Partial Breaking of Global Supersymmetry in d=1

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It is follows from Stefano's talk that we do not know how to construct the N=2 BI superfields action within the standard approach. The main problems are

- The corresponding N=4 linear supermultiplet is infinite dimensional
- In the standard approach the Lagrangian involves infinite number of different terms
- In is unclear how to find the closed form of the action and the equations of motion

We propose to use nonlinear realization approach to find

- The nonlinear variant of the conditions selecting irreducible supermultiplet
- The component equations of motion

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Let us demonstrate the basic steps of our construction on the simplest example of the particle in D=3.
If we are going to consider particle moving in D=3 we have to take into account the following facts

- Two translations (orthogonal to the particle trajectory) have to be spontaneously broken
- Any action depending on time derivatives of two goldstone fields $q, \bar{q}$ will possess invariance with respect to all translations in D=3

$$S = \int dt \mathcal{L}(\dot{q}, \dot{\bar{q}})$$

- One have to impose additional symmetry to fix the action completely. This additional symmetry is the Lorenz symmetry in D=3 space.

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We will use the standard D=3 Poincaré algebra

$$[M_{ab}, M_{cd}] = \epsilon_{ac} M_{bd} + \epsilon_{bd} M_{ac} + \epsilon_{ad} M_{bc} + \epsilon_{bc} M_{ad} ,$$
$$[M_{ab}, P_{cd}] = \epsilon_{ac} P_{bd} + \epsilon_{bd} P_{ac} + \epsilon_{ad} P_{bc} + \epsilon_{bc} P_{ad}$$

To get a convenient $d = 1$ form let us define the following generators

$$P = P_{11} + P_{22}, \quad Z = P_{11} - P_{22} - 2iP_{12}, \; \overline{Z} = P_{11} - P_{22} + 2iP_{12},$$
$$J = \frac{i}{4}\left(M_{11} + M_{22}\right),$$
$$T = \frac{i}{4}\left(M_{11} - M_{22} - 2iM_{12}\right), \; \overline{T} = \frac{i}{4}\left(M_{11} - M_{22} + 2iM_{12}\right).$$

Being rewritten in terms of these generators the D=3 Poincaré algebra acquires the familiar $d = 1$ form

$$[J, T] = T, \; \left[J, \overline{T}\right] = -\overline{T}, \; \left[T, \overline{T}\right] = -2J,$$
$$\begin{cases} [J, Z] = Z, \\ \left[J, \overline{Z}\right] = -\overline{Z}, \end{cases} \quad \begin{cases} [T, P] = -Z, \\ \left[T, \overline{Z}\right] = -2P, \end{cases} \quad \begin{cases} \left[\overline{T}, P\right] = \overline{Z}, \\ \left[\overline{T}, Z\right] = 2P, \end{cases}$$

From $d = 1$ point of view we will treat the generators $\{Z, \overline{Z}\}$ as the generators of the central charges and $\{T, \overline{T}\}$ as the generators of spontaneously broken D=3 rotations. In addition we will put the $so(1,1)$ generator J in the stability subgroup and choose the parametrization of our coset as

$$g = e^{itP}\, e^{i\left(qZ + \bar{q}\overline{Z}\right)}\, e^{i\left(\lambda T + \bar{\lambda}\overline{T}\right)}.$$

With our definitions the important Cartan forms read

$$\omega_p = \frac{1}{1 - \Lambda\overline{\Lambda}}\left[\left(1 + \Lambda\overline{\Lambda}\right) dt + 2i\left(\Lambda d\bar{q} - \overline{\Lambda} dq\right)\right],$$

$$\omega_Z = \frac{1}{1 - \Lambda\overline{\Lambda}}\left[dq - \Lambda^2 d\bar{q} + i\Lambda dt\right],\; \bar{\omega}_Z = \frac{1}{1 - \Lambda\overline{\Lambda}}\left[d\bar{q} - \overline{\Lambda}^2 dq - i\overline{\Lambda} dt\right],$$

where

$$\Lambda = \frac{\tanh(\sqrt{\lambda\bar{\lambda}})}{\sqrt{\lambda\bar{\lambda}}}\lambda, \quad \overline{\Lambda} = \frac{\tanh(\sqrt{\lambda\bar{\lambda}})}{\sqrt{\lambda\bar{\lambda}}}\bar{\lambda}.$$

On this step we have 4 bosonic fields: $q,\bar{q}$ and $\Lambda,\bar{\Lambda}$. Thus we need some additional constraints to reduce the number of independent field to two. Within nonlinear realization approach we used the Cartan'a forms which are invariant under left action of the D=3 Poincaré group. Therefore, one may impose the following *invariant* constraints

$$\omega_Z=\bar{\omega}_Z=0 \quad \Rightarrow \left\{ \begin{array}{l} \dot{q}=-i\frac{\Lambda}{1+\Lambda\bar{\Lambda}} \\ \dot{\bar{q}}=i\frac{\bar{\Lambda}}{1+\Lambda\bar{\Lambda}} \end{array} \right.$$

The last non-zero Cartan's form (besides the form for stability subgroup generator J) now reads

$$\omega_p=\sqrt{1-4\dot{q}\dot{\bar{q}}}\; dt$$

In addition, we also know the transformation properties of our fields(coordinates)

- Automorphism group $\left(g_0=e^{i\left(\alpha T+\bar{\alpha}\bar{T}\right)}\right)$: $\quad \left\{ \begin{array}{l} \delta t=-2i\left(\alpha\bar{q}-\bar{\alpha}q\right),\; \delta q=-i\alpha\, t \\ \delta\Lambda=\alpha-\bar{\alpha}\Lambda^2,\; \delta\bar{\Lambda}=\bar{\alpha}-\alpha\bar{\Lambda}^2. \end{array} \right.$

Thus we conclude that

- By imposing invariant constraints $\omega_Z = \bar{\omega}_Z = 0$ we expressed the fields $\Lambda, \overline{\Lambda}$ in terms of $\dot{q}, \dot{\bar{q}}$ - (Inverse Higgs phenomenon -E.Ivanov,V.Ogievetsky)
- We can immediately construct the invariant action:

$$S_1 = \int \omega_p = \int dt \sqrt{1 - 4\dot{q}\dot{\bar{q}}}$$

- to have a proper behaviour of the action one has to add additional (also invariant) action):

$$S_2 = \int dt$$

- The full action:

$$S = S_2 - S_1 = \int dt \left(1 - \sqrt{1 - 4\dot{q}\dot{\bar{q}}}\right)$$

is perfectly invariant under full D=3 Poincaré symmetry.

Thus

- One has to take into account all symmetries
- In such cases by imposing the constraints on the Cartan's form one may
  - reduce the number of the essential fields
  - find the equations of motion
  - construct the invariant action
- The invariant equations of motion follow from the nullifying the rest of the coset forms:

$$\omega_T = \bar{\omega}_T = 0$$

- The final equations of motion are free ones

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If we will add some supersymmetries to the previous picture, some of them have to be also spontaneously broken - those ones which anticommute on "central charges" $Z,\overline{Z}$. The simplest case corresponds $N=4\to N=2$ breaking in D=3. Indeed, the standard relations

$$\left\{Q_a,\overline{Q}_b\right\}=P_{ab},\quad [M_{ab},\mathcal{Q}_c]=\epsilon_{ac}\mathcal{Q}_b+\epsilon_{bc}\mathcal{Q}_a,$$

where $\mathcal{Q}_a$ in the second commutator denotes any of the spinor generators $\{Q_a,\overline{Q}_a\}$.

To get a convenient $d=1$ form let us define the following generators

$$Q=Q_1-iQ_2,\ \overline{Q}=\overline{Q}_1+i\overline{Q}_2,\quad S=\overline{Q}_1-iQ_2,\ \overline{S}=Q_1+iQ_2$$

Being rewritten in terms of these generators the $N=2, D=3$ super Poincaré algebra acquires the familiar $d=1$ form

$$\left\{Q,\overline{Q}\right\}=2P,\ \left\{S,\overline{S}\right\}=2P,\quad \{Q,S\}=2Z,\ \left\{\overline{Q},\overline{S}\right\}=2\overline{Z},$$

$$\begin{cases}[J,Q]=\frac{1}{2}Q,\ \left[J,\overline{Q}\right]=-\frac{1}{2}\overline{Q},\left[T,\overline{Q}\right]=-S,\left[\overline{T},Q\right]=\overline{S},\\ [J,S]=\frac{1}{2}S,\ \left[J,\overline{S}\right]=-\frac{1}{2}\overline{S},\left[T,\overline{S}\right]=-Q,\left[\overline{T},S\right]=\overline{Q}.\end{cases}$$

We will choose the generators $S, \overline{S}$ to be spontaneously broken ones and, therefore, we will parameterize our coset as

$$g = e^{itP}\, e^{\theta Q + \bar\theta \overline{Q}}\, e^{\psi S + \bar\psi \overline{S}}\, e^{i\left(qZ + \bar q \overline{Z}\right)}\, e^{i\left(\lambda T + \bar\lambda \overline{T}\right)}.$$

Now we have

- $N = 2, d = 1$ superspace with the coordinates $(t, \theta, \bar\theta)$
- bosonic $(q, \bar q, \lambda, \bar\lambda)$ and fermionic $(\psi, \bar\psi)$ $N = 2$ superfields

Now, similarly to the bosonic case, one has to reduce the number of the superfields by imposing the constraints on the Cartan's forms.

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With our definitions the important Cartan forms read

$$\omega_p = \frac{1}{1-\Lambda\bar{\Lambda}}\left[\left(1+\Lambda\bar{\Lambda}\right)\triangle t + 2i\left(\Lambda d\hat{\bar{q}} - \bar{\Lambda} d\hat{q}\right)\right],$$

$$\omega_Z = \frac{1}{1-\Lambda\bar{\Lambda}}\left[d\hat{q} - \Lambda^2 d\hat{\bar{q}} + i\Lambda\triangle t\right], \; \bar{\omega}_Z = \frac{1}{1-\Lambda\bar{\Lambda}}\left[d\hat{\bar{q}} - \bar{\Lambda}^2 d\hat{q} - i\bar{\Lambda}\triangle t\right],$$

$$\omega_Q = \frac{1}{\sqrt{1-\Lambda\bar{\Lambda}}}\left[d\theta + i\Lambda d\bar{\psi}\right], \; \bar{\omega}_Q = \frac{1}{\sqrt{1-\Lambda\bar{\Lambda}}}\left[d\bar{\theta} - i\bar{\Lambda} d\psi\right],$$

$$\omega_S = \frac{1}{\sqrt{1-\Lambda\bar{\Lambda}}}\left[d\psi + i\Lambda d\bar{\theta}\right], \; \bar{\omega}_S = \frac{1}{\sqrt{1-\Lambda\bar{\Lambda}}}\left[d\bar{\psi} - i\bar{\Lambda} d\theta\right],$$

where

$$\triangle t = dt - i\left(\theta d\bar{\theta} + \bar{\theta} d\theta + \psi d\bar{\psi} + \bar{\psi} d\psi\right), \; d\hat{q} = dq - 2i\psi d\theta, \; d\hat{\bar{q}} = d\bar{q} - 2i\bar{\psi} d\bar{\theta},$$

and

$$\Lambda = \frac{\tanh(\sqrt{\lambda\bar{\lambda}})}{\sqrt{\lambda\bar{\lambda}}}\lambda, \quad \bar{\Lambda} = \frac{\tanh(\sqrt{\lambda\bar{\lambda}})}{\sqrt{\lambda\bar{\lambda}}}\bar{\lambda}.$$

Using the semi-covariant differentials of the coordinates $\triangle t, d\theta, d\bar\theta$ one may define the semi-covariant derivatives $\nabla_\theta, \bar\nabla_\theta, \nabla_t$

$$d\mathcal{F} = d\theta\nabla_\theta\mathcal{F} + d\bar\theta\bar\nabla_\theta\mathcal{F} + \triangle t\nabla_t\mathcal{F} \Rightarrow \begin{cases} \nabla_t = E^{-1}\partial_t \\ \nabla_\theta = D - i\left(\bar\psi D\psi + \psi D\bar\psi\right)\nabla_t \\ \bar\nabla_\theta = \overline{D} - i\left(\bar\psi\overline{D}\psi + \psi\overline{D}\bar\psi\right)\nabla_t \end{cases}$$

where

$$\begin{cases} D = \frac{\partial}{\partial\theta} - i\bar\theta\partial_t, \\ \overline{D} = \frac{\partial}{\partial\bar\theta} - i\theta\partial_t, \end{cases} \Rightarrow \left\{D, \overline{D}\right\} = -2i\partial_t,$$

and

$$E = 1 + i\left(\dot\psi\bar\psi + \dot{\bar\psi}\psi\right) \Rightarrow E^{-1} = 1 - i\left(\nabla_t\psi\bar\psi + \nabla_t\bar\psi\psi\right).$$

The fully covariant derivatives have to be defined with respect to Cartan's forms $\omega_P, \omega_Q, \omega_{\overline{Q}}$:

$$d\mathcal{F} = \omega_Q\mathcal{D}_\theta\mathcal{F} + \omega_{\overline{Q}}\overline{\mathcal{D}}_\theta\mathcal{F} + \omega_P\mathcal{D}_t\mathcal{F}$$

The semi-covariant derivatives obey rather complicate relations

$$\{\nabla_\theta, \bar\nabla_\theta\} = -2i\left(1 + \nabla_\theta\psi\bar\nabla_\theta\bar\psi + \bar\nabla_\theta\psi\nabla_\theta\bar\psi\right)\nabla_t,$$
$$\{\nabla_\theta, \nabla_\theta\} = -4i\nabla_\theta\bar\psi\nabla_\theta\psi\nabla_t, \quad \{\bar\nabla_\theta, \bar\nabla_\theta\} = -4i\bar\nabla_\theta\bar\psi\bar\nabla_\theta\psi\nabla_t,$$
$$[\nabla_t, \nabla_\theta] = -2i\left(\nabla_\theta\bar\psi\nabla_t\psi + \nabla_\theta\psi\nabla_t\bar\psi\right)\nabla_t.$$

Similarly to the previous case, one may find the transformations properties of the coordinates and superfields under action of our group:

- Unbroken SUSY $\left(g_0 = e^{\epsilon Q + \bar\epsilon\bar Q}\right)$: $\quad \left\{\delta\theta = \epsilon,\ \delta\bar\theta = \bar\epsilon,\ \delta t = i\left(\epsilon\bar\theta + \bar\epsilon\theta\right),\right.$
- Broken SUSY $\left(g_0 = e^{\varepsilon S + \bar\varepsilon\bar S}\right)$: $\quad \left\{\ \delta t = i\left(\varepsilon\bar\psi + \bar\varepsilon\psi\right),\quad \delta\psi = \varepsilon,\ \delta q = 2i\varepsilon\theta,\right.$
- Automorphism group $\left(g_0 = e^{i\left(\alpha T + \bar\alpha\bar T\right)}\right)$:

$$\left\{\begin{array}{l}\delta t = -2i\left(\alpha\bar q - \bar\alpha q\right) + 2\alpha\bar\theta\bar\psi - 2\bar\alpha\theta\psi,\\ \delta\theta = -i\alpha\bar\psi, \delta q = -i\alpha\left(t - i\theta\bar\theta + i\psi\bar\psi\right),\ \delta\psi = -i\alpha\bar\theta,\\ \delta\Lambda = \alpha - \bar\alpha\Lambda^2,\ \delta\overline\Lambda = \bar\alpha - \alpha\overline\Lambda^2.\end{array}\right.$$

Outline

Now, to reduce the number of independent superfields one has to impose the covariant constraints on Cartan's forms which we choose to be

$$\omega_Z = 0,\ \bar{\omega}_Z = 0 \Rightarrow$$
$$\bar{\nabla}_\theta q = 0,\ \nabla_\theta \bar{q} = 0,\ \nabla_\theta q + 2\mathrm{i}\psi = 0,\ \bar{\nabla}_\theta \bar{q} + 2\mathrm{i}\bar{\psi} = 0.$$
$$\nabla_t q = -i\frac{\Lambda}{1+\Lambda\bar{\Lambda}},\ \nabla_t \bar{q} = i\frac{\bar{\Lambda}}{1+\Lambda\bar{\Lambda}}. \quad (1)$$

One may note, that $q,\ \bar{q}$ obey covariantized chirality conditions.
Moreover, using the algebra of covariant derivatives, one may find additional constraints:

$$\nabla_\theta^2 q + 2\mathrm{i}\nabla_\theta\psi = 0 \Rightarrow 2\mathrm{i}\nabla_\theta\psi\left(1 - \nabla_\theta\bar{\psi}\nabla_t q\right) = 0.$$

Second bracket could not be zero, therefore, like to q, ψ is also covariantly chiral one.
In principle, from these relations one may express all $\theta, \bar{\theta}$-derivatives of $\psi, \bar{\psi}$ through time derivatives of $q, \bar{q}$:

$$\nabla_\theta\psi = \bar{\nabla}_\theta\bar{\psi} = 0,\ \bar{\nabla}_\theta\psi = \frac{2}{1+\sqrt{1-4\nabla_t q\nabla_t\bar{q}}}\nabla_t q,\ \nabla_\theta\bar{\psi} = \frac{2}{1+\sqrt{1-4\nabla_t q\nabla_t\bar{q}}}\nabla_t\bar{q}$$

Thus we have constructed $N = 2, d = 1$ supermultiplet with $(2, 2, 0)$ components contest.
To find covariant equations of motion one has to put equal to zero all coset Cartan's forms corresponding to spontaneously broken generators.
As the result we will have

$$\ddot{q} = \ddot{\bar{q}} = 0, \quad \dot{\psi} = \dot{\bar{\psi}} = 0.$$

Thus, we can see that the equations of motion are trivial ones, but the invariant action is rather complicated! Let us note, that it can not be formulated (in the contrast with the bosonic case) in terms of Cartan's forms, because it shifted by full derivatives under transformations from our group. How to find the superfiled action?

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The unique candidate to be a superfield action is

$$S = \int dt d\theta d\bar{\theta} F(\nabla_t q \cdot \nabla_t \bar{q}) \psi \bar{\psi} = \int dt d\theta d\bar{\theta} F(\dot{q} \cdot \dot{\bar{q}}) \psi \bar{\psi}.$$

(Note that $E\psi\bar{\psi} = E^{-1}\psi\bar{\psi} = 1 \cdot \psi\bar{\psi}$).
The function $F(\nabla_t q \cdot \nabla_t \bar{q})$ has to be fixed to insure the invariance with respect to spontaneously broken supersymmetry, or with the respect of automorphism group transformations. More easy way is to fix the bosonic part of the action to be the same as it was in a pure bosonic case. This condition gives

$$F = (1 + \sqrt{1 - 4\nabla_t q \cdot \nabla_t \bar{q}}).$$

What about equations of motion which follow from this action?

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The equations of motion which follow from the superfiled action look awful

$$\frac{d}{dt}\left(\frac{\dot{\bar q}}{\sqrt{1-4\dot q\dot{\bar q}}} - \frac{2\mathrm{i}(\dot\psi\bar\psi+\dot{\bar\psi}\psi)\dot{\bar q}}{(1-4\dot q\dot{\bar q})^{3/2}} - \frac{4\psi\bar\psi\dot\psi\dot{\bar\psi}\dot{\bar q}}{(1-4\dot q\dot{\bar q})^{3/2}} - \frac{24\psi\bar\psi\dot\psi\dot{\bar\psi}\dot q\dot{\bar q}^2}{(1-4\dot q\dot{\bar q})^{5/2}}\right) = 0,$$

$$-2\mathrm{i}\left(1+\frac{1}{\sqrt{1-4\dot q\dot{\bar q}}}\right)\dot{\bar\psi} + 8\frac{\dot q\dot{\bar q}\dot\psi\dot{\bar\psi}\bar\psi}{(1-4\dot q\dot{\bar q})^{3/2}} - 4\frac{\psi\bar\psi\ddot{\bar\psi}\dot q\dot{\bar q}}{(1-4\dot q\dot{\bar q})^{3/2}} =$$

$$= \left[\frac{2\mathrm{i}\bar\psi}{(1-4\dot q\dot{\bar q})^{3/2}} + 4\psi\bar\psi\dot{\bar\psi}\frac{2+4\dot q\dot{\bar q}}{(1-4\dot q\dot{\bar q})^{5/2}}\right]\left(\ddot q\dot{\bar q}+\dot q\ddot{\bar q}\right).$$

We may compare them with equations of motion, which follow from nullifying the forms:

$$\frac{1}{\sqrt2}\frac{(1+\sqrt{1-4\dot q\dot{\bar q}})^{1/2}}{(1-4\dot q\dot{\bar q})^{3/4}}\dot{\bar\psi} + \frac{\mathrm{i}}{\sqrt2}\dot\psi\dot{\bar\psi}\bar\psi\frac{1+2\dot q\dot{\bar q}+\sqrt{1-4\dot q\dot{\bar q}}}{(1+\sqrt{1-4\dot q\dot{\bar q}})^{1/2}(1-4\dot q\dot{\bar q})^{7/4}} = 0,$$

which immediately results in the equation

$$\dot{\bar\psi} = 0$$

Outline

1. Preliminaries: main idea and bosonic case
   - Main idea
   - Partial breaking of the translations in D=3
   - Nonlinear realization of D=3 Poincaré group
2. Adding supersymmetry: $N = 4 \to N = 2$
   - Superalgebra $N = 2, D = 3$ super Poincaré
   - Cartan's forms
   - Constraints
   - Action
   - Equations of motion
   - **Action from linear realizations**
3. Adding higher supersymmetries
   - Still d=1
   - Towards N=2 BI

The construction of the Goldstone superfield action is the most difficult part of the PBGS approach. The generic methods of nonlinear realizations which nicely work in the case of standard internal symmetry and space-time groups prove to be not too helpful when trying to employ them for constructing PBGS invariants. All the known Goldstone superfield Lagrangians are of the Chern-Simons or WZNW type, in the sense that they are not tensors with respect to the hidden supersymmetry transformations. The latter shift them by a full derivative, thus leaving the action invariant up to surface terms. As the result, one cannot directly apply the powerful method of covariant Cartan forms for constructing invariant actions.

A way around this difficulty was proposed J. Bagger and A. Galperin and used to construct the $N = 1$, $D = 4$ superfield actions providing the PBGS description of $N = 2$ D3-superbrane and super 3-brane. It is based on the idea of embedding the basic Goldstone superfield into some *linear* multiplet of the underlying supersymmetry group. Initially this multiplet comprises a set of independent worldvolume superfields. After imposing appropriate covariant constraints one succeeds in expressing all these superfields in terms of the basic Goldstone ones. One of the superfields of the initial linear representation is shifted by a full derivative under the broken supersymmetry transformations and so can be chosen as the Goldstone superfields Lagrangian.

Our basic objects will be the fermionic and bosonic $N = 2,\ d = 1$ superfields $\xi, \bar{\xi}$ and ρ. We assume that ρ is a real superfield while $\xi, \bar{\xi}$ are related to it by

$$\xi = -\frac{1}{2}\overline{D}\rho\,, \qquad \bar{\xi} = \frac{1}{2}D\rho\,,$$

After introducing an additional scalar superfield Φ one can realize an extra $N = 2, d = 1$ supersymmetry on the spinors $\xi, \bar{\xi}$ and scalar Φ (the second supersymmetry with the generators $S, \overline{S}$ is supposed to be spontaneously broken):

$$\delta\xi = \epsilon\left(1 - \overline{D}D\Phi\right)\,, \quad \delta\bar{\xi} = \bar{\epsilon}\left(1 + D\overline{D}\Phi\right)\,, \quad \delta\Phi = \epsilon\bar{\xi} - \bar{\epsilon}\xi\,.$$

The transformation law of the scalar superfield ρ reads:

$$\delta\rho = 2\left(\theta\bar{\epsilon} - \bar{\theta}\epsilon + D\Phi\epsilon + \overline{D}\Phi\bar{\epsilon}\right)\,.$$

Due to the explicit presence of $\theta, \bar{\theta}$ in this transformation law, the brackets of the manifest and second $N = 2$ supersymmetries yield a constant shift of the superfield ρ, and therefore a central charge Z appears in the anticommutators of these two $N = 2$ supersymmetries.

The field Φ can be treated as the Lagrangian density and the action

$$S = \int dt d^2\theta \; \Phi$$

is manifestly invariant with respect to both $N=2$ supersymmetries. To express Φ in terms of the Goldstone superfields $\xi, \bar\xi$ we use the method proposed by E.Ivanov and A.Kapustnikov. In the present case the basic steps are

- Let us find the finite transformations of the second supersymmetry for the basic superfields $\xi, \bar\xi, \Phi$

$$\Delta\xi = \epsilon\left(1 - \overline{D}D\Phi\right) + \frac{1}{2}\epsilon\bar\epsilon\partial_t\xi \;, \quad \Delta\bar\xi = \bar\epsilon\left(1 + D\overline{D}\Phi\right) - \frac{1}{2}\epsilon\bar\epsilon\partial_t\bar\xi \;,$$
$$\Delta\Phi = \epsilon\bar\xi - \bar\epsilon\xi + \epsilon\bar\epsilon\left(1 + \frac{1}{2}\left[D, \overline{D}\right]\Phi\right) \;.$$

- Let us substitutes the Goldstone fermions $(-\psi, -\bar\psi)$ for the parameters $(\epsilon, \bar\epsilon)$ in the r.h.s. of these transformations to define the new superfields

$$\tilde\xi = \xi - \psi\left(1 - \overline{D}D\Phi\right) + \frac{1}{2}\psi\bar\psi\partial_t\xi\ , \quad \tilde{\bar\xi} = \bar\xi - \bar\psi\left(1 + D\overline{D}\Phi\right) - \frac{1}{2}\psi\bar\psi\partial_t\bar\xi\ ,$$
$$\tilde\Phi = \Phi - \psi\bar\xi + \bar\psi\xi + \psi\bar\psi\left(1 + \frac{1}{2}\left[D, \overline{D}\right]\Phi\right)\ .$$

- One may check that these tilde superfields transform independently of each another and homogeneously under the spontaneously broken supersymmetry.
- As the last step one puts these superfields equal to zero

$$\tilde\xi = 0\ , \qquad \tilde{\bar\xi} = 0\ , \quad \tilde\Phi = 0\ .$$

The solution of these equations is given by

$$\Phi = \frac{2\xi\bar{\xi}}{1+\sqrt{1-4D\xi\,\overline{D}\bar{\xi}}} = \frac{\psi\bar{\psi}}{1+\overline{D}D\,(\psi\bar{\psi})}\,, \qquad (2)$$

$$\psi = \frac{\xi}{1-\overline{D}D\Phi} + \frac{\frac{1}{2}\xi\bar{\xi}\partial_t\xi}{\left(1-\overline{D}D\Phi\right)^3}\,, \quad \bar{\psi} = \frac{\bar{\xi}}{1+D\overline{D}\Phi} - \frac{\frac{1}{2}\xi\bar{\xi}\partial_t\bar{\xi}}{\left(1+D\overline{D}\Phi\right)^3}\,.$$

Note that the Goldstone fermions $\psi, \bar{\psi}$ automatically obey the covariant chirality conditions which are equivalent to the ordinary chirality conditions for $\xi, \bar{\xi}$.

Thus, we constructed the superfield action and established the relations between linear and non-linear realizations.

Outline

- The case with $N=8 \to N=4$ can be considered in the same manner. The final component equations of motion are still free ones.
- The case with $N=16 \to N=8$ (analog of $N=2$ BI theory) contains some peculiarities
  - The consideration within non-linear realization approach can be done in the same way.
  - Adding the superfields corresponding to automorphism group provides possibility to split the irreducibility constraints and the equations of motion (which was impossible to do previously).
  - Despite the rather complicated intermediate calculations the final component equations of motion are still free ones.
  - These equations of motion can be obtained by putting all coset Cartan's forms equal to zero
- It is unclear how to construct the linear realization of $N=16$ supersymmetry (the minimal off-shell supermultiplet should contain 128+128 components)

- Thus, we do not know how to relate the linear and non-linear realizations and how to construct the superfield action.

So, we have almost the same problems as in the case of $N=2$ BI theory.
But now we have some additional information

- We know full set of covariant irreducibility constraints
- We know the component equations of motion which are free ones
- Moreover, one may prove the theorem that within nonlinear realizations approach in all cases the equations of motion will be free ones

Outline

Based on the our observations in $d = 1$ we are proposing to do the following

- Extend the coset in the case of $N = 4 \to N = 2$ in $D = 4$ by the generators of 6-d Lorentz group
- Construct the corresponding Cartan's forms
- Impose the irreducibility constraints - they will be just the constraints on the Cartan's forms written in terms of full covariant derivatives
- Put equal to zero all coset forms and to find the component equations of motion