Scalar Fields with Higher Derivatives in Supergravity and Cosmology

Workshop "Breaking of supersymmetry and Ultraviolet Divergences in extended Supergravity", INFN

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Based on work with Jean-Luc Lehners and Burt Ovrut

arXiv:1207.3798
arXiv:1302.0840
work in progress

See also

MK, Lehners, Ovrut
arXiv:1208.0752, arXiv:1212.2185

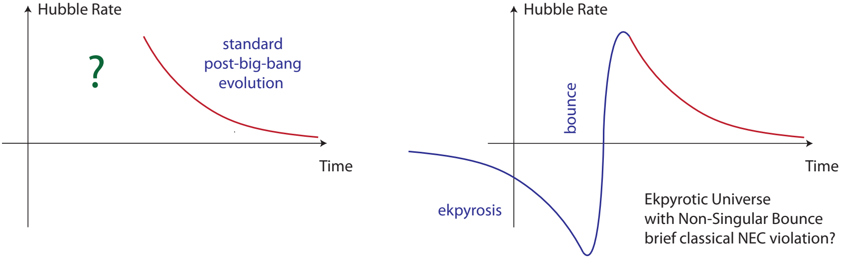
Khoury, Lehners, Ovrut
arXiv:1012.3748, arXiv:1103.0003

Baumann, Green
arXiv:1109.0293

Sasaki, Yamaguchi, Yokoyama
arXiv:1205.1353

Farakos, Kehagias
arXiv:1207.4767

Higher-derivative scalars in cosmology



Classical non-singular bounces (Buchbinder,Khoury,Ovrut)
require $\dot{H} = -\frac{1}{2}(\rho + P) > 0$
How to achieve stable NEC violation $\rho + P < 0$?
Higher-derivative theories:

- ghost condensate (Arkani-Hamed,Cheng,Luty,Mukohyama)
- galileon theories (Horndeski;Nicolis,Rattazzi,Trincherini;. . .)

Other situations of interest for higher-derivative kinetic terms in cosmology: DBI inflation, k-inflation (Silverstein,Tong;Armendariz-Picon,Damour,Mukhanov)

Can these models be realized in $\mathcal{N} = 1$ supergravity?

Work with chiral superfields Φ in superspace: $\bar{\mathcal{D}}_{\dot{\alpha}}\Phi = 0$

Contain three components:

$A \equiv \Phi\big\vert_{\theta=\bar{\theta}=0}$ Complex scalar	$\chi_\alpha \equiv \frac{1}{\sqrt{2}} D_\alpha \Phi\big\vert_{\theta=\bar{\theta}=0}$ Spin $\frac{1}{2}$ fermion	$F \equiv -\frac{1}{4} D^2\Phi\big\vert_{\theta=\bar{\theta}=0}$ Auxiliary field

Usual supersymmetric Lagrangian built from chiral superfields:

$$\int \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\ \Phi^\dagger\Phi = -\,\partial A \cdot \partial A^* + F^* F$$

$$= -\,\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\xi)^2 + F^* F$$

(without fermionic terms, $A = \frac{1}{\sqrt{2}}(\phi + \mathrm{i}\xi)$)

What is the supersymmetric extension of $X^2 \propto (\partial\phi)^4$?

$(\partial^\mu\phi)(\partial_\mu\phi)(\partial^\nu\phi)(\partial_\nu\phi)$ $\rightsquigarrow$ 2 extra fields, 2 extra spacetime derivatives

or 4 extra superspace derivatives due to $-2i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu = \{D_\alpha, \bar{D}_{\dot\alpha}\}$

In global susy, only 2 clean extensions of $(\partial\phi)^4$:

1) $\int d^4\theta\, D^\alpha\Phi D_\alpha\Phi\bar{D}_{\dot\alpha}\Phi^\dagger\bar{D}^{\dot\alpha}\Phi^\dagger$ (Khoury,Lehners,Ovrut 1012.3748)

2) $\int d^4\theta\, (\Phi^\dagger - \Phi)^2\partial^m\Phi^\dagger\partial_m\Phi$ (Baumann,Green 1109.0293)

In local susy,

1) leads to minimal coupling to gravity
2) contains derivative couplings to gravity (of the form $\xi^2(\partial\phi)^2R$), and propagating F field

$\rightsquigarrow$ We use the first term, which is the unique clean, minimally-coupled extension of $(\partial\phi)^4$: $\mathcal{L} = -\frac{1}{8}\int d^2\Theta 2\mathcal{E}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger + h.c.$

$$-\frac{1}{8}\int \mathrm{d}^2\Theta 2\mathcal{E}(\bar{\mathcal{D}}^2 - 8R)\left[\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger\, T(\Phi)\right]_{\text{0f}} + h.c.$$
$$= 16e\left((\partial A)^2(\partial A^*)^2 - 2\partial A\cdot\partial A^* FF^* + (FF^*)^2\right)T(\Phi)|$$

(MK,Lehners,Ovrut 1207.3798)

- Scalars appear in combination $(\partial A)^2(\partial A^*)^2$, and not $(\partial A\cdot\partial A^*)^2$ as one might have expected
- F still auxiliary (note: we could have obtained $AA^*\partial F\cdot\partial F^*$, but didn't)
- Equation for F is now cubic – hence there exist three branches of the theory
- For the bosonic part, only the top component is non-zero
  $\rightarrow$ can multiply by arbitrary scalar function T of Φ and its spacetime derivatives
  $\rightarrow$ can obtain a supergravity extension of any term containing $(\partial\phi)^4$ as a factor
  e.g. can obtain sugra version of $P(X,\phi)$ where $X \equiv -\frac{1}{2}(\partial\phi)^2$

Complete supergravity extension for X and X^2

$$\mathcal{L} = \int \mathrm{d}^2\Theta 2\mathcal{E}\left[\frac{3}{8}(\bar{\mathcal{D}}^2 - 8R)e^{-K(\Phi^i,\Phi^{\dagger k*})/3} + W(\Phi^i)\right] + h.c.$$
$$-\frac{1}{8}\int \mathrm{d}^2\Theta 2\mathcal{E}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}\Phi^i\mathcal{D}\Phi^j\bar{\mathcal{D}}\Phi^{\dagger k*}\bar{\mathcal{D}}\Phi^{\dagger l*}T_{ijk*l*} + h.c.$$

K	Kähler Potential
W	Superpotential
$T_{ijk^*l^*}$	Target Space Tensor, Spacetime Scalar

Weyl rescaling and elimination of auxiliary fields

Weyl rescaling $\mathcal{L} \xrightarrow{\text{WEYL}} \mathcal{L}_{\text{Weyl}}$

$$e_n{}^a \xrightarrow{\text{WEYL}} e_n{}^a \mathrm{e}^{K/6} \,, \quad \chi^i \xrightarrow{\text{WEYL}} \chi^i \mathrm{e}^{-K/12} \,, \quad \psi_m \xrightarrow{\text{WEYL}} \psi_m \mathrm{e}^{K/12}$$

Disentangle M and F fields from supergravity and scalar multiplets:

$$N = M + K_{,A^{k*}} F^{k*}$$

Solve for the auxiliary fields:

$$b_m = \frac{\mathrm{i}}{2}(\partial_m A^i K_{,A^i} - \partial_m A^{k*} K_{,A^{k*}})$$
$$N = -3\mathrm{e}^{K/3} W$$

cf. (Wess,Bagger)

After Weyl re-scaling & eliminating b_m, M & setting $g_{ij^*} \equiv K_{,A^i A^{k*}}$

$$\begin{aligned}\frac{1}{e}\mathcal{L}_{\text{Weyl}} \quad = \quad & -\frac{1}{2}\mathcal{R} - g_{ik*}\partial A^i \cdot \partial A^{k*} + g_{ik*}e^{K/3}F^i F^{k*} \\ & + e^{2K/3}[F^i (D_A W)_i + F^{k*}(D_A W)^*_{k*}] + 3e^K W W^* \\ & + 16(\partial A^i \cdot \partial A^j)(\partial A^{k*} \cdot \partial A^{l*})\, T_{ijk*l*\text{Weyl}}| \\ & -32\, e^{K/3} F^i F^{k*}(\partial A^j \cdot \partial A^{l*})\, T_{ijk*l*\text{Weyl}}| \\ & +16 e^{2K/3} F^i F^j F^{k*} F^{l*}\, T_{ijk*l*\text{Weyl}}|\end{aligned}$$

Equation of motion for F^i

$$g_{ik*}F^i + e^{K/3}(D_A W)^*_{k*} + 32F^i(e^{K/3}F^j F^{l*} - \partial A^j \cdot \partial A^{l*})T_{ijk*l*\text{Weyl}}| = 0$$

algebraic and cubic $\rightarrow$ 3 branches

1) "Ordinary" branch: small corrections to two-derivative & potential terms
2) New branches: cannot be reached dynamically due to infinite potential barrier from resubstitution of F (higher-derivative terms generate potential)

Potential generated by higher-derivative terms

$W = 0$, single chiral superfield $T_{1111} \,|\equiv \mathcal{T}$

$\rightsquigarrow$ e.o.m. for F: $F\Big(K_{,AA^*} + 32\mathcal{T}(e^{K/3}|F|^2 - |\partial A|^2)\Big) = 0$

$\rightsquigarrow$ either $F = 0$, or

$$|F_{\text{new}}|^2 = -\frac{1}{32\mathcal{T}}e^{-K/3}K_{,AA*} + e^{-K/3}\,|\partial A|^2$$

Reinsertion into $\mathcal{L}$:

$$\begin{aligned}\frac{1}{e}\mathcal{L}_{W=0,F_{\text{new}}} \quad &= \quad -\frac{1}{2}\mathcal{R} + 16\mathcal{T}\Big((\partial A)^2(\partial A^*)^2 - (\partial A \cdot \partial A^*)^2\Big) \\ &\quad -\frac{1}{64\mathcal{T}}(K_{,AA^*})^2\end{aligned}$$

- The potential diverges for $\mathcal{T} \to 0$
  $\rightsquigarrow$ new branch not connected to ordinary branch
- Ordinary kinetic term has disappeared

$W \neq 0$: Solving the e.o.m. for F perturbatively yields the

Ordinary branch Lagrangian

$$
\begin{aligned}
\frac{1}{e}\mathcal{L}_{\mathrm{ordinary},\mathcal{T}\to 0} = &-\frac{1}{2}\mathcal{R} - K_{,AA^*}|\partial A|^2 - e^K(K^{,AA^*}|D_A W|^2 - 3|W|^2) \\
&- 32\, e^K K^{,AA^*}|D_A W|^2 K^{,AA^*}|\partial A|^2\,\mathcal{T} \\
&+ 16\,(\partial A)^2(\partial A^*)^2\,\mathcal{T} \\
&+ 16 e^{2K}(K^{,AA^*}|D_A W|^2)^2\,(K^{,AA^*})^2\mathcal{T}
\end{aligned}
$$

- Corrections to kinetic terms
- Higher-derivative terms induce a new term to the potential:

$$
\begin{aligned}
V \quad = \quad & e^K(K^{,AA^*}|D_A W|^2 - 3|W|^2) \\
& -16(e^K K^{,AA^*}|D_A W|^2)^2\,(K^{,AA^*})^2\mathcal{T}_{\mathrm{no\,der.}}
\end{aligned}
$$

Example for small higher-derivative terms

Example:

$\mathcal{T} = c(K_{,AA^*})^2$, $W = \Phi$

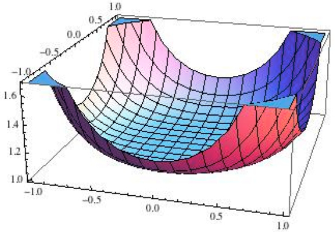
Uncorrected potential:

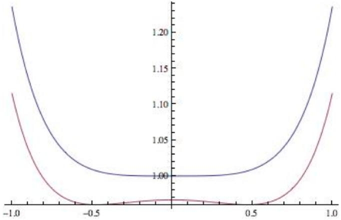
$V = 1 + (\phi^2 + \xi^2)^2 + \cdots$

Corrections:

$-c[1 + (\phi^2 + \xi^2) + \cdots]$

$\rightsquigarrow$ Mexican hat for $c > 0$





Example: DBI action

$$
\begin{aligned}
&\frac{1}{e}\mathcal{L}_{\mathrm{DBI}} \\
=\quad & -\frac{1}{f(A,A^*)}\left(\sqrt{\det(g_{mn}+f(A,A^*)\,\partial_m A\partial_n A^*)}-1\right) \\
=\quad & -\frac{1}{f}\left(\sqrt{1+2f\,|\partial A|^2+f^2\,|\partial A|^4-f^2\,(\partial A)^2(\partial A^*)^2}-1\right) \\
=\quad & -|\partial A|^2+(\partial A)^2(\partial A^*)^2\frac{f}{1+f\,|\partial A|^2+\sqrt{(1+f\,|\partial A|^2)^2-f^2\,(\partial A)^2(\partial A^*)^2}}
\end{aligned}
$$

can be easily studied in supergravity with our method!

Note: for time-dependent backgrounds

$$\frac{1}{e}\mathcal{L}_{\mathrm{DBI}}=-\frac{1}{f}\left(\sqrt{1-2f|\dot{A}|^2}-1\right)$$

→ warp factor f sets speed limit $|\dot{A}|^2 \leq \frac{1}{2f}$

→ DBI inflation (Silverstein, Tong)

For DBI in supergravity:

$$16T = \frac{f(\Phi,\Phi^\dagger)}{1 + f\partial\Phi\cdot\partial\Phi^\dagger e^{K/3} + \sqrt{(1 + f\partial\Phi\cdot\partial\Phi^\dagger e^{K/3})^2 - f^2(\partial\Phi)^2(\partial\Phi^\dagger)^2 e^{2K/3}}}$$

(including factors that compensate for the Weyl transformation)

Then

$$\begin{aligned}\frac{1}{e}\mathcal{L} \quad = \quad & -\frac{1}{2}\mathcal{R} + 3e^K|W|^2 \\ & -\frac{1}{f}\left(\sqrt{1 + 2f\,\partial A\cdot\partial A^* + f^2\,(\partial A\cdot\partial A^*)^2 - f^2\,(\partial A)^2(\partial A^*)^2} - 1\right) \\ & + e^{K/3}|F|^2 + \; e^{2K/3}\Big(F(D_A W) + F^*(D_A W)^*\Big) \\ & -32\, e^{K/3}|F|^2\partial A\cdot\partial A^*\,\mathcal{T} + 16 e^{2K/3}|F|^4\,\mathcal{T}\end{aligned}$$

Solve F perturbatively in ordinary branch:

f small

$$F \approx -e^{K/3}(D_A W)^* \quad \rightsquigarrow \quad V_{\text{non-rel.}} = e^K\left(|D_A W|^2 - 3|W|^2\right)$$

f large

$$F \approx -\left(\frac{(D_A W)^{*2}}{4f\, D_A W}\right)^{1/3}$$

$$\rightsquigarrow \quad V_{\text{rel.}} \approx \frac{3}{2}\frac{e^K |D_A W|^2}{\left(4f\, e^K |D_A W|^2\right)^{1/3}} - 3e^K|W|^2 \approx -3e^K|W|^2$$

→ Potential becomes negative, a general feature for these higher-derivative supergravity theories

→ In particular: DBI inflation is impossible!

⋆ Remedy: additional chiral supermultiplets (MK,Lehners,Ovrut 1208.0752)

Higher-derivative Galileon Lagrangians in $D = 4$ (Horndeski 1974)

$$\mathcal{L}_3 = -\frac{1}{2}(\partial\phi)^2 \Box\phi$$
$$\mathcal{L}_4 = -\frac{1}{2}(\partial\phi)^2 \left((\Box\phi)^2 - \phi^{,\mu\nu}\phi_{,\mu\nu}\right)$$
$$\mathcal{L}_5 = -\frac{1}{2}(\partial\phi)^2 \left((\Box\phi)^3 - 3\Box\phi\phi^{,\mu\nu}\phi_{,\mu\nu} + 2\phi^{,\mu\nu}\phi_{,\mu}\phi_{,\nu}\right)$$

Invariance under "Galilean" shift symmetry $\phi \to \phi + c + b_\mu x^\mu$
Applications:

* dS solutions in absence of Λ (Silva,Koyama De Felice,Tsujikawa;Chow,Khoury)
* Vainshtein-type screening mechanism (Deffayet,Dvali Gabadadze,Vainshtein)
* Violations of NEC $\to$ non-singular bouces, Galilean genesis, ... (Creminelli,Nicolis,Trincherini;Buchbinder,Khoury,Ovrut Lehners,Reneaux-Petel;Cai,Easson,Brandenberger)

Speciality: e.o.m. contain max 2 derivatives on each field $\to$ no ghosts

What about Galileons in $\mathcal{N} = 1$ supergravity?

Replace $\phi \to \sqrt{2}A$ & take real part $\qquad (A = \frac{1}{\sqrt{2}}(\phi + \mathrm{i}\xi))$

e.g. $L_3^{\mathbb{C}} = -\frac{1}{\sqrt{2}}(\partial A)^2 \Box A + h.c. \quad \rightsquigarrow$ 2nd order e.o.m.

$$(\Box A)^2 - A^{,\mu\nu} A_{,\mu\nu} = 0, \qquad (\Box A^*)^2 - A^{*,\mu\nu} A^*_{,\mu\nu} = 0$$

In terms of the real scalars ϕ and ξ, the Lagrangian and equations of motion are

$$L_3^{\mathbb{C}} = -\frac{1}{2}\left((\partial\phi)^2 \Box\phi - (\partial\xi)^2 \Box\phi - 2(\partial\phi \cdot \partial\xi)\Box\xi\right)$$

$$\begin{aligned} 0 =& (\Box\phi)^2 - \phi^{,\mu\nu}\phi_{,\mu\nu} - (\Box\xi)^2 + \xi^{,\mu\nu}\xi_{,\mu\nu} \\ 0 =& \Box\phi\Box\xi - \phi^{,\mu\nu}\xi_{,\mu\nu} \end{aligned}$$

Note that other actions, involving both A and A^* in a single term, do not lead to second-order equations of motion.

$$\text{e.g.}\quad \tilde{L}_3^{\mathbb{C}} = -\frac{1}{\sqrt{2}}\Box A^*(\partial A)^2 + h.c.$$

$$= -\frac{1}{2}\left((\partial\phi)^2\Box\phi - (\partial\xi)^2\Box\phi + 2(\partial\phi\cdot\partial\xi)\Box\xi\right)$$

$$0 = (\Box\phi)^2 - \phi^{,\mu\nu}\phi_{,\mu\nu} - \xi^{,\mu\nu}\xi_{,\mu\nu} - \xi_{,\mu}\xi_{,\nu}{}^{\nu\mu}$$

$$0 = \Box\xi\Box\phi + \xi_{,\mu}\phi_{,\nu}{}^{\nu\mu}$$

Clearly, these are higher-order in time and, thus, by Ostrogradsky's theorem, lead to the appearance of ghosts. (Ostrogradsky 1850)

Complex Galileons: replace $\phi \to \sqrt{2}A$ & take real part ($A = \frac{1}{\sqrt{2}}(\phi + \mathrm{i}\xi)$)

e.g. $L_3^{\mathbb{C}} = -\frac{1}{\sqrt{2}}(\partial A)^2 \Box A + h.c.$ $\rightsquigarrow$ 2nd order e.o.m.

Construct SUSY extension: $\int \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}(\partial^\mu \Phi)(\partial_\mu \Phi)\Phi$ is zero.

Terms involving A & A^*, e.g. $\tilde{L}_3^{\mathbb{C}} = -\frac{1}{\sqrt{2}}\partial A \cdot \partial A^* \Box A + h.c.$
always imply 3rd order e.o.m.

$\to$ Possible SUSY extensions:

1) $\int \mathrm{d}^4\theta \partial^\mu \Phi \partial_\mu \Phi^\dagger \Phi = -A\Box A \Box A^* - \Box A^*(\partial A)^2$

2) $\int \mathrm{d}^4\theta \partial^\mu \Phi \partial_\mu \Phi \Phi^\dagger = \Box A^*(\partial A)^2$

All others related through partial integration

Only 2) leads to Galileon term for ϕ (MK,Lehners,Ovrut 1302.0840)

$$\rightsquigarrow L_3^{SUSY} \equiv -\frac{1}{\sqrt{2}} \int \mathrm{d}^4\,\theta \partial^\mu \Phi \partial_\mu \Phi \Phi^\dagger + h.c.$$

$$= -\frac{1}{2}\left((\partial\phi)^2 \Box\phi - (\partial\xi)^2 \Box\phi + 2(\partial\phi \cdot \partial\xi)\Box\xi\right)$$

$\to$ 3rd order e.o.m.

Only time-derivative terms are associated with the ghost:

$$L_{2+3}^{SUSY} \equiv L_2^{SUSY} + c_3 L_3^{SUSY} = \tfrac{1}{2}\dot{\phi}^2 + \tfrac{1}{2}\dot{\xi}^2 + c_3\dot{\xi}^2\ddot{\phi}$$

Perturbations about time-dependent background:

$$\phi = \bar{\phi}(t) + \delta\phi(x^\mu), \qquad \xi = \bar{\xi}(t) + \delta\xi(x^\mu)$$

$$\rightsquigarrow L_{2+3\ \mathrm{quad}}^{SUSY} = \tfrac{1}{2}(\dot{\delta\phi})^2 + \tfrac{1}{2}(1 + 2c_3\ddot{\bar{\phi}})(\dot{\delta\xi})^2 + 2c_3\dot{\bar{\xi}}\,\dot{\delta\xi}\ddot{\delta\phi}$$

Diagonalize using $\dot{\delta b} \equiv \dot{\delta\xi} + \frac{2c_3\dot{\bar{\xi}}}{1+2c_3\ddot{\bar{\phi}}}\ddot{\delta\phi}$

$$\rightsquigarrow L_{2+3\ \mathrm{quad}}^{SUSY} = \tfrac{1}{2}(\dot{\delta\phi})^2 + \tfrac{1}{2}(1 + 2c_3\ddot{\bar{\phi}})\left((\dot{\delta b})^2 - \frac{4c_3^2\dot{\bar{\xi}}^2}{(1+2c_3\ddot{\bar{\phi}})^2}(\ddot{\delta\phi})^2\right)$$

Dispersion relation: $p_0^2\left(1 + \frac{4c_3^2\dot{\bar{\xi}}^2}{(1+2c_3\ddot{\bar{\phi}})}p_0^2\right) = 0 \qquad (m^2 = -p^2 = {p_0}^2)$

assuming $|c_3\ddot{\bar{\phi}}| \ll 1 \quad \rightarrow \quad$ scale of ghost: $(c_3\dot{\bar{\xi}})^{-1}$

cut-off $\Lambda < (c_3\dot{\bar{\xi}})^{-1}$, $|\dot{\bar{\xi}}| < \Lambda^2 \Rightarrow |\dot{\bar{\xi}}| < \frac{1}{|c_3|^{2/3}}$

$\rightsquigarrow$ For general backgrounds, c_3 has to be small for consistency.

Globally supersymmetric extension of L_3 term:

(Khoury,Lehners,Ovrut 1103.0003)

$$L_3^{\text{SUSY}} \equiv -\frac{1}{\sqrt{2}}\int \mathrm{d}^4\,\theta \partial^\mu \Phi \partial_\mu \Phi \Phi^\dagger + h.c.$$

$$= -\frac{1}{2}\Big((\partial\phi)^2 \Box\phi - (\partial\xi)^2 \Box\phi + 2(\partial\phi \cdot \partial\xi)\Box\xi\Big)$$

$\mathcal{N} = 1$ supergravity extension for conformal Galileon (MK,Lehners,Ovrut unpublished)

$$\mathcal{L}_{\text{c.g.}}^{\text{SUGRA}} = -\frac{1}{8}\int \mathrm{d}^2\Theta 2\mathcal{E}(\bar{\mathcal{D}}^2 - 8R)\Big[-3\mathrm{e}^{-K(\Phi,\Phi^\dagger)/3} + T\mathcal{D}^\beta \Phi \mathcal{D}_\beta \Phi \bar{\mathcal{D}}_{\dot\beta}\Phi^\dagger \bar{\mathcal{D}}^{\dot\beta}\Phi^\dagger$$

$$+ c_3 T_3 (\mathcal{D}^\beta \Phi \mathcal{D}_\beta \Phi \bar{\mathcal{D}}_{\dot\beta}\bar{\mathcal{D}}^{\dot\beta}\Phi^\dagger + H.c.) + W(\Phi)\Big] + H.c.$$

Conformal Galileons: (Nicolis,Rattazzi,Trincherini 08,09)

SUGRA Galileon component expansion

$$\frac{1}{e}\mathcal{L}_{\text{c.g.}}^{\text{SUGRA}}\Big|_{0f} = -\frac{1}{8}\int d^2\Theta 2\mathcal{E}(\bar{\mathcal{D}}^2 - 8R)\Big[-3e^{-K(\Phi,\Phi^\dagger)/3} + T\mathcal{D}^\beta\Phi\mathcal{D}_\beta\Phi\bar{\mathcal{D}}_{\dot\beta}\Phi^\dagger\bar{\mathcal{D}}^{\dot\beta}\Phi^\dagger$$

$$+ c_3 T_3(\mathcal{D}^\beta\Phi\mathcal{D}_\beta\Phi\bar{\mathcal{D}}_{\dot\beta}\bar{\mathcal{D}}^{\dot\beta}\Phi^\dagger + H.c.) + W(\Phi)\Big]\Big|_{0f} + H.c.$$

$$\begin{aligned}
=&e^{-K/3}\Big(-\frac{1}{2}\mathcal{R} - \frac{1}{3}MM^* + \frac{1}{3}b^a b_a\Big) + 3\frac{\partial^2 e^{-K/3}}{\partial A\partial A^*}\left(|\partial A|^2 - |F|^2\right)\\
&+ ib^m\Big(\partial_m A\frac{\partial e^{-K/3}}{\partial A} - \partial_m A^*\frac{\partial e^{-K/3}}{\partial A^*}\Big) + MF\frac{\partial e^{-K/3}}{\partial A}\\
&+ M^*F^*\frac{\partial e^{-K/3}}{\partial A^*} - WM^* - W^*M + \partial W F + \partial W^* F^*\\
&+ 16\Big((\partial A)^2(\partial A^*)^2 - 2|F|^2|\partial A|^2 + |F|^4\Big)\mathcal{T} + c_3[\ldots]\\
&+ c_3 16\Big(F^* F^{,a}A_{,a} + FF^{*,a}A^*{}_{,a}\Big)\mathcal{T}_3 \qquad \Big[\mathcal{T}_3 \equiv T_3\big|\Big]
\end{aligned}$$

Weyl rescaling $\mathcal{L}_{\text{c.g.}}^{\text{SUGRA}} \xrightarrow{\text{WEYL}} \mathcal{L}^{W}$

$$e_n{}^a \xrightarrow{\text{WEYL}} e_n{}^a \mathrm{e}^{K/6}\,, \quad \chi \xrightarrow{\text{WEYL}} \chi \mathrm{e}^{-K/12}\,, \quad \psi_m \xrightarrow{\text{WEYL}} \psi_m \mathrm{e}^{K/12}$$

Disentangle M and F fields from supergravity and scalar multiplets:

$$M = N - K_{,A^*}F^* + 16c_3F^*\left[|\partial A|^2 - |F|^2\right]\mathcal{T}_3^W$$

Solve for the auxiliary fields:

$$\begin{aligned} b_m =& \frac{1}{2}\mathrm{i}(A_{,m}K_{,A} - A^*{}_{,m}K_{,A^*}) - c_3 16\mathrm{i}\mathrm{e}^{K/3}|F|^2(A_{,m} - A^*_{,m})\mathcal{T}_3^W \\ &- c_3 8\mathrm{i}\Big((\partial A)^2 A^*{}_{,m} - (\partial A^*)^2 A_{,m}\Big)\mathcal{T}_3^W \\ N =& -3\mathrm{e}^{K/3}W \end{aligned}$$

$$
\begin{aligned}
\frac{1}{e}\mathcal{L}^W\Big|_{\text{0f}} = &-\frac{1}{2}\mathcal{R} - K_{,AA^*}|\partial A|^2 + K_{,AA^*}\mathrm{e}^{K/3}|F|^2 + 3\mathrm{e}^K|W|^2 \\
&+ \mathrm{e}^{2K/3}(D_A W)F + \mathrm{e}^{2K/3}(D_A W)^* F^* \\
&+ 16\Big[(\partial A)^2(\partial A^*)^2 - 2\mathrm{e}^{K/3}|F|^2|\partial A|^2 + \mathrm{e}^{2K/3}|F|^4\Big]\,\mathcal{T}^W \\
&+ c_3 8\Big[(\partial A)^2 e^{am}\mathcal{D}_m A^*{}_{,a} + (\partial A^*)^2 e^{am}\mathcal{D}_m A_{,a}\Big]\mathcal{T}_3^W \\
&+ c_3 16\mathrm{e}^{K/3}\Big[F^* F^{,a} A_{,a} + F F^{*,a} A^*{}_{,a}\Big]\mathcal{T}_3^W \\
&+ c_3[\ldots] + \mathcal{O}({c_3}^2)
\end{aligned}
$$

The equation of motion for F:

$$F_{,a}\Xi^a + F\Omega_F + F|F|^2\Omega_{F|F|^2} + |F|^2\Omega_{|F|^2} + F^2\Omega_{F^2} + F|F|^4\Omega_{F|F|^4} + \Omega = 0$$

If $W = 0$, the e.o.m. reduces to: $F_{,a}\Xi^a + F\Omega_F + F|F|^2\Omega_{F|F|^2} = 0$

$$\begin{aligned}\Xi_a =& c_3 16\sqrt{2}\mathrm{i}\xi_{,a}\mathcal{T}_3^W \\ \Omega_F =& K_{,AA^*} - 32\mathcal{T}^W|\partial A|^2 + c_3[\ldots] \\ \Omega_{F|F|^2} =& 2^5\mathcal{T}^W\mathrm{e}^{K/3} + c_3[\ldots]\end{aligned}$$

Solve perturbatively: $F = F_0 + c_3 F_1$

Order zero:

$$F_0\Big(K_{,AA^*}+32\mathcal{T}^W(\mathrm{e}^{K/3}|F_0|^2-|\partial A|^2)\Big)=0$$

$\rightsquigarrow$ $F_0=0$ or $|F_0|^2=-\frac{1}{32\mathcal{T}^W}\mathrm{e}^{-K/3}K_{,AA^*}+\mathrm{e}^{-K/3}|\partial A|^2$

Ordinary branch

Order one:

$$F_0{,}^a\Xi_a+F_0[\ldots]+F_1\Big(K_{,AA^*}+32\mathcal{T}^W(\mathrm{e}^{K/3}|F_1|^2-|\partial A|^2)\Big)=0$$

Insert $F=0$:

$$\begin{aligned}\frac{1}{e}\mathcal{L}^W=&-\frac{1}{2}\mathcal{R}-K_{,AA^*}|\partial A|^2+16\,(\partial A)^2(\partial A^*)^2\,\mathcal{T}^W\\&+8c_3[(\partial A)^2e^{am}\mathcal{D}_mA^*{}_{,a}+(\partial A^*)^2e^{am}\mathcal{D}_mA_{,a}]\,\mathcal{T}_3^W\\&+\frac{4}{3}c_3(\partial A)^2(\partial A^*)^2(K_{,A}+K_{,A^*})\,\mathcal{T}_3^W\\&-4c_3|\partial A|^2[(\partial A)^2K_{,A}+(\partial A^*)^2K_{,A^*}]\,\mathcal{T}_3^W\end{aligned}$$

$\rightsquigarrow$ new h.-d. terms

The equation of motion for F if $W \neq 0$:

$$-\dot{F}\dot{\Xi} + F\Omega_F + F|F|^2\Omega_{F|F|^2} + |F|^2\Omega_{|F|^2} + F^2\Omega_{F^2} + \Omega = 0$$

Solve perturbatively: $F = F_0 + c_3 F_1$

Order zero:

$$K_{,AA^*}F_0 + \mathrm{e}^{K/3}(D_A W)^* + 32F_0\left(\mathrm{e}^{K/3}|F_0|^2 - |\partial A|^2\right)\mathcal{T}^W = 0$$

small $\mathcal{T}^W$, ordinary branch: $F_0 = 0$

$$\begin{aligned} F_0 \approx &- K^{,AA^*}\mathrm{e}^{K/3}(D_A W)^* + 32\mathcal{T}^W \mathrm{e}^{4K/3}(K^{,AA^*})^4(D_A W)^{*2} D_A W \\ &- 32\mathcal{T}^W \mathrm{e}^{K/3}(K^{,AA^*})^2(D_A W)^*|\partial A|^2 + \mathcal{O}(\mathcal{T}^{W\,2}) \end{aligned}$$

Order one: quite a mess!

If $\mathcal{T} = 0$: $\quad F_1 K_{,AA^*} + F_0{}^{,a}\Xi_a + f(F_0) = 0$

...work in progress!

- Square of ordinary kinetic term, *i.e.* $(\partial\phi)^4$ has a unique clean, minimally-coupled extension to supergravity
- Can obtain supergravity version of any term containing $(\partial\phi)^4$ as a factor, hence can write out supergravity versions of $P(X,\phi)$ theories such as DBI actions
- Generic features:
  – three branches
  – potential without superpotential
  – corrections to both ordinary kinetic term and to potential
  – potential negative when higher-derivative terms are important
- The latter is the case for DBI inflation – remedy: couple to additional chiral supermultiplets

- Ghost condensates can exist in supergravity (MK,Lehners,Ovrut 1212.2185) (cf. Burt Ovrut's talk)
- Supersymmetric extension of Galileon term $(\partial\phi)^2\Box\phi$ does not lead to 2nd order e.o.m. $\rightarrow$ is on the same footing as other higher-derivative terms, valid only in an effective setting
- Supergravity extension of Galileons lead to a non-algebraic higher-order equation for the formerly auxiliary field F that we solved perturbatively
- Work in progress: construct a new ekpyrotic model with (supersymmetric) galileons in such a way that the ghosts can be tamed

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