The simplicity of maximally supersymmetric field theories

Martin Cederwall

Breaking of supersymmetry and ultraviolet divergences in extended supergravity
Frascati, March 28, 2013

Plan

0. Background
1. From superspace to pure spinors
   - On-shell superfields
   - BRST operator
   - Cohomologies and linearised fields
   - Towards an action
2. Pure spinors
   - Pure spinor space (D=10, D=11)
   - Non-minimal variables and integration
3. Supersymmetric actions (D=10 SYM, D=11 SG)
   - Linearised action
   - Batalin–Vilkovisky formalism
   - Operators and full actions
   - Gauge fixing and the b operator
4. Higher derivatives and D=10 Born–Infeld
5. Conclusions

Maximally supersymmetric models have on-shell supermultiplets. There is no finite set of auxiliary fields.

Examples:

- $D = 10$ super-Yang–Mills theory
- $N = (2{,}0)$ model in $D = 6$
- IIB supergravity in $D = 10$
- $D = 11$ supergravity
- BLG model in $D = 3$
- Dimensional reductions of above

How does one formulate an action principle preserving manifest supersymmetry? This is of course desirable, especially for examining quantum properties.

Pure spinors provide an answer (in the cases self-dual fields are not present).

Pure spinor arise naturally as a "book-keeping device" in the traditional superspace formulation of maximally supersymmetric gauge and gravity theories.

I will start with the canonical example, a $D = 10$ vector multiplet, to show how this works, and then continue to $D = 11$ supergravity.

$D = 10$ SYM has a formulation as gauge theory on superspace. One introduces

$$A_M(x, \theta) = E_M{}^A A_A(x, \theta)$$

where $M = (m, \mu)$, $A = (a, \alpha)$.

$$A_M(x,\theta) = E_M{}^A A_A(x,\theta)$$

$E_M{}^A$ encodes the background supergeometry, which for simplicity can be thought of as flat. The superspace torsion is a gamma matrix,

$$\{D_\alpha, D_\beta\} = -T^a_{\alpha\beta} D_a = -2\gamma^a_{\alpha\beta} D_a$$

Demanding $F_{\alpha\beta} = 0$ implies the equations of motion for the component fields.

More precisely:

$\gamma_a^{\alpha\beta} F_{\alpha\beta} = 0$ is a conventional constraint, relating the superfield $A_a(x,\theta)$ to the vector which also exists at order θ in $A_\alpha(x,\theta)$.

$\gamma_{abcde}^{\alpha\beta} F_{\alpha\beta} = 0$ is the equation of motion, which puts the theory on-shell.

More precisely:

$\gamma_a^{\alpha\beta} F_{\alpha\beta} = 0$ is a conventional constraint, relating the superfield $A_a(x, \theta)$ to the vector which also exists at order θ in $A_\alpha(x, \theta)$.

$\gamma_{abcde}^{\alpha\beta} F_{\alpha\beta} = 0$ is the equation of motion, which puts the theory on-shell.

The lesson is:

Everything is contained in A_α, the lowest-dimensional superfield.

The linearised field equations are $\gamma_{abcde}^{\alpha\beta} D_\alpha A_\beta = 0$.

From this the component equations of motion arise by checking the superspace Bianchi identities.

Consider introducing a bosonic spinor λ^α, and forming

$$q = \lambda^\alpha D_\alpha$$

If λ is pure, *i.e.*, if $(\lambda\gamma^a\lambda) = 0$, then $q^2 = 0$.

(Remember: $\{D_\alpha, D_\beta\} = -2\gamma^a_{\alpha\beta}D_a$.)

Form the fermionic scalar field $\Psi = \lambda^\alpha A_\alpha$.

The linearised field equations are

$$q\Psi = \lambda^\alpha D_\alpha \cdot \lambda^\beta A_\beta \propto (\lambda\gamma^{abcde}\lambda)(D\gamma_{abcde}A) = 0$$

The equations of motion are $q\Psi = 0$.

The gauge symmetry $\delta_\Lambda A_\alpha = D_\alpha \Lambda$ is written $\delta_\Lambda \Psi = q\Lambda$.

So, physical states (with linearised field equations) are identified as cohomology of q.

We think of λ as a ghost variable, with gh# 1, and thus $\Psi(x, \theta, \lambda)$ is a field of gh# 1.

There is cohomology also at other ghost numbers, *i.e.*, at other powers of λ. The interpretation is as ghosts and antifields.

gh # =	1	0	−1	−2	−3
dim = 0	c				
$\frac{1}{2}$	•	•			
1	•	A_a	•		
$\frac{3}{2}$	•	χ^α	•	•	
2	•	•	•	•	•
$\frac{5}{2}$	•	•	χ^*_α	•	•
3	•	•	A^{*a}	•	•
$\frac{7}{2}$	•	•	•	•	•
4	•	•	•	c^*	•
$\frac{9}{2}$	•	•	•	•	•

Supergravity is formulated as Cartan geometry on superspace (analogous statements true for other supersymmetric gauge theories).

Coordinates: $Z^M = (x^m, \theta^\mu)$.

Vielbein: $E^A = dZ^M E_M{}^A$.

Spin connection 1-form (Lorentz valued): $\Omega_A{}^B$.

Torsion 2-form: $T^A = DE^A = dE^A + E^B \wedge \Omega_B{}^A$.

Curvature 2-form: $R_A{}^B = d\Omega_A{}^B + \Omega_A{}^C \wedge \Omega_C{}^B$.

Bianchi identities: $DT^A = E^B \wedge R_B{}^A$, $DR_A{}^B = 0$.

$(M = (m, \mu)$, $A = (a, \alpha)$.)

[Cremmer, Ferrara 1980; Brink, Howe 1980]

Too many superfields. Conventional constraints remove all independent superfield except the lowest-dimensional one, $E_\mu{}^a$.

They are used to set all of the dimension-0 torsion to zero, except

$$T_{\alpha\beta}{}^c = 2\gamma^c_{\alpha\beta} + \tfrac{1}{2}U^c{}_{e_1e_2}\gamma^{e_1e_2}_{\alpha\beta} + \tfrac{1}{5!}V^c{}_{e_1\ldots e_5}\gamma^{e_1\ldots e_5}_{\alpha\beta}$$

Too many superfields. Conventional constraints remove all independent superfield except the lowest-dimensional one, $E_\mu{}^a$.

They are used to set all of the dimension-0 torsion to zero, except

$$T_{\alpha\beta}{}^c = \underbrace{2\gamma^c_{\alpha\beta}}_{\text{standard}} + \frac{1}{2}U^c{}_{e_1e_2}\gamma^{e_1e_2}_{\alpha\beta} + \frac{1}{5!}V^c{}_{e_1\dots e_5}\gamma^{e_1\dots e_5}_{\alpha\beta}$$

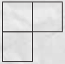
Too many superfields. Conventional constraints remove all independent superfield except the lowest-dimensional one, $E_\mu{}^a$.

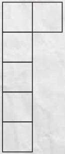
They are used to set all of the dimension-0 torsion to zero, except

$$T_{\alpha\beta}{}^c = 2\gamma^c_{\alpha\beta} + \tfrac{1}{2}U^c{}_{e_1e_2}\gamma^{e_1e_2}_{\alpha\beta} + \tfrac{1}{5!}V^c{}_{e_1\ldots e_5}\gamma^{e_1\ldots e_5}_{\alpha\beta}$$

$\uparrow$

standard





Too many superfields. Conventional constraints remove all independent superfield except the lowest-dimensional one, $E_\mu{}^a$.

They are used to set all of the dimension-0 torsion to zero, except

$$T_{\alpha\beta}{}^c = 2\gamma^c_{\alpha\beta} + \frac{1}{2}U^c{}_{e_1 e_2}\gamma^{e_1 e_2}_{\alpha\beta} + \frac{1}{5!}V^c{}_{e_1\dots e_5}\gamma^{e_1\dots e_5}_{\alpha\beta}$$

If U and V are set to 0, the torsion BI imply the equations of motion.

All physical fields are contained in the supergeometry. For example,

$$T_{a\beta}{}^\gamma \sim H_{a e_1 e_2 e_3}(\gamma^{e_1 e_2 e_3})_\beta{}^\gamma - \frac{1}{8}H^{e_1 e_2 e_3 e_4}(\gamma_{a e_1 e_2 e_3 e_4})_\beta{}^\gamma$$

A similar BRST operator $q = \lambda^\alpha D_\alpha$ can be used, now acting on a linearised field

$$\Phi^a = \lambda^\alpha E_\alpha{}^a$$

Again, if $(\lambda\gamma^a\lambda) = 0$, q is nilpotent. The linearised supergravity equations of motion are encoded in $q\Phi^a \approx 0$ (implying the vanishing of U and V), and $\delta_\xi \Phi^a = q\xi^a$ contains the linearised diffeomorphisms and local supersymmetry.

Φ^a is considered modulo a "shift symmetry" $\Phi^a \to \Phi^a + (\lambda\gamma^a\rho)$.

An alternative linearised superspace description of the $D = 11$ supergravity is provided by the 3-form potential on superspace. Again, its lowest-dimensional part, $C_{\alpha\beta\gamma}$, contains all the fields, including the geometric degrees of freedom.

Then one forms $\Psi = \lambda^\alpha\lambda^\beta\lambda^\gamma C_{\alpha\beta\gamma}$, and $q\Psi = 0$ implies the linearised equations of motion.

I will not show a table of cohomologies. They contain all fields, ghosts and antifields, as for SYM.

The field Ψ is more fundamental than Φ^a, since it contains the 3-form potential, while Φ^a only contains $H = dC$.

One would expect a relation of the form $\Phi^a = R^a\Psi$. I will come back to this.

The only thing lacking for an off-shell formulation and an action is a measure. If it can be formed, a linearised action is

$$S = \int [dZ](\Psi Q \Psi + \ldots)$$

gh # =	1	0	-1	-2	-3
dim = 0	c				
$\frac{1}{2}$	•	•			
1	•	A_a	•		
$\frac{3}{2}$	•	χ^α	•	•	
2	•	•	•	•	•
$\frac{5}{2}$	•	•	χ^*_α	•	•
3	•	•	A^{*a}	•	•
$\frac{7}{2}$	•	•	•	•	•
4	•	•	•	c^*	•
$\frac{9}{2}$	•	•	•	•	•

The only thing lacking for an off-shell formulation and an action is a measure. If it can be formed, a linearised action is

$$S = \int [dZ](\Psi Q \Psi + \ldots)$$

In the SYM case, a “measure” picking the coefficient of the cohomology at $\lambda^3\theta^5$ has correct gh# and dimension.

The corresponding top cohomology in $D = 11$ SG is at $\lambda^7\theta^9$.

Such a measure is however degenerate, and and action based on it does not give correct equations of motion.

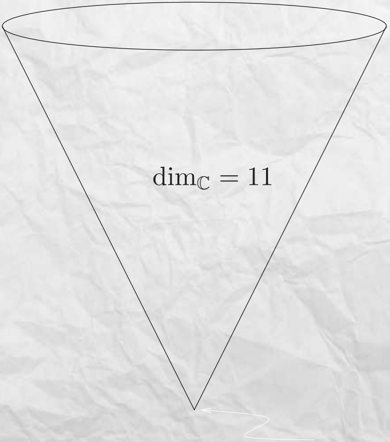
The solution to the pure spinor constraint $(\lambda\gamma^a\lambda) = 0$ requires λ to be complex.

In $D = 10$, the space of pure spinors is $11_{\mathbb{C}}$-dimensional (out of the original 16),

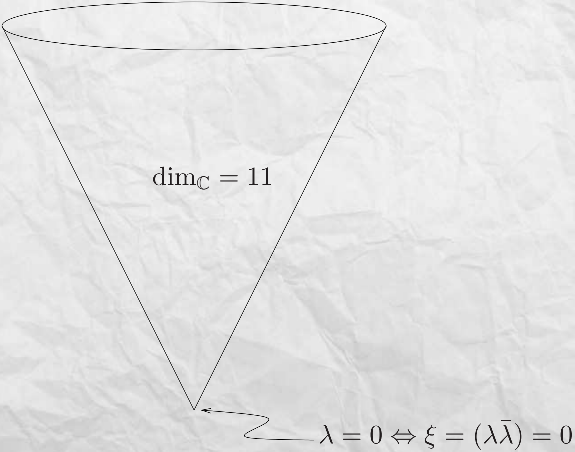
and in $D = 11$ $23_{\mathbb{C}}$-dimensional (out of the original 32).

A $D = 11$ pure spinor does *not* contain two 10-dimensional ones.

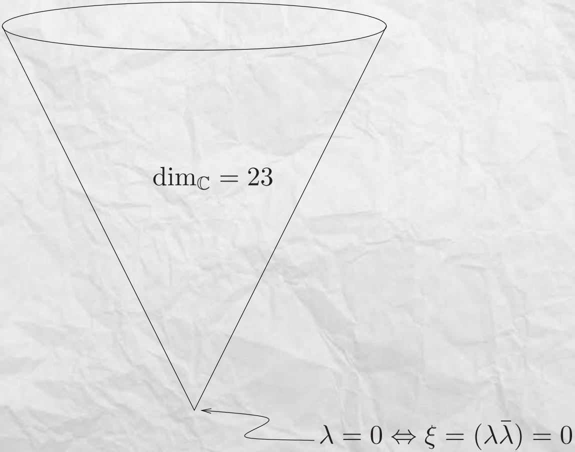
$D = 10$:



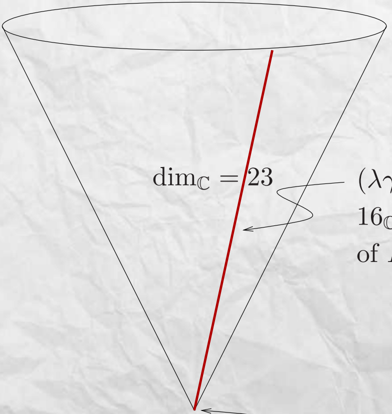
$D = 10$:


$\dim_{\mathbb{C}} = 11$
$\lambda = 0 \Leftrightarrow \xi = (\lambda\bar{\lambda}) = 0$

$D = 11$:


$\dim_{\mathbb{C}} = 23$
$\lambda = 0 \Leftrightarrow \xi = (\lambda\bar{\lambda}) = 0$

$D = 11$:



$(\lambda\gamma^{ab}\lambda) = 0 \Leftrightarrow \eta = (\lambda\gamma^{ab}\lambda)(\bar{\lambda}\gamma_{ab}\bar{\lambda}) = 0$
$16_{\mathbb{C}}$-dimensional subspace
of $D = 12$ pure spinors

$\lambda = 0 \Leftrightarrow \xi = (\lambda\bar{\lambda}) = 0$

We would like to integrate over the complex manifolds, with some natural measure that effectively reproduces the top cohomology mentioned before.

We must include $\bar{\lambda}_\alpha$ as a variable. But in order not to destroy cohomology, it must be accompanied by a fermionic variable r_α, and the new non-minimal variables must be included in the BRST operator:

$$Q = q + \bar{\partial} = \lambda^\alpha D_\alpha + r_\alpha \frac{\partial}{\partial \bar{\lambda}_\alpha}$$

r obeys $(\bar{\lambda}\gamma^a r) = 0$, and has as many indep. components as λ. It can be thought of as the differential $d\bar{\lambda}_\alpha$ (hence the notation $\bar{\partial}$).

[Berkovits]

The most elegant way to understand the measure is to think of $\Psi(x, \theta; \lambda, \bar{\lambda}, r)$ as a cochain,

$$\Psi = \sum_{k=0}^{11 \text{ or } 23} \psi^{\alpha_1 \ldots \alpha_k}(x, \theta; \lambda, \bar{\lambda}) d\bar{\lambda}_{\alpha_1} \wedge \ldots \wedge d\bar{\lambda}_{\alpha_1}$$

There is a unique Calabi–Yau metric on the pure spinor space. The corresponding holomorphic top form Ω can be used to define integration:

$$\int [dZ] F = \int [dx] \int [d\theta] \int \Omega \wedge F$$

[Berkovits; Cederwall]

$$\int [dZ] F = \int [dx] \int [d\theta] \int \Omega \wedge F$$

This integration is obviously non-degenerate, and it turns out to be BRST-equivalent to the naive top cohomology.

In $D = 10$, $\Omega \sim \lambda^{-3} d^{11} \lambda$.

In $D = 11$, $\Omega \sim \lambda^{-7} d^{23} \lambda$.

The correct integration of a minimal cohomology $\lambda^3 \theta^5$ or $\lambda^7 \theta^9$ is obtained by insertion of a regulator

$$e^{-\{Q,(\bar{\lambda}\theta)\}} = e^{-(\lambda\bar{\lambda})-(d\bar{\lambda}\theta)}$$

(note: $5 = 16 - 11$, $9 = 32 - 23$!),

which also makes integration convergent at $\lambda = \infty$ (alternatively, a basis for cohomology with these properties is chosen).

A linearised action is

$$S = \int [dZ]\Psi Q \Psi \ .$$

In order to introduce interaction, the concept of cohomology (which is inherently linear) must be generalised. The appropriate language is the Batalin–Vilkovisky formalism. This is already hinted at by the fact that ghosts and antifields are included in the cohomology.

The action itself is the generator of "gauge transformations", generated as $\delta X = (S, X)$, where $(\cdot, \cdot)$ is the antibracket. In a component formalism:

$$(A, B) = \int [dx] \left(A \frac{\overleftarrow{\delta}}{\delta\phi^A(x)} \frac{\overrightarrow{\delta}}{\delta\phi^\star_A(x)} B - A \frac{\overleftarrow{\delta}}{\delta\phi^\star_A(x)} \frac{\overrightarrow{\delta}}{\delta\phi^A(x)} B \right) .$$

The governing equation generalising $Q^2 = 0$ is the BV master equation $(S, S) = 0$.

[Batalin, Vilkovisky 1981]

For the pure spinor superfield Ψ, the antibracket takes the simple form

$$(A, B) = \int A \frac{\overleftarrow{\delta}}{\delta\Psi(Z)} [dZ] \frac{\overrightarrow{\delta}}{\delta\Psi(Z)} B .$$

[Cederwall 2009]

The full BV action for $D = 10$ super-Yang–Mills (and its dimensional reductions) is the Chern–Simons-like action

$$S = \int [dZ] \mathrm{Tr}\, \left(\tfrac{1}{2} \Psi Q \Psi + \tfrac{1}{3} \Psi^3 \right) \ .$$

implicit in [Berkovits 2001,2005; Cederwall, Nilsson, Tsimpis 2001]

Note that there is only a 3-point coupling; the quartic interaction arises on elimination of "auxiliary fields".

Still, the full gauge symmetry (and more) is present.

An analogous formulation exists for the Bagger–Lambert–Gustavsson and Aharony–Bergman–Jafferis–Maldacena models in $D = 3$.

The simplification there is even more radical: The component actions contain 6-point couplings, but the pure spinor superfield actions only have minimal coupling (*i.e.*, 3-point interactions).

[Cederwall, 2008]

But I would like to turn to supergravity.

Remember that we had the two fields

Ψ (fermionic, gh# 3, containing C), and

Φ^a (fermionic, gh# 1, containing H).

A reasonable 3-point coupling is

$$S_3 \propto \int [dZ](\lambda\gamma_{ab}\lambda)\Psi\Phi^a\Phi^b$$

It has correct ghost number and dimension.

The factor $(\lambda\gamma_{ab}\lambda)$ also provides the antisymmetry $[ab]$ and invariance under $\Phi^a \to \Phi^a + (\lambda\gamma^a\rho)$.

Note the similarity with $\int C \wedge H \wedge H$, which it can be shown to contain.

All that is needed now is to find an operator R^a such that $[Q, R^a] \approx 0$. Then the master equation will be satisfied to this order.

It is possible to relate the fields Ψ and Φ^a through an operator R^a of non-trivial cohomology as

$$\Phi^a = R^a \Psi \ .$$

where

$$R^a = \eta^{-1}(\bar{\lambda}\gamma^{ab}\bar{\lambda})\partial_b + \ldots$$

where the ellipsis represents terms with r and r^2 and more singular behaviour in $\eta = (\lambda\gamma_{ab}\lambda)(\bar{\lambda}\gamma^{ab}\bar{\lambda})$.

[Cederwall 2009,2010]

One may expect that an expansion around flat space would be non-polynomial. This is however not the case. Checking the master equation to higher order in the field involves commutators of R^a's. The R^a's don't commute, but "almost".

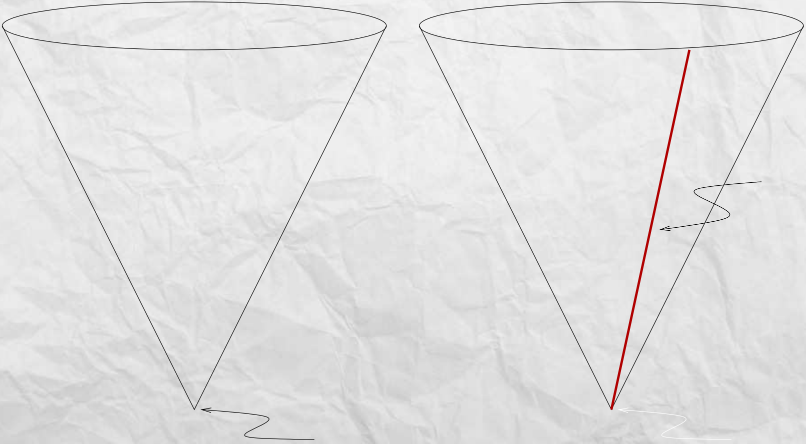
The master equation is *exactly* satisfied by

$$S = \int [dZ] \left[\tfrac{1}{2}\Psi Q \Psi + \tfrac{1}{6}(\lambda\gamma_{ab}\lambda)(1 - \tfrac{3}{2}T\Psi)\Psi R^a \Psi R^b \Psi \right]$$

where T is a nilpotent operator ($TATB = 0$).

This is a complete description of $D = 11$ supergravity, respecting all local symmetries.

Operators in $D = 10$ will typically be singular at $\lambda = 0$ (the tip of the pure spinor cône), while in $D = 11$ they are singular on the subspace $\eta = 0$ of $D = 12$ pure spinors.



An important example of operators is the b operator, or “b-ghost”.

In the pure spinor formalism, there is no constraint corresponding to $p^2 = 0$ (or Virasoro). This is a consequence of the on-shell property of the supermultiplets. Therefore, there is no bc ghost system.

In superstring or superparticle models, gauge fixing can be achieved by imposing $b\Psi = 0$. Here, the b operator is not fundamental, but composite. Just like the R^a operator already encountered, it can be derived in terms of non-minimal variables. The defining property is

$$\{b, Q\} = \Box$$

which makes Q (the kinetic operator) invertible on a field with $b\Psi = 0$. The propagator becomes $b\Box^{-1}$.

[Berkovits]

The b operator in $D = 11$ has been derived,

$$b = \tfrac{1}{2}\eta^{-1}(\bar{\lambda}\gamma_{ab}\bar{\lambda})(\lambda\gamma^{ab}\gamma^{i}D)\partial_i + \ldots$$

[Cederwall, Karlsson 2012]

where the ellipsis denotes terms with $\eta^{-(k+1)}r^k$, $k \leq 3$.

There is no need for introduction of extra non-minimal *fields* (antighost, Nakanishi–Lautrup) on gauge fixing. These are automatically included in the gauge-fixed Ψ. Good for calculations.

Normally, gauge fixing in the BV framework involves expressing the antifields in terms of fields using a gauge fixing fermion. That procedure is not available to us, when the self-conjugate Ψ contains both fields and antifields.

The formalism presented gives the opportunity to investigate deformations, *e.g.*. higher-derivative terms, in a systematic way.

This has been used for $D = 10$ SYM in order to obtain the complete F^4 terms

[Cederwall, Nilsson, Tsimpis]

The formalism presented gives the opportunity to investigate deformations, *e.g.*. higher-derivative terms, in a systematic way.

This has been used for $D = 10$ SYM in order to obtain the complete F^4 terms

[Cederwall, Nilsson, Tsimpis]

$$
\begin{aligned}
L = & -\tfrac{1}{4}F^{Aij}F^{A}_{ij} + \tfrac{1}{2}\chi^{A}\not{D}\chi^{A} \\
& - 6\alpha'^2 M_{ABCD}\Big[\mathrm{tr}(F^{A}F^{B}F^{C}F^{D}) - \tfrac{1}{4}\mathrm{tr}(F^{A}F^{B})\mathrm{tr}(F^{C}F^{D}) \\
& - 2F^{A\,i}{}_{k}F^{B\,jk}(\chi^{C}\gamma_{i}D_{j}\chi^{D}) + \tfrac{1}{2}F^{A\,il}D_{l}F^{B\,jk}(\chi^{C}\gamma_{ijk}\chi^{D}) \\
& + \tfrac{1}{180}(\chi^{A}\gamma^{ijk}\chi^{B})(D_{l}\chi^{C}\gamma_{ijk}D^{l}\chi^{D}) + \tfrac{3}{10}(\chi^{A}\gamma^{ijk}\chi^{B})(D_{i}\chi^{C}\gamma_{j}D_{k}\chi^{D}) \\
& + \tfrac{7}{60}f^{D}{}_{EF}F^{A\,ij}(\chi^{B}\gamma_{ijk}\chi^{C})(\chi^{E}\gamma^{k}\chi^{F}) \\
& - \tfrac{1}{360}f^{D}{}_{EF}F^{A\,ij}(\chi^{B}\gamma^{klm}\chi^{C})(\chi^{E}\gamma_{ijklm}\chi^{F})\Big] + O(\alpha'^3)\ .
\end{aligned}
$$

The formalism presented gives the opportunity to investigate deformations, *e.g.*. higher-derivative terms, in a systematic way.

This has been used for $D = 10$ SYM in order to obtain the complete F^4 terms

[Cederwall, Nilsson, Tsimpis]

and for the generic deformations of $D = 11$ supergravity

[Howe; Cederwall, Gran, Nielsen, Nilsson; Howe, Tsimpis,...]

This earlier work was done at the level of field equations (as superspace constraints).

An action principle and a master equation give more powerful tools.

The F^4 terms of $D = 10$ SYM were obtained by replacing the constraint $F_{\alpha\beta} = 0$ by $F_{\alpha\beta} \sim \chi^2 F$. The expression was unique modulo trivial terms, and led to deformed equations of motion, which could be integrated to terms in a component action

Analogously, deformation of $T_{\alpha\beta}{}^a = 2\gamma^a_{\alpha\beta}$ is necessary for higher-derivative terms in maximal supergravity.

When we now have an action, it is useful to work at that level, letting the master equation do the job.

The F^4 deformation of SYM was reconsidered in [Cederwall, Karlsson 2011].

Much in the spirit of the construction of operators we have already encountered, we construct "physical operators" of negative ghost number, the effect of which is to form fields "starting with" a certain physical field, say χ^α or F_{ab}, from the pure spinor field Ψ.

Using the properties of such operators we could show that a deformation with

$$S_4 \sim \int [dZ] \Psi (\lambda \gamma^a \hat{\chi}) \Psi (\lambda \gamma^b \hat{\chi}) \Psi \hat{F}_{ab} \Psi$$

provides the F^4 deformation of super-Maxwell theory.

The situation is even better. It turns out that the action

$$S = \int [dZ] \left(\Psi Q \Psi + k\alpha'^2 \Psi(\lambda\gamma^a\hat{\chi})\Psi(\lambda\gamma^b\hat{\chi})\Psi\hat{F}_{ab}\Psi \right)$$

satisfies the master equation $(S, S) = 0$ to all orders, and we conjecture that it is gives the full $D = 10$ BI dynamics.

In a non-abelian situation, it provides the complete answer for the totally symmetric part in adjoint indices.

It would of course be informative to see how the non-polynomial equations of motion for component fields arise. We have not been able to do this in detail.

5. Conclusions

The framework described resolves the issue of supersymmetric actions for maximally supersymmetric theories.

The interaction terms are generically much simpler and of lower order than in a component language; for supergravity and abelian BI to the extent that the actions becomes polynomial.

5. Conclusions

The framework described resolves the issue of supersymmetric actions for maximally supersymmetric theories.

The interaction terms are generically much simpler and of lower order than in a component language; for supergravity and abelian BI to the extent that the actions becomes polynomial.

Although the supergravity action respects the local symmetries, the geometric picture is lost, and background invariance is not manifest (the BRST operator contains information of the background).

5. Conclusions

The framework described resolves the issue of supersymmetric actions for maximally supersymmetric theories.

The interaction terms are generically much simpler and of lower order than in a component language; for supergravity and abelian BI to the extent that the actions becomes polynomial.

Although the supergravity action respects the local symmetries, the geometric picture is lost, and background invariance is not manifest (the BRST operator contains information of the background).

Still, the formalism is suitable for perturbation theory and amplitude calculations. The presence of an action provides consistent vertices (3-point, and "very little" 4-point) without the consistency checks necessary in the first-quantised formalism of [Green, Björnsson; Björnsson]. The D=11 formalism seems however to be less connected to KLT or "double-copy".

5. Conclusions

The framework described resolves the issue of supersymmetric actions for maximally supersymmetric theories.

The interaction terms are generically much simpler and of lower order than in a component language; for supergravity and abelian BI to the extent that the actions becomes polynomial.

Although the supergravity action respects the local symmetries, the geometric picture is lost, and background invariance is not manifest (the BRST operator contains information of the background).

Still, the formalism is suitable for perturbation theory and amplitude calculations. The presence of an action provides consistent vertices (3-point, and "very little" 4-point) without the consistency checks necessary in the first-quantised formalism of [Green, Björnsson; Björnsson]. The D=11 formalism seems however to be less connected to KLT or "double-copy".

[Cederwall, Karlsson 2012, see talk by Anna Karlsson]

We do not yet know how to implement U-duality.

The superspace formalism may be useful in models with less supersymmetry.