrm{e}^{K/3}\ .
\end{aligned}
$$

The N=1 Supergravity Ghost Condensate:

Taking $K(\Phi, \Phi^\dagger) = -\Phi\Phi^\dagger, \qquad \tau = 1$

the sum of these two terms is the N=1 supergravity extension of the prototype scalar ghost condensate $P(X) = -X + X^2$ given by

$$\mathcal{L}^{SUGRA}_{T=1/16,\mathrm{Weyl}} = \frac{1}{8}\Big[\int \mathrm{d}^2\Theta 2\mathcal{E}(\bar{\mathcal{D}}^2 - 8R)\Big(3\mathrm{e}^{\Phi\Phi^\dagger/3} - \frac{1}{2^4}(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger)\Big)\Big]_{\mathrm{Weyl}} + \mathrm{h.c.}$$

The purely scalar part of this supergravity Lagrangian is simply

$$\frac{1}{e}\mathcal{L}^{SUGRA}_{T=1/16,\mathrm{Weyl}} = -\frac{1}{2}\mathcal{R} + |\partial A|^2 + (\partial A)^2(\partial A^*)^2 + \ldots$$

For $A = \frac{1}{\sqrt{2}}(\phi + \mathrm{i}\xi)$ this becomes

$$\frac{1}{e}\mathcal{L}^{SUGRA}_{T=1/16,\mathrm{Weyl}} = -\frac{1}{2}\mathcal{R} + \frac{1}{2}(\partial\phi)^2 + \frac{1}{4}(\partial\phi)^4$$
$$+\frac{1}{2}(\partial\xi)^2 + \frac{1}{4}(\partial\xi)^4 - \frac{1}{2}(\partial\phi)^2(\partial\xi)^2 + (\partial\phi\cdot\partial\xi)^2 + \ldots$$

The Einstein and gravitino equations can be solved in a FRW spacetime $ds^2 = -dt^2 + a(t)^2 \eta_{ij} dx^i dx^j$ with

$$a(t) = e^{\pm \frac{1}{\sqrt{12}} t}, \quad \psi_m = 0$$

The $\phi,\ \xi,\ \text{and}\ \chi$ equations continue to admit the ghost condensate vacuum of the form

$$\phi = t, \quad \xi = 0, \quad \chi = 0$$

To assess stability, expand in the small fluctuations

$$\phi = t + \delta\phi(x^m), \quad \xi = \delta\xi(x^m), \quad \chi = \delta\chi(x^m)$$

To quadratic order, the result is

$$\begin{aligned} \frac{1}{e} \mathcal{L}^{SUGRA}_{T=1/16,\mathrm{Weyl}} &= (\dot{\delta\phi})^2 + 0 \cdot \delta\phi^{,i} \delta\phi_{,i} \\ &+ \ 0 \cdot (\dot{\delta\xi})^2 + \delta\xi^{,i} \delta\xi_{,i} \\ &+ \ldots \end{aligned}$$

1) $\delta\phi$ kinetic term: As previously, has a gradient instability in the NEC violating region. $\Rightarrow$In the global SUSY case, this was solved by adding the term

$$-\frac{1}{2^{11}} D\Phi D\Phi \bar{D}\Phi^\dagger \bar{D}\Phi^\dagger \left(\{D,\bar{D}\}\{D,\bar{D}\}(\Phi+\Phi^\dagger)\right)^2\Big|_{\theta\theta\bar{\theta}\bar{\theta}}$$

to the Lagrangian. In the supergravity case, this is easily generalized to

$$-\frac{1}{8}\int d^2\Theta 2\mathcal{E}(\bar{\mathcal{D}}^2-8R)(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger T_\phi)+\text{h.c.}$$

where

$$T_\phi=\frac{\kappa}{2^9}\left(\{\mathcal{D}^\alpha,\bar{\mathcal{D}}_{\dot\alpha}\}\{\mathcal{D}_\alpha,\bar{\mathcal{D}}^{\dot\alpha}\}(\Phi+\Phi^\dagger)\right)^2$$

and κ is any real number (chosen arbitrarily to be $\kappa=-1/4$ in the global SUSY case). Setting F=M=0, its bosonic contribution to the Lagrangian is

$$-\frac{1}{8e}\Big[\int d^2\Theta 2\mathcal{E}(\bar{\mathcal{D}}^2-8R)\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger T_\phi\Big]_{\text{Weyl}}+\text{h.c.}$$

$$=\kappa(\Box\phi)^2\left((\partial\phi)^4+(\partial\xi)^4-2(\partial\phi)^2(\partial\xi)^2+4(\partial\phi\cdot\partial\xi)^2\right)$$

Adding this to the original scalar Lagrangian $\frac{1}{e}\mathcal{L}^{SUGRA}_{T=1/16,\mathrm{Weyl}}$, the metric and ϕ solutions of their equations of motion change--unlike in the global SUSY case. Expanded perturbatively in small κ , they become

$$\begin{aligned} \langle\dot{\phi}\rangle^2 &= 1 - 3\kappa + \mathcal{O}(\kappa^2), \\ \langle H\rangle^2 &= \frac{1}{12} + \frac{1}{4}\kappa + \mathcal{O}(\kappa^2) \end{aligned}$$

That is, there is a shift in the condensate/FRW solution without altering its fundamental features. However, expanded around this new vacuum $\Rightarrow$

$$\mathcal{L}^{\mathrm{SUGRA}} = \frac{1}{2}(3\langle\dot{\phi}\rangle^2 - 1)(\dot{\delta\phi})^2 + \frac{1}{2\cancel{a^2}}(1 - \langle\dot{\phi}\rangle^2)\delta\phi^{,i}\delta\phi_{,i} + \kappa(\Box\delta\phi)^2 + \ldots$$

which, for $\kappa < 0$, softens the gradient instability-- as anticipated.

2) $\delta\xi$ kinetic term: Has vanishing time and wrong sign spatial kinetic terms. In global SUSY, this is cured by adding the higher-derivative terms

$$+\frac{8}{16^2}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\Big(\{D,\bar{D}\}(\Phi-\Phi^\dagger)\{D,\bar{D}\}(\Phi^\dagger-\Phi)\Big)\Big|_{\theta\theta\bar{\theta}\bar{\theta}}$$
$$-\frac{4}{16^3}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\Big(\{D,\bar{D}\}(\Phi+\Phi^\dagger)\{D,\bar{D}\}(\Phi-\Phi^\dagger)\Big)\Big(\{D,\bar{D}\}(\Phi+\Phi^\dagger)\{D,\bar{D}\}(\Phi^\dagger-\Phi)\Big)\Big|_{\theta\theta\bar{\theta}\bar{\theta}}$$

to the Lagrangian. In the supergravity case, this is easily generalized to

$$-\frac{1}{8}\int \mathrm{d}^2\Theta 2\mathcal{E}(\bar{\mathcal{D}}^2-8R)(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger T_\xi)+\text{h.c.}$$

where

$$\begin{aligned} T_\xi &= +2^{-5}\{\mathcal{D}^\alpha,\bar{\mathcal{D}}_{\dot{\alpha}}\}(\Phi-\Phi^\dagger)\{\mathcal{D}_\alpha,\bar{\mathcal{D}}^{\dot{\alpha}}\}(\Phi^\dagger-\Phi) \\ &\quad -2^{-10}\Big(\{\mathcal{D}^\alpha,\bar{\mathcal{D}}_{\dot{\alpha}}\}(\Phi+\Phi^\dagger)\{\mathcal{D}_\alpha,\bar{\mathcal{D}}^{\dot{\alpha}}\}(\Phi-\Phi^\dagger)\Big)^2 \end{aligned}$$

Setting F=M=0, its bosonic contribution is

$$-\frac{1}{8e}\Big[\int \mathrm{d}^2\Theta 2\mathcal{E}(\bar{\mathcal{D}}^2-8R)\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger T_\xi\Big]_{\text{Weyl}}+\text{h.c.}$$
$$=-2(\partial\phi)^4(\partial\xi)^2-(\partial\phi)^4(\partial\phi\cdot\partial\xi)^2$$

The addition of these terms does not alter the supergravity ghost condensate vacuum given above. Expanding around this vacuum, the ξ fluctuations are

$$\frac{1}{e}\mathcal{L}^{\rm SUGRA} = \cdots + \left(-\frac{1}{2} + \frac{1}{2}\langle\dot{\phi}\rangle^2 + 2\langle\dot{\phi}\rangle^4 - \langle\dot{\phi}\rangle^6\right)(\dot{\delta\xi})^2 + \left(\frac{1}{2} + \frac{1}{2}\langle\dot{\phi}\rangle^2 - 2\langle\dot{\phi}\rangle^4\right)\delta\xi^{,i}\delta\xi_{,i} + \dots$$

$$= \cdots + \left(1 - \frac{9}{2}\kappa + \mathcal{O}(\kappa^2)\right)\left((\dot{\delta\xi})^2 - \delta\xi^{,i}\delta\xi_{,i}\right) + \dots$$

$\Rightarrow$ the scalar $\delta\xi$ kinetic energy is rendered Lorentz covariant and stable by the addition of these terms. By suitably choosing the coefficients, this KE can be made canonical.

3) $\delta\chi$ kinetic term: Ghost free with gradient "instability". Can be cured with the context of supergravitational Galileons but re-grow a ghost! Won't discuss here.

The ghost condensate vacuum of this theory breaks N=1 supersymmetry spontaneously in a specific way.

The SUSY transformations of the fermions in the ghost condensate vacuum are

$$\delta\chi = \mathrm{i}\sqrt{2}\sigma^m\bar{\zeta}\partial_m A = \mathrm{i}\sigma^0\bar{\zeta}c$$

$$\delta\psi_m = 2\mathcal{D}_m\zeta$$

Redefining

$$\psi_{m\alpha} = \tilde{\psi}_{m\alpha} - \frac{2\mathrm{i}}{(\partial\phi)^2}\mathcal{D}_m(\phi_{,n}\sigma^n_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}})$$

$\Rightarrow$

$$\delta\tilde{\psi}_m = 0$$

This identifies χ as the Goldstone fermion and $\tilde{\psi}_{m\alpha}$ as the physical gravitino. Since $m_{3/2} = e^{K/2}|W|$ then

$$W = 0 \Longrightarrow \quad m_{3/2} = 0$$

consistent with an explicit calculation.

Specifically- using various identities, redefining the gravitino as above and evaluating on the ghost-condensate FRW background, we find that

$$\begin{aligned}\frac{1}{e}\mathcal{L}^{SUGRA}_{T=1/16,\mathrm{Weyl}} = \cdots + \frac{1}{2}\varepsilon^{klmn}\left(\bar{\tilde{\psi}}_k\bar{\sigma}_l\tilde{\mathcal{D}}_m\tilde{\psi}_n - \tilde{\psi}_k\sigma_l\tilde{\mathcal{D}}_m\bar{\tilde{\psi}}_n\right)\\ + \frac{\mathrm{i}}{2}\left(\chi\sigma^m\mathcal{D}_m\bar{\chi} + \bar{\chi}\bar{\sigma}^m\mathcal{D}_m\chi\right)\\ + \mathrm{i}\phi^{,m}\phi_{,n}\left(\bar{\chi}\bar{\sigma}^n(\mathcal{D}_m\chi) + \chi\sigma^n(\mathcal{D}_m\bar{\chi})\right) + \ldots\end{aligned}$$

$\Rightarrow$ canonical gravitino kinetic term, Lorentz violating ghost-free/gradient unstable χ kinetic term, and vanishing masses for both $\tilde{\psi}_m$ and χ.