High energy closed string scattering on a stack of N Dp branes

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Foreword

- This talk is based on the work done together with
  - Giuseppe D'Appollonio, Rodolfo Russo and Gabriele Veneziano, JHEP 1011 (2010) 100, arXiv:1008:4773 [hep-th].
  - and work done this year.

Plan of the talk

Introduction

- ACV studied the high energy scattering of two closed strings [Amati, Ciafaloni and Veneziano, 1987....].
- At leading order in the impact parameter, the effective geometry turns out to be the Aichelburg-Sexl shock-wave metric produced by an energetic massless particle.
- In this talk we consider the scattering of a massless closed string against a maximally supersymmetric stack of N Dp branes.
- It is a cleaner process because the external metric is not produced by the other particle, but it is given by the presence of the Dp branes.
- In string theory all calculations are done in flat Minkowski space-time with suitable boundary conditions.
- The curved space-time structure, generated by the Dp brane, emerges from string scattering amplitudes.

Dp-branes in (super)gravity

- In (super)gravity, a Dp brane corresponds to a solution of the (super)gravity classical equations of motion.
- Consider the bulk action involving the metric, the dilaton and a RR field (low-energy 10-dim string effective action), given by ($a(p) = \frac{3-p}{2}$):

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[R - \frac{1}{2} (\nabla\phi)^2 - \frac{1}{2(p+2)!} \mathrm{e}^{a(p)\phi} \left(F_{p+2}\right)^2 \right]$$

  and the boundary action (DBI brane action), given by

$$S_{boundary} = -\frac{T_p}{\kappa_{10}} \left[\int d^{p+1}x \ \mathrm{e}^{-\frac{1}{2}a(p)\phi} \sqrt{-\det g_{\alpha\beta}} + \int C_{p+1} \right]$$

- One writes the equations of motion for the metric $g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa_{10} h_{\mu\nu}$, the dilaton ϕ and the RR $(p+1)$-form field

- and one finds a classical solution given by:

$$ds^2 = [H(r)]^{2A}\left(\eta_{\alpha\beta}dx^\alpha dx^\beta\right) + [H(r)]^{2B}\left(\delta_{ij}dx^i dx^j\right)$$

$$A = -\frac{7-p}{16} \quad , \quad B = \frac{p+1}{16} \quad ; \quad r^2 \equiv \delta_{ij}x^i x^j$$

$$\mathrm{e}^{-\phi(x)} = [H(r)]^{\frac{p-3}{4}} \quad , \quad \mathcal{C}_{01\ldots p}(x) = \left([H(r)]^{-1} - 1\right)$$

- $H(r)$ is an harmonic function given by

$$H(r) = 1 + \left(\frac{R_p}{r}\right)^{7-p} \; ; \; R_p^{7-p} = \frac{2\kappa_{10}T_pN}{(7-p)\Omega_{8-p}} \; ; \; \Omega_q = \frac{2\pi^{\frac{q+1}{2}}}{\Gamma(\frac{q+1}{2})}$$

N is the number of Dp branes.

- R_p is a parameter analogous to the Schwarzschild radius $R_S^{D-3} = \frac{16\pi G_N\sqrt{s}}{(D-2)\Omega_{D-2}}$ of ACV (In $D = 4$ $R_S = 2G_NE$).
In this case it is independent from the energy.

- In string theory everything is determined in terms of α' and g_s

$$\kappa_{10} = \frac{(2\pi)^{7/2}}{\sqrt{2}} g_s (\alpha')^2 \quad ; \quad T_p = \frac{\sqrt{\pi}}{(2\pi\sqrt{\alpha'})^{p-3}}$$

$$R_p^{7-p} = \frac{2\kappa_{10} T_p N}{(7-p)\Omega_{8-p}} = \frac{g_s N (2\pi\sqrt{\alpha'})^{7-p}}{(7-p)\Omega_{8-p}}$$

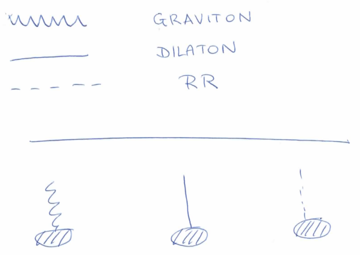
- Dp branes are non-perturbative states of string theory.
- Their mass per unit volume and their RR charge are given by

$$M_p = \frac{T_p}{\kappa_{10}} N = \frac{(2\pi\sqrt{\alpha'})^{1-p}}{2\pi\alpha' g_s} N \quad ; \quad \mu_p = \sqrt{2} T_p N$$

- There is an alternative (diagrammatical) way of obtaining the previous classical solution.
- It consists in computing the one-point function for the metric, the dilaton and the RR field respectively, in an action that is the sum of the previous bulk and boundary actions:

$$\langle \Phi(x)\ \mathrm{e}^{i(S_{bulk}+S_{boundary})}\rangle$$

- The term with one boundary action gives the leading term for $r \to \infty$ of the classical solution.
- The term with two boundaries gives the next to the leading term [M. Bertolini, M. Frau, A. Lerda, R. Marotta, R. Russo, DV (2000)] and so on. For Schwarzschild see [M. Duff (1973)].
- The sum over the boundaries gives the complete classical solution.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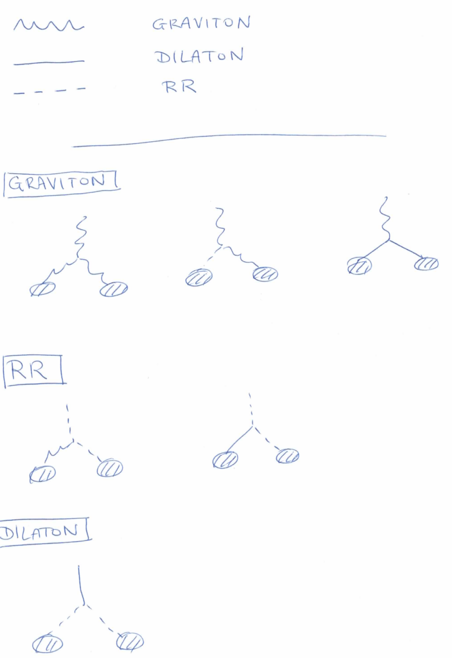
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The classical deflection angle in brane background

- We want to compute the deflection angle of a massless probe moving in the metric created by a stack of N Dp branes.
- Consider a general metric of the kind:

$$ds^2 \equiv g_{\mu\nu}(x)dx^\mu dx^\nu = -\alpha(r)dt^2 + \beta(r)(dr^2 + r^2 d\theta^2)$$

  where we have neglected coordinates that are not involved in the geodesic where only t, r and θ vary.
- The deflection angle can be best derived from the action of a massless point-particle in this metric:

$$S = \frac{1}{2}\int \frac{d\tau}{e}\dot{x}^\mu \dot{x}^\nu g_{\mu\nu}(x) = \frac{1}{2}\int \frac{d\tau}{e}\left(-\dot{t}^2\alpha(r) + \beta(r)\left(\dot{r}^2 + r^2\dot{\theta}^2\right)\right)$$

  e is the einbein and S is invariant under arbitrary reparametrizations of the world line coordinate τ.
- The conjugate momenta are given by:

$$p_t \equiv \frac{\partial L}{\partial \dot{t}} = -\frac{\dot{t}\alpha}{e} \;\; ; \;\; p_r \equiv \frac{\partial L}{\partial \dot{r}} = \frac{\beta(r)\dot{r}}{e} \;\; ; \;\; p_\theta = \frac{\partial L}{\partial \dot{\theta}} = \frac{\dot{\theta}r^2\beta(r)}{e}$$

- The Eq. of motion for e gives:

$$\beta(r)\dot{r}^2 + \beta(r)r^2\dot{\theta}^2 = \alpha(r)\dot{t}^2$$

- Since the Lagrangian does not depend explicitly on either t or θ there are two conserved quantities: the energy and the angular momentum

$$E = -\alpha(r)\dot{t} \quad ; \quad J = \beta(r)r^2\dot{\theta}$$

where a dot denotes derivative with respect to τ and we have taken $e = 1$.

- Combining the three previous equations we get

$$\frac{\dot{\theta}}{\dot{r}} \equiv \frac{d\theta}{dr} = \frac{J}{\beta r^2}\frac{1}{\sqrt{\frac{E^2}{\alpha\beta} - \frac{J^2}{\beta^2 r^2}}} = \frac{b}{r^2}\frac{1}{\sqrt{\frac{\beta}{\alpha} - \frac{b^2}{r^2}}}$$

where $b \equiv J/E$ is the impact parameter.

- Integrating it, one gets the deflection angle:

$$\Theta_p = 2\int_{r*}^{\infty} \frac{dr}{r^2} \frac{b}{\sqrt{\frac{\beta}{\alpha} - \frac{b^2}{r^2}}} - \pi = 2\int_{r*}^{\infty} \frac{dr}{r^2} \frac{b}{\sqrt{1 + \left(\frac{R_p}{r}\right)^{7-p} - \frac{b^2}{r^2}}} - \pi$$

  $r*$ is the turning point i.e. the largest root of the equation $1 + \left(\frac{R_p}{r}\right)^{7-p} - \frac{b^2}{r^2} = 0$.
- The result depends only on α/β.
- It is therefore invariant under a r-dependent rescaling of the whole metric.
- Therefore, we can work alternatively in either the string or the Einstein frame.
- The integral can be done exactly for the cases $p = 5, 6$:

$$\tan\frac{\Theta_6}{2} = \frac{R_6}{2b} \quad ; \quad \Theta_5 = \frac{\pi}{\sqrt{1 - \left(\frac{R_5}{b}\right)^2}} - \pi$$

- Θ_5 diverges when $b \sim R_5$: the probe particle is captured.

- For the case $p = 3$ we get instead:

$$\Theta_3 = 2\sqrt{1+k_3^2}K(k_3) - \pi \;\; ; \;\; k_3 = -1 + \frac{b}{2R_3}\sqrt{\left(\frac{b}{R_3}\right)^2 - 1}$$

$$K(k) = \int_0^1 \frac{dv}{\sqrt{(1-v^2)(1-k^2v^2)}} = \frac{\pi}{2}\sum_{n=0}^{\infty}\left(\frac{(2n)!}{(n!)^2 2^{2n}}\right)^2 k^{2n}$$

  K is the complete elliptic integral of first kind.
- For general p we have not been able to write the deflection angle in a closed form.
- We have computed the leading and the next to the leading behaviour for large impact parameter:

$$\Theta_p = \sqrt{\pi}\left[\frac{\Gamma(\frac{8-p}{2})}{\Gamma(\frac{7-p}{2})}\left(\frac{R_p}{b}\right)^{7-p} + \frac{1}{2}\frac{\Gamma(\frac{15-2p}{2})}{\Gamma(6-p)}\left(\frac{R_p}{b}\right)^{2(7-p)} + \dots\right]$$

- Also for the deflection angle there is an alternative (diagrammatical) way of computing it.

- Consider the dilaton ϕ, compute the two-point Green function:

$$\langle \phi(x)\phi(y) e^{i(S_{bulk}+S_{boundary})} \rangle$$

  and from it extract the scattering amplitude
  $\mathcal{A}_n(E,t)$ (with n boundaries)
  for the elastic scattering of a dilaton on a Dp brane.
- The two dilatons have respectively momentum p_1 and p_2.
- Along the directions of the world-volume of a Dp brane there is conservation of energy and momentum:

$$(p_1 + p_2)_{||} = 0 \quad ; \quad p_1^2 = p_2^2 = 0$$

- The scattering is described by two Mandelstam variables:

$$t = -(p_{1\perp} + p_{2\perp})^2 = -4E^2 \sin^2 \frac{\Theta}{2} \;\; ; \quad s = E^2 = |p_{1\perp}|^2 = |p_{2\perp}|^2$$

  $\Theta =$ the angle between the $(9-p)$-dim vectors $p_{1\perp}$ and $-p_{2\perp}$.

- At high energy and low transfer momentum we consider the following kinematical configuration:

$$p_1 \simeq (E, \underbrace{0 \ldots 0}; E, \mathbf{p}_1) \quad ; \quad p_2 \simeq (-E, \underbrace{0 \ldots 0}; -E, \mathbf{p}_2)$$

$\mathbf{p}_1, \mathbf{p}_2$ are $(8-p)$-dim vectors orthogonal to the $(p+1)$ direction.

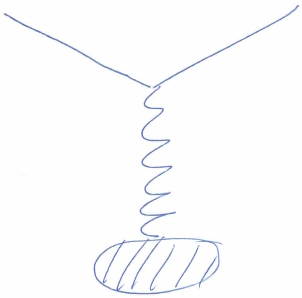
- The T-matrix ($S = 1 + iT = 1 + i\frac{\mathcal{A}}{2E}$) with one boundary is equal to

$$T_1(E,t) \equiv \frac{\mathcal{A}_1(E,t)}{2E} = \frac{2\pi^{\frac{9-p}{2}} R_p^{7-p}}{\Gamma(\frac{7-p}{2})} \cdot \frac{E}{(-t)}$$

corresponding to the diagram with a graviton exchange.

- A T-matrix that diverges with energy violates unitarity.
- In our case unitarity is restored by summing over the number of boundaries.
- Going to impact parameter space

$$T(E,b) \equiv \int \frac{d^{8-p}\mathbf{q}}{(2\pi)^{8-p}} \, \mathrm{e}^{i\mathbf{b}\cdot\mathbf{q}} \, T(E,t)$$



- one gets:

$$iT_1(E,b) = i\left(\frac{R_p^{7-p}\sqrt{\pi}E}{2b^{6-p}}\cdot\frac{\Gamma(\frac{6-p}{2})}{\Gamma(\frac{7-p}{2})}\right)$$

- The T-matrix with two boundaries is equal to

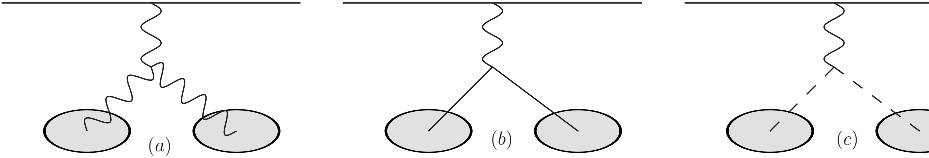
$$iT_2(E,b) \;\;=\;\; -\frac{1}{2}(T_1)^2 + i\sqrt{\pi}E\; b\;\left(\frac{R_p}{b}\right)^{2(7-p)}\cdot\frac{\Gamma(\frac{13-2p}{2})}{4\Gamma(6-p)}$$

- Up to two boundaries we get the following S matrix:

$$S = 1 + iT_1 - \frac{1}{2}(T_1)^2 + iT_2^{(1)} \sim \mathrm{e}^{iT_1(E,b)+iT_2^{(1)}+\dots}$$

- The terms that diverge with the energy exponentiate in a phase and in this way unitarity is recovered.


(a)
(b)


(a)
(b)
(c)

- In general

$$T_n(E,b) \sim A_n^{(n)} E^n + A_n^{(n-1)} E^{n-1} + \cdots + A_n^{(1)} E + 0(1) + \ldots$$

- $A_n^{(n)}(E,b)$ contributes to the leading eikonal, while $A_n^{(n-1)}(E,b)$ to the next to the leading eikonal and so on.
- To restore unitarity all terms divergent with energy must exponentiate.
- From the eikonal one can compute the deflection angle.
- Assuming that all terms (up to two boundaries) divergent with the energy exponentiate, we get:

$$S(E,b) \equiv \mathrm{e}^{2i\delta(E,b)} = \mathrm{e}^{i\sqrt{\pi}\,\frac{Eb}{2}\left[\left(\frac{R_p}{b}\right)^{7-p}\cdot\frac{\Gamma(\frac{6-p}{2})}{\Gamma(\frac{7-p}{2})}+\left(\frac{R_p}{b}\right)^{2(7-p)}\cdot\frac{\Gamma(\frac{13-2p}{2})}{2\Gamma(6-p)}+\ldots\right]}$$

- Going back to momentum space, we get:

$$\int d^{8-p}b\ \mathrm{e}^{i(-\mathbf{b}\cdot\mathbf{q}+2\delta(E,b))}$$

- For large impact parameter we have the saddle point equation:

$$\mathbf{q} = \frac{\mathbf{b}}{b} \, \frac{\partial(2\delta(E,b))}{\partial b}$$

- From which we compute the deflection angle:

$$\Theta_p = -\frac{\widehat{\mathbf{b}} \cdot \mathbf{q}}{E} = -\frac{1}{E} \, \frac{\partial(2\delta(E,b))}{\partial b}$$
$$= \sqrt{\pi}\left[\left(\frac{R_p}{b}\right)^{7-p} \cdot \frac{\Gamma(\frac{8-p}{2})}{\Gamma(\frac{7-p}{2})} + \frac{1}{2}\left(\frac{R_p}{b}\right)^{2(7-p)} \frac{\Gamma(\frac{13-2p}{2})}{\Gamma(6-p)} + \dots\right]$$

- The deflection angle is obtained from the phase shift $2\delta(E,b)$.
- It agrees with the classical calculation for large impact parameter!!

- In conclusion, we have presented two alternative ways to compute the classical solution and the deflection angle in the metric of the classical solution.
- Both methods are based on curved space Lagrangians.
- In the following, we will extend the previous results to string theory.
- In string theory we start using the method based on the amplitudes.
- Unlike field theory, every calculation is done in flat Minkowski space-time with suitable boundary conditions.
- From the string amplitudes, however, we recover the curved space-time structure that instead in field theory is put by hand.

String theory

- In string theory the boundary action becomes the boundary state.
- It is a closed string state that describes a Dp brane and that creates a boundary.
- It is given by:

$$|B\rangle \equiv \frac{T_p}{2}|B_X\rangle|B_\psi\rangle$$

- The bosonic part of the boundary state is equal to

$$|B_X\rangle = \delta^{d-p-1}(\hat{q}^i - y^i)\left(\prod_{n=1}^{\infty} e^{-\frac{1}{n}\alpha_{-n}S\cdot\widetilde{\alpha}_{-n}}\right)|0\rangle_\alpha|0\rangle_{\widetilde{\alpha}}|p=0\rangle$$

$S \equiv (\eta_{\alpha\beta}; -\delta_{ij})$

- Using the boundary state and the vertex operators for open and closed strings one can compute any amplitude involving scattering of strings on Dp branes.

- In particular, from the boundary state one can compute the large distance behaviour of the various fields of the classical solution as follows:

$$\langle\psi|D|B\rangle \quad ; \quad D = \frac{\alpha'}{4\pi}\int \frac{d^2 z}{|z|^2} z^{L_0 - a}\bar{z}^{\tilde{L}_0 - a}$$

$\langle\psi|$ is the string state corresponding to the field of the classical solution
[M. Frau, A. Lerda, I. Pesando, R. Russo, S. Sciuto and DV, 1997].

- The next to the leading behaviour is expected to come from the one-point function with two boundary states:

$$\mathcal{N}\sum_{\alpha,\beta} {}_{\alpha,\beta}\langle B| \int d^2 z_1 \; W(z_1, \bar{z}_1) \; D \; |B\rangle_{\alpha,\beta}$$

W is the vertex operator corresponding to the massless closed string. But, in string theory the external state must be on shell.

- The explicit calculation gave zero after the sum over the spin structures [R. Marotta, I. Pesando, PDV (1998), unpublished].

- The disk amplitude for the elastic scattering of a massless state on the Dp brane is given by:

$$\mathcal{A}_1(E,t) \sim \langle 0| \int \frac{d^2 z_1 d^2 z_2}{dV_{abc}} W_1(z_1,\bar{z}_1) W_2(z_2,\bar{z}_2)|B\rangle$$

$$= -\frac{\pi^{\frac{9-p}{2}} R_p^{7-p}}{\Gamma(\frac{7-p}{2})} \mathcal{K}(p_1,\epsilon_1;p_2,\epsilon_2) \frac{\Gamma(-\alpha' E^2)\Gamma(-\frac{\alpha'}{4}t)}{\Gamma(1-\alpha' E^2 - \frac{\alpha'}{4}t)}$$

[Ademollo et al, 1974, Klebanov and Thorlacius, 1995; Klebanov and Hashimoto, 1996, Garousi and Myers, 1996]

- At high energy

$$\mathcal{K}(p_1,\epsilon_1;p_2,\epsilon_2) = (\alpha' E^2)^2 Tr(\epsilon_1 \epsilon_2^t)$$

- The poles in the *t*-channel correspond to exchanges of closed strings, while those in the *s*-channel correspond to exchanges of open strings:

$$2 + \frac{\alpha'}{2} t = 2m \ ; \ \ m = 1,2,\ldots \ \ ; \ \ 1 + \alpha' E^2 = n \ ; \ \ n = 1,2\ldots$$

- Massless closed string vertex operator (picture 0):

$$W^{(0)}(z,\bar{z}) \sim \epsilon_{\mu\nu}\left(i\partial x^{\mu}(z)+\frac{\alpha'}{2}p\cdot\psi(z)\psi^{\mu}(z)\right)$$
$$\times\left(i\bar{\partial}x^{\mu}(\bar{z})+\frac{\alpha'}{2}p\cdot\tilde{\psi}(\bar{z})\tilde{\psi}^{\nu}(\bar{z})\right)\mathrm{e}^{ip\cdot x(z,\bar{z})}$$

- Massless closed string vertex operator (picture -1):

$$W^{(-1)}(z,\bar{z}) \sim \epsilon_{\mu\nu}\mathrm{e}^{-\phi(z)-\tilde{\phi}(\bar{z})}\psi^{\mu}(z)\tilde{\psi}^{\nu}(\bar{z})\ \mathrm{e}^{ip\cdot x(z,\bar{z})}$$

- Regge behaviour at high energy $(\alpha' s >> \alpha' t \sim 0)$ $(s \equiv E^2)$:

$$T_1(E,t) \equiv \frac{\mathcal{A}_1(E,t)}{2E} = \frac{R_p^{7-p}\pi^{\frac{9-p}{2}}}{\Gamma(\frac{7-p}{2})} \; \frac{\pi e^{-i\frac{\alpha'}{4}t}(\alpha' s)^{1+\frac{\alpha'}{4}t}}{2E \; \sin(\pi\frac{\alpha'}{4}(-t)) \; \Gamma(1+\frac{\alpha' t}{4})}$$

- It has a real and an imaginary part.
- The real part describes the scattering of the closed string on the Dp brane, while the imaginary part describes the absorption of the closed string by the Dp brane.
- When $\alpha' \to 0$ the real part reduces to the field theoretical result (graviton exchange).
- For $\alpha' \neq 0$ we have the graviton exchange dressed with string corrections.
- Assuming that also the imaginary part exponentiates, we get the absorption amplitude:

$$S^{abs}(E,b) \;=\; e^{-\frac{\pi}{2\Gamma\left(\frac{7-p}{2}\right)}\sqrt{\frac{\pi\alpha' s}{\ln \alpha' s}}\left(\frac{R_p}{l_s(s)}\right)^{7-p} e^{-\frac{b^2}{l_s^2(s)}}} \;\; ; \;\; l_s(s) = l_s\sqrt{\ln \alpha' s}$$

that is a purely stringy effect, negligible for $b >> l_s(s)$.

- To compute the next to the leading behaviour in the expansion for large impact parameter we need to compute the annulus diagram.
- The annulus diagram is given by:

$$A_2 = \mathcal{N} \int d^2 z_1 d^2 z_2 \sum_{\alpha,\beta} {}_{\alpha\beta}\langle B| W_1^{(0)}(z_1, \bar{z}_1) W_2^{(0)}(z_2, \bar{z}_2) D |B\rangle_{\alpha,\beta}$$

  $\mathcal{N}$ is a normalization factor and $\sum_{\alpha,\beta}$ is the sum over the spin structures.
- The sum over the spin structures can be explicitly performed obtaining in practice only the contribution of the bosonic degrees of freedom without the bosonic partition function.
- The final result is rather explicit.
  [Pasquinucci, 1997 and Lee and Rey, 1997]

- In the closed string channel the coefficient of the term with $\mathrm{Tr}(\epsilon_1 \epsilon_2^T)$ (relevant at high energy) of the annulus is equal to:

$$\mathcal{A}_2(s,t) = \frac{\pi^3(\alpha' s)^2}{\Gamma^2\left(\frac{7-p}{2}\right)} \frac{R_p^{14-2p}}{(2\alpha')^{\frac{7-p}{2}}} \times \left[2\int_0^\infty \frac{d\lambda}{\lambda^{\frac{5-p}{2}}} \int_0^{\frac{1}{2}} d\rho_1 \int_0^{\frac{1}{2}} d\rho_2 \int_0^1 d\omega_1 \int_0^1 d\omega_2\, \mathcal{I}\right]$$

- where

$$\mathcal{I} \equiv \mathrm{e}^{-\alpha' s V_s - \frac{\alpha'}{4} t V_t} \quad ; \quad z_{1,2} \equiv \mathrm{e}^{2\pi(-\lambda\rho_{1,2} + i\omega_{1,2})}$$

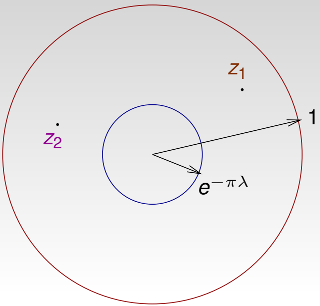
- and

$$V_s = -2\pi\lambda\rho^2 + \log \frac{\Theta_1(i\lambda(\zeta+\rho)|i\lambda)\Theta_1(i\lambda(\zeta-\rho|)i\lambda)}{\Theta_1(i\lambda\zeta+\omega)|i\lambda)\Theta_1(i\lambda\zeta-\omega)|i\lambda)}$$

and

$$V_t = 8\pi\lambda\rho_1\rho_2 + \log \frac{\Theta_1(i\lambda\rho+\omega)|i\lambda)\Theta_1(i\lambda\rho-\omega)|i\lambda)}{\Theta_1(i\lambda\zeta+\omega)|i\lambda)\Theta_1(i\lambda\zeta-\omega)|i\lambda)}$$

$\rho \equiv \rho_1 - \rho_2 \;;\; \zeta = \rho_1 + \rho_2 \;;\; \omega \equiv \omega_1 - \omega_2$



$$z_1 = e^{2\pi(-\lambda\rho_1 + i\omega_1)}$$

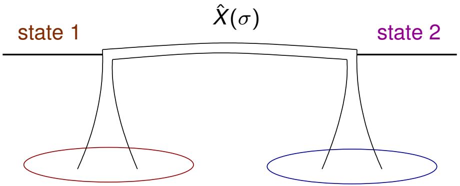
$$z_2 = e^{2\pi(-\lambda\rho_2 + i\omega_2)}$$

with

$$0 \leq \omega_1, \omega_2 \leq 1$$

$$0 \leq \rho_1, \rho_2 \leq \frac{1}{2}$$

World-sheet picture



Space-time picture

- The high energy behaviour ($E \to \infty$) of the annulus diagram can be studied, by the saddle point technique, looking for points where V_s vanishes.
- This happens for $\lambda \to \infty$ and $\rho \to 0$.
- Performing the calculation one gets the leading term for $E \to \infty$:

$$\frac{\mathcal{A}_2^{(3)}(E,t)}{2E} \to \frac{i}{2} \prod_{i=1}^{2} \left[\int \frac{d^{8-p}\mathbf{k_i}}{(2\pi)^{8-p}} \frac{\mathcal{A}_1(E,t_i)}{2E} \right]$$

$$\times \delta^{(8-p)}\left(\sum_{i=1}^{2} k_i - q\right) \; V_2(t_1,t_2,t) \quad ; \quad t_i \equiv -\mathbf{k}_i^2 \quad ; \quad t = -\mathbf{q}^2$$

where

$$V_2(t_1,t_2,t) = \frac{\Gamma(1+\frac{\alpha'}{2}(t_1+t_2-t))}{\Gamma^2(1+\frac{\alpha'}{4}(t_1+t_2-t))}$$

- Also the next to the leading term can be extracted from the annulus.
- Both the leading and the next to the leading terms reduce to the ones already computed In the field theory when $\alpha' \to 0$.

Parameters and approximations

- The scattering amplitude depends on the following parameters:

$$E, b, R_p, l_s \equiv \sqrt{\alpha' \hbar} \text{ (dimensional)} \quad ; \quad g_s, N \text{ (dimensionless)}$$

- The Regge limit implies that $\sqrt{\alpha'} E >> 1$ and $\alpha' t \sim 0$ that implies large impact parameter: $b >> R_p > l_s$.
- Contribution to δ from open and closed string loops:

$$\delta(n, m) \sim \frac{Eb}{\hbar} \left(\frac{R_p}{b} \right)^{n(7-p)} \left(\frac{G_{10} E}{b^7} \right)^m$$

- We want to neglect closed string loops with respect to open string loops: no quantum gravity effects.
- No limit on E if N is arbitrarily large.
- String effects will give corrections $\sim \frac{\alpha'}{b^2}$.

The leading eikonal operator

- In order to find the complete leading eikonal operator we go back to V_2 and write it in a more suggestive way, in terms of the $(8-p)$-dim bosonic transverse oscillators:

$$V_2(t_1, t_2, t) = \langle 0| \prod_{i=1}^{2} \left[\int_0^{2\pi} \frac{d\sigma_i}{2\pi} : \mathrm{e}^{i\mathbf{k}_i \cdot X(\sigma_i)} : \right] |0\rangle$$

  where

$$\hat{X}(\sigma) = i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \left(\frac{\alpha_n}{n} e^{in\sigma} + \frac{\tilde{\alpha}_n}{n} e^{-in\sigma} \right)$$

  without the zero mode part.
- The two vacuum states correspond to the two external massless states (states with no bosonic excitations: $(\epsilon_{\mu\nu}\psi^{\mu}_{-\frac{1}{2}}\tilde{\psi}^{\nu}_{-\frac{1}{2}}|0\rangle)$.

- Then the leading order from the annulus can be written as follows:

$$\frac{\mathcal{A}_2^{(3)}(E,t)}{2E} \to \frac{i}{2}\prod_{i=1}^{2}\left[\int \frac{d^{8-p}\mathbf{k}_i}{(2\pi)^{8-p}}\frac{\mathcal{A}_1(E,-\mathbf{k}_i^2)}{2E}\right]\delta^{(8-p)}(\sum_{i=1}^{2}\mathbf{k}_i-\mathbf{q})$$
$$\times\langle 0|\prod_{i=1}^{2}\left[\int_0^{2\pi}\frac{d\sigma_i}{2\pi}:\mathrm{e}^{i\mathbf{k}_i\cdot X(\sigma_i)}:\right]|0\rangle$$

where the two vertex operators correspond to the two leading Reggeons exchanged in the two t-channels: t_1 and t_2.

- It can be naturally generalized to the leading term coming from a surface with h boundaries:

$$\frac{\mathcal{A}_h^{(h+1)}(s,t)}{2E} \sim \frac{i^{h-1}}{h!}\prod_{i=1}^{h}\left[\int \frac{d^{8-p}\mathbf{k}_i}{(2\pi)^{8-p}}\frac{\mathcal{A}_1(s,-\mathbf{k}_i^2)}{2E}\right]$$
$$\times\delta^{(8-p)}(\sum_{i=1}^{h}\mathbf{k}_i-\mathbf{q})\ \langle 0|\prod_{i=1}^{h}\left[\int_0^{2\pi}\frac{d\sigma_i}{2\pi}:\mathrm{e}^{i\mathbf{k}_i\cdot X(\sigma_i)}:\right]|0\rangle$$

- Going to impact parameter space

$$i\frac{\mathcal{A}_h^{(h+1)}(s,\mathbf{b})}{2E} = \int \frac{d^{8-p}\mathbf{q}}{(2\pi)^{8-p}}\; e^{i\mathbf{bq}}\; i\frac{\mathcal{A}_h^{(h+1)}(s,t)}{2E}$$

$$= \frac{i^h}{h!}\prod_{i=1}^{h}\left[\int \frac{d^{8-p}\mathbf{k}_i}{(2\pi)^{8-p}}\frac{\mathcal{A}_1(s,-\mathbf{k}_i^2)}{2E}\right]$$

$$\langle 0|\prod_{i=1}^{h}\left[\int_0^{2\pi}\frac{d\sigma_i}{2\pi} : \mathrm{e}^{i\mathbf{k}_i(\mathbf{b}+\hat{X}(\sigma_i))} :\right]|0\rangle$$

- Summing all contributions:

$$\sum_{h=1}^{\infty}\frac{\mathcal{A}_h^{(h+1)}(s,\mathbf{b})}{2E} \sim \langle 0|\frac{1}{i}\left[e^{2i\hat{\delta}(s,b)} - 1\right]|0\rangle$$

we get the leading eikonal operator

$$2\hat{\delta}(s,b) \quad = \quad \int_0^{2\pi}\frac{d\sigma}{2\pi}\int \frac{d^{8-p}\mathbf{k}}{(2\pi)^{8-p}}\frac{\mathcal{A}_1(s,-\mathbf{k}^2)}{2E} : e^{i\mathbf{k}(\mathbf{b}+\hat{\mathbf{X}}(\sigma))} :$$

- The final result (that includes all string corrections) is obtained from the field theoretical one with the substitution:

$$\mathbf{b} \Longrightarrow \mathbf{b} + \hat{\mathbf{X}} \;\; ; \;\; \hat{X}(\sigma) = i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \left(\frac{\alpha_n}{n} e^{in\sigma} + \frac{\tilde{\alpha}_n}{n} e^{-in\sigma} \right)$$

and normal ordering.

- We have constructed the leading eikonal operator (acting in the space of the closed string states) that, when saturated with two massless states (vacuum of bosonic oscillators), reproduces the leading term of their scattering amplitude at high energy.
- It is natural to expect that, when it is saturated with any couple of physical states, it will also reproduce the leading term of their amplitude at high energy.

- For the states of the leading Regge trajectory it has been shown that the quantity

$$\int \frac{d^{8-p}\mathbf{k}}{(2\pi)^{8-p}} \frac{\mathcal{A}_1(E,-\mathbf{k}^2)}{2E} \delta^{(8-p)}(\mathbf{k}-\mathbf{q}) \ \langle 0| \int_0^{2\pi} \frac{d\sigma}{2\pi} : e^{i\mathbf{k}\cdot X(\sigma)} : |\lambda\rangle$$

  reproduces the high energy behaviour of the disk amplitude involving a massless state ($\langle 0|$) and a state of the leading Regge trajectory ($|\lambda\rangle$). Not completely correct, but in a non-trivial way. [W. Black and C. Monni, arXiv:1107.4321].
- The annulus diagram for a massless state and an excited state of the leading Regge trajectory has also been computed [M. Bianchi and P. Teresi, arXiv:1108.1071].
- Comparison with the eikonal operator gave agreement, but again not completely correct.
- For a massive state the longitudinal polarizations get enhanced at high energy. This has not been properly taken care in all previous calculations.

- The leading eikonal operator can be expanded in string corrections:

$$2\hat{\delta} = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{\partial^n T_1(E,b)}{\partial b_{i_1} \partial b_{i_2} \ldots \partial b_{i_n}} : \overline{X^{i_1} X^{i_2} \ldots X^{i_n}} :$$

where

$$T_1 = \frac{\sqrt{\pi}}{2} \, Eb \, \frac{\Gamma(\frac{6-p}{2})}{\Gamma(\frac{7-p}{2})} \left(\frac{R_p}{b}\right)^{7-p} \quad ; \quad b^2 \equiv \sum_{i=p+1}^{9-p} b^i b^i$$

and

$$: \overline{X^{i_1} X^{i_2} \ldots X^{i_n}} :\equiv \int_0^{2\pi} \frac{d\sigma}{2\pi} : X^{i_1}(\sigma) X^{i_2}(\sigma) \ldots X^{i_n}(\sigma) :$$

$$\hat{X}(\sigma) = i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \left(\frac{\alpha_n}{n} e^{in\sigma} + \frac{\tilde{\alpha}_n}{n} e^{-in\sigma} \right)$$

- The first string correction is given by:

$$2\,\hat{\delta}(s, \mathbf{b} + \hat{\mathbf{X}}) \sim \frac{1}{2E}\left[\mathcal{A}_1(s,b) + \frac{1}{2}\frac{\partial^2 \mathcal{A}_1(s,b)}{\partial b^i \partial b^j}\,\overline{\hat{X}^i \hat{X}^j} + \ldots\right]$$

- Since

$$\frac{1}{4E}\,\frac{\partial^2 \mathcal{A}_1(s,b)}{\partial b_i \partial b_j} = Q_\perp(s,b)\,\left[\delta_{ij} - \frac{b_i b_j}{b^2}\right] + \,Q_{\|}(s,b)\,\frac{b_i b_j}{b^2}$$

where $(s \equiv E^2)$

$$Q_\perp(s,b) = \frac{1}{4\sqrt{s}}\,\frac{1}{b}\,\frac{d\mathcal{A}_1(s,b)}{db} \;\;;\;\; Q_{\|}(s,b) = \frac{1}{4\sqrt{s}}\,\frac{d^2\mathcal{A}_1(s,b)}{db^2}$$

- The leading term in $\frac{R_p}{b}$ is equal to:

$$Q_\perp(s,b) \equiv -\frac{E}{2b}\Theta_p = -\frac{\sqrt{\pi}}{2}\sqrt{s}\frac{\Gamma\left(\frac{8-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)}\,\frac{R^{7-p}}{b^{8-p}}$$

$$Q_{\|}(s,b) = -(7-p)\;Q_\perp(s,b)$$

- From the elastic scattering of a massless closed string on a Dp brane we have constructed the leading eikonal operator.
- From it we have derived all the field theoretical results (for $\alpha' \to 0$) plus string corrections corresponding to tidal forces acting on the string as an extended object.
- We believe that it includes the string corrections not only for the elastic scattering of massless states, but also for the scattering of any string state going into another arbitrary string state.
- Why only the bosonic oscillators?
- The scattering amplitudes have been computed in flat Minkowski space-time.
- On the other hand, the classical deflection angle for a massless particle has been computed from the action of a massless particle in the external Dp brane metric.
- Consider the string action in the external Dp brane metric.
- As in the case of a point particle, also in this case we recover all the properties already obtained from the scattering amplitudes....but this time from a curved space-time formalism.

Closed string in the background of a Dp brane: I

- Let us start from the D brane metric

$$ds^2 = \left(1 + \left(\frac{R_p}{r}\right)^{7-p}\right)^{-\frac{1}{2}} \left(-dt^2 + \sum_{a=1}^{p} dx^a dx^a\right)$$
$$+ \left(1 + \left(\frac{R_p}{r}\right)^{7-p}\right)^{\frac{1}{2}} \sum_{i=p+1}^{9} dy^i dy^i$$

where $r^2 = \sum_{i=p+1}^{9} y^i y^i$.

- Let us denote y^{p+1} by z, which we identify with the direction of the initial probe string, and define:

$$u = t + z \quad ; \quad v = t - z$$

- Rewrite the metric in these coordinates and go to the limit $r >> R$:

$$ds^2 = -dudv + \frac{1}{4}\left(\frac{R_p}{r}\right)^{7-p}\left(du^2 + dv^2\right) + \sum_{a=1}^{p} dx^a dx^a$$

$$+ \sum_{p+2}^{9} dy^i dy^i + \ldots \quad ; \quad r^2 = z^2 + \rho^2 = \frac{1}{4}(u-v)^2 + \rho^2$$

where the dots represent subleading terms in R/r.

- We perform a boost on the light-cone coordinates:

$$u = \lambda U \quad ; \quad v = \lambda^{-1} V$$

- The metric becomes:

$$ds^2 = -dUdV + \frac{1}{4}\left(\frac{R_p}{r}\right)^{7-p}\left(\lambda^2 dU^2 + \lambda^{-2} dV^2\right) + \sum_{a=1}^{p} dx^a dx^a$$

$$+ \sum_{p+2}^{9} dy^i dy^i + \ldots \quad ; \quad r^2 = \frac{1}{4}(\lambda^2 U - \lambda^{-2} V)^2 + \rho^2$$

- In the limit of large λ (Penrose limit), we can approximate the above metric as:

$$ds^2 = -dUdV + \frac{1}{4}\lambda R_p^{7-p} f dU^2 + \sum_1^p dx^a dx^a + \sum_2^{9-p} dy^i dy^i$$

$$f = \frac{2^{7-p}\,\lambda}{\left(\lambda^2 U^2 + 4\rho^2\right)^{\frac{7-p}{2}}}$$

- In this limit, the function f tends to zero at any finite value of U and has a finite integral over U.
- In other words it tends to $\delta(U)g(\rho)$ where $g(\rho)$ can be computed from the integral:

$$\int_{-\infty}^{+\infty} \frac{dx}{(x^2 + 4\rho^2)^{\frac{7-p}{2}}} = (2\rho)^{p-6}\sqrt{\pi}\frac{\Gamma(\frac{6-p}{2})}{\Gamma(\frac{7-p}{2})}$$

- The limiting metric for large r/R_p and large λ thus becomes:

$$ds^2 = -dUdV + \lambda\rho\left(\frac{R_p}{\rho}\right)^{7-p}\frac{\sqrt{\pi}}{2}\frac{\Gamma(\frac{6-p}{2})}{\Gamma(\frac{7-p}{2})}\delta(U)dU^2 + \sum_{a=1}^{p} dx^a dx^a + \sum_{i=p+2}^{9} dy^i dy^i$$

- The action of a string in this metric is given (in the light-cone gauge $U = \alpha' E' \tau$) by:

$$S - S_0 = \frac{(\alpha' E')^2}{4\pi\alpha'}\int_0^{2\pi} d\sigma \int d\tau\, \lambda\, \delta(\alpha' E' \tau)\rho\left(\frac{R}{\rho}\right)^{7-p}\frac{\sqrt{\pi}\ \Gamma(\frac{6-p}{2})}{2\ \Gamma(\frac{7-p}{2})}$$

$$= \frac{1}{2\pi}\int_0^{2\pi} d\sigma\, T_1(\mathbf{b} + X(\sigma, 0), E) \ \ ; \ \ E \equiv \lambda E'$$

that agrees with the leading eikonal obtained from the scattering amplitudes at high energy at leading order in $\frac{R_p}{b}$ and all orders in $\frac{\alpha'}{b^2}$.

- To go beyond the leading term in $\frac{R_p}{b}$, we used a system of coordinates adapted to the geodetics followed by the string centre of mass (Fermi coordinates).
- We found that the next to the leading term in $\frac{R_p}{b}$ does not agree with what one gets from the eikonal operator.
- In particular, one finds string excitations along the world-volume of the Dp brane that are absent in the eikonal operator.
- To find out which approach is the correct one, we are computing string scattering amplitudes (disk and annulus) where an incoming graviton produces a massive string state.
- Take into account the fact that the longitudinal (along the motion of the massive particle) polarization v^μ is enhanced at high energy:

$$p_2^\mu = \left(-E, 0_p; 0_{8-p}, -\sqrt{E^2 - M^2}\right); v^\mu = \left(\frac{\sqrt{E^2 - M^2}}{M}, 0_p; 0_{8-p}, \frac{E}{M}\right)$$

$$e_K^\mu = \delta_K^\mu \quad ; \quad K = 1 \dots 8 \ ; \quad p_2 \cdot v = p_2 \cdot e_K = 0$$

Truncation of the physical spectrum at high energy

Bosonic physical states at the first massive level are given by:

- **GS light-cone**

$$|\zeta_{ij}\rangle = \alpha^i_{-1}|j\rangle \Longrightarrow 64 \;\; states$$
$$Q^a_{-1}|b\rangle \Longrightarrow 64 \;\; states$$

$i, j = 1 \ldots 8$ $(a, b = 1 \ldots 8)$ are vector (spinor) indices of $SO(8)$.

- **RNS light-cone**

$$|\zeta_{ij}\rangle = A^i_{-1} B^j_{-\frac{1}{2}}|0\rangle \Longrightarrow 64 \;\; states$$
$$B^i_{-\frac{3}{2}}|0\rangle \Longrightarrow 8 \;\; states$$
$$B^i_{-\frac{1}{2}} B^j_{-\frac{1}{2}} B^k_{-\frac{1}{2}}|0\rangle \Longrightarrow 56 \;\; states$$

$i, j, k = 1 \ldots 8$ are vector indices of $SO(8)$.

- **Covariant formalism**

$$|T^{IJ}\rangle = \left(\alpha_{-1}^{I}\psi_{-\frac{1}{2}}^{J} + \alpha_{-1}^{J}\psi_{-\frac{1}{2}}^{I} - \frac{2}{d-1}\eta^{IJ}\eta^{HK}\alpha_{-1}^{H}\psi_{-\frac{1}{2}}^{K}\right)|0, M_1\rangle$$
$$\Longrightarrow 44 \;\; states$$
$$|V^{IJK}\rangle = \psi_{-\frac{1}{2}}^{I}\psi_{-\frac{1}{2}}^{J}\psi_{-\frac{1}{2}}^{K}|0, M_1\rangle \Longrightarrow 84 \;\; states$$

$I, J, K, H = 1\ldots 9$ are vector indices of $SO(9)$.

- They satisfy the physical conditions:

$$G_{\frac{1}{2}}|T^{IJ}\rangle = G_{\frac{3}{2}}|T^{IJ}\rangle = 0 \;\; ; \;\; G_{\frac{1}{2}}|V^{IJK}\rangle = G_{\frac{3}{2}}|V^{IJK}\rangle = 0$$

- Decomposing the 9-dim indices $(I = i, v); (J = j, v)$ in 8-dim indices + a longitudinal one (v):

$$|T^{ij}\rangle \Longrightarrow 36 \;\; states \quad ; \quad |T^{iv}\rangle + |T^{vv}\rangle \Longrightarrow 8 \;\; states$$
$$|V^{ijk}\rangle \Longrightarrow 56 \;\; states \quad ; \quad |V^{ijv}\rangle \Longrightarrow 28 \;\; states$$

- The scattering amplitude for the production of a massive state of the first massive level, from a graviton scattering on a Dp brane, shows that the 64 states, given in the GS formalism by $A_{-1;i_1}|i_2\rangle$, have an extra factor of E^2 with respect to the 64 states $Q_{-1,i_1}|i_2\rangle$.
- At high energy, the states with only bos. osc. $A_{i_1,-m_1}\dots A_{i_n.-m_n}|i\rangle$ have an extra factor E^2 with respect to the other states.

Closed string in the background of a Dp brane: II

- Start from the metric of a Dp brane:

$$ds^2 = \alpha(r) \left(-dt^2 + \sum_{\alpha=1}^{p} (dx^\alpha)^2 \right) + \beta(r) \left(dr^2 + r^2 (d\theta^2 + \sin^2\theta d\Omega^2_{7-p}) \right)$$

  where

$$\beta(r) = 1/\alpha(r) = \sqrt{H(r)} \quad ; \quad H(r) = 1 + \left(\frac{R_p}{r} \right)^{7-p}$$

- It is convenient to go to a system of coordinates u, v, z, x^α, y^r adapted to the geodesic followed by the string centre of mass.
- In these coordinates the geodesic followed by the string centre of mass is a straight line parallel to the u-axis ($u(\tau) = \alpha' E \tau$), while all other coordinates v, z, x^α, y^r vanish.

- The new coordinates are given in terms of the old ones by

$$dv = -dt + bd\theta + Cdr \,, \qquad dz = d(\theta + \bar{\theta}(u))$$

$$du = \pm\frac{\beta dr}{C} \,, \qquad C(r) = \sqrt{\frac{\beta(r)}{\alpha(r)} - \frac{b^2}{r^2}} \,.$$

- In these coordinates the metric becomes:

$$ds^2 = 2dudv - \alpha dv^2 + 2b\alpha dvdz + r^2\alpha C^2 dz^2 + \alpha dx^\alpha dx^\alpha$$
$$+\beta r^2 \sin^2(z - \bar{\theta}) d\Omega^2_{7-p}$$

- Expanding transversally to a given geodesic, it is convenient to use Fermi coordinates.
- They are defined by following the geodesic tangent to the vielbein vectors.
- We work therefore with the Fermi coordinates x^+, x^-, x^a $(a = 1 \ldots 8)$ instead of the previous coordinates u, v, z, x^α, y^r.

- In terms of the Fermi coordinates the metric around the reference geodesic takes the following form (after the Penrose limit $x^{\pm} \to \lambda^{\pm} x^{\pm}$) :

$$\begin{aligned} ds^2 &= 2dx^+dx^- + dx^a dx^b \delta_{ab} + (dx^+)^2 \Big[R_{+a+b} x^a x^b \\ &+ \frac{1}{3} D_c R_{+a+b} x^c x^a x^b \\ &+ \left(\frac{1}{3} R_{+aAb} R^A_{c+d} + \frac{1}{12} D_c D_d R_{+a+b} \right) x^a x^b x^c x^d + \ldots \Big] \end{aligned}$$

$a, b = 1 \ldots 8$ [M. Blau and S. Weiss, 2008].

- We consider the string action in this metric:

$$S = -\frac{1}{4\pi\alpha'} \int d\tau \int_0^{2\pi} d\sigma \; \eta^{\alpha\beta} \; \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}(X)$$

plus the Virasoro conditions: $(\dot{X} \pm X')^2 = 0$.

- Choosing the light-cone gauge $X^+(\sigma,\tau) = \alpha' E\tau$ ($d\tau = \frac{dX^+}{\alpha' E}$), we get

$$
\begin{aligned}
S - S_0 &= \frac{E}{2}\int_{-\infty}^{+\infty} dX^+ \int_0^{2\pi} \frac{d\sigma}{2\pi}\Big[R_{+a+b}X^a X^b \\
&+ \frac{1}{3} D_c R_{+a+b} X^c X^a X^b \\
&+ \left(\frac{1}{3} R_{+aAb} R^A_{c+d} + \frac{1}{12} D_c D_d R_{+a+b}\right) X^a X^b X^c X^d + \ldots\Big] \\
&= \frac{E}{2}\sum_{n=2}^{\infty} c_{a_1 a_2 \ldots a_n} \overline{X^{a_1} X^{a_2} \ldots X^{a_n}}
\end{aligned}
$$

At high energy $\tau = \frac{x^+}{\alpha' E} \to 0$. $a_1 \ldots a_n$ along 8 directions. (not r).

- To be compared with the string corrections to the leading eikonal phase

$$2\hat{\delta}(E,b) - T_1(E,b) = \sum_{n=2}^{\infty} \frac{1}{n!} \frac{\partial^n T_1(E,b)}{\partial b_{i_1} \partial b_{i_2} \ldots \partial b_{i_n}} : \overline{X^{i_1} X^{i_2} \ldots X^{i_n}} :$$

$i_1 \ldots i_n$ run over the $8-p$ transverse directions (no r).

- Why $S - S_0$ should be equal to $2\hat{\delta}(E,b) - T_1(E,b)$?
- The vertex operator V describes the interaction of the string with the external field (corresponding to a state of the string).
- The probability amplitude for the emission of the external field between two arbitrary string states is given by:

$$\langle n|V|m\rangle$$

- Analogously, the high-energy scattering on a Dp brane is given by:

$$\langle n|\mathrm{e}^{\hat{\delta}(E,b)-T_1(E,b)}|m\rangle = \langle n|\mathrm{e}^{i(S-S_0)}|m\rangle$$

- For all cases that we have checked, it turns out that the two exponentials are equal as operators, but only at the leading order in $\frac{R_p}{b}$.
- When some of the indices are longitudinal the corresponding $c = 0$ at the leading order in $\frac{R_p}{b}$.

- Example

$$\int_{-\infty}^{+\infty} dx^+ R_{+z+z} = \int_{-\infty}^{+\infty} du \frac{\partial_u^2 \sqrt{G_{zz}}}{\sqrt{G_{zz}}}$$

$$= 2\int_0^{\infty} dr \frac{\partial_r \left(\frac{dr}{du}\partial_r \sqrt{G_{zz}}\right)}{\sqrt{G_{zz}}} \sim \frac{\sqrt{\pi}}{b}(7-p)\left(\frac{R_p}{b}\right)^{7-p} \frac{\Gamma(\frac{8-p}{2})}{\Gamma(\frac{7-p}{2})} + \dots$$

where

$$\frac{dr}{du} = \sqrt{1 - \frac{b^2\alpha^2}{r^2}} \;\; ; \;\; \alpha = \frac{1}{\sqrt{1 + \left(\frac{R_p}{r}\right)^{7-p}}} \;\; ; \;\; \sqrt{G_{zz}} = \frac{r}{\sqrt{\alpha}}\frac{dr}{du}$$

- In conclusion, at leading order in $\frac{R_p}{b}$ and to all orders in $\frac{\alpha'}{b^2}$ (string corrections), the string excitations computed from the leading eikonal and from the string in the background of the brane are identical.

Conclusion and outlook

- We have seen how from the string scattering amplitudes in flat space-time one recovers properties of curved space-time.
- In particular, from the elastic scattering of a massless closed string on a stack of N Dp branes at high energy and low transfer momentum we have computed the deflection angle of a probe particle moving in the metric of the Dp brane.
- The result reproduces the leading and the next to the leading contributions for large impact parameter computed from classical gravity in the metric of a stack of N Dp branes.
- String corrections (tidal excitations) to the leading eikonal (all orders in $\frac{\alpha'}{b^2}$) have been computed and they agree with what one obtains from the classical action of the string in the metric of the Dp branes. But only at the leading order in $\frac{R_p}{b}$.
- Up to now, we have not (directly) seen any effect from the dilaton and the RR field (except in the classical solution).

- They seem to be irrelevant at high energy because, unlike the graviton, their coupling does not grow with the energy.
- The most important thing is to check if the eikonal operator correctly describes the high energy behaviour (in the Regge limit) of all string scattering amplitudes (involving arbitrary incoming and outgoing string states). For both the disk and the annulus.
- If this will turn out to be true, this will imply that, at high energy, the string spectrum is truncated to only those states that in the GS light-cone formalism involve only the bosonic oscillators: $A_{i_1,-m_1} \dots A_{i_n.-m_n}|i\rangle$.
- Extend this approach to less supersymmetric Dp branes.
- In the case of the D3 branes, this approach may shed some light on the AdS/CFT correspondence.