Issues in N=4, N=8 supergravity, UV properties dualities, amplitudes

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Frascati, March 26, 2013

S. Ferrara, RK, A. Van Proeyen, 1209.0418
E. Bergshoeff, F. Coomans, RK, C.S. Shahbazi, A. Van Proeyen, 1303.5662
J.J. Carrasco, RK, 1303.5663
RK, Ortin 1205.4437 ; Gunaydin, RK 1303.3540
J.J. Carrasco, RK, R. Roiban, A. Tseytlin, 1303.6219

My talk is complimentary to the talks of Toine,
Eric, John Joseph

I am using many of their slides,
sometimes repeat or add additional comments

What do we know about quantum situation in perturbative supergravity

- Null results on UV infinities in D=4 quantum supergravity calculations, N=8 L=3,4, N=4, L=3 (Bern, Dixon, Carrasco, Roiban; Bern, Davies, Dennen, Huang; Tourkine,Vanhove)

- No information on N=8, L=5 and N=4, L=4 yet

- The "on shell supersymmetric" candidate counterterms (RK; Howe, Lindstrom) break genuine local supersymmetry (as well as supergravity duality (RK), e. g. $E_{7(7)}$)
Elimination of auxiliary fields leads to models which require a deformation of supersymmetry (Chemissany, Ferrara, RK, Shabhazi)

- To support duality with higher derivatives, the theory must be deformed to Born-Infeld type N-extended supergravities (RK, Bossard, Nicolai, Broedel, Carrasco, Ferrara, Roiban,Chemissany, Ortin)

- There are obstacles on the way to do the same for N = 4,…,8 supergravity (RK, Ortin)

Talk today on most recent work

Conjecture on a hidden N=4 superconformal symmetry of N=4 supergravity
DBI-VA models with 16+16 deformed supersymmetries/ Hidden supersymmetry and Duality
No-go on Bossard-Nicolai supersymmetric N=8 $E_{7(7)}$ deformation
N=4 supergravity, U(1) anomaly and S-matrix

N=2 superconformal action with higher derivatives which upon gauge-fixing extra local symmetries produces N=2 pure supergravity deformed by R^4

$$\int d^4\theta \left(S^2 + \lambda \frac{W^2}{S^2} \mathbb{T}\left(\frac{\overline{W^2}}{\overline{S^2}} \right) \right) .$$

de Wit, Katmadas, van Zalk, 2011

Simple prepotential for pure supergravity

S is the chiral compensator superfield (gauge multiplet),
W is a chiral Weyl superfield

The first legitimate genuine N=2 supersymmetric R^4 candidate counterterm!

Derived using N=2 off shell superconformal calculus.

It was known, in principle, that it is possible to do it, however, having the explicit answer was really important!

What is the difference between genuine and "on shell" supersymmetry?

Classical action is invariant under local supersymmetry for generic field configurations

$$\int d^4x \frac{\delta S_0}{\delta\varphi(x)} \delta_0\varphi(x) = 0 \qquad \frac{\delta S_0}{\delta\varphi(x)} \neq 0$$

The counterterms have local supersymmetry under classical supersymmetry transformations under condition that the fields satisfy classical EOM

$$\int d^4x \frac{\delta S_{ct}}{\delta\varphi(x)} \delta_0\varphi(x) = 0 \quad \text{iff} \quad \frac{\delta S_0}{\delta\varphi(x)} = 0$$

Do the on shell counterterms for N=8, …, 4 supergravities have genuine local supersymmetry ?

Relevant difference ? What does it mean for the higher derivative superinvariant to be genuine versus "on shell" ?

In the past nobody asked these questions

Why today we may ask and answer this question: What is the difference between genuine and "on shell" supersymmetry?

- The reason to ask this question today is that we are looking for all possible ways to explain the UV finite loop computations and this might be one of the explanations
- The second reason is that now we were able to answer this question, which would not be possible until recently, when de Wit et al decided to construct an explicit genuine N=2 supersymmetric version of R^4 with the purpose to prove that these terms do not contribute to the entropy of supersymmetric black holes.
- In the past only R^2 terms were interesting since in N=2 supergravity + matter, these corrections do not vanish and actually they deform the entropy formula

- The benefit of having such an explicit N=2 supersymmetric R^4 is that we were able to make a conclusive statement about the difference between the genuine and "on shell" candidate counterterms. This required a specific computation which led us to a simple conclusion: the supersymmetry has to be deformed.

The action and the transformation laws deform

EXACT

$$\delta\psi_\mu^i = D_\mu\epsilon^i - \frac{1}{16}\gamma^{ab}T_{ab}^{-}\varepsilon^{ij}\gamma_\mu\epsilon_j - \gamma_\mu\eta^i$$

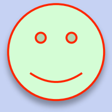
Deformation of the supergravity local N=2 supersymmetry after S-supersymmetry gauge-fixing and expanding near the classical solution for auxiliary fields

Order by order

$$T_{ab}^{-} = 4\mathcal{F}_{ab}^{-} + \lambda[\partial^4 T^3]_{ab}^{-}$$

The deformation of the gravitino supersymmetry due to higher derivative term is

$$\Delta\psi_\mu = -4\lambda[\partial^4\mathcal{F}^3]_\mu{}^\nu\gamma_\nu\,\epsilon^i + \ldots$$



N=4,…,8 on shell superspace and the corresponding candidate counterterms, truncated to N=2 do not have terms

which we found to be required in genuine N=2 superspace and in genuine higher derivative superinvariants.

On shell superspace candidate counterterms in N=4,…,8 break the N=2 part of local supersymmetry. The candidate counterterms have to be constructed in a deformed N=8,…,4 superspace. The available ones are not legitimate.

This is one possible explanation of various previously unexpected UV finite amplitudes

Also, the light-cone superspace counterterms were never constructed

Finally, duality!

UV properties of supergravity and duality

$E_{7(7)}$ in N=8, SL(2,R)xSU(4) in N=4

- A conservation of the Noether-Gaillard-Zumino duality current was proposed as a possible explanation of the L=3 UV finiteness of the D=4 N=8 and N=4 supergravities

- Issue with N=4 1-loop Marcus anomaly of a U(1) subgroup of duality symmetry (A comment on di Vecchia, Ferrara and Girardello earlier anomaly computation: different context, different anomaly!)

- If these explanations are valid, they predict

  also higher loop UV finiteness in these theories

- More studies required and more loop computations

A conjecture of quantum superconformal symmetry

Ferrara, RK, Van Proeyen, 2012

Known facts

- N=4 pure supergravity is a gauge-fixed version of the superconformal action of six N=4 Maxwell supermultiplets interacting with the Weyl multiplet.

- The model has full local superconformal symmetry. N=4 pure supergravity is recovered when some of these symmetries are gauge-fixed.

A conjecture to explain the 3-loop UV finiteness

The conjecture is that the UV properties of N=4 supergravity respect the prediction of the underlying full superconformal symmetry.

Explains the 3-loop UV finiteness and predicts the same for higher loops

Superconformal coupling of 6 N=4 vector multiplets to the N=4 Weyl multiplet

Bosonic action

$$e^{-1} L^{bos}_{s.c.} = -\frac{1}{4} F^{+I}_{\mu\nu} \eta_{IJ} F^{+J}_{\mu\nu} \frac{\phi^1 - \phi^2}{\Phi} - \frac{1}{4} \mathcal{D}_\mu \phi^I_{ij} \eta_{IJ} \mathcal{D}_\mu \phi^{ijJ}$$
$$- F^{+I}_{\mu\nu} \eta_{IJ} T^{\mu\nu}_{ij} \phi^{ijJ} \frac{1}{\Phi} - \frac{1}{2} T_{\mu\nu ij} \phi^{ijJ} \eta_{IJ} T^{\mu\nu}_{kl} \phi^{klJ} \frac{\Phi^*}{\Phi}$$
$$- \frac{1}{48} \phi^I_{ij} \eta_{IJ} \phi^{ijJ} \left(E^{kl} E_{kl} + 4 D_a \phi^\alpha D^a \phi_\alpha - 12 f_\mu{}^\mu \right) + \frac{1}{8} \phi^I_{ij} \eta_{IJ} \phi^{klJ} D^{ij}{}_{kl} + h.c.$$

The conformal boost gauge field $f_\mu{}^a$ is a function of a curvature $f_\mu{}^\mu = -\frac{1}{6} R(\omega)$

$$\Phi = \phi^1 + \phi^2 \ , \qquad \Phi^* = \phi_1 - \phi_2 \ , \qquad \phi^\alpha \phi_\alpha = 1$$

Classical superconformal action after gauge-fixing

Weyl symmetry, local SU(4), local U(1), (S-supersymmetry, K-conformal boosts)

$$\phi^I_{ij} \eta_{IJ} \phi^{ijJ} = -\frac{6}{\kappa^2} \qquad \varphi_M{}^I(x) = \frac{1}{2\kappa} \delta_M{}^I \qquad \mathrm{Im}(\phi_1 - \phi_2) = 0$$

Is N=4 Cremmer-Scherk-Ferrara supergravity

Ferrara, RK, Van Proeyen

$$\frac{1}{4} R - \frac{1}{8} \frac{\partial\tau \partial\bar{\tau}}{(\mathrm{Im}\tau)^2} + \frac{i}{4} \tau F^{+I}_{\mu\nu} \delta_{IJ} F^{+J\mu\nu} + h.c.$$

Other explanations of N=4 D=4 3-loop finiteness

RK assumptions

- Marcus U(1) global anomaly is not relevant for UV finiteness
- Noether-Gaillard-Zumino SL(2,R)xSU(4) duality current conservation

Prediction: N=4 L=4 is UV finite

Tourkine, Vanhove, Pierre’s talk :

2-loop string theory computation, non-renormalization theorem

Bossard, Howe, Stelle assumptions, Guillaume’s and Kelly’s talks

- Marcus U(1) anomaly is not relevant for 3-loop finiteness
- SL(2,R)xSU(4) duality invariance of the counterterm
- There exists an off-shell formulation with 16 local supersymmetries
- Off-shell quantization formalism exist

Prediction: N=4 L=4 is UV infinite

More on a SC conjecture, what can go wrong?

- The superconformal symmetry may be subject to anomalies (studied by Grisaru and de Wit for N=1 case). We argue that in N=4 case there are no consistent local superconformal anomalies.

- Our prediction from N=4 local superconformal symmetry may be eventually confirmed or invalidated either by the 4-loop N=4 supergravity computation, or by our own efforts to make new constructions (part of the talk on DBI-VA 16+16 deformed supersymmetries).

- A simple counter-argument comes from higher dimensions where known divergences occur already in supergravity theories at low loop orders. The half-maximal D=6 supergravity is UV divergent at the 2 loop order. One has to explain how the D=4 superconformal argument does not forbid such 2-loop divergences in the 16-supercharge half-maximal theory. The classical level theory in D=6 should be able to be promoted to a compensated conformal supergravity theory, just as it can in D=4. And one would have similar difficulties writing compensated counterterms in that case.

N=4 consistent local superconformal anomaly

N=1: local superconformal anomalies
satisfying Wess-Zumino consistency condition can be constructed using superconformal tensor calculus

de Wit, Grisaru 1987

N=4: there is no such a construction, essentially for same reasons as absence of superconformal counterterms

D=4 and D=6 and Superconformal Methods

- In D=4 there is an N=4 SC algebra, one can use Superconformal Methods to construct the SC classical action. However, there is no SC calculus which would allow to build higher SC order actions. This is the reason for our conjecture, explaning the 3-loop finiteness of D=4 L=3 N=4 supergravity.

- In D=6 SC methods are available for a half-supersymmetric model of interest, a (2,0) model with 16 supersymmetries and self-dual tensor multiplets. (1,1) model with 16 supersymmetries has no underlying superconformal symmetry.

- The SC field eqs. are available for , a (2,0) model but there is no action.
  Bergshoeff, Sezgin, Van Proeyen, 1999

- This is the reason why the D=6 UV divergence of the 2-loop half-maximal supergravity is not in contradiction with our conjecture in D=4 where the N=4 supergravity action is a gauge-fixed version of the N=4 superconformal action. In D=6 there is an action of half-maximal supergravity, but it is not a gauge-fixed action of the superconformal action.

Dirac-Born-Infeld-Volkov-Akulov and Deformation of Supersymmetry

Bergshoeff, Coomans, RK, Shahbazi, Van Proeyen, 1303.5662

Hidden Supersymmetry and Duality and the Role of Goldstino

Carrasco, RK, 13.035663

Let us deform extended supersymmetries and understand their relation to UV properties of N=8, N=4 supergravities

Bottom up deformation starting with free supersymmetric Maxwell action	⟺	Top down gauge-fixed kappa-symmetric actions

Hidden supersymmetry, duality and non-linear deformation of supersymmetry

What was known in the past:

One model with manifest N=2 supersymmetry and U(1) self-duality,

Kuzenko, Theisen,… up to W^8

One model with manifest N=2 supersymmetry and an extra hidden supersymmetry,

Bellucci, Ivanov, Krivonos up to W^{10}

Recent work: many new models with N=2 and U(1) duality, any order of deformation but no information on hidden supersymmetry

Broedel, Carrasco, Ferrara, RK, Roiban

New Dirac-Born-Infeld-Volkov-Akulov model solves many puzzles of duality and supersymmetry

Dirac-Born-Infeld-Volkov-Akulov and Deformation of Supersymmetry

Bergshoeff, Coomans, RK, Shahbazi, Van Proeyen

Maximal DBI-VA action with explicit 16 +16 non-linear supersymmetries in D=10, 6, 4, Complete to all order of deformation of the supersymmetric Maxwell multiplet.

Based on D-branes with kappa-symmetry gauge-fixed

Half-maximal DBI-VA action with explicit 8 +8 non-linear supersymmetries in D= 6, 4, complete to all order of deformation of the supersymmetric Maxwell multiplet.

Based on V-branes with kappa-symmetry gauge-fixed

Example

DBI-VA model with 8+8 supersymmetries, complete to all orders of deformation (from gauge-fixing of kappa-symmetric V-branes)

$$S^{V3} = \frac{1}{\alpha^2} \int d^4x \left\{ 1 - \sqrt{-\det(G_{\mu\nu} + \alpha \mathcal{F}_{\mu\nu})} \right\} , \qquad \mu = 0,1,2,3$$

$$G_{\mu\nu} = \eta_{mn} \Pi^m_\mu \Pi^n_\nu + \delta_{IJ} \Pi^I_\mu \Pi^J_\nu , \qquad m = 0,1,2,3 , \quad I = 1,2$$

$$\Pi^m_\mu = \delta^m_\mu - \alpha^2 \bar\lambda \Gamma^m \partial_\mu \lambda , \qquad \Pi^I_\mu = \partial_\mu \phi^I - \alpha^2 \bar\lambda \Gamma^I \partial_\mu \lambda , \qquad \mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} - b_{\mu\nu}$$

$$b_{\mu\nu} = -\alpha \bar\lambda \Gamma_m \partial_\mu \lambda \, \Pi^m_\nu - \alpha \bar\lambda \Gamma_I \partial_\mu \lambda \, \Pi^I_\nu - (\mu \leftrightarrow \nu) .$$

Truncated to pure vectors gives an N=0 Born-Infeld model

$$S^{V3}|_{\phi=\lambda=0} = \frac{1}{\alpha^2} \int d^4x \left\{ 1 - \sqrt{-\det(\eta_{\mu\nu} + \alpha F_{\mu\nu})} \right\}$$

When the 2-form is truncated consistently, the action is the Volkov-Akulov Goldstino action

$$S^{V3}|_{\mathcal{F}_{\mu\nu} = \Pi^I_\mu = 0} = \frac{1}{\alpha^2} \int d^4x \left\{ 1 - \det(\delta^\mu_\nu - \alpha^2 \bar\lambda \Gamma^\mu \partial_\nu \lambda) \right\}$$

8 Maxwell multiplet supersymmetries, complete to all orders of deformation

$$\delta_\epsilon \phi^I = -\tfrac{1}{2}\alpha\bar{\epsilon}\big(\mathbb{1} - \mathcal{G}(\mathcal{F})\,\Lambda(-\mathcal{F})\big)\Gamma^I\lambda + \xi^\mu_\epsilon \partial_\mu \phi^I \,,$$

$$\delta_\epsilon \lambda = -\tfrac{1}{2\alpha}\left(\mathbb{1} - \mathcal{G}(\mathcal{F})\,\Lambda(\mathcal{F})\right)\epsilon + \xi^\mu_\epsilon \partial_\mu \lambda \,,$$

$$\delta_\epsilon A_\mu = \tfrac{1}{2}\bar{\epsilon}\big(\mathbb{1} - \mathcal{G}(\mathcal{F})\,\Lambda(-\mathcal{F})\big)\big(\Gamma_\mu + \Gamma_I \partial_\mu \phi^I\big)\lambda$$

$$- \tfrac{1}{2}\alpha^2\bar{\epsilon}\big(\tfrac{1}{3}\mathbb{1} - \mathcal{G}(\mathcal{F})\,\Lambda(-\mathcal{F})\big)\Gamma_m\lambda\bar{\lambda}\Gamma^m\partial_\mu\lambda + \xi^\rho_\epsilon F_{\rho\mu}$$

where

$$\xi^\mu_\epsilon \equiv -\tfrac{1}{2}\alpha\bar{\lambda}\Gamma^\mu\left(\mathbb{1} + \mathcal{G}(\mathcal{F})\,\Lambda(\mathcal{F})\right)\epsilon \,.$$

$$\mathcal{G}(\mathcal{F}) = \frac{\sqrt{|G|}}{\sqrt{|G + \alpha\mathcal{F}|}} = \left[\det\left(\delta_\mu{}^\nu + \alpha\mathcal{F}_{\mu\rho}G^{\rho\nu}\right)\right]^{-1/2} \,,$$

$$\Lambda(\mathcal{F}) = \frac{1}{4!\sqrt{|G|}}\sum_{k=0}^{2}\frac{\alpha^k}{2^k k!}\hat{\Gamma}^{\mu_1\nu_1\cdots\mu_k\nu_k}\mathcal{F}_{\mu_1\nu_1}\cdots\mathcal{F}_{\mu_k\nu_k}\,\varepsilon^{\mu_0\ldots\mu_3}\hat{\Gamma}_{\mu_0\ldots\mu_3}$$

8 Volkov-Akulov supersymmetries, complete to all order of deformation

$$\delta_\zeta \phi^I = \alpha\bar{\zeta}\Gamma^I\lambda + \alpha\bar{\lambda}\Gamma^\mu\zeta\partial_\mu\phi^I$$

$$\delta_\zeta \lambda = \alpha^{-1}\zeta + \bar{\lambda}\Gamma^\mu\zeta\partial_\mu\lambda \,,$$

$$\delta_\zeta A_\mu = -\bar{\zeta}\big(\Gamma_\mu + \Gamma_I\partial_\mu\phi^I\big)\lambda +$$

$$\alpha^2\tfrac{1}{3}\bar{\zeta}\Gamma_m\lambda\bar{\lambda}\Gamma^m\partial_\mu\lambda + \alpha\bar{\lambda}\Gamma^\rho\zeta F_{\rho\mu}$$

Why we were able to construct all these new actions with deformed explicit 16+ 16 global supersymmetries and were is the problem of deforming local supersymmetry in N=4,…,8 supergravity?

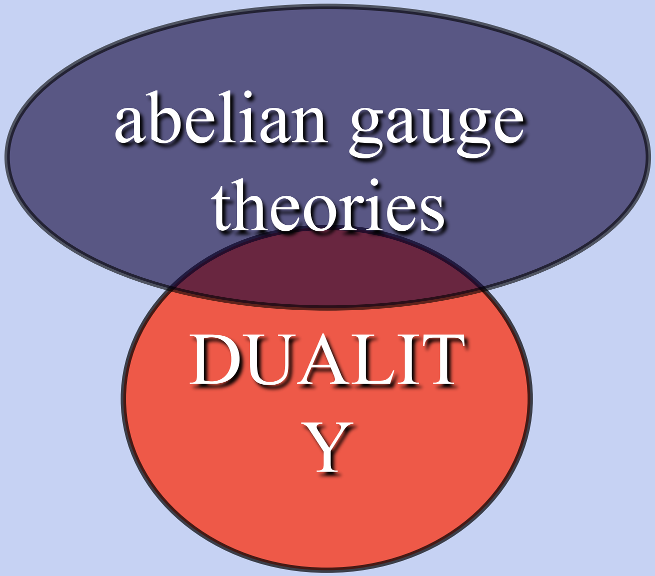
Our tool, local fermionic kappa-symmetry on D-branes, is valid only in a background supergravity, satisfying classical field equations

M. Cederwall, A. von Gussich, B. E. Nilsson and A. Westerberg, *The Dirichlet super three-brane in ten-dimensional type IIB supergravity*, Nucl.Phys. **B490** (1997) 163–178, arXiv:hep-th/9610148 [hep-th]

M. Cederwall, A. von Gussich, B. E. Nilsson, P. Sundell and A. Westerberg, *The Dirichlet super p-branes in ten-dimensional type IIA and IIB supergravity*, Nucl.Phys. **B490** (1997) 179–201, arXiv:hep-th/9611159 [hep-th]

E. Bergshoeff and P. Townsend, *Super D-branes*, Nucl.Phys. **B490** (1997) 145–162, arXiv:hep-th/9611173 [hep-th]

Therefore this tool in its current form is not useful for deformation of supergravity, but was working nicely for global supersymmetry!



2001 BIK derives nice recursive formula for N=2 "Born-Infeld" action gives to order W^10 open question if this last term satisfied duality...

JJM Carrasco, RK 2013

Action with 8 manifest and 8 hidden supersymmetries, up to W^{10}

Bellucci, Ivanov, and Krivonos '01

$$\mathcal{S}_{\rm BI} = \mathcal{S}_{\rm free} + \mathcal{S}_{\rm int,10}$$

$$\begin{aligned}
\mathcal{S}_{\rm int,10} = \int d^{12}\mathcal{Z}\Big\{ & \frac{1}{8}\mathcal{W}^2\overline{\mathcal{W}}^2\lambda + \lambda^2\left(\frac{1}{72}\mathcal{W}^3\Box\left[\overline{\mathcal{W}}^3\right] + \frac{1}{16}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right] + \frac{1}{16}\mathcal{W}^2\overline{\mathcal{W}}^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]\right) \\
& + \lambda^3\left(\frac{\mathcal{W}^4\Box\left[\Box\left[\overline{\mathcal{W}}^4\right]\right]}{1152} + \frac{1}{48}\mathcal{W}^3\Box\left[\overline{\mathcal{W}}^3\mathcal{D}^4\left[\mathcal{W}^2\right]\right] + \frac{1}{48}\overline{\mathcal{W}}^3\Box\left[\mathcal{W}^3\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]\right] + \frac{1}{32}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]^2\right. \\
& \left. + \frac{3}{32}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right] + \frac{1}{32}\mathcal{W}^2\overline{\mathcal{W}}^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]^2\right) \\
& + \lambda^4\left(-\frac{\mathcal{W}^5\Box\left[\Box\left[\Box\left[\overline{\mathcal{W}}^5\right]\right]\right]}{28800} + \frac{1}{576}\overline{\mathcal{W}}^4\Box\left[\Box\left[\mathcal{W}^4\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]\right]\right] + \frac{1}{576}\Box\left[\mathcal{W}^4\right]\Box\left[\overline{\mathcal{W}}^4\mathcal{D}^4\left[\mathcal{W}^2\right]\right] + \frac{1}{48}\mathcal{W}^3\Box\left[\overline{\mathcal{W}}^3\mathcal{D}^4\left[\mathcal{W}^2\right]^2\right]\right. \\
& + \frac{1}{48}\overline{\mathcal{W}}^3\Box\left[\mathcal{W}^3\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]^2\right] + \frac{1}{36}\overline{\mathcal{W}}^3\Box\left[\mathcal{W}^3\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]\right]\mathcal{D}^4\left[\mathcal{W}^2\right] + \frac{1}{64}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]^3 + \frac{1}{576}\overline{\mathcal{W}}^4\Box\left[\mathcal{W}^3\right]\mathcal{D}^4\left[\Box\left[\mathcal{W}^3\right]\right] \\
& + \frac{1}{16}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right] + \frac{1}{32}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]\right]\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right] + \frac{1}{16}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]^2 \\
& \left. + \frac{1}{64}\mathcal{W}^2\overline{\mathcal{W}}^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]^3 + \frac{1}{576}\mathcal{W}^4\Box\left[\overline{\mathcal{W}}^3\right]\overline{\mathcal{D}}^4\left[\Box\left[\overline{\mathcal{W}}^3\right]\right] + \frac{1}{32}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\right]\right)\Big\} \qquad (1.2)
\end{aligned}$$

Broedel, JJMC, Ferrara, Kallosh, Roiban '12

duality satisfied if

$$\int \mathrm{d}^8\mathcal{Z}\left(\mathcal{W}^2+\mathcal{M}^2\right) - \int \mathrm{d}^8\overline{\mathcal{Z}}\left(\overline{\mathcal{W}}^2+\overline{\mathcal{M}}^2\right) = 0$$

$$\mathrm{i}\,\mathcal{M} \;\equiv\; 4\,\frac{\mathrm{d}}{\mathrm{d}\mathcal{W}}\,\mathcal{S}[\mathcal{W},\overline{\mathcal{W}}].$$

$$0 = \frac{1}{3}I_2 + \frac{1}{36}I_{3a} + \frac{1}{2}I_{3b} + 2I_{3c} + \frac{1}{720}I_{4a} + \frac{1}{12}I_{4b} + \frac{1}{12}I_{4c} + \frac{1}{6}I_{4d} + \frac{1}{6}I_{4e} + \frac{1}{18}I_{4f} + \frac{1}{18}I_{4g} + \frac{1}{18}I_{4h} + \frac{1}{2}I_{4i} + \frac{1}{2}I_{4j}$$

JJMC, Kallosh '13

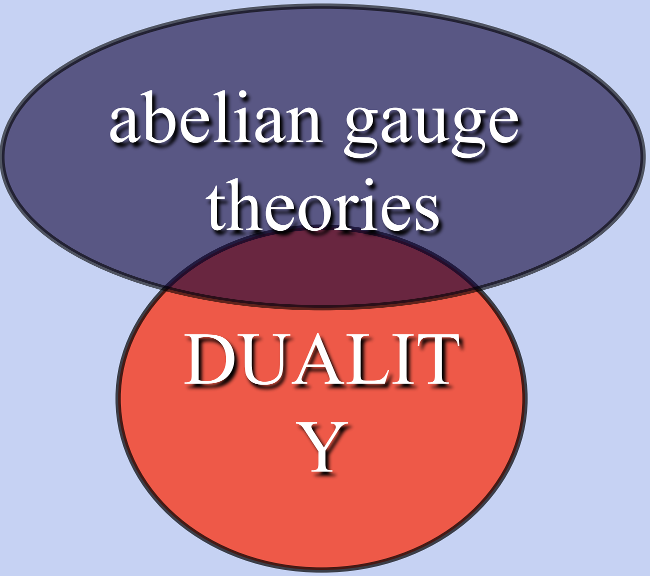
somewhat tedious, but straightforward integration by parts identities shows all terms vanishes....

$$I_{4i} = \lambda^4 \int d^{12}\mathcal{Z} \Big(+7\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right] - 4\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\mathcal{D}^4\left[\mathcal{W}^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]\right]$$
$$+3\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\right]\right] - 3\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\right]$$
$$- 3\mathcal{W}^2\overline{\mathcal{W}}^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]^2\right]\Big)$$

+ 13 others

exists much more satisfying constructive proof...

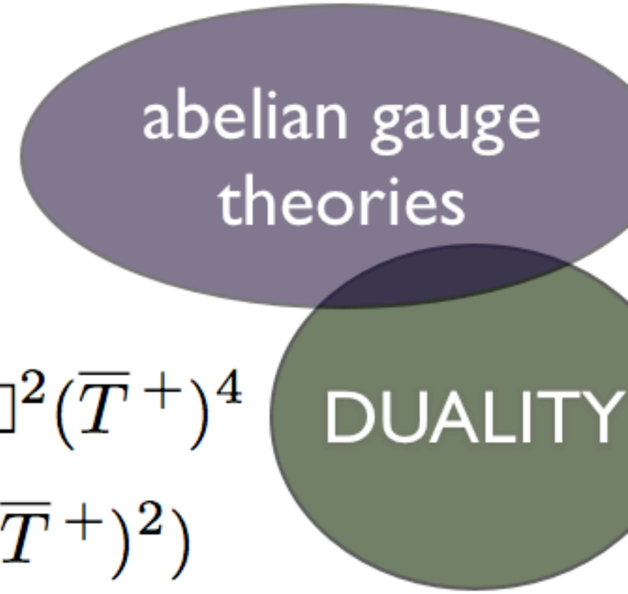
JJMC, Kallosh '13



find duality-conserving sources of deformation...

JJMC, Kallosh ’13

through W^8



$$\mathcal{I}(T^-,\overline{T}^+) = \int \mathrm{d}^{12}\mathcal{Z}\Bigg(\lambda\, a_0\, (T^-)^2(\overline{T}^+)^2$$

$$+ \lambda^2\, a_1\, (T^-)^3\Box(\overline{T}^+)^3 + \lambda^3\, a_2 (T^-)^4\Box^2(\overline{T}^+)^4$$

$$+ \lambda^3\, a_3\, (T^-)^2(\overline{T}^+)^2\overline{\mathcal{D}}^4((T^-)^2)\mathcal{D}^4((\overline{T}^+)^2)$$

$$+ \mathcal{O}(\lambda^4)\Bigg)$$

Broedel, JJMC, Ferrara, Kallosh, Roiban '12

$$\mathcal{I}_{\lambda^4}(T^-,\overline{T}^+) = \lambda^4 \int \mathrm{d}^{12}\mathcal{Z}\Bigg(a_4\, (T^-)^5\Box^3(\overline{T}^+)^5$$

$$+\ a_5\, (T^-)^3\mathcal{D}^4((\overline{T}^+)^2)\Box(\overline{T}^+)^3\overline{\mathcal{D}}^4((T^-)^2)\Bigg)$$

JJMC, Kallosh '13

Manifest N=2 self-duality is valid for models with generic a_i

Only when

$$\vec{a}_{0,1,2,3,4,5} = (-2^{-4},\, -2^{-6}\, 3^{-2}\,,\, -2^{-12}\, 3^{-2}\,,\, 2^{10}\,,\, 2^{-5}\, 3^{-2}\, 5^{-2}\,,\, 2^{-3}\, 3^{-2})$$

There is hidden second N=2 supersymmetry

What is special about Born-Infeld and the possibility to embed it into a theory with supersymmetry + hidden supersymmetry?

1. From infinite number of self-dual models the BI model is now known to have supersymmetric 16 + 16 supersymmetries, whereas other models do not have it, why?

2. What is the role of Goldstino in self-duality? What has Born-Infeld to do with Goldstino?

3. What is the relation between Volkov-Akulov and Komargoski-Seiberg?

Answers in examples

$$t = \frac{1}{4}F^2\,, z = \frac{1}{4}F\tilde{F}$$

$$\mathcal{L}_{\rm BI} = g^{-2}(1 - \sqrt{1 + 2g^2 t - g^4 z^2}\,)$$

$$\mathcal{L}_{\rm BI} = -t + \frac{1}{2}g^2\left(t^2+z^2\right) - \frac{1}{2}g^4 t\left(t^2+z^2\right) + \frac{1}{8}g^6\left(t^2+z^2\right)\left(5t^2+z^2\right) - \frac{1}{8}g^8 t\left(t^2+z^2\right)\left(7t^2+3z^2\right) + \cdots$$

$$\mathcal{L}_{\rm BN} = -t + \frac{1}{2}g^2\left(t^2+z^2\right) - \frac{1}{2}g^4\, t\,\left(t^2+z^2\right) + \frac{1}{4}g^6\left(t^2+z^2\right)\left(3t^2+z^2\right) - \frac{1}{8}g^8\, t\,\left(t^2+z^2\right)\left(11t^2+7z^2\right)$$

A relation between BI and BN models $t(t', z'), z(t', z')$

Not a local change of field variables $A'_\mu = A'_\mu(A_\mu)$

We now know that BI model (from all other self-dual models) can be embedded into a DBI-VA model with 16 supersymmetries and 16 VA type supersymmetries, but other self-dual models, not related by local field redefinition, do not have hidden supersymmetry.

Source of deformation is manifestly duality invariant, it depends on a doublet F and G so that

the action of $E_{7(7)}$ embedded into $Sp(56,\mathbb{R})$ is given by

$$\delta\mathcal{F} = \delta \begin{pmatrix} F^{ij} \\ G_{ij} \end{pmatrix} = \begin{pmatrix} 2\Lambda^{[i}{}_k \delta^{j]}{}_l & \Sigma^{ijkl} \\ \Sigma_{ijkl} & 2\Lambda_{[i}{}^k \delta_{j]}{}^l \end{pmatrix} \begin{pmatrix} F^{kl} \\ G_{kl} \end{pmatrix} . \tag{2.1}$$

where the $\Lambda^i{}_j$ are infinitesimal transformations of the maximal, (non-compact) subgroup $SL(8,\mathbb{R})$ (i.e. $\Lambda^i{}_i = 0$) and where the off-diagonal infinitesimal parameters satisfy

$$\Sigma^{ijkl} = \frac{1}{4!}\varepsilon^{ijklmnpq}\Sigma_{mnpq} . \tag{2.2}$$

F^{ij} and G_{ij} transform separately contravariantly and covariantly, respectively, in the **28** of $SL(8,\mathbb{R})$. Together, they transform as components of a symplectic vector in the **56** of $E_{7(7)} \subset Sp(56,\mathbb{R})$.

In all successful examples of new models of U(1) duality the manifestly U(1) invariant source of deformation depends on a doublet F and G

$$\delta \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} 0 & B \\ -B & 0 \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix}$$

We have shown that the existense of such sources of deformation is perfectly consistent with N=2 global supersymmetry

A no-go for Bossard-Nicolai proposal for a deformation of the twisted self-duality constraint in N=8 supergravity

Gunaydin and RK

- In the work with Ortin in 2012 we have shown that Bossard-Nicolai proposal of a deformation of the twisted self-duality constraint in N=8 supergravity is inconsistent with the undeformed N=8 superspace construction of Brink and Howe, 1981. We have found that the supersymmetric construction of the "source of $E_{7(7)}$ deformation" which requires the doubling of vectors (56 instead of 28), contradicts the superspace Bianchi Identities. Our conclusion was that we need to study the N=8 superspace deformation to make the further progress.

- P. Howe / H. Nicolai contemplation
- the incorporation of higher orders in the superspace formulation ("a formidable task") would have to start from scratch with the modification of the torsion and curvature constraints and the Bianchis.

- One would introduce superfields for all the component fields (except the vierbein + gravitino which are in the superspace vielbein), in particular include the super-56-bein from the start, and then just see what the Bianchis give. The 56 vector field strengths would just be elevated to superfields…

- Is it possible to realize this proposal ?

Bossard-Nicolai proposal on how to deform models to preserve duality symmetry in presence of non-linear (higher derivative corrections) in the advanced form given by Carrasco, RK, Roiban

Deformation of the twisted self-duality constraint in N=8 supergravity

$$F^m + J^m{}_n \tilde{F}^n = G^{mn} \frac{\delta \mathcal{I}}{\delta F^n} + \Omega^{mn} \frac{\delta \mathcal{I}}{\delta \tilde{F}^n} \,, \qquad m = 1, ..., 56$$

The the source of deformation, depending of the duality doublet which has all other symmetries of the theory

$$\mathcal{I}(F^m) = \mathcal{I}[F^{\mathrm{a}}, F^{\bar{\mathrm{a}}}]$$

$$F^m \equiv (F^{\mathrm{a}}, F^{\bar{\mathrm{a}}}) \,, \qquad \mathrm{a} = 1, ..., 28 \qquad \bar{\mathrm{a}} = 1, ..., 28$$

28 F^a and 28 G_a must be independent !

- Assume that the BN source of deformation is available in N=8 supergravity. This means that there is a Lorentz and general covariant and supersymmetric action which depends on fields of N=8 supergravity, but the number of vectors is doubled.

- Consider the linearized approximation of the source of deformation depending on a duality doublet.
- When twisted self-duality is applied to the source of deformation, it reduces to the CT which depends only on 28 vectors

$$F^m + J^m{}_n \tilde{F}^n = 0$$

- To double the vectors in a way consistent with N=8 supersymmetry one has to enlarge the CPT-self-conjugate doubleton supermultiplet of SU(2,2|8)

$SL(2,\mathbb{C})$	E_0	$SU(8)$	$U(1)$	Fields
$(0,0)$	1	$\mathbf{70}$	0	$\phi^{[ijkl]}$
$(\frac{1}{2},0)$	$\frac{3}{2}$	$\mathbf{56}$	1	$\lambda_+^{[ijk]} \Leftrightarrow \lambda_\alpha^{[ijk]}$
$(0,\frac{1}{2})$	$\frac{3}{2}$	$\overline{\mathbf{56}}$	-1	$\lambda_{-[ijk]} \Leftrightarrow \lambda_{\dot\alpha[ijk]}$
(1,0)	2	$\mathbf{28}$	2	$F_{\mu\nu}^{+[ij]} \Leftrightarrow F_{(\alpha\beta)}^{[ij]}$
(0,1)	2	$\overline{\mathbf{28}}$	-2	$F_{\mu\nu[ij]}^{-} \Leftrightarrow F_{(\dot\alpha\dot\beta)[ij]}$
$(\frac{3}{2},0)$	$\frac{5}{2}$	$\mathbf{8}$	3	$\partial_{[\mu}\psi_{\nu]}^{+i} \Leftrightarrow \psi_{(\alpha\beta\gamma)}^{i}$
$(0,\frac{3}{2})$	$\frac{5}{2}$	$\bar{\mathbf{8}}$	-3	$\partial_{[\mu}\psi_{\nu]i}^{-} \Leftrightarrow \psi_{(\dot\alpha\dot\beta\dot\gamma)i}$
$(2,0)$	3	1	4	$R_{(\alpha\beta\gamma\delta)}$
$(0,2)$	3	1	-4	$R_{(\dot\alpha\dot\beta\dot\gamma\dot\delta)}$

We studied all possibilities to double the vectors consistent with N=8 supersymmetry and conclusion is

- At the linearized level one can construct various candidates for the sources of deformation. They inevitably cross the barrier of spin 2 and have multiple gravitons.
- No known non-linear completion with general covariance and non-linear local supersymmetry.
- Examples

$SL(2,\mathbb{C})$	$SU(8)$	Fields
$(2,0)$	**70**	$R_{(\alpha\beta\gamma\delta)}$
$(5/2,0)$	**56**	$\psi^{[ijk]}_{(\alpha\beta\gamma\delta\epsilon)}$
$(3,0)$	**28**	$R^{[ij]}_{(\alpha\beta\gamma\delta\epsilon\lambda)}$
$(7/2,0)$	**8**	$\psi^{i}_{(\alpha\beta\gamma\delta\epsilon\kappa\lambda)}$
$(4,0)$	**1**	$R_{(\alpha\beta\gamma\delta\epsilon\kappa\lambda\sigma)}$
$(3/2,0)$	$\overline{\mathbf{56}}$	$\psi_{(\dot\alpha\dot\beta\dot\gamma)[ijk]}$
(1,0)	$\overline{\mathbf{28}}$	$F^{-}_{\mu\nu[ij]} \Leftrightarrow F_{(\dot\alpha\dot\beta)[ij]}$
(1/2,0)	$\overline{\mathbf{8}}$	$\lambda_{\alpha i}$
$(0,0)$	**1**	ϕ

$SL(2,\mathbb{C})$	E_0	$SU(10)$	$U(1)$	Fields
$(0,0)$	1	**252**	0	$\phi^{[ijklm]}$
$(\frac{1}{2},0)$	$\frac{3}{2}$	**210**	1	$\lambda_{+}^{[ijkl]} \equiv \lambda_{\alpha}^{[ijkl]}$
$(0,\frac{1}{2})$	$\frac{3}{2}$	$\overline{\mathbf{210}}$	-1	$\lambda_{-[ijkl]} \equiv \lambda_{\dot\alpha[ijkl]}$
(1,0)	2	**120**	2	$F^{+[ijk]}_{\mu\nu} \equiv F^{[ijk]}_{(\alpha\beta)}$
(0,1)	2	$\overline{\mathbf{120}}$	-2	$F^{-}_{\mu\nu[ijk]} \equiv F_{(\dot\alpha\dot\beta)[ijk]}$
$(\frac{3}{2},0)$	$\frac{5}{2}$	**45**	3	$\partial_{[\mu}\psi^{+[ij]}_{\nu]} \equiv \psi^{[ij]}_{(\alpha\beta\gamma)}$
$(0,\frac{3}{2})$	$\frac{5}{2}$	$\overline{\mathbf{45}}$	-3	$\partial_{[\mu}\psi^{-}_{\nu][ij]} \equiv \psi_{(\dot\alpha\dot\beta\dot\gamma)[ij]}$
$(2,0)$	3	**10**	4	$R^{i}_{(\alpha\beta\gamma\delta)}$
$(0,2)$	3	$\overline{\mathbf{10}}$	-4	$R_{(\dot\alpha\dot\beta\dot\gamma\dot\delta)i}$
$(\frac{5}{2},0)$	$\frac{7}{2}$	1	5	$R_{(\alpha\beta\gamma\delta\epsilon)}$
$(0,\frac{5}{2})$	$\frac{7}{2}$	1	5	$R_{(\dot\alpha\dot\beta\dot\gamma\dot\delta\dot\epsilon)}$

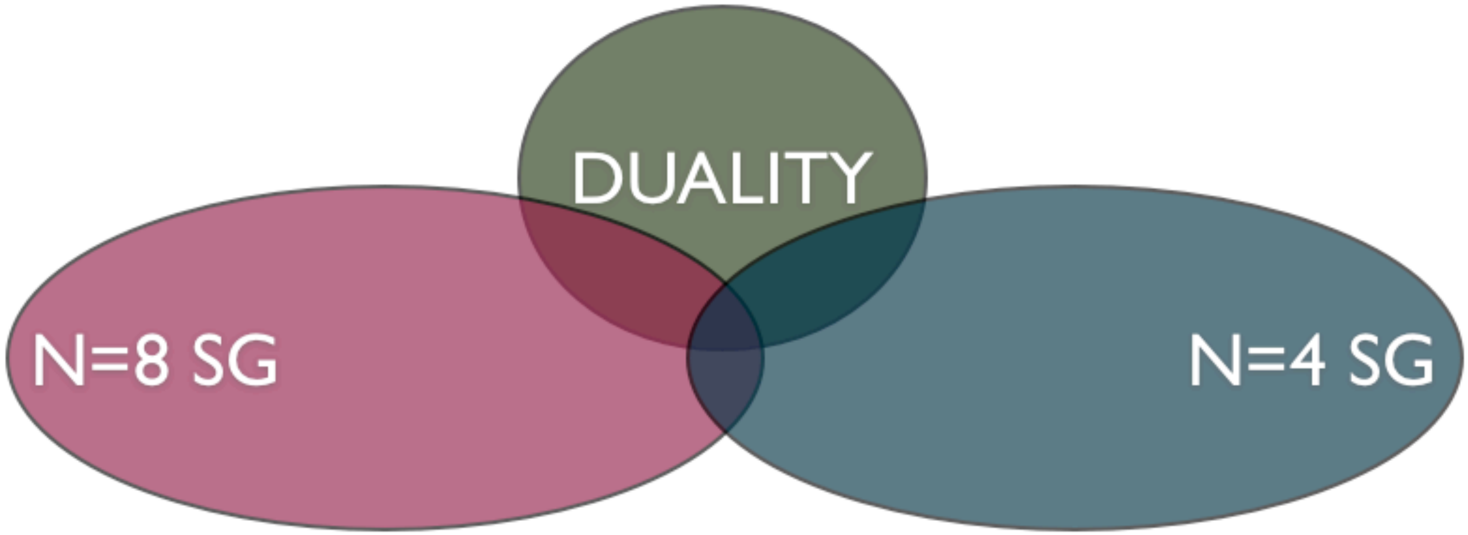
CPT-conjugate doubleton multiplet of SU(2,2|10)

We find therefore an obstruction to the N=8 supergravity deformation proposal, which was designed by BN to rescue the E77 duality current conservation induced by the L-loop counterterm deformation.

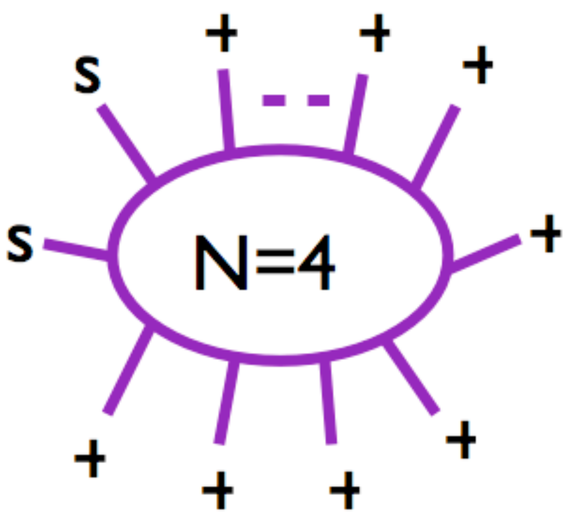
Deformation of N=8 supergravity requires higher spins and multiple gravitons, which presents a concrete obstacle to this proposal and was not realized before.

Thus, if the $E_{7(7)}$ duality current conservation is the reason for the 3-loop finiteness of N=8 supergravity, we claim that it is broken also by the higher loop candidate counterterms which predicts all-loop finiteness of N=8.

This claim can be falsified either by explicit computations or by a new proposal invalidating the “no-go” for the BN one.



New stuff in N=4 SG



Is there a way to line this up and identify / clarify known U(1) anomaly in N=4 SG?

YES

JJMC, Kallosh, Roiban, Tseytlin

On the $U(1)$ duality anomaly and the S-matrix of $\mathcal{N} = 4$ supergravity

J.J.M. Carrasco[a], R. Kallosh[b], R. Roiban[c] and A.A. Tseytlin[d,1]

We started with an observation that computing amplitudes in the double-copy method we find some 1-loop amplitudes in N=4 supergravity which have the following properties

1. Some N=4 supersymmetric 4-point amplitudes are not supersymmetric partners of a 4-graviton amplitudes !!!

In N=8 all 4-point amplitudes are partners of 4-graviton amplitude

2. They have strange non-vanishing soft scalar limits

3. They seem to be somehow related to Marcus U(1) anomaly

N=4 supergravity interacting with n_v matter multiplets

Correct choice of charge q under U(1) leads to anomalous part of effective action

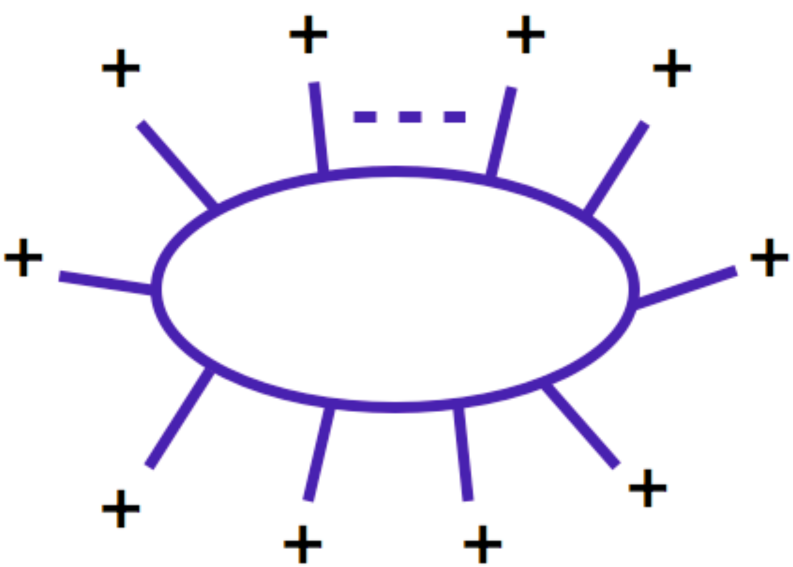
$$\Gamma_{\rm an}^{\mathcal{N}=4,n_{\rm v}} = \frac{1}{2}(2+n_{\rm v})\Gamma_{\rm an}^{\mathcal{N}=4,n_{\rm v}=0} = \frac{2+n_{\rm v}}{4(4\pi)^2}\int RR^*\nabla^{-2}\nabla_\mu a^\mu$$

Amplitude considerations reproduces this exactly (n_v corresponding to including n_v scalars in N=0 YM)

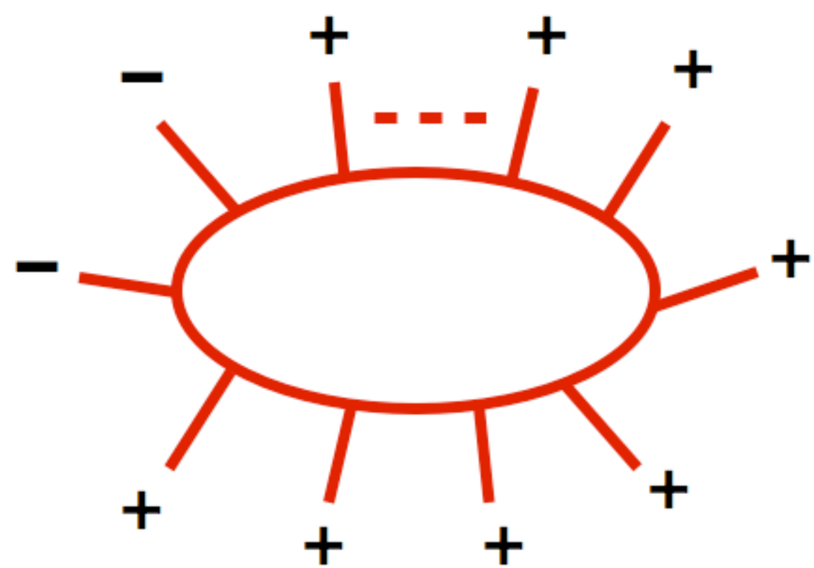
JJMC, Kallosh, Roiban, Tseytlin

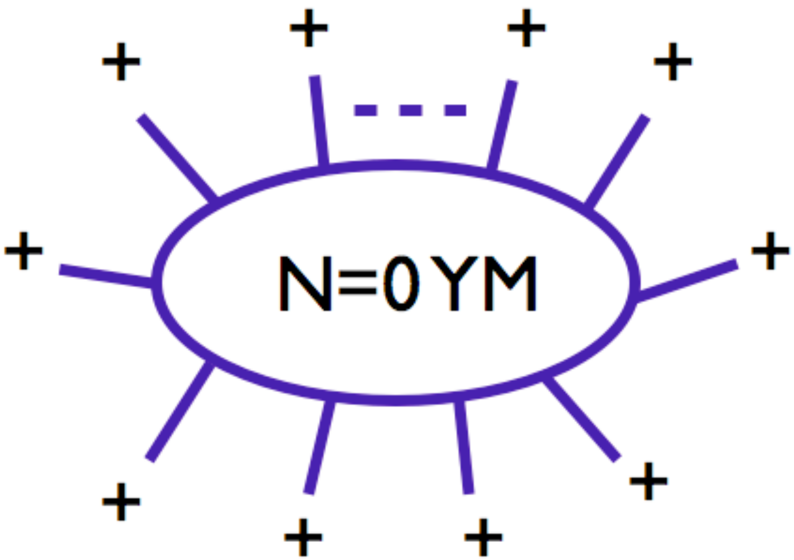
This means something new at N=4 SG

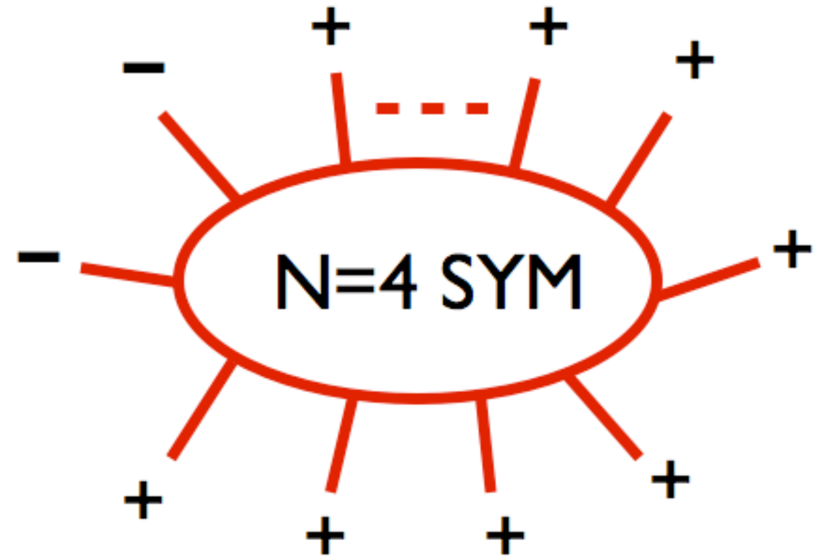
N=0 YM



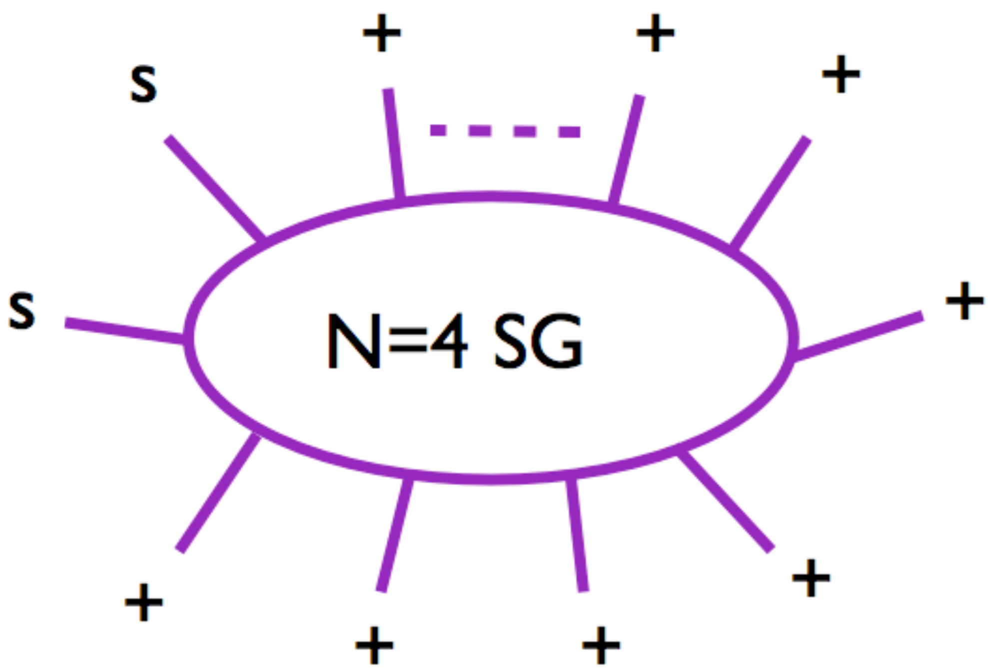
N=4 SYM







N=4 SG:



One class of these new amplitudes is local and therefore can be easily given in terms of superfields

$$\frac{1}{(4\pi)^2}\Big(\int d^8\theta \sum_n d_n W^n + \int d^8\bar{\theta} \sum_n d_n \overline{W}^n\Big)$$

so that the action is hermitian for real coefficients d_n. They are to be determined by explicit calculations. The component expansion of the chiral superfield $W(x,\theta)$ is

$$W(x,\theta) = \tau + \theta^\alpha_A \chi^A_\alpha + \theta^\alpha_A \theta^\beta_B F_{\alpha\beta\, CD}\epsilon^{ABCD} + \theta^\alpha_A\theta^\beta_B\theta^\gamma_C \psi_{\alpha\beta\gamma\, D}\epsilon^{ABCD} + \theta^\alpha_A\theta^\beta_B\theta^\gamma_C\theta^\delta_D C_{\alpha\beta\gamma\delta}\epsilon^{ABCD}, \quad (3.18)$$

In helicity superamplitudes language d_n is identified

$$\mathcal{M}_{\Gamma^{(1)}_{n+2}}(h_1^{++}, h_2^{++}, \bar{t}_3 \ldots \bar{t}_{n+2}) = 2i\, s_n\, n! \left(\frac{\kappa}{2}\right)^{n+2} [12]^4$$

$$s_n = \frac{1}{2(4\pi)^2}\frac{1}{n}$$

$$d_{n+2} = \frac{1}{(n+2)(n+1)} s_{n+2} = \frac{1}{2(4\pi)^2}\frac{1}{n(n+1)(n+2)}$$

The second class of U(1) breaking ampltudes is non-local, first discovered in YM double-copy computation

$$\mathcal{M}_4^{(1);\mathcal{N}=4\,\mathrm{PSG}}(1,2,3,4) = -\frac{i}{(4\pi)^2}\frac{1}{[31][14]}\frac{[32][24]\,\langle 2\,1\rangle}{[21]}\delta^{(8)}\Big(\sum_{i=1}^{4}\tilde{\eta}_{i,A}\tilde{\lambda}_i\Big)$$

Can also be given using N=4 supergravity superfields

$$f_4\frac{\delta\Big(\sum_{i=1}^4 p_i\Big)}{s_{12}s_{13}s_{23}}\Big(\int d^8\bar\theta\, \mathrm{C}_{\alpha\beta\gamma\delta}(p_1,\bar\theta)D^{\alpha\dot\alpha}D^{\beta\dot\beta}\overline{W}(p_2,\bar\theta)D^{\gamma}{}_{\dot\alpha}D^{\delta}{}_{\dot\beta}\overline{W}(p_3,\bar\theta)\overline{W}(p_4,\bar\theta)$$
$$+\int d^8\theta\,\bar{\mathrm{C}}_{\dot\alpha\dot\beta\dot\gamma\dot\delta}(p_1,\bar\theta)D^{\alpha\dot\alpha}D^{\beta\dot\beta}W(p_2,\theta)D^{\dot\gamma}{}_{\alpha}D^{\dot\delta}{}_{\beta}W(p_3,\theta)W(p_3,\theta)\Big)\,. \quad (3.20)$$

Proof of supersymmetry is based on absence of spin 5/2 physical states in N=8 supergravity

$f_4 \neq 0$ Follows from double-copy amplitude computations!

Whether such anomalous amplitudes affect the UV behavior of the theory (which, at four loops, will be unambiguously determined by an explicit calculation currently in progress, as described in a talk here by Tristan Dennen) remains an open question.

In conclusion, trying to understand the UV puzzles of recent extended supergravity loop computations, we came across various interesting phenomena

One of this is duality symmetry first discovered for the Born-Infeld model by Schrodinger in 1935

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Contributions to Born's New Theory of the Electromagnetic Field

Proc. R. Soc. Lond. A 1935 **150**, 465-477
doi: 10.1098/rspa.1935.0116

By E. Schrödinger

In studying Born's theory I came across a further representation, which is so entirely different from all the aforementioned, and presents such curious analytical aspects, that I desired to have it communicated.

For the first time in 1935 Schrodinger noticed duality symmetry and used the duality doublet to make it manifest