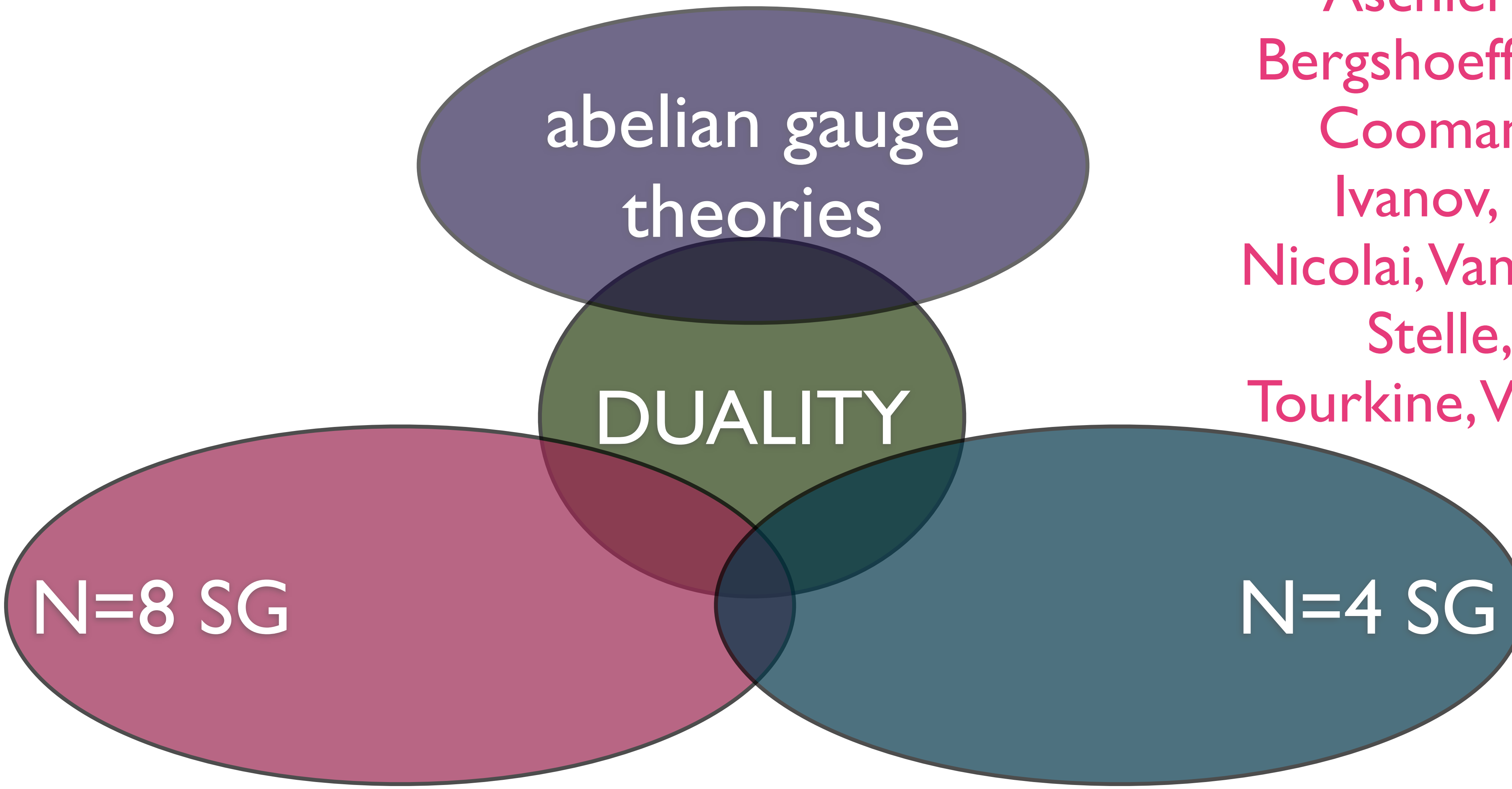
Based on work with Bern, Broedel, Chiodaroli, Dennen, Dixon, Johansson, Gunaydin, Ferrara, Kallosh, Roiban, and Tseytlin

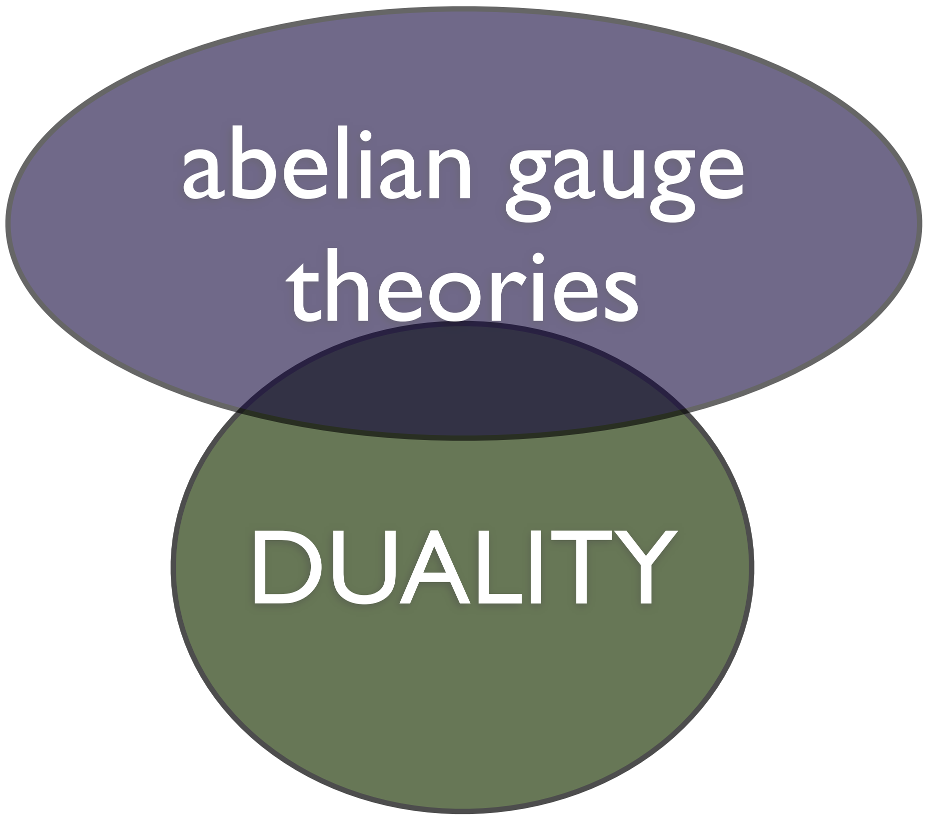
Interplay with work by Aschieri, Bellucci, Bergshoeff, Bossard, Coomans, Howe, Ivanov, Krivonos, Nicolai, Van Proeyen, Stelle, Shahbazi, Tourkine, Vanhove,...



John Joseph M Carrasco

Stanford Institute for Theoretical Physics

Breaking of supersymmetry and Ultraviolet Divergences in extended Supergravity (BUDS)



Status update on U(1) duality satisfaction of BIK N=2 Born-Infeld action from 2001

JJMC, Kallosh '13

easy to see duality invariance of W^(10) term

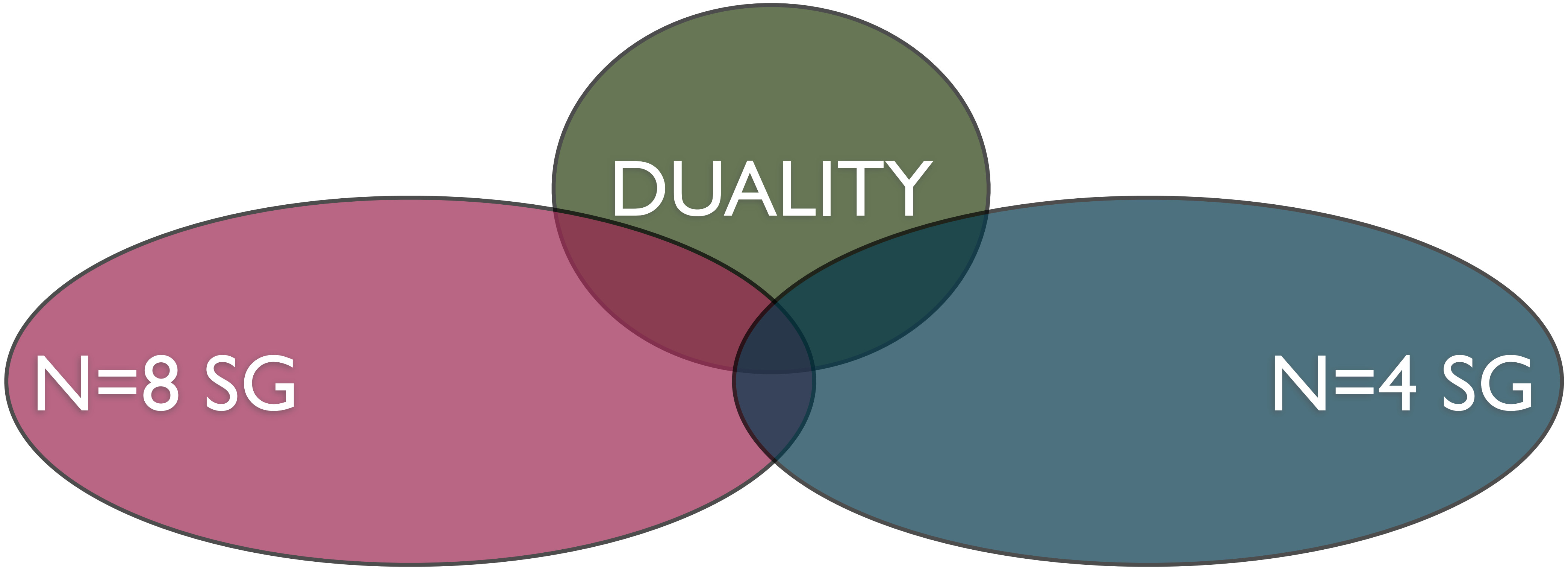
hints that hidden supersymmetry => duality invariance (see Toine, Eric, Renata talks)

Bossard, Nicolai '11

JJMC, Kallosh, Roiban '11

Chemissany, Kallosh, Ortin '12

Broedel, JJMC, Ferrara, Kallosh, Roiban '12



similarities and differences between N=4 SG and N=8 SG

possibility of matter-couplings

existence of anomalies

goal: convince you calculation of amplitudes can help clarify

JJMC, Chioradelli, Gunyadin, Roiban '12

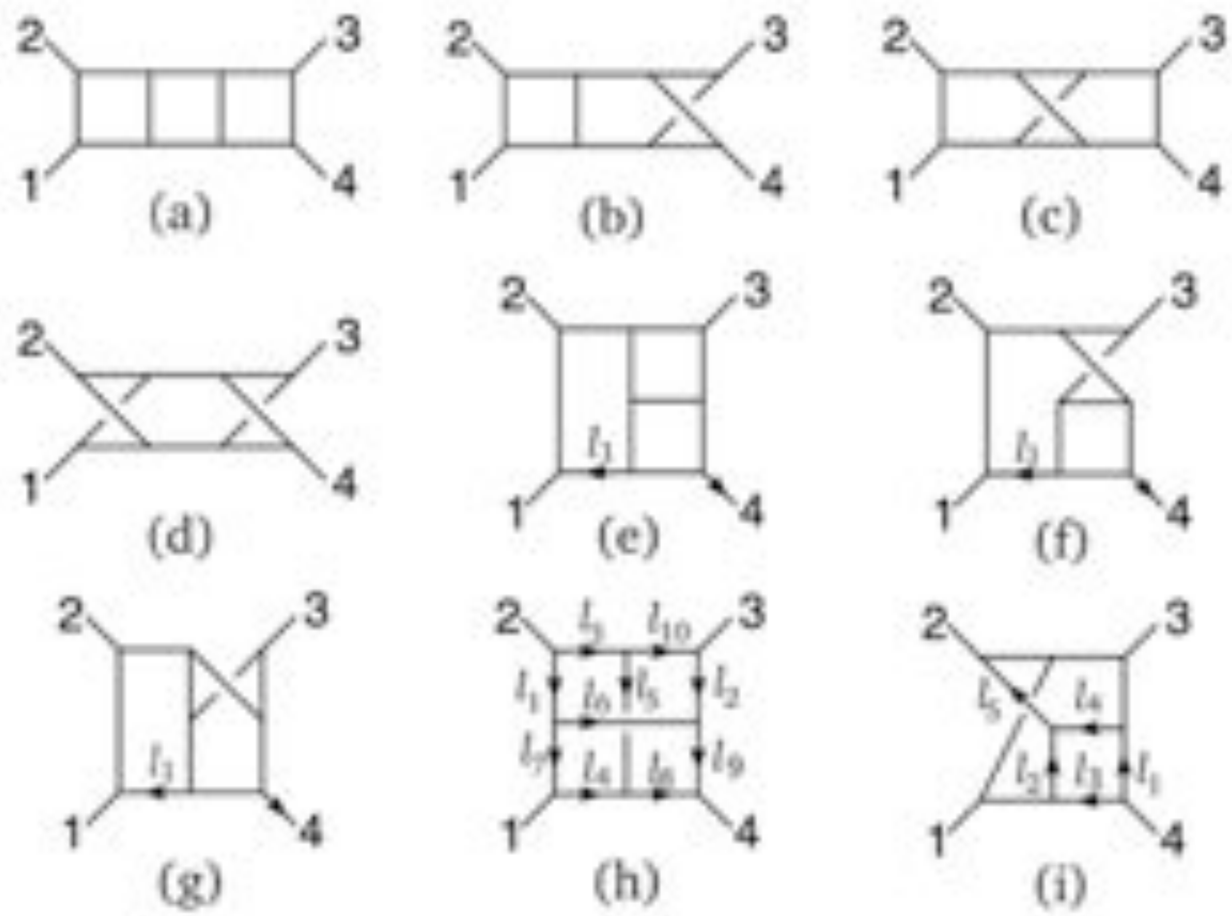
JJMC, Kallosh, Roiban, Tseytlin '13

Gluons for (almost) nothing and gravitons for free!

motivate calculating with color-kinematics & double copy

(see also talks by Tristan and Henrik)

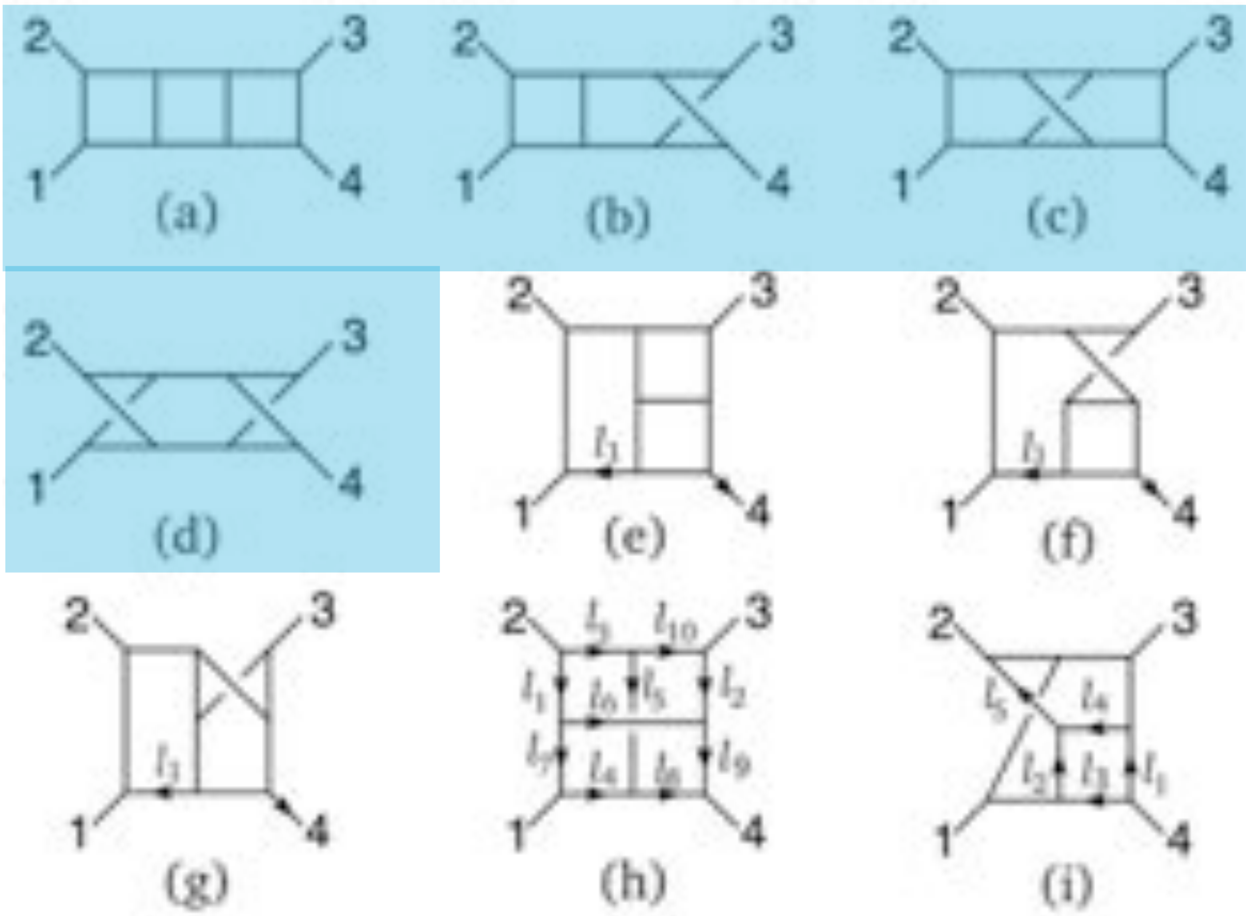
Bern, JJMC, Dixon, Kosower, Johansson, Roiban '07



Original solution of three-loop four-point N=4 sYM and N=8 sugra

Integral	$\mathcal{N}=4$ Yang-Mills	$\mathcal{N}=8$ Supergravity
(a)–(d)	s^2	$[s^2]^2$
(e)–(g)	$s(l_1+k_4)^2$	$[s\,(l_1+k_4)^2]^2$
(h)	$s(l_1+l_2)^2+t(l_3+l_4)^2$ $-\,sl_5^2-tl_6^2-st$	$(s(l_1+l_2)^2+t(l_3+l_4)^2-st)^2-s^2(2((l_1+l_2)^2-t)+l_5^2)l_5^2$ $-t^2(2((l_3+l_4)^2-s)+l_6^2)l_6^2-s^2(2l_7^2l_2^2+2l_1^2l_9^2+l_2^2l_9^2+l_1^2l_7^2)$ $-\,t^2(2l_3^2l_8^2+2l_{10}^2l_4^2+l_8^2l_4^2+l_3^2l_{10}^2)+2stl_5^2l_6^2$
(i)	$s(l_1+l_2)^2-t(l_3+l_4)^2$ $-\frac{1}{3}(s-t)l_5^2$	$(s(l_1+l_2)^2-t(l_3+l_4)^2)^2$ $-\,(s^2(l_1+l_2)^2+t^2(l_3+l_4)^2+\frac{1}{3}stu)l_5^2$

Bern, JJMC, Dixon, Kosower, Johansson, Roiban '07



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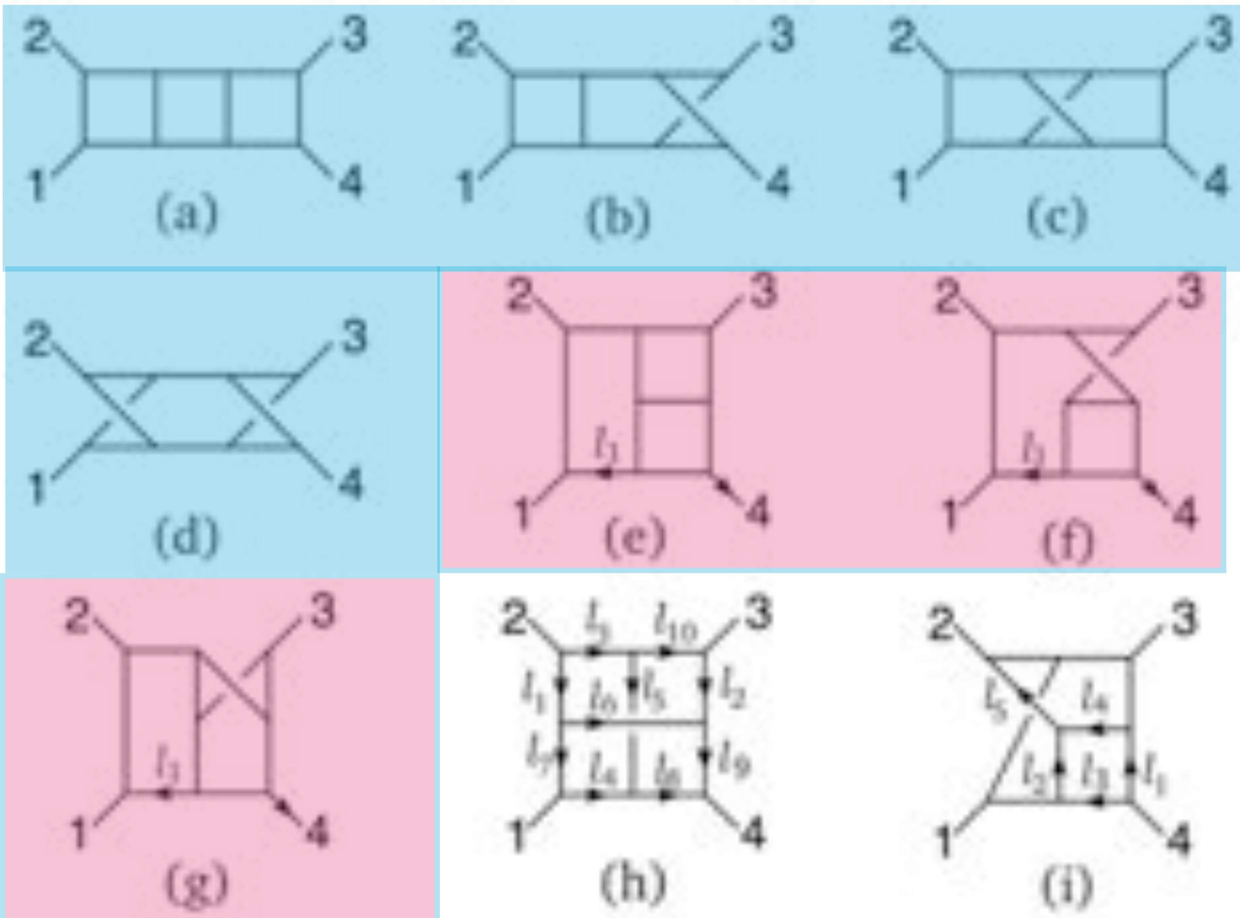
(h)	$s(l_1+l_2)^2+t(l_3+l_4)^2 - s l_5^2 - t l_6^2 - st$	$(s(l_1+l_2)^2+t(l_3+l_4)^2-st)^2 - s^2(2((l_1+l_2)^2-t)+l_5^2)l_5^2 - t^2(2((l_3+l_4)^2-s)+l_6^2)l_6^2 - s^2(2l_7^2l_2^2+2l_1^2l_9^2+l_2^2l_9^2+l_1^2l_7^2) - t^2(2l_3^2l_8^2+2l_{10}^2l_4^2+l_8^2l_4^2+l_3^2l_{10}^2)+2stl_5^2l_6^2$
(i)	$s(l_1+l_2)^2-t(l_3+l_4)^2 -\frac{1}{3}(s-t)l_5^2$	$(s(l_1+l_2)^2-t(l_3+l_4)^2)^2 - (s^2(l_1+l_2)^2+t^2(l_3+l_4)^2+\frac{1}{3}stu)l_5^2$

Bern, JJMC, Dixon, Kosower, Johansson, Roiban ’07



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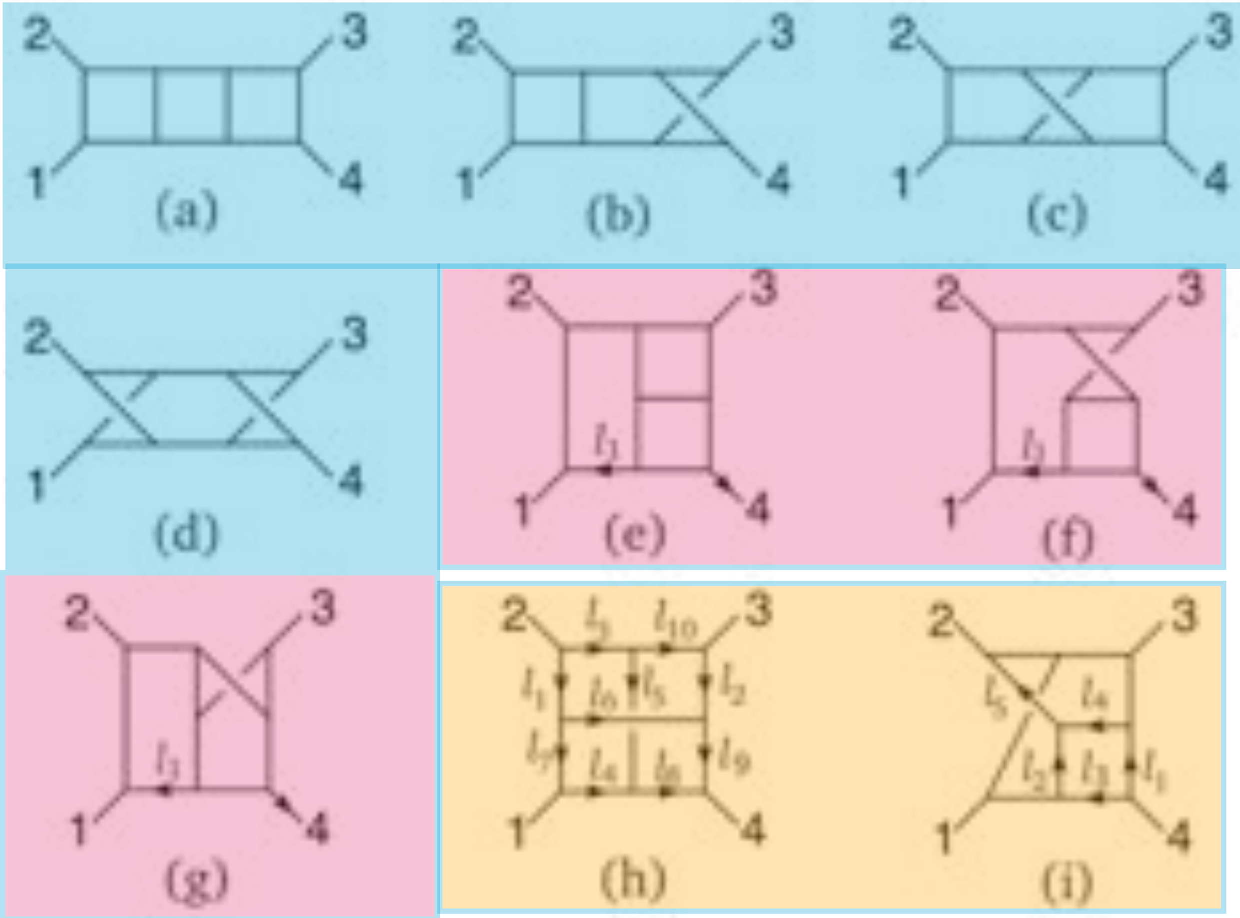
(i)	$s(l_1+l_2)^2 - t(l_3+l_4)^2 - \frac{1}{3}(s-t)l_5^2$	$(s(l_1+l_2)^2 - t(l_3+l_4)^2)^2 - (s^2(l_1+l_2)^2+t^2(l_3+l_4)^2+\frac{1}{3}stu)l_5^2$

Bern, JJMC, Dixon, Kosower, Johansson, Roiban '07



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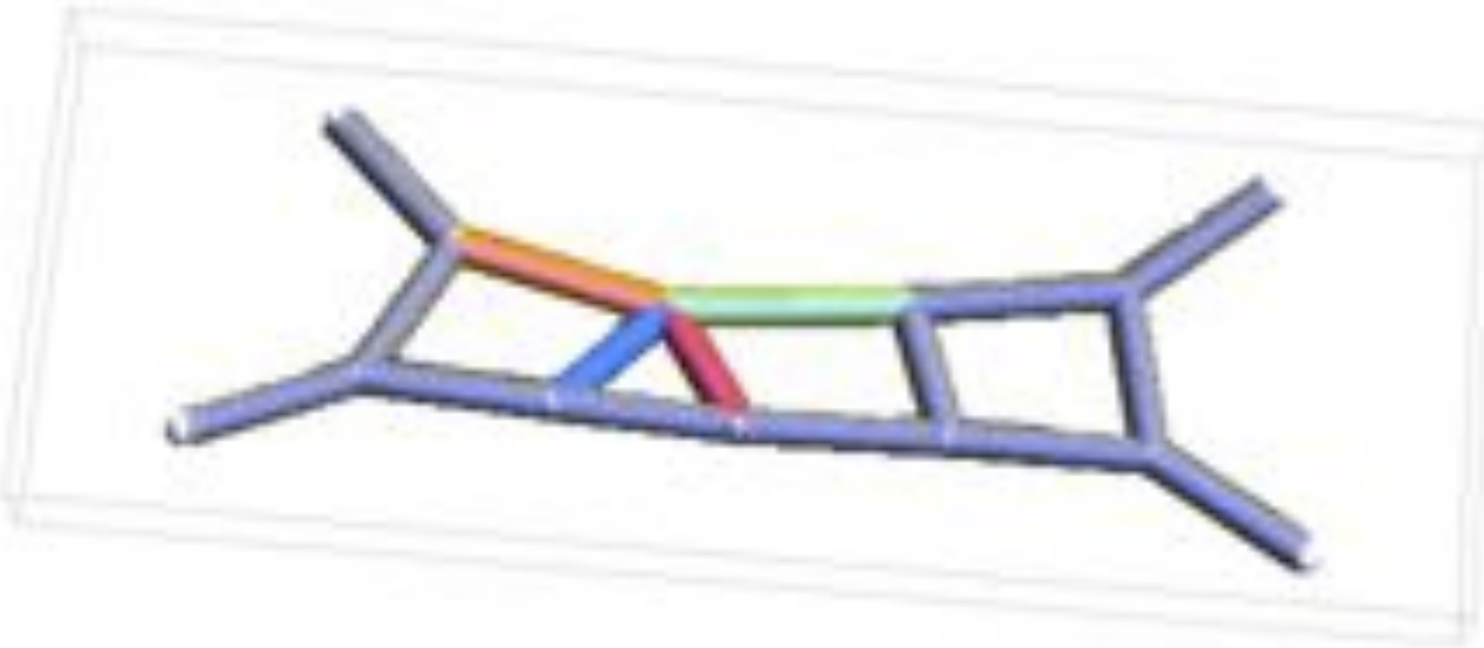
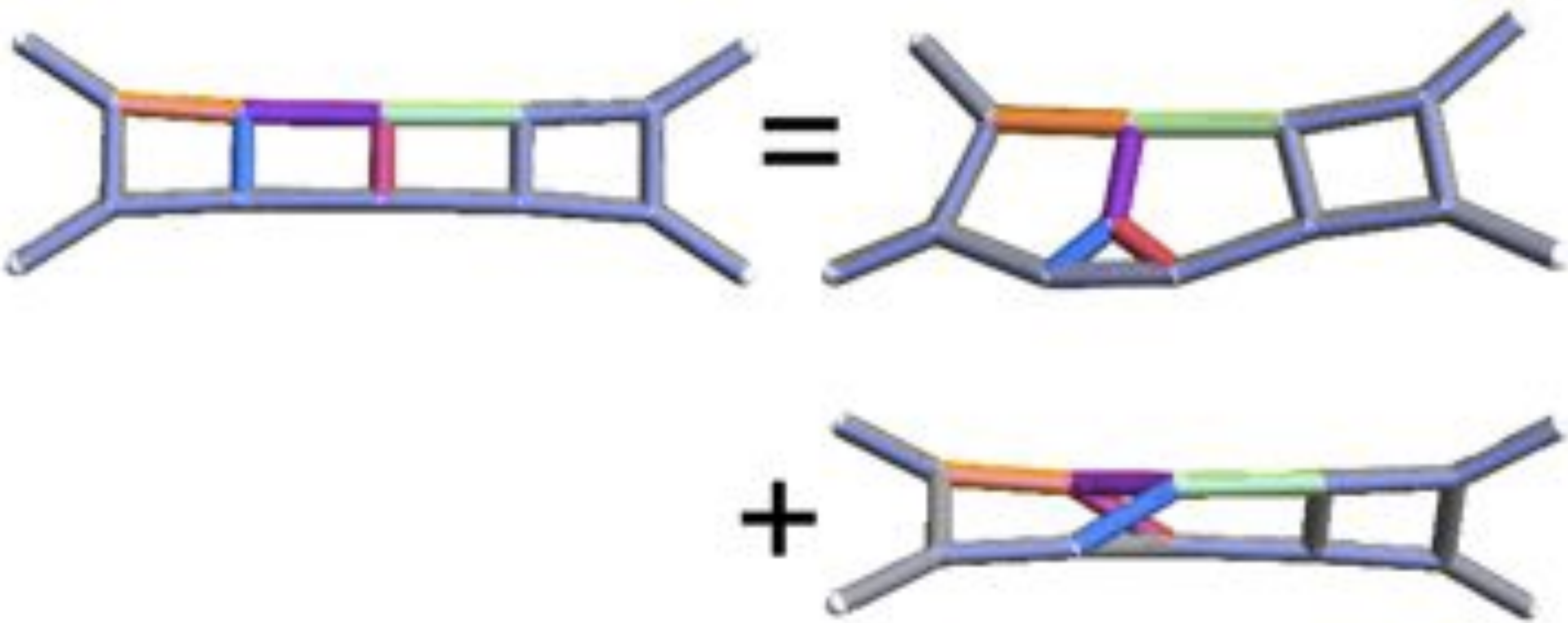
(i)	$s(l_1+l_2)^2-t(l_3+l_4)^2 - \frac{1}{3}(s-t)l_5^2$	$(s(l_1+l_2)^2-t(l_3+l_4)^2)^2 - (s^2(l_1+l_2)^2+t^2(l_3+l_4)^2+\frac{1}{3}stu)l_5^2$

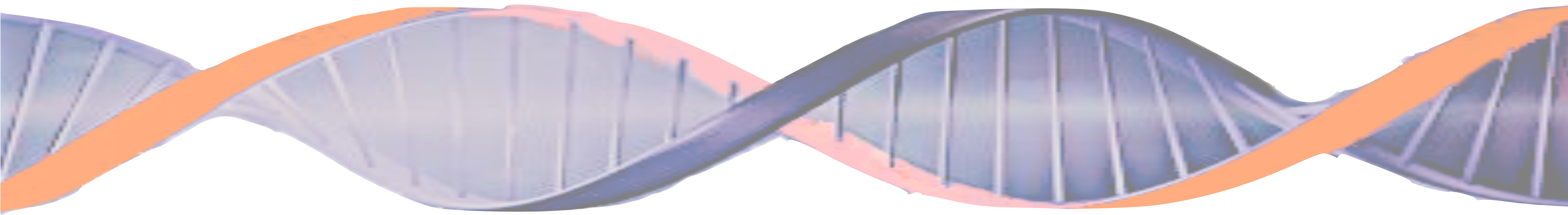
BCJ '08, '10





“DECODING THE DNA OF GRAVITY” BCJ ’08, ’10

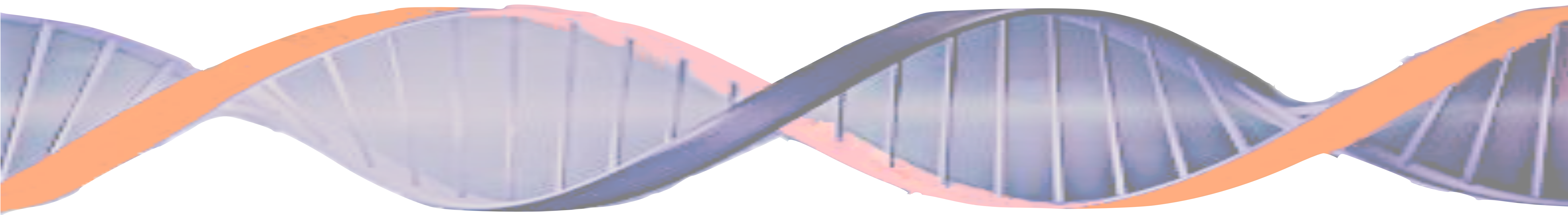
$$\frac{(-i)^L}{g^{n-2+2L}}\mathcal{A}^{\text{loop}} = \sum_{\mathcal{G}\in\text{cubic}} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S(\mathcal{G})} \frac{n(\mathcal{G})c(\mathcal{G})}{D(\mathcal{G})}$$



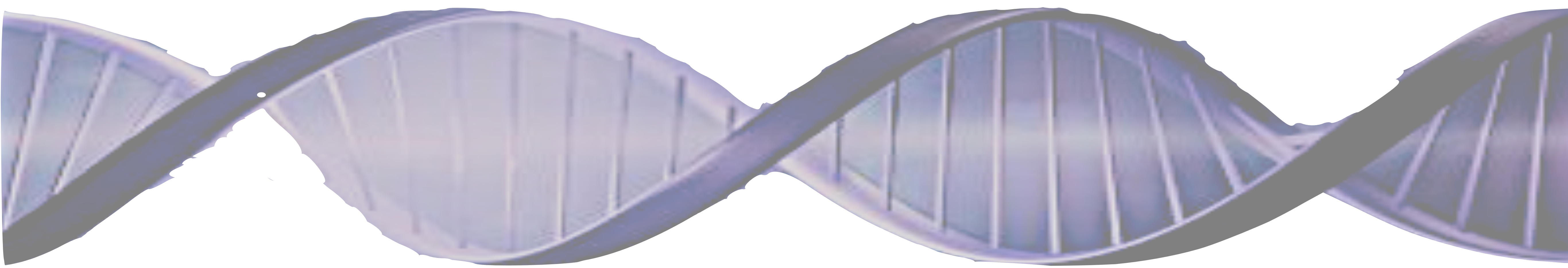
LOOP LEVEL DOUBLE COPY

“DECODING THE DNA OF GRAVITY” BCJ ’08, ’10

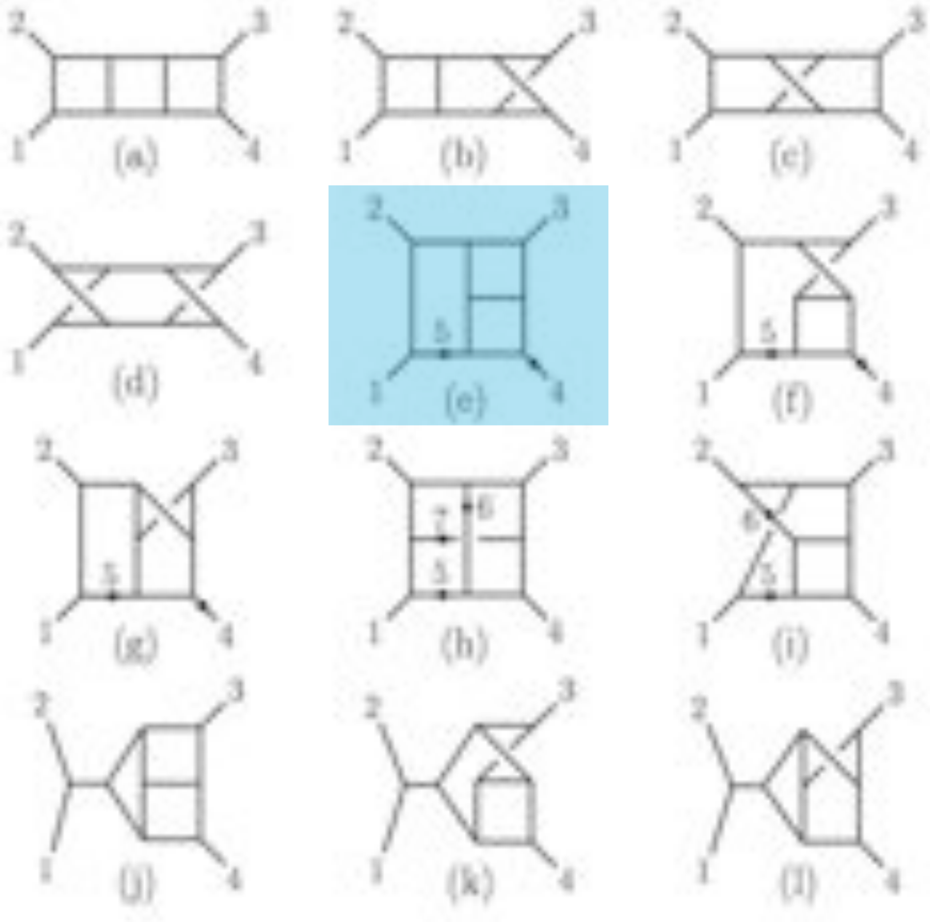
$$\frac{(-i)^L}{g^{n-2+2L}}\mathcal{A}^{\text{loop}} = \sum_{\mathcal{G}\in\text{cubic}} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S(\mathcal{G})} \frac{n(\mathcal{G})c(\mathcal{G})}{D(\mathcal{G})}$$



LOOP LEVEL DOUBLE COPY



$$\frac{(-i)^{L+1}}{(\kappa/2)^{n-2+2L}}\mathcal{M}^{\text{loop}} = \sum_{\mathcal{G}\in\text{cubic}} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S(\mathcal{G})} \frac{n(\mathcal{G})\tilde{n}(\mathcal{G})}{D(\mathcal{G})}$$



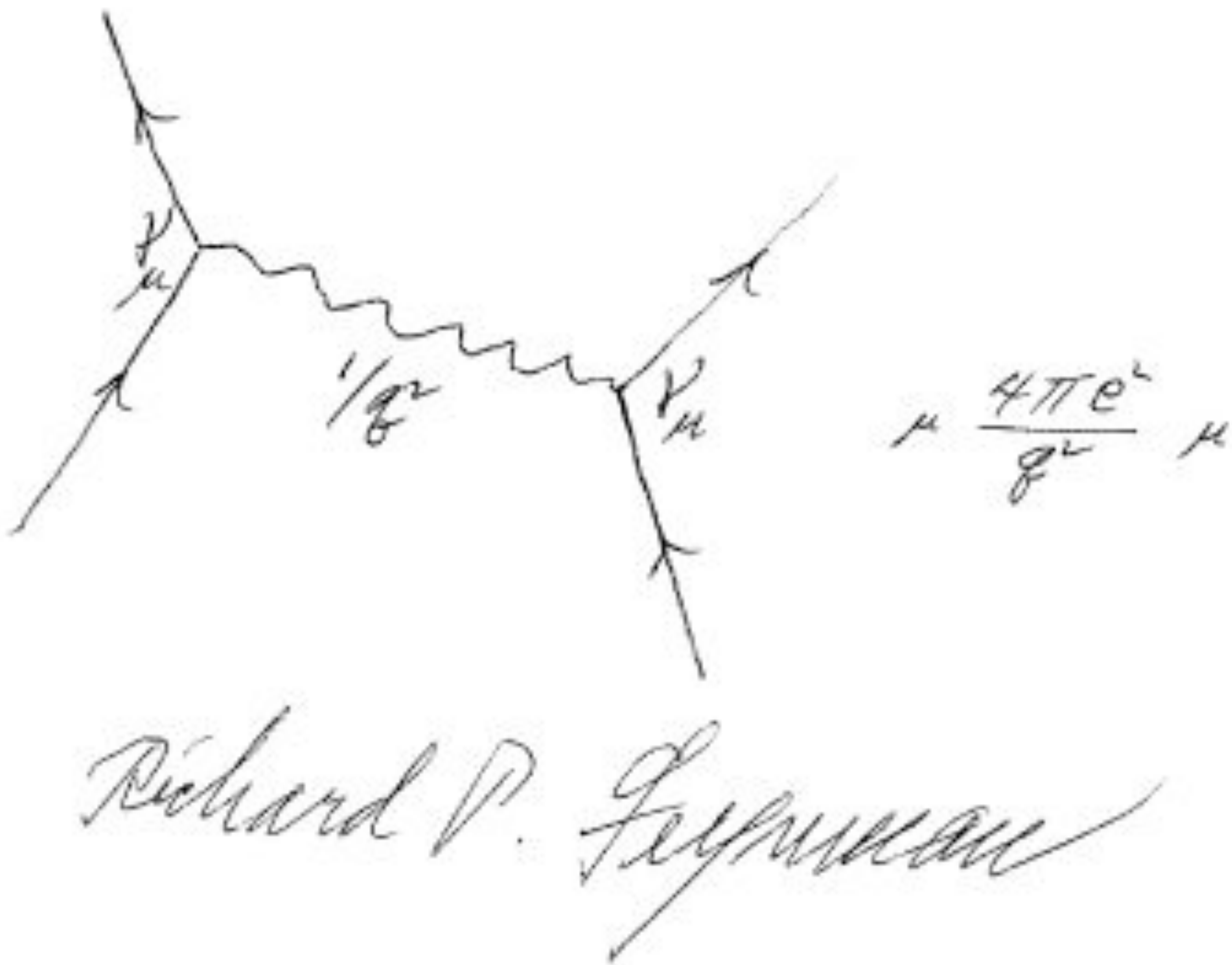
Only need maximal cut information of (e) graph to build full amplitude!

Integral $I^{(x)}$	$\mathcal{N}=4$ Super-Yang-Mills ($\sqrt{\mathcal{N}=8}$ supergravity) numerator
(a)–(d)	s^2
(e)–(g)	$\big(s\,(-\tau_{35}+\tau_{45}+t)-t\,(\tau_{25}+\tau_{45})+u\,(\tau_{25}+\tau_{35})-s^2\big)/3$
(h)	$\big(s\,(2\tau_{15}-\tau_{16}+2\tau_{26}-\tau_{27}+2\tau_{35}+\tau_{36}+\tau_{37}-u)$ $+t\,(\tau_{16}+\tau_{26}-\tau_{37}+2\tau_{36}-2\tau_{15}-2\tau_{27}-2\tau_{35}-3\tau_{17})+s^2\big)/3$
(i)	$\big(s\,(-\tau_{25}-\tau_{26}-\tau_{35}+\tau_{36}+\tau_{45}+2t)$ $+t\,(\tau_{26}+\tau_{35}+2\tau_{36}+2\tau_{45}+3\tau_{46})+u\,\tau_{25}+s^2\big)/3$
(j)-(l)	$s(t-u)/3$

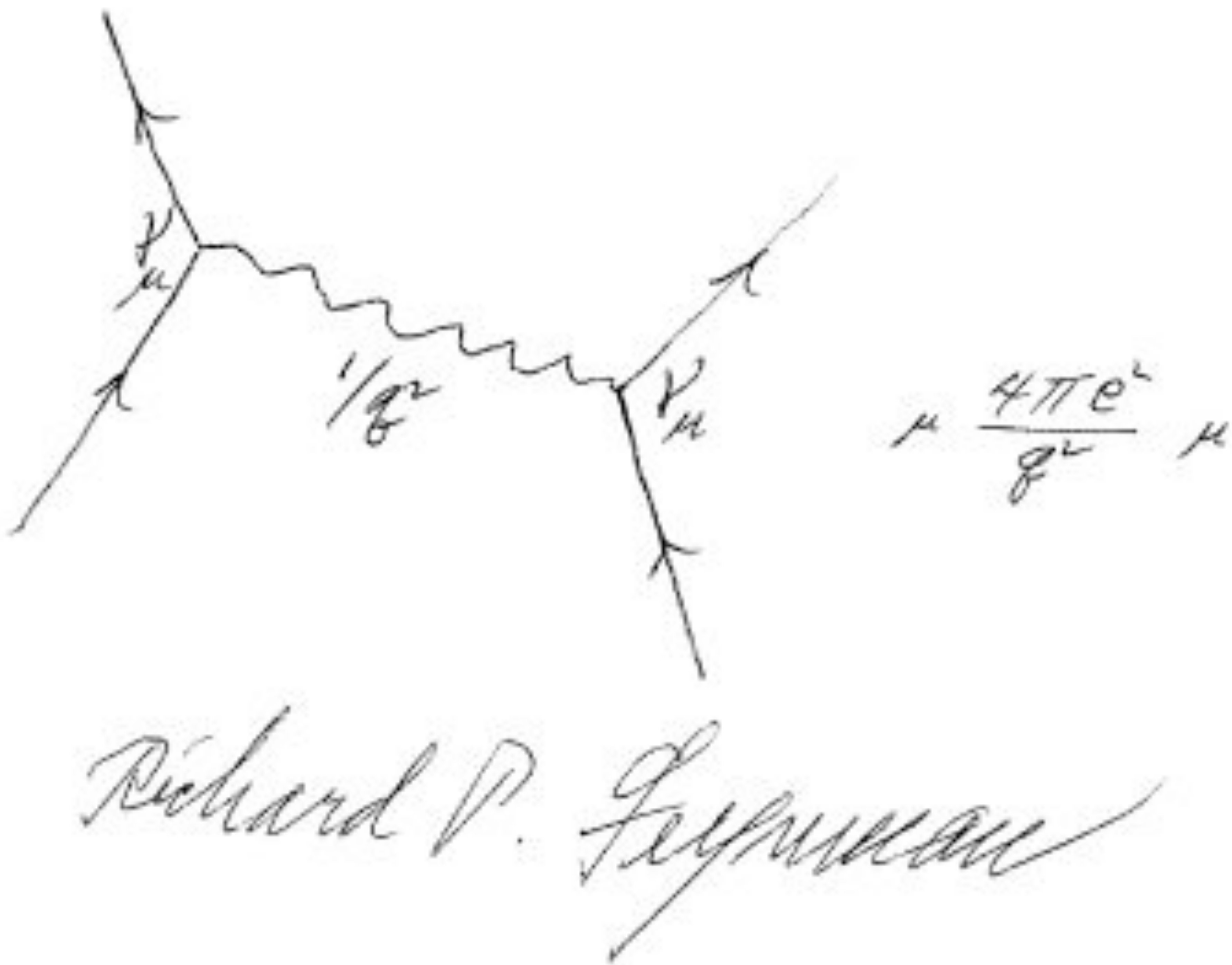
Aside: on cuts

(where do we get our data?)

TEXTBOOK APPROACH

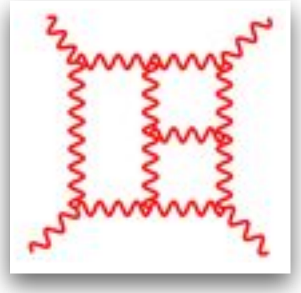


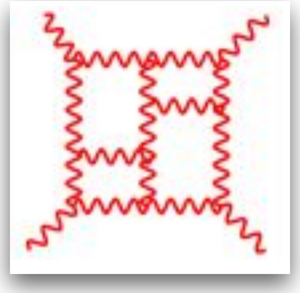
TEXTBOOK APPROACH

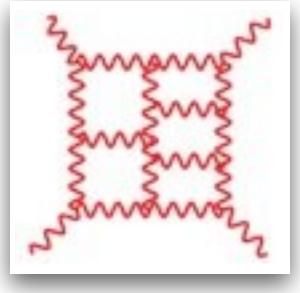


Simple graph rules for constructing scattering amplitudes

JUST THE GRAVITON....

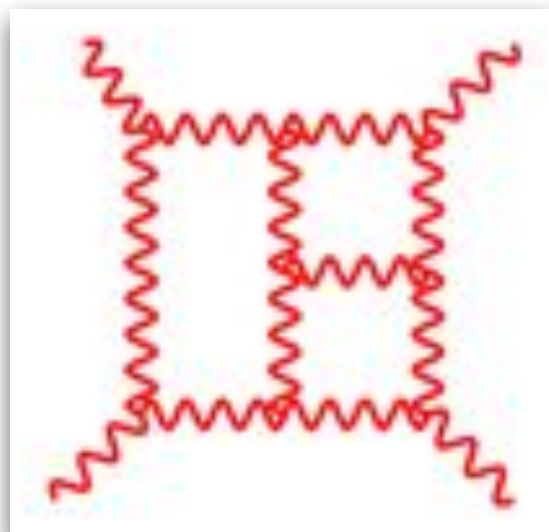
3 loops  $\sim 10^{20}$ TERMS

4 loops  $\sim 10^{26}$ TERMS

5 loops  $\sim 10^{31}$ TERMS

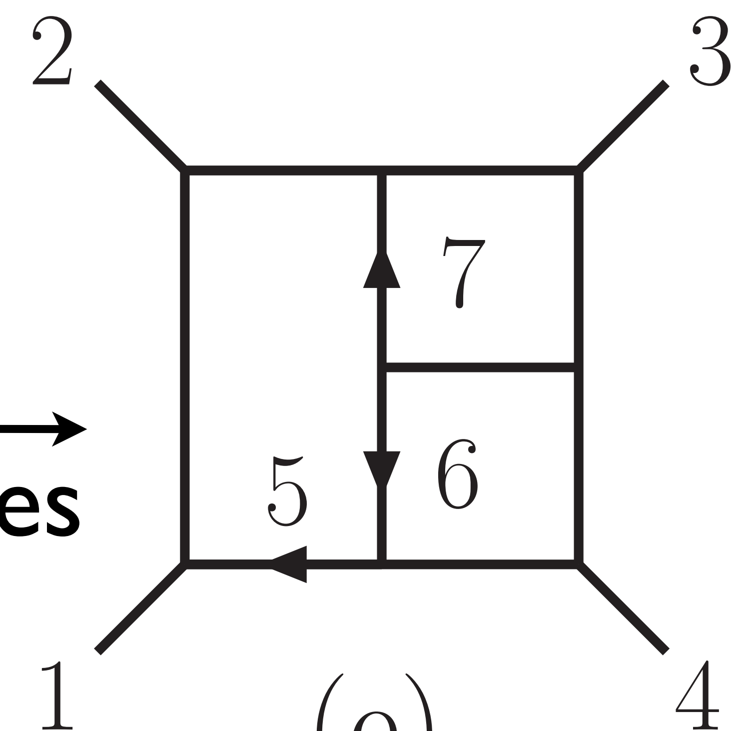
BUT FINAL EXPRESSIONS ARE TRACTABLE

MOST SYMMETRIC 4D THEORY, N=8 SUGRA



~10[20]
TERMS

add all other particles

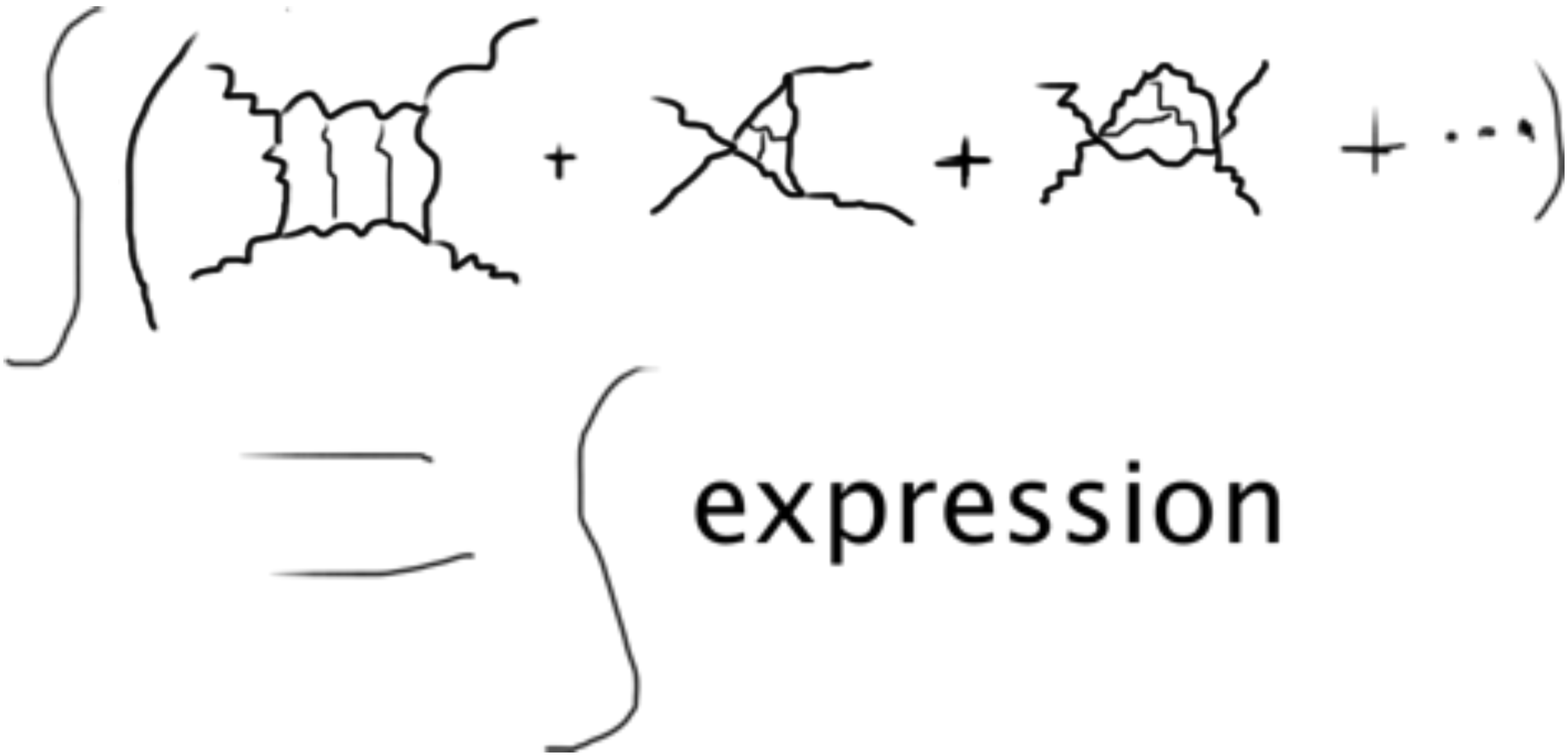


(e)

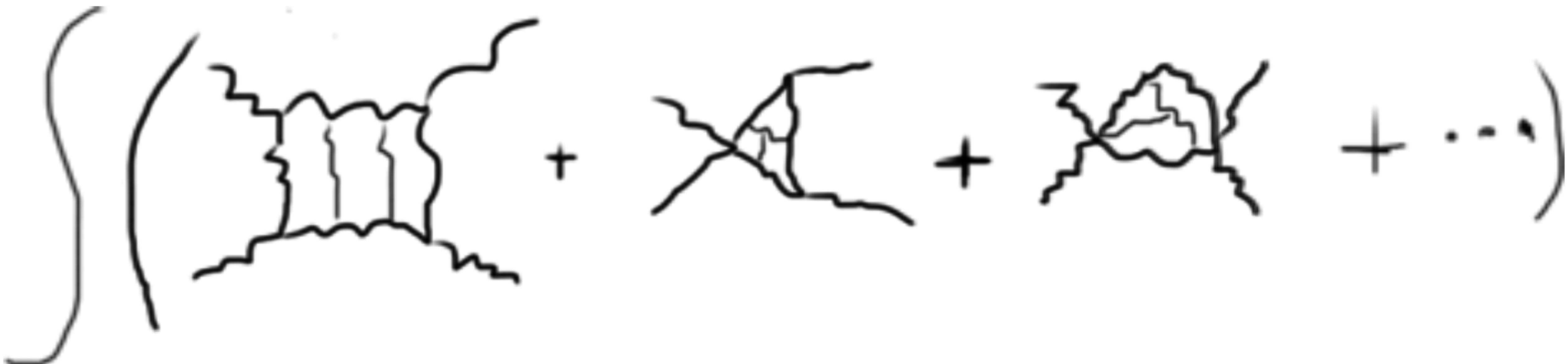
$$\propto \int stu\mathcal{M}_4^{(0)} \frac{\left(s\,(k_4+l_5)^2\right)^2}{d\circ(e) \equiv (l_5^2 l_6^2 l_7^2 (k_1-l_5)^2 \cdots)}$$

Bern, JJMC, Dixon, Kosower, Roiban '07

NECESSARY & SUFFICIENT



NECESSARY & SUFFICIENT



$$= \int \sum_{g} \frac{n(g)}{d(g)}$$

SUFFICIENT

$\mathcal{U}_c \circ \Big($  $+ \cdots \Big)$

$$= \mathcal{U}_c \circ \sum_g \frac{n(g)}{d(g)}$$

$\forall\, \mathcal{U}_c \in$ unitarity cuts

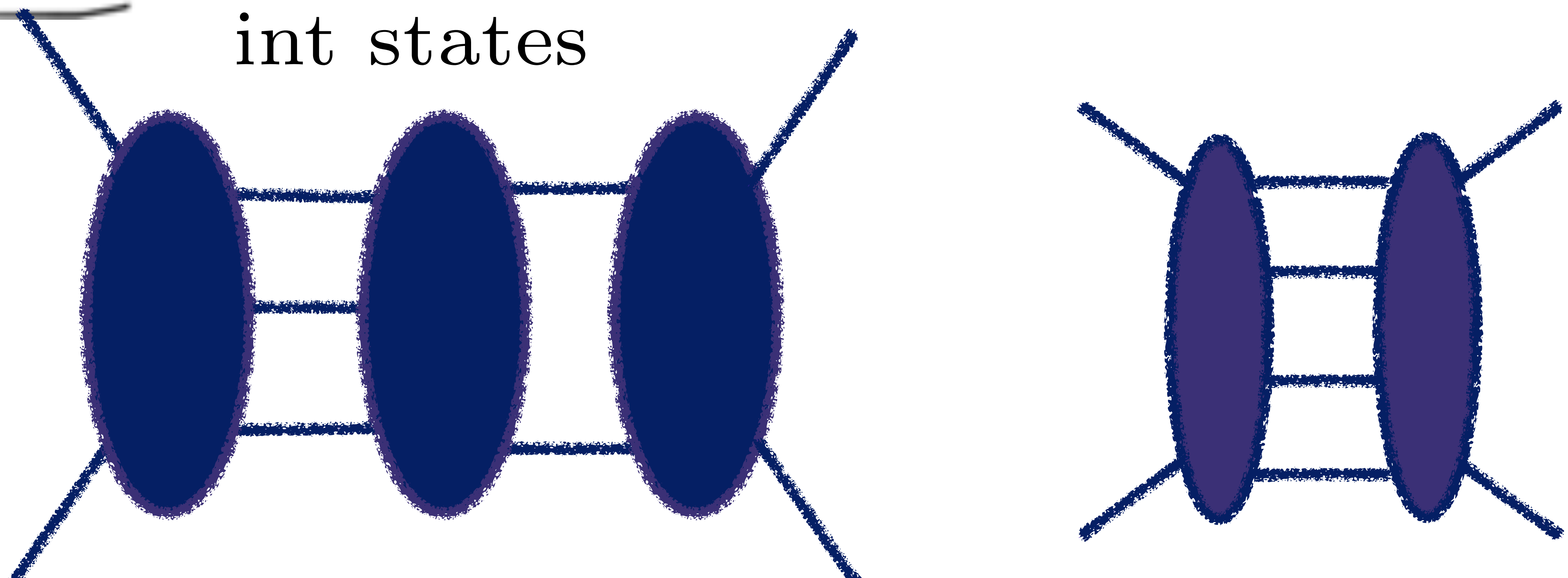
Bern, Dixon, Dunbar, and Kosower (‘94,’95)

Bern, Dixon, and Kosower (‘96)

Britto, Cachazo, and Feng (’04)

TREE-LEVEL CUT

$$\equiv \sum_{\text{int states}} \mathcal{M}^{(0)}_{U_c,1} \mathcal{M}^{(0)}_{U_c,2} \cdots \mathcal{M}^{(0)}_{U_c,m}$$



Scared by state sums?

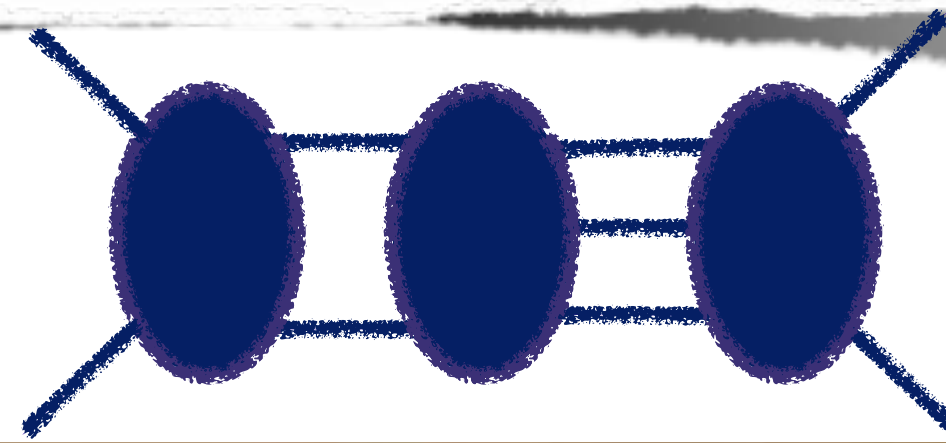
$2^8 = 256$ massless states, ~ expansion of $(x+y)^8$

$$\mathcal{N}=8: \quad 1 \longleftrightarrow 8 \longleftrightarrow 28 \longleftrightarrow 56 \longleftrightarrow 70 \longleftrightarrow 56 \longleftrightarrow 28 \longleftrightarrow 8 \longleftrightarrow 1$$

$$\text{helicity}: \quad -2 \quad -\tfrac{3}{2} \quad -1 \quad -\tfrac{1}{2} \quad 0 \quad \tfrac{1}{2} \quad 1 \quad \tfrac{3}{2} \quad 2$$

$$h^- \quad \psi_i^- \quad v_{ij}^- \quad \chi_{ijk}^- \quad s_{ijkl} \quad \chi_{ijk}^+ \quad v_{ij}^+ \quad \psi_i^+ \quad h^+$$

SUSY $\longleftrightarrow$



$$\mathcal{N}=4 \text{ SYM}: \quad 1 \quad 4 \quad 6 \quad 4 \quad 1$$

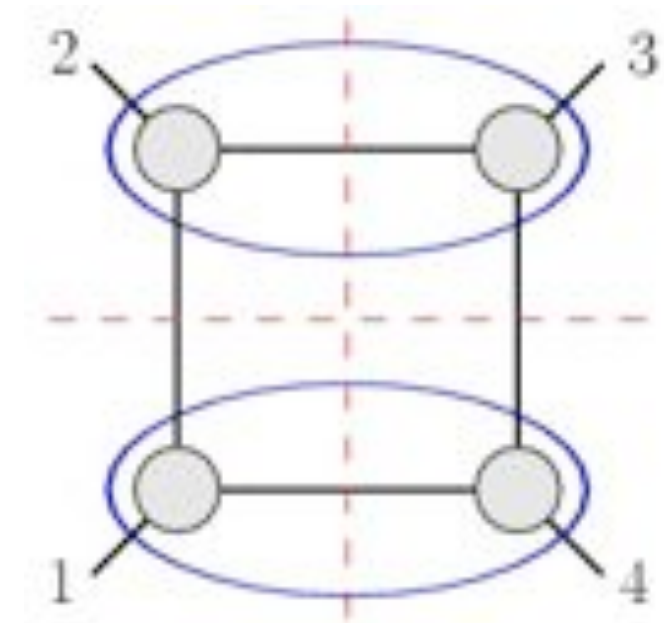
$$g^- \quad \lambda_A^- \quad \phi_{AB} \quad \lambda_A^+ \quad g^+$$

all in adjoint representation

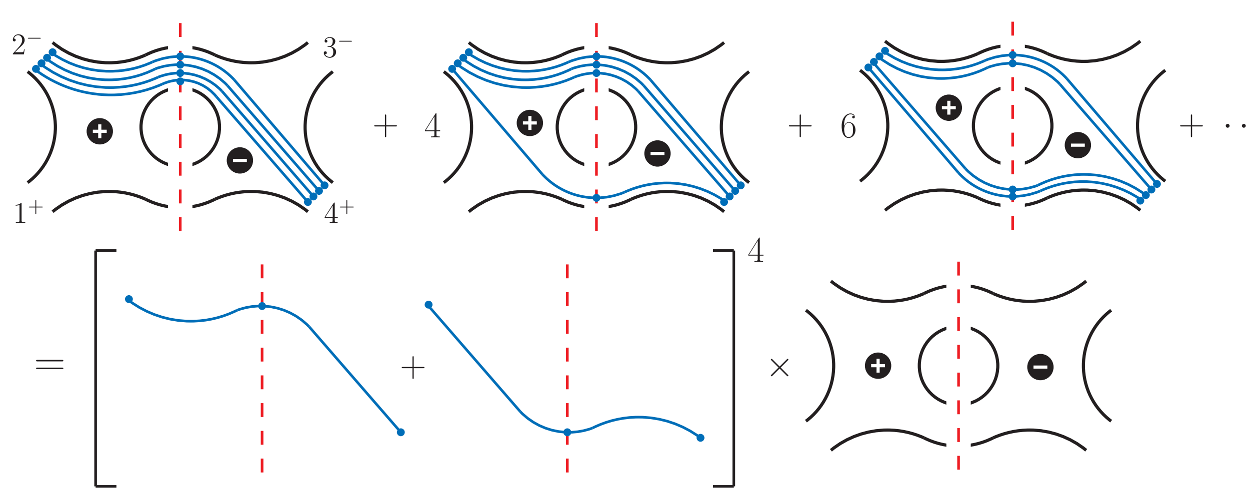
$2^4 = 16$ states ~ expansion of $(x+y)^4$

$$\Pi_i\, \delta(l_i^2) \sum_{\mathcal{N}=4} \big(A_3(-l_1, k_1, -l_2)\, A_3(l_1, -l_3, -$$
$$\times\, A_3(-l_6, l_3, k_2)\, A_3(l_6, l_2, -l_7)\, A_3(l_7, k_4, l_5)\big)$$
$$= (k_1 + k_2)^2$$

Don't be!

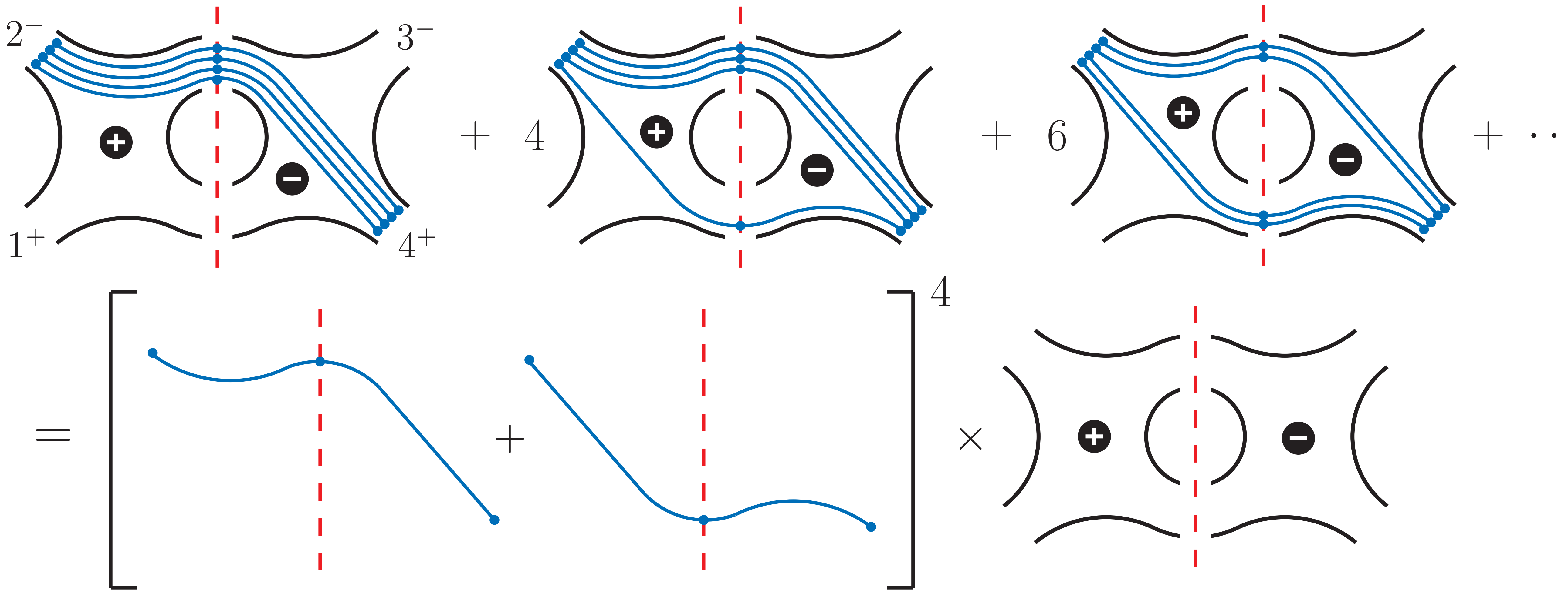


4D state-sums completely under control for N<=4 sYM



Bern, JJMC, Ita, Johansson, Roiban '09

N=4 sYM



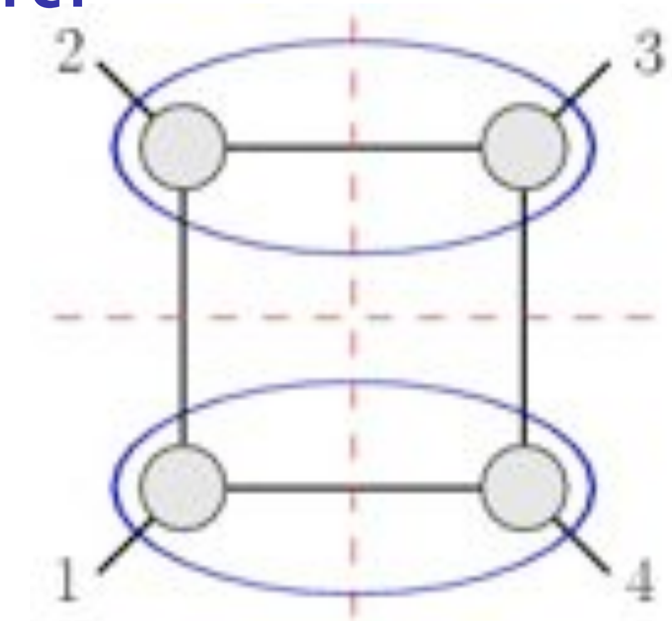
(state sums follow the gluons)

$$\mathcal{S}_{\mathcal{N}=4} = (A + B + C + \ldots)^4 \ \longrightarrow$$

$$\mathcal{S}_{\mathcal{N}<4} = (A + B + C + \ldots)^{\mathcal{N}} (A^{4-\mathcal{N}} + B^{4-\mathcal{N}} + C^{4-\mathcal{N}} + \ldots)$$

Bern, JJMC, Ita, Johansson, Roiban ‘09

Higher dimensional cuts also important!



(as tree multiplicity increases expressions can be unwieldy)

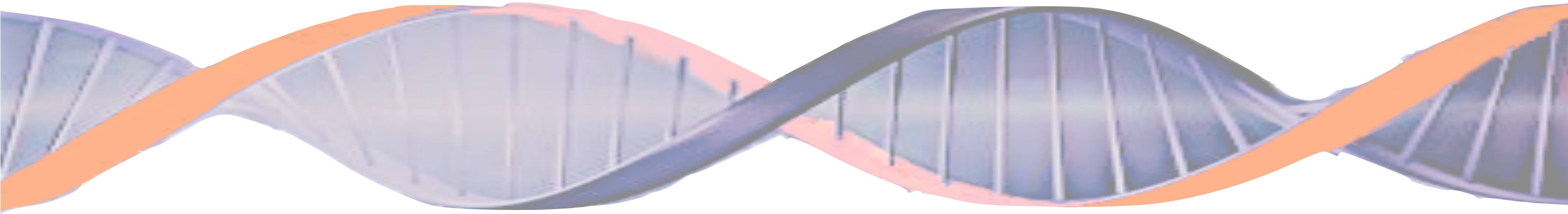
Workhorse: N=1 in 10D

Also very useful: N=2 in 6D

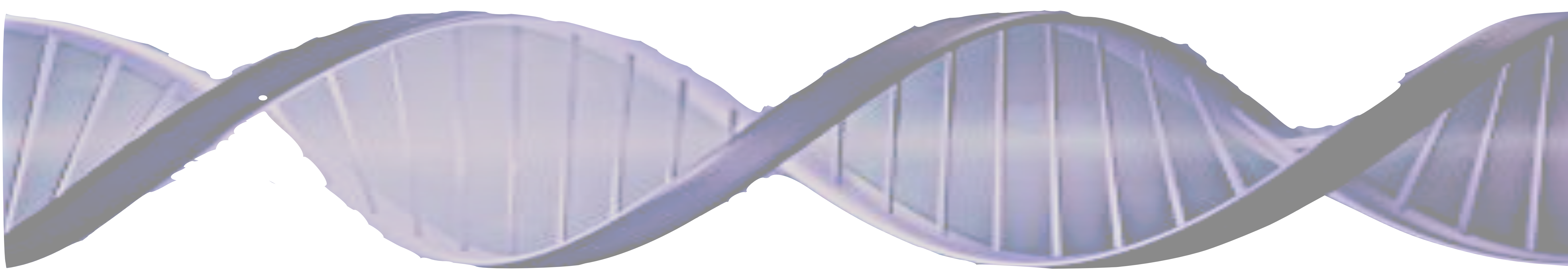
Cheung, O'Connell;
Dennen, Huang, Siegel; Boels;
Bern, JJMC, Dennen, Huang, Ita

Best: Recycling known D-dimensional amplitudes

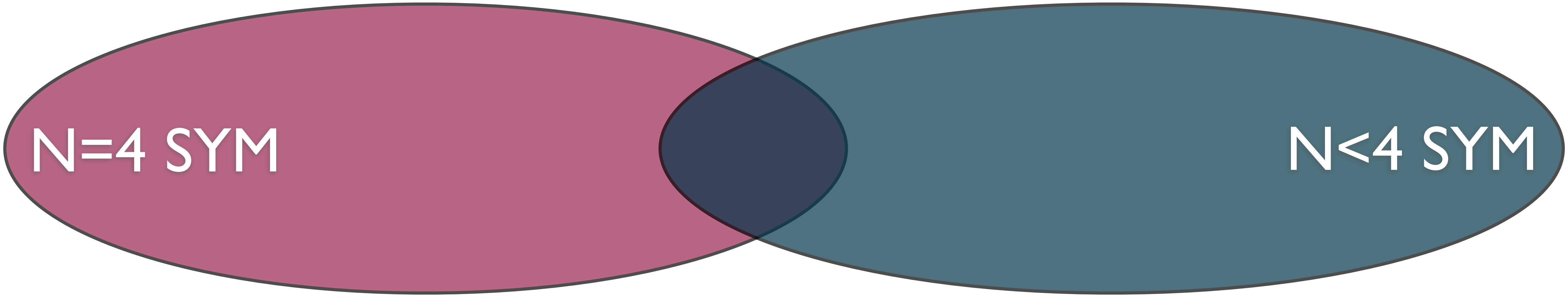
EVERYONE



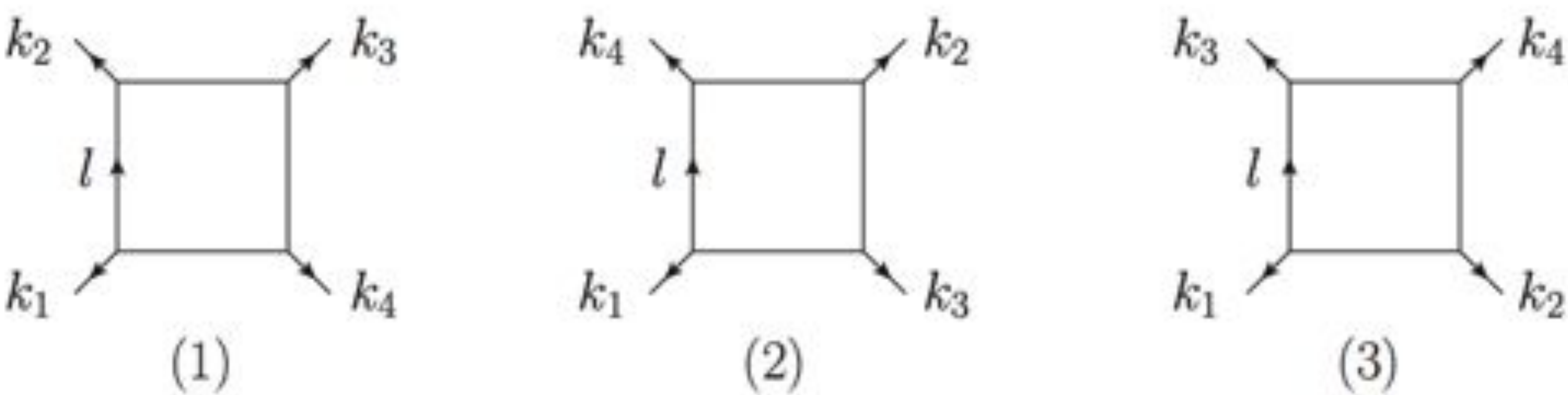
Back to Color-Kinematics / Double-copy



How does color-kinematics help us with less SUSY?



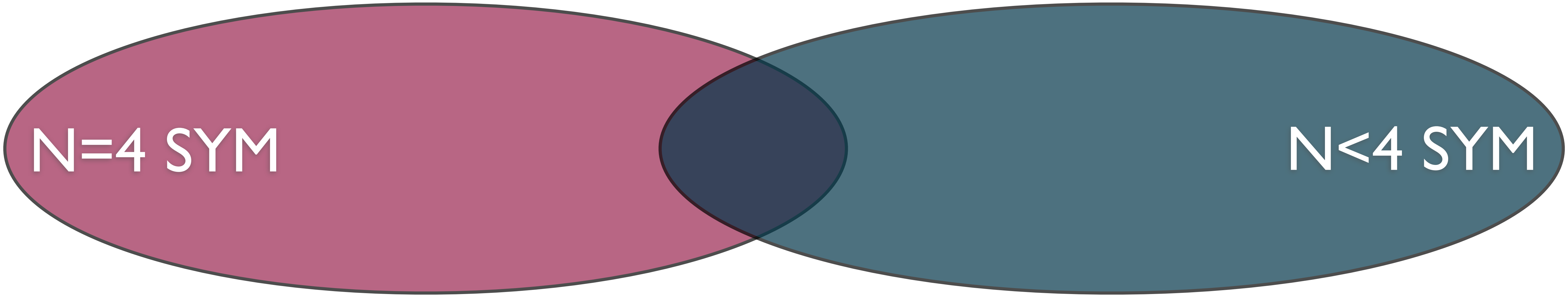
Look at N=4 Super Yang-Mills 4-pt1-loop



Green, Schwarz, Brink

$$\alpha = 1$$

How does color-kinematics help us with less SUSY?



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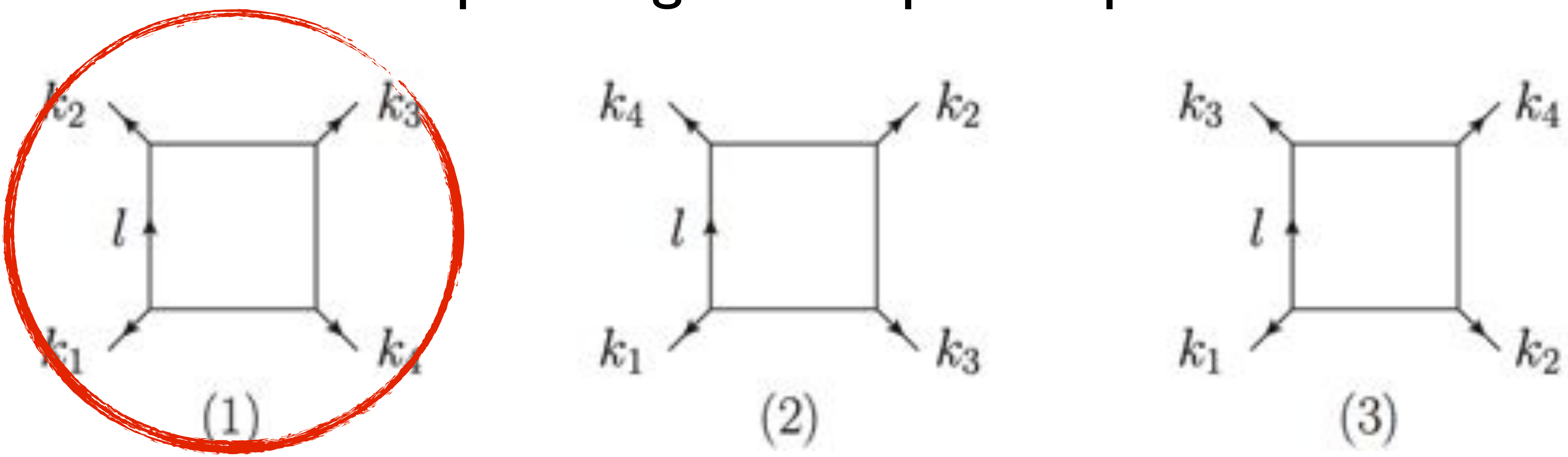
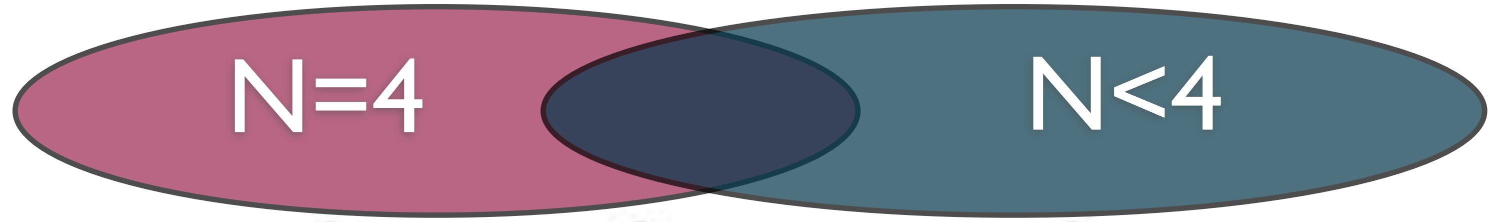


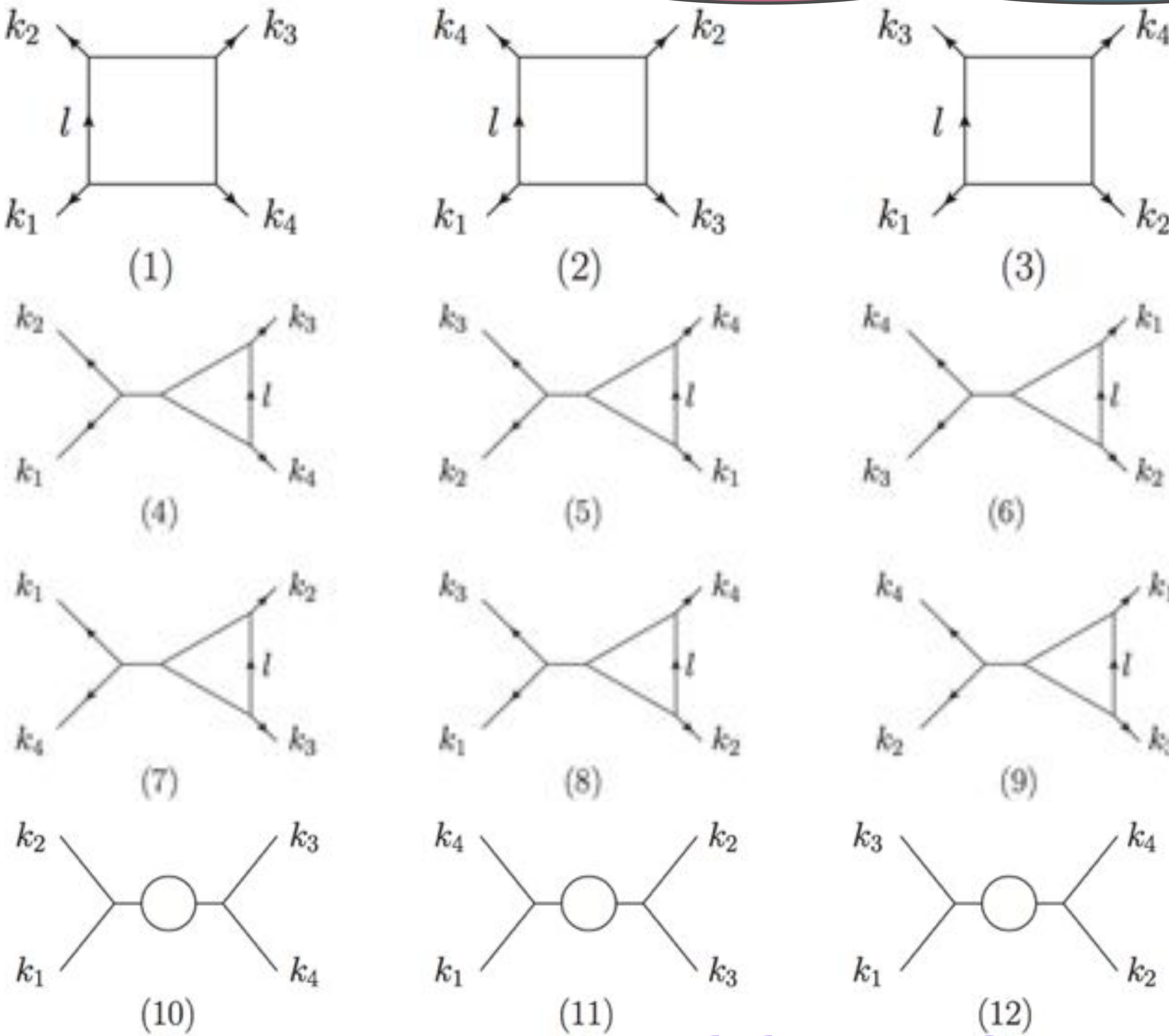
Symmetric

Green, Schwarz, Brink

Numerator ansatz by powercounting? a constant: $\alpha = 1$

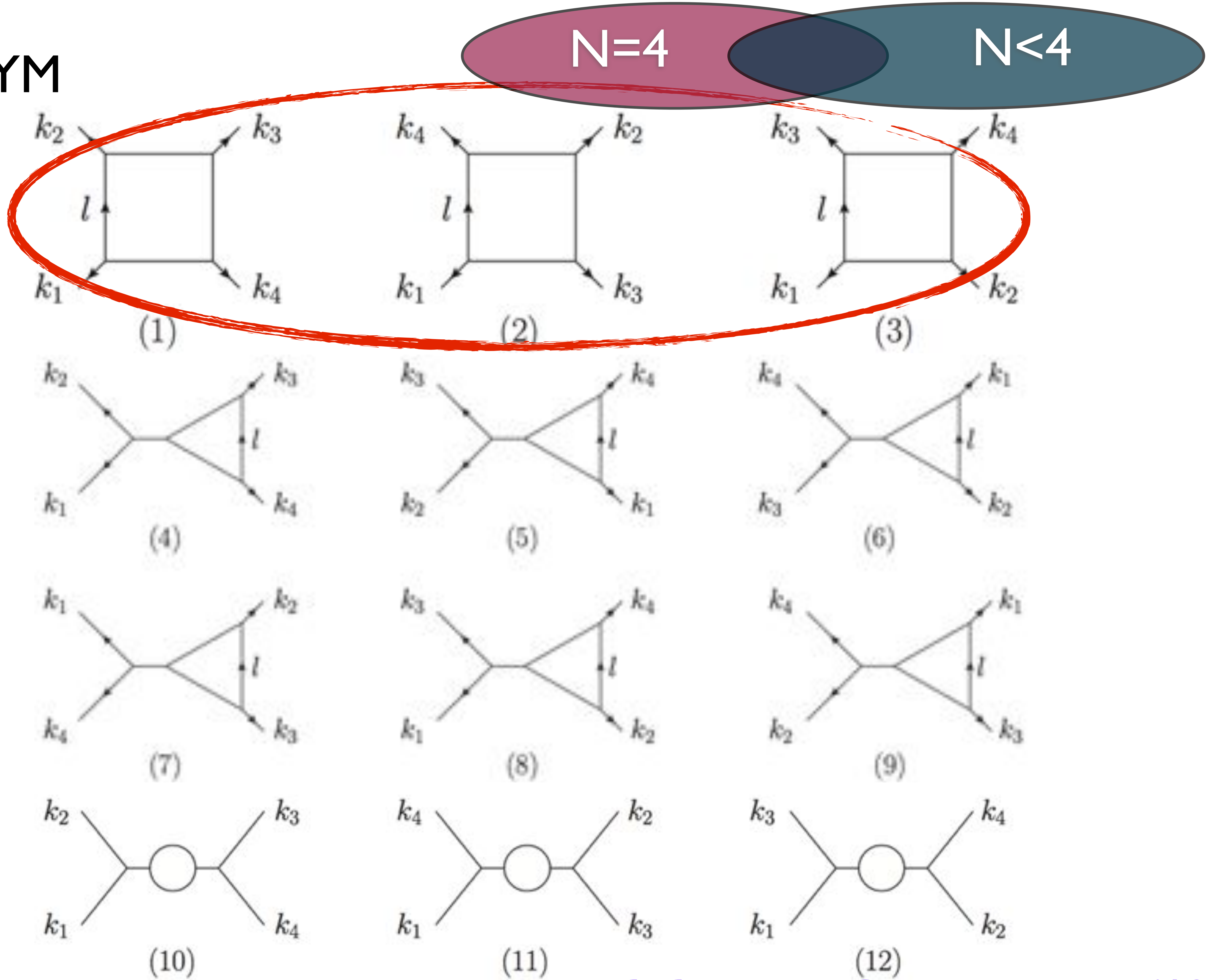
N<4 SYM





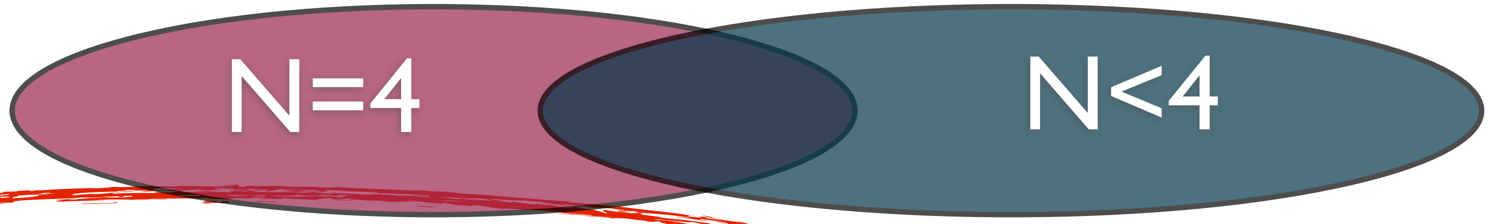
JJMC, Chiodaroli, Gunaydin, Roiban '12

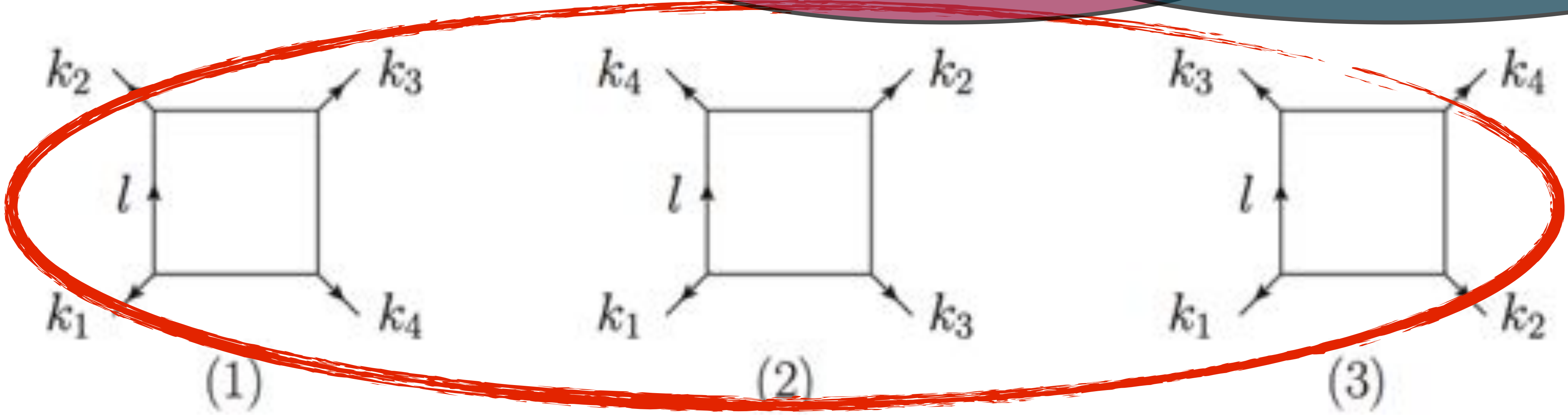
N<4 SYM



JJMC, Chiodaroli, Gunaydin, Roiban '12

N<4 SYM





Asymmetric

but problem! local gauge-inv. numerators don't work

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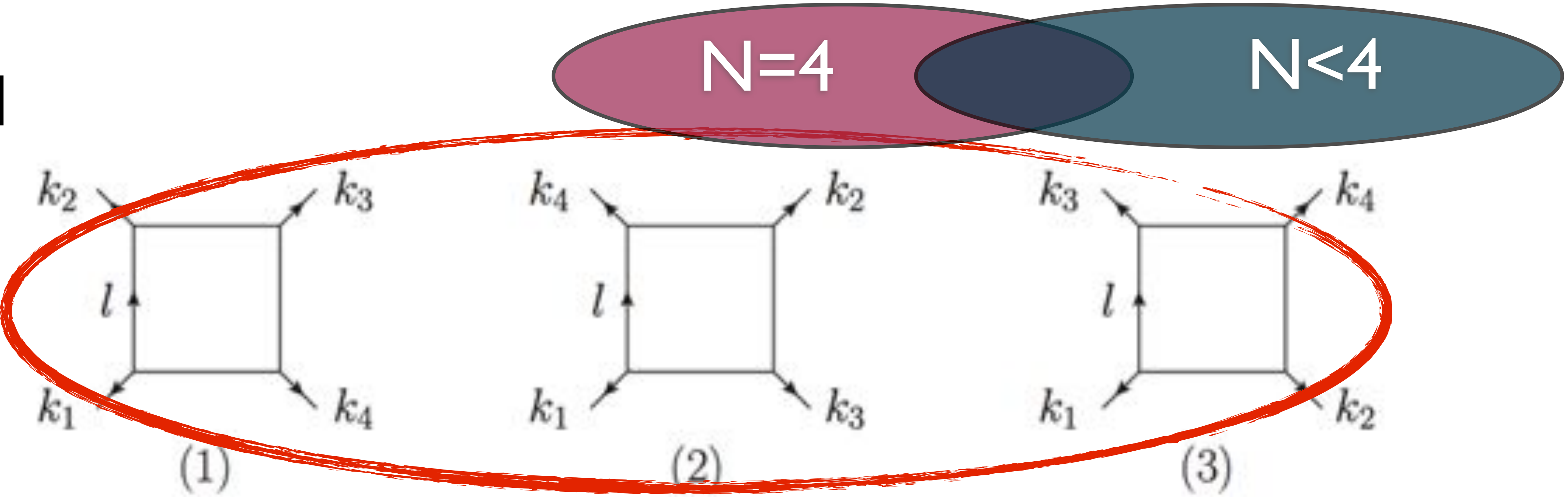
N<4 SYM



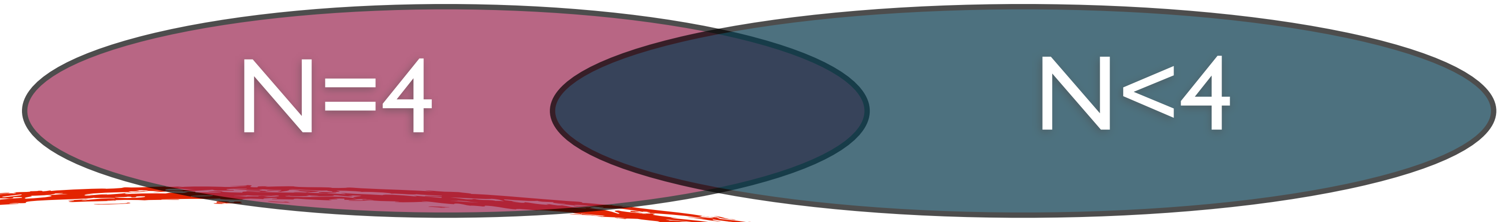
Asymmetric

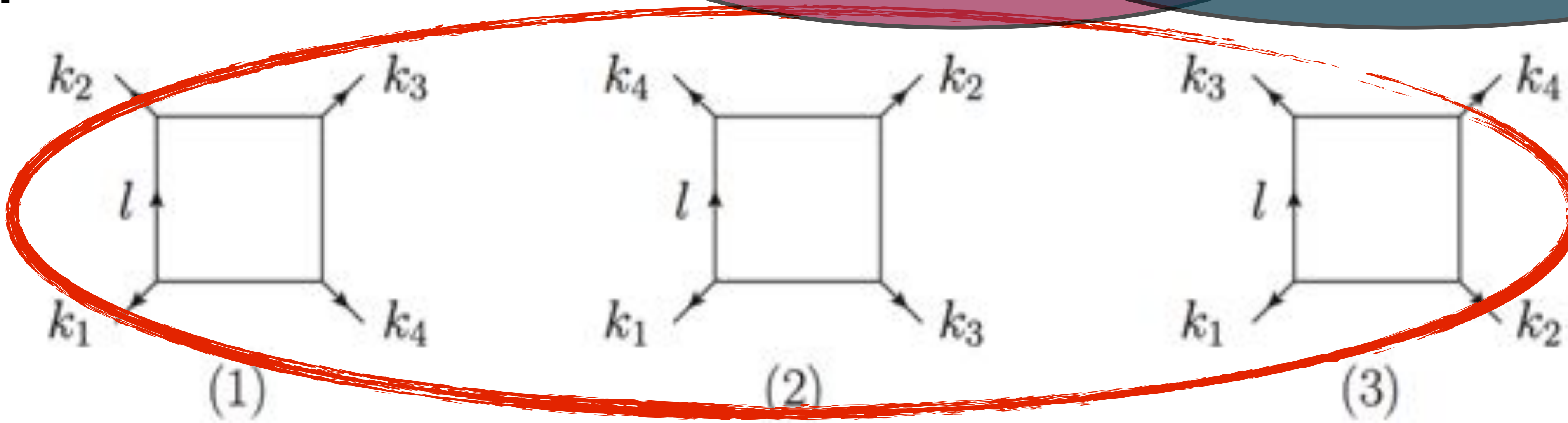
but problem! local gauge-inv. numerators don't work

(Lessons from N=4 5-pt multiloop, and symmetric tree through 6-pt: *divide by [Gram Det]^k* — *JJMC, Johansson* / *Broedel, JJMC*)

JJMC, Chiodaroli, Gunaydin, Roiban '12

N<4 SYM





Asymmetric

gauge invariant numerators,
requires non-locality in ext legs

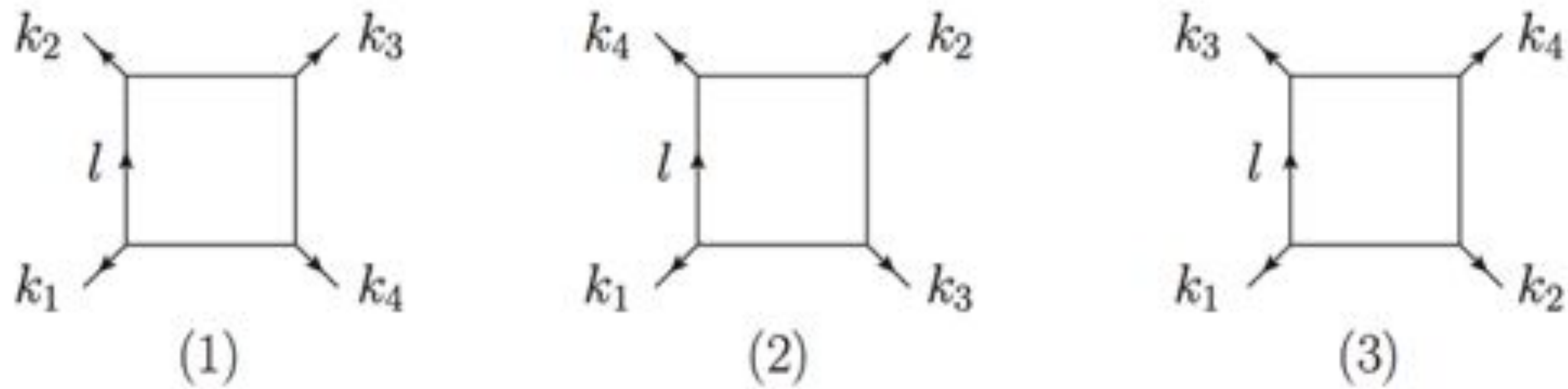
Parity odd components!

Numerator ansatz for $\mathcal{N} \geq 1$ SYM

$$N(g) = \frac{1}{(stu)^2}\Big(P_{g;(6,2)}(\tau_{l,k_1}, \tau_{l,k_2}, \tau_{l,k_3}; s, t)$$
$$+ 4i\varepsilon(k_1, k_2, k_3, l)\, P_{g;(4,0)}(\tau_{l,k_1}, \tau_{l,k_2}, \tau_{l,k_3}; s, t)\Big)$$

JJMC, Chiodaroli, Gunaydin, Roiban '12

Universal structure for $\mathcal{N} \geq 1$ SYM



$$\boxed{N_i^{\mathcal{N}} = N_i^{\mathcal{N}=4} - (4-\mathcal{N})N_i^{\text{chiral}}} \qquad N_{1,2,3}^{\mathcal{N}=4} = 1$$

$$N_1^{\text{chiral}} = \frac{1}{s}(\tau_{l,k_1} - \tau_{l,k_2}) + \frac{2}{stu}(s\tau_{l,k_1}\tau_{l,k_2} - u\tau_{l,k_1}\tau_{l,k_3} - t\tau_{l,k_2}\tau_{l,k_3})$$

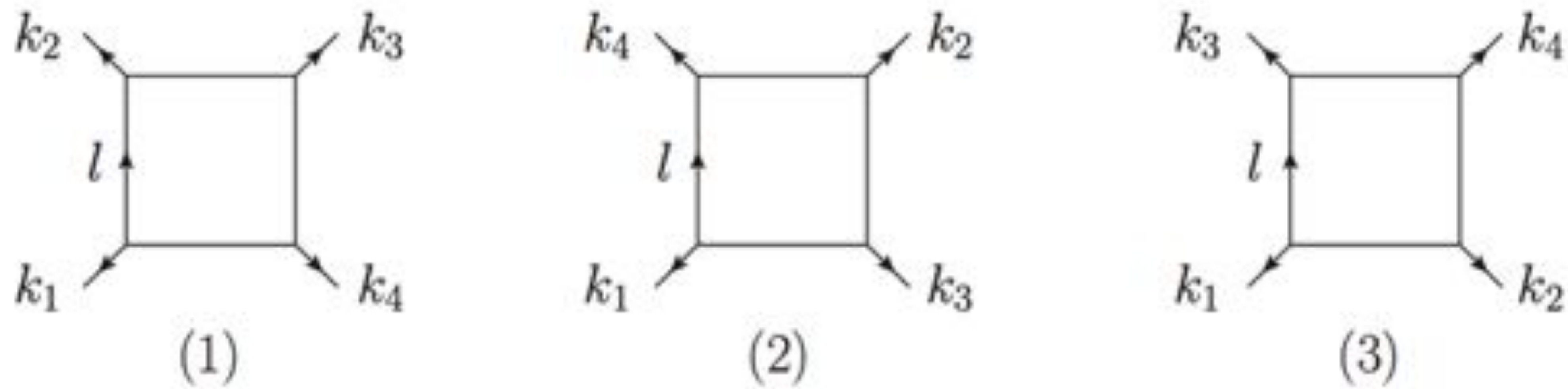
$$N_2^{\text{chiral}} = \frac{1}{s^2}(t\tau_{l,k_2} + s\tau_{l,k_3} - u\tau_{l,k_1}) + \frac{2}{stu}(s\tau_{l,k_1}\tau_{l,k_2} - u\tau_{l,k_1}\tau_{l,k_3} - t\tau_{l,k_2}\tau_{l,k_3}) - \frac{2i}{s^2}\varepsilon_{k_1,k_2,k_3,l}$$

$$N_3^{\text{chiral}} = -\frac{1}{u}(\tau_{l,k_3} + \tau_{l,k_1}) + \frac{2}{stu}(s\tau_{l,k_1}\tau_{l,k_2} - u\tau_{l,k_1}\tau_{l,k_3} - t\tau_{l,k_2}\tau_{l,k_3})$$

others given by Jacobi

JJMC, Chiodaroli, Gunaydin, Roiban '12

Universal structure for $\mathcal{N} \geq 1$ SYM



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$$N_2^{\text{chiral}} = \frac{1}{s^2}(t\tau_{l,k_2} + s\tau_{l,k_3} - u\tau_{l,k_1}) + \frac{2}{stu}(s\tau_{l,k_1}\tau_{l,k_2} - u\tau_{l,k_1}\tau_{l,k_3} - t\tau_{l,k_2}\tau_{l,k_3}) - \frac{2i}{s^2}\varepsilon_{k_1,k_2,k_3,l}$$

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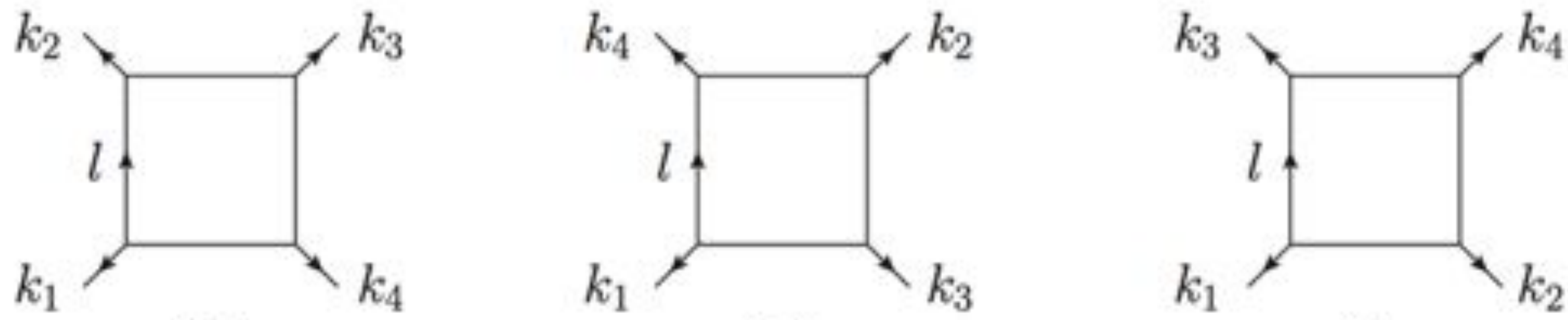
others given by Jacobi

JJMC, Chiodaroli, Gunaydin, Roiban '12

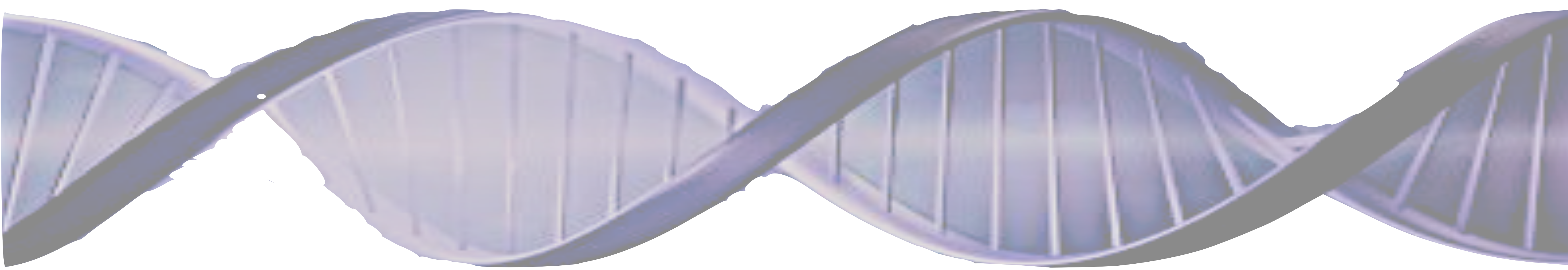
Universal structure for factorizable $\mathcal{N} \geq 1$ SG



(1) (2) (3)

$$\boxed{N_i^{\mathcal{N}} = N_i^{\mathcal{N}=4} - (4-\mathcal{N}) N_i^{\text{chiral}}}$$

$$N_{1,2,3}^{\mathcal{N}=4} = 1$$



$$\frac{(-i)^{L+1}}{(\kappa/2)^{n-2+2L}} \mathcal{M}^{\text{loop}} = \sum_{\mathcal{G} \in \text{cubic}} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S(\mathcal{G})} \frac{n(\mathcal{G}) \tilde{n}(\mathcal{G})}{D(\mathcal{G})}$$

JJMC, Chiodaroli, Gunaydin, Roiban '12

$$\mathcal{M}^{(1)}_{4,\mathcal{N}=1\times\mathcal{N}=1} = \mathcal{M}^{(1)}_{4,\mathcal{N}=8} + i\left(\frac{\kappa}{2}\right)^4 M_{\text{tree}}\Big[6\,\frac{tu}{s^2}(s^2 - \frac{3(1+f)}{4}tu)\,I_4^{D=6-2\epsilon}(t,u)$$
$$+\,\frac{9}{4}\,(1+f)\frac{tu}{s^2}(t-u)(I_2(u)-I_2(t)) + \frac{9}{2}\frac{1+(3-D)f}{D-2}(I_2(u)+I_2(t))\Big]\Big|_{f\to 1}$$

$$\mathcal{M}^{(1)}_{4,\mathcal{N}=2\times\mathcal{N}=1} = \mathcal{M}^{(1)}_{4,\mathcal{N}=8} + i\left(\frac{\kappa}{2}\right)^4 M_{\text{tree}}\Big[\frac{tu}{s^2}(5s^2 - 3(1+f)tu)\,I_4^{D=6-2\epsilon}(t,u)$$
$$+\,\frac{3}{2}\,(1+f)\frac{tu}{s^2}(t-u)(I_2(u)-I_2(t)) + 3\frac{1+(3-D)f}{D-2}(I_2(u)+I_2(t))\Big]\Big|_{f\to 1}$$

$$\mathcal{M}^{(1)}_{4,\mathcal{N}=2\times\mathcal{N}=2} = \mathcal{M}^{(1)}_{4,\mathcal{N}=8} + i\left(\frac{\kappa}{2}\right)^4 M_{\text{tree}}\Big[4\,\frac{tu}{s^2}(s^2 - \frac{1+f}{2}tu)\,I_4^{D=6-2\epsilon}(t,u)$$
$$+\,(1+f)\frac{tu}{s^2}(t-u)(I_2(u)-I_2(t)) + 2\frac{1+(3-D)f}{D-2}(I_2(u)+I_2(t))\Big]\Big|_{f\to 1}$$

(same as $\mathcal{M}^{(1)}_{4,\mathcal{N}=4\times\mathcal{N}=0+2\phi}$ **)**

JJMC, Chiodaroli, Gunaydin, Roiban '12

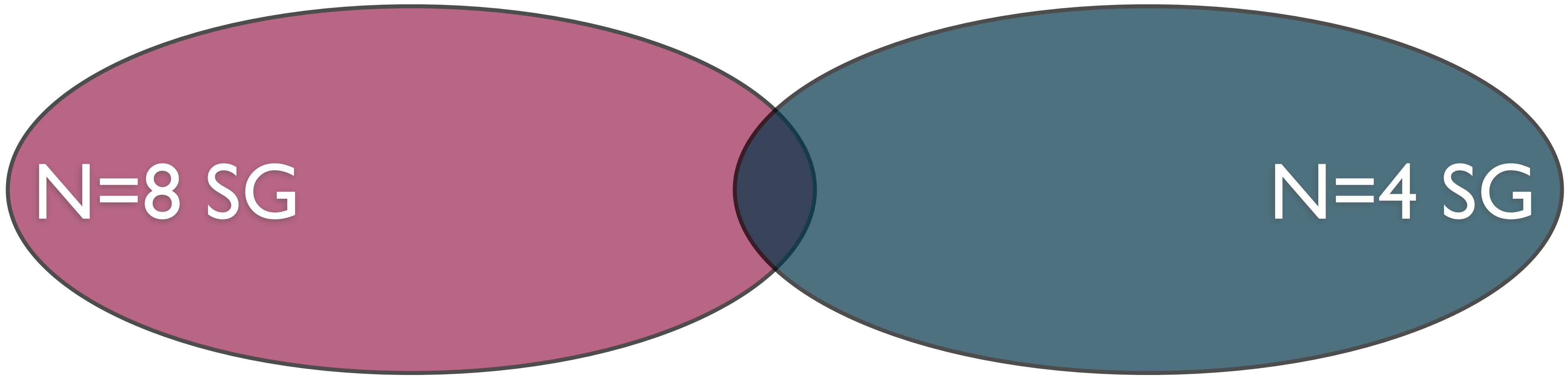
Many interesting lessons from this:

- observe a unified structure in gauge and gravity amplitudes

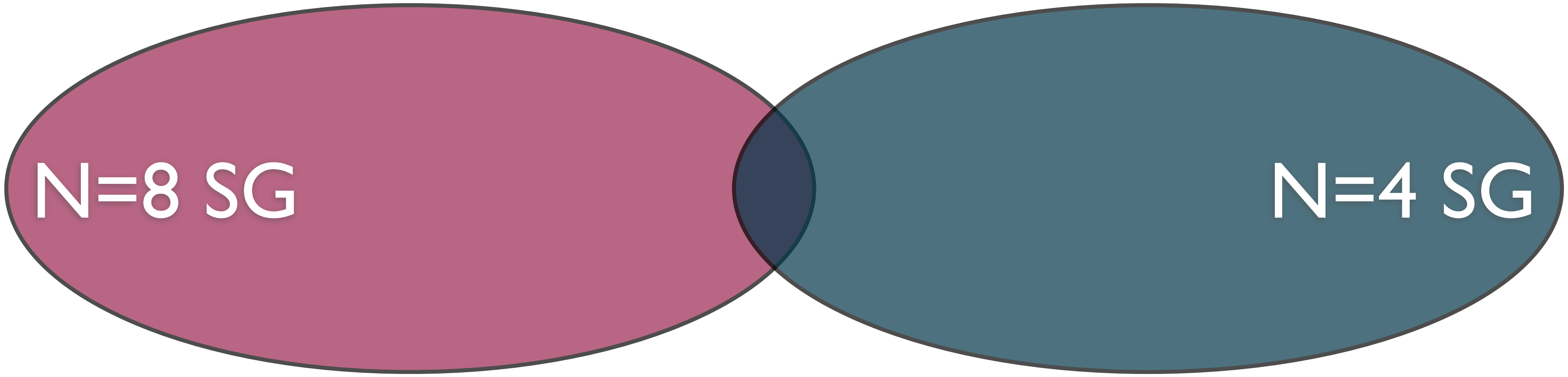
- four-graviton amplitudes are insensitive to the precise nature of the matter couplings

- everywhere we look two different presentations of N=4 SG are the same

- just because something integrates to zero for gauge theories doesn't mean we can ignore it in double-copying to gravity

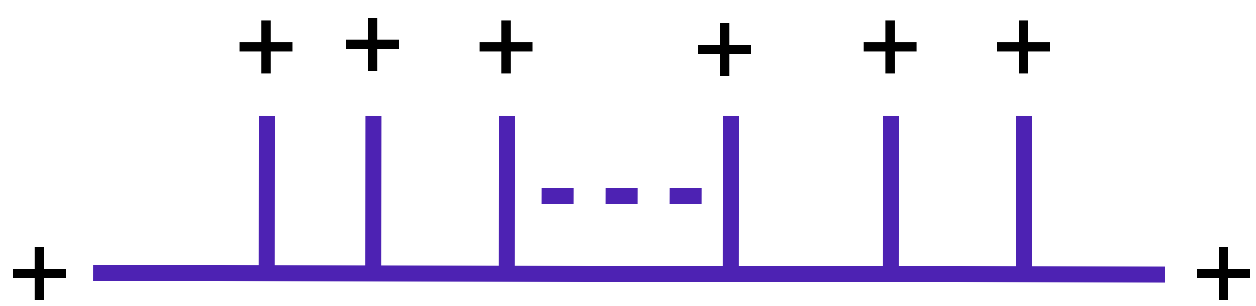


difference structurally
between
N>4 SG and N=4 SG

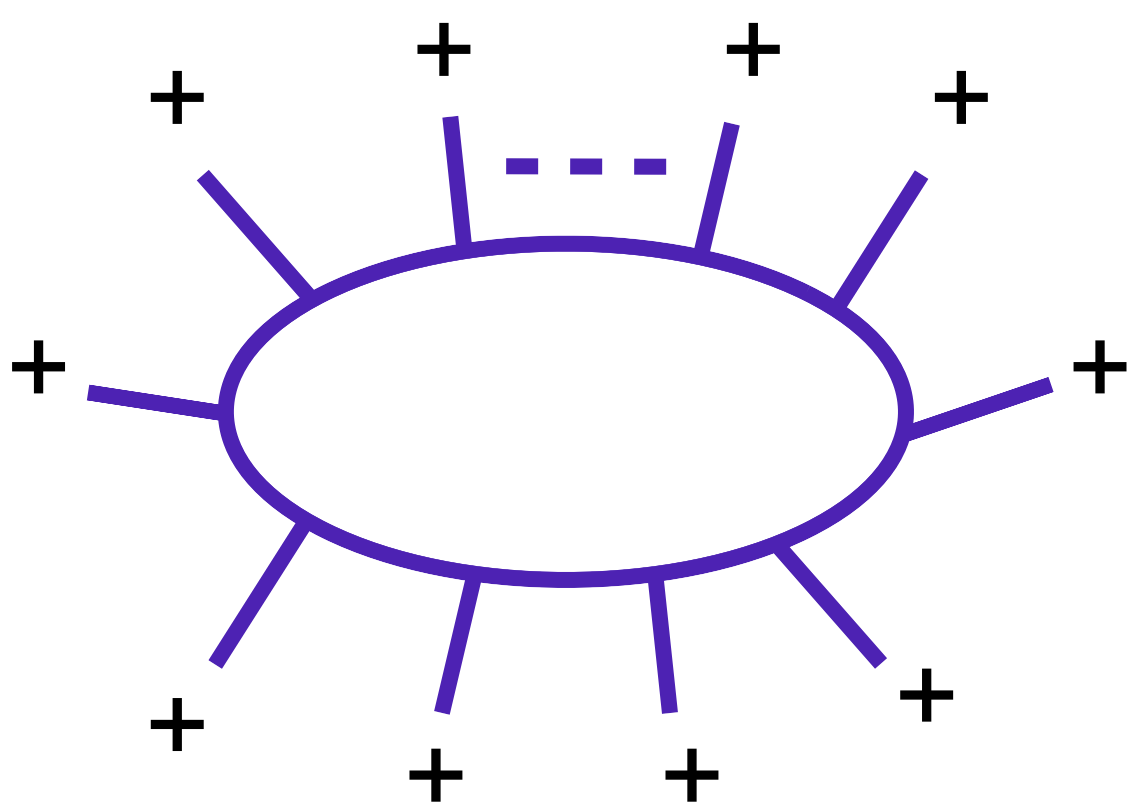


N=4 SG first time double copied from N=0 YM something new with N=0 YM amplitudes?

YES!

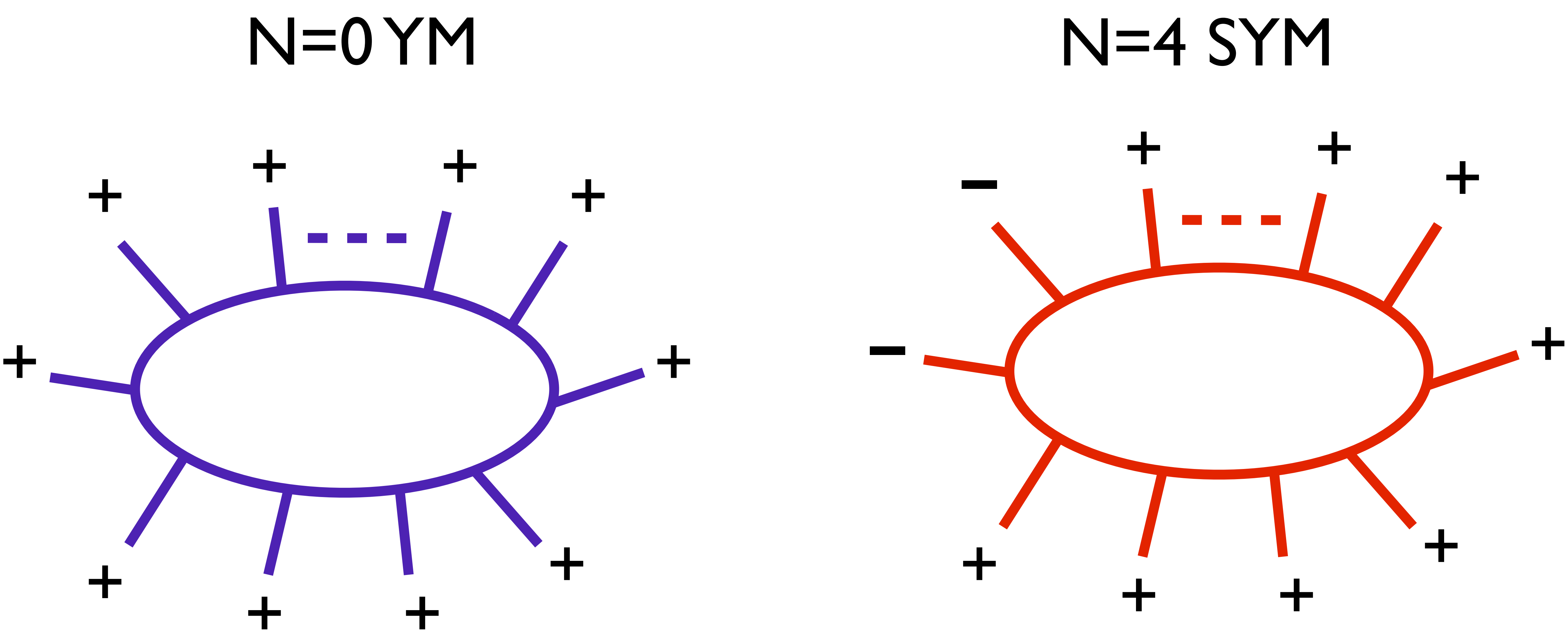


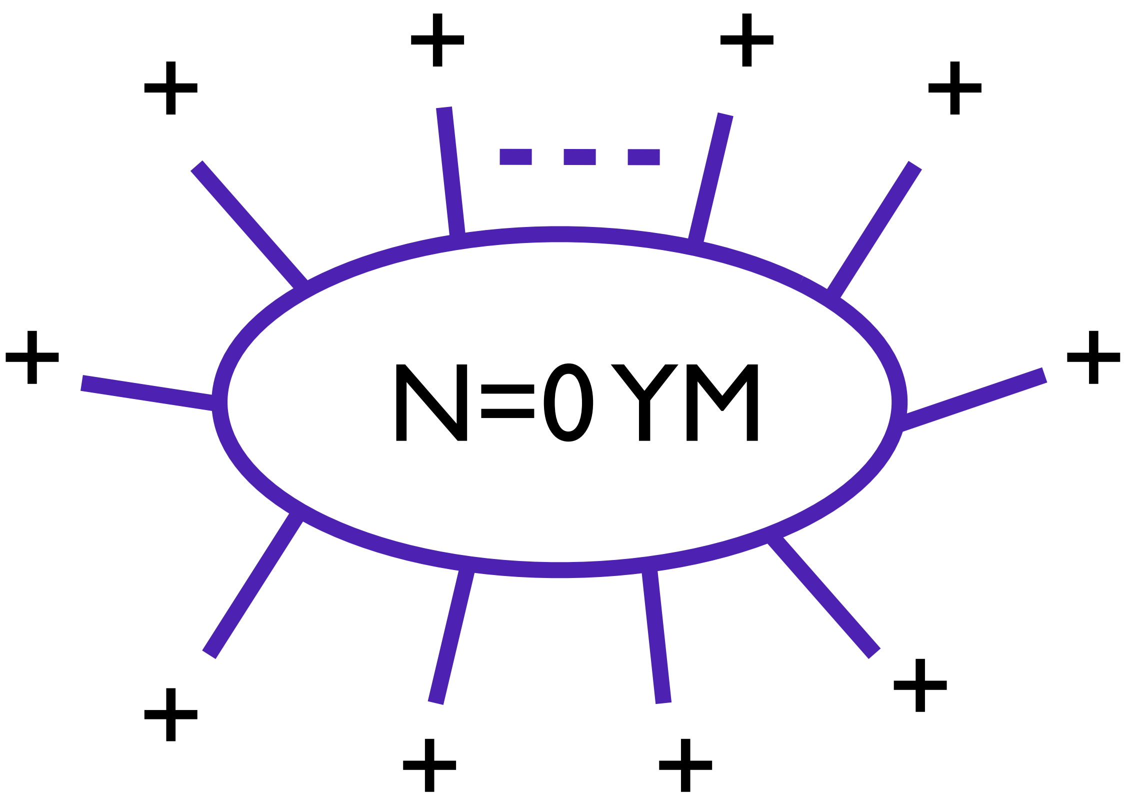
+ FRIENDS = 0

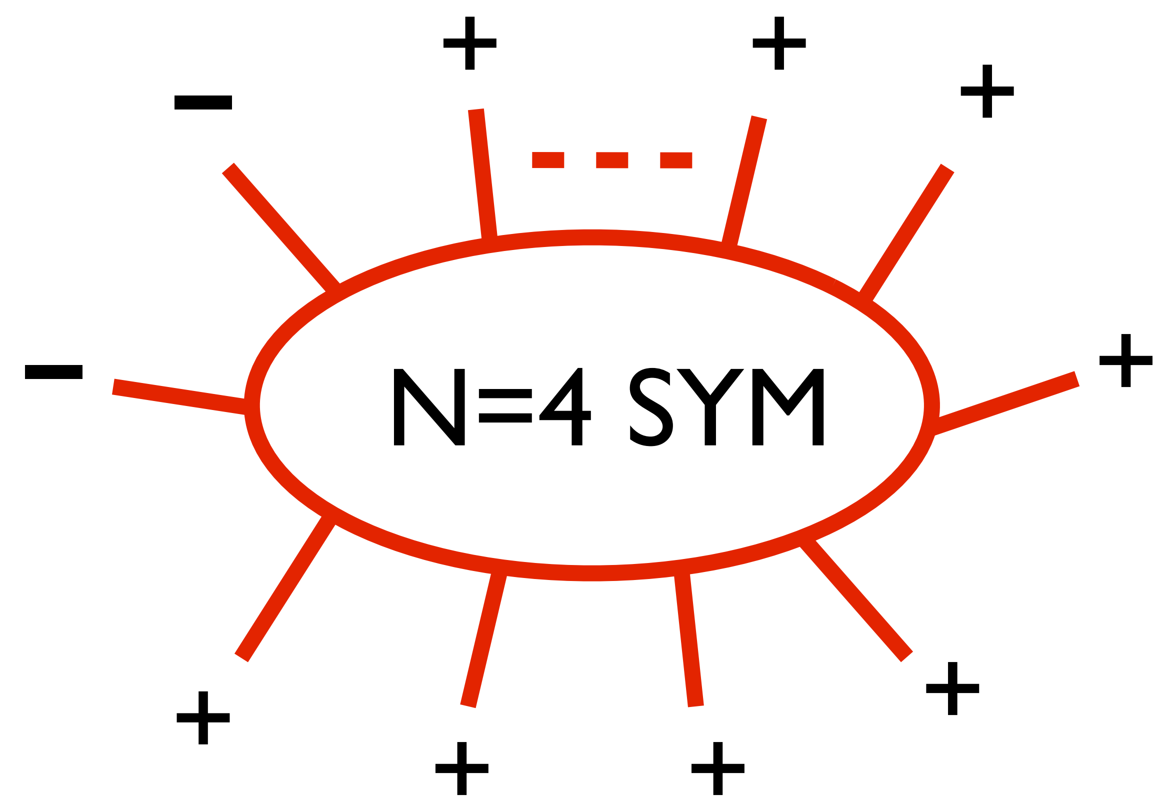


+ FRIENDS = finite contribution

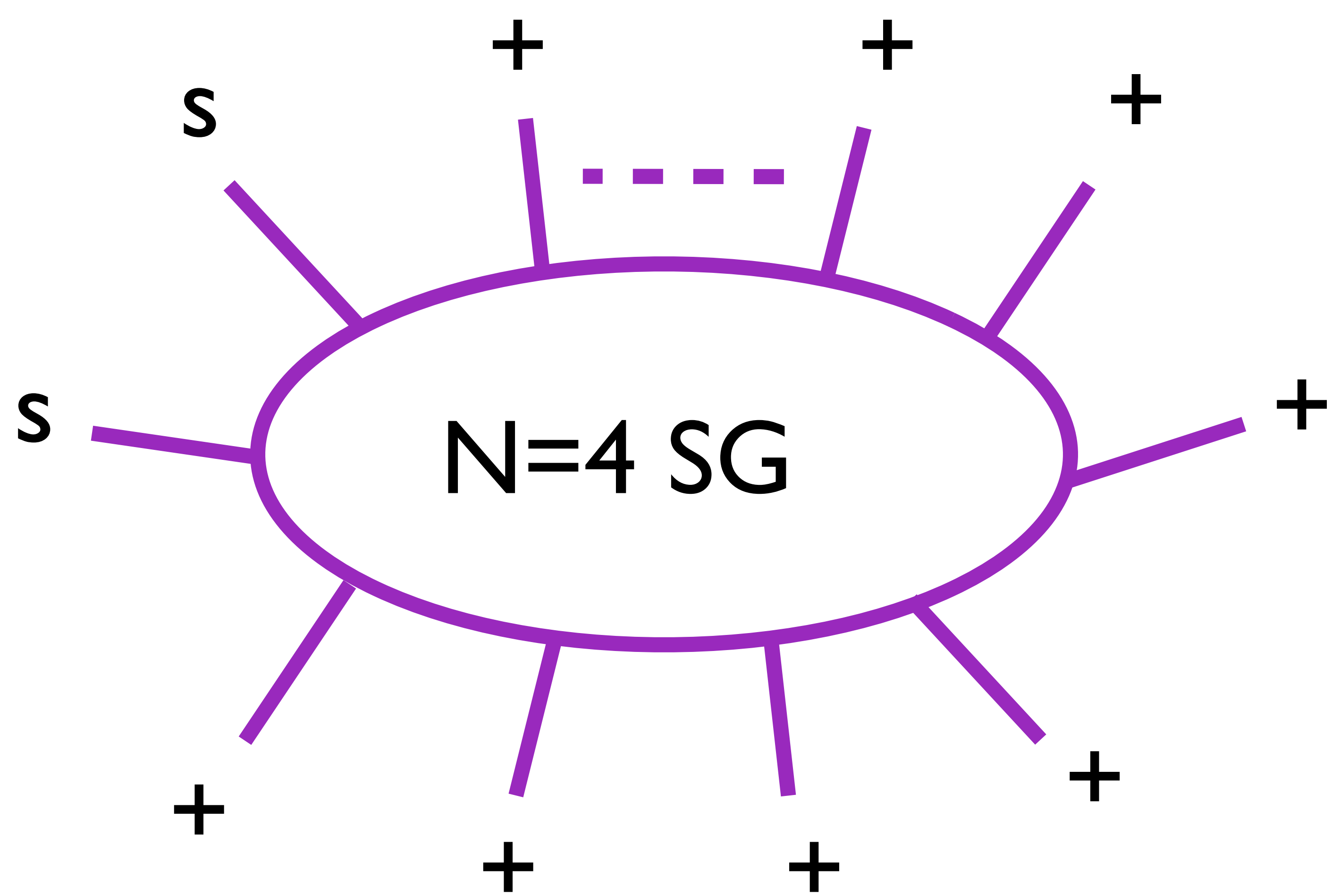
This means something new at N=4 SG

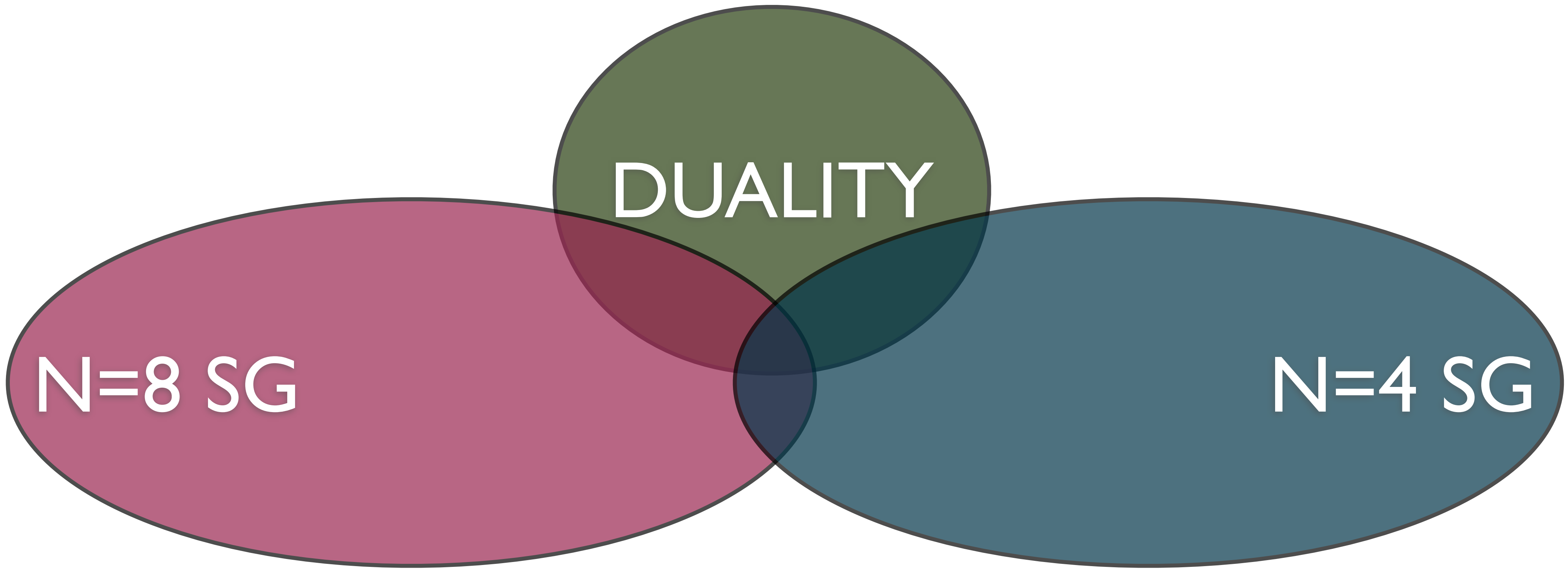




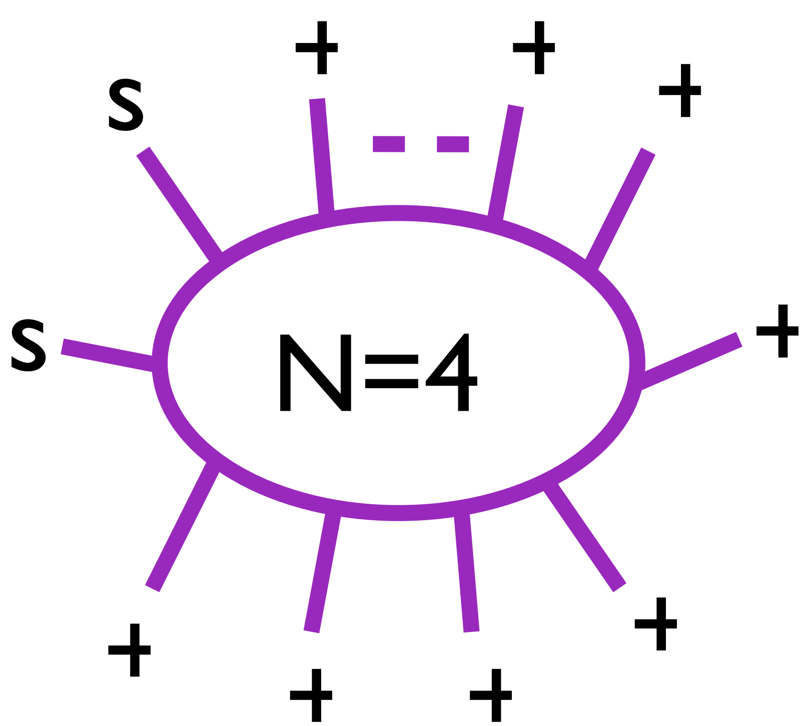


N=4 SG:

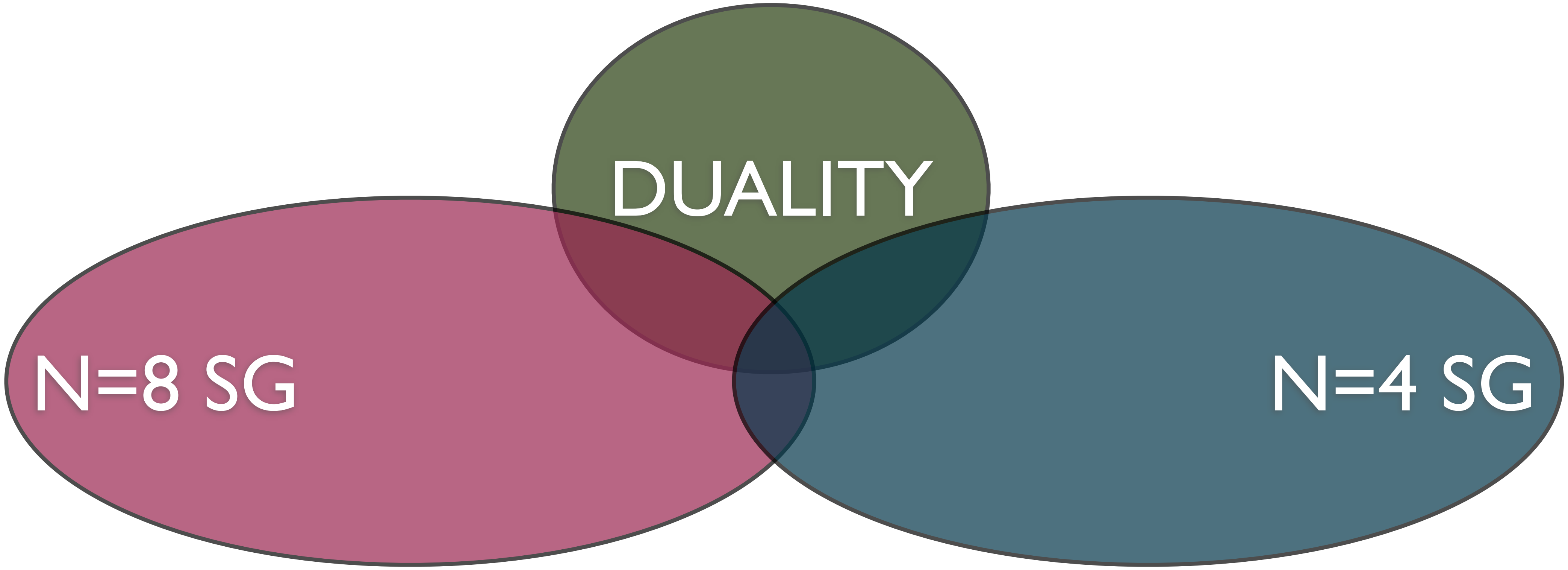




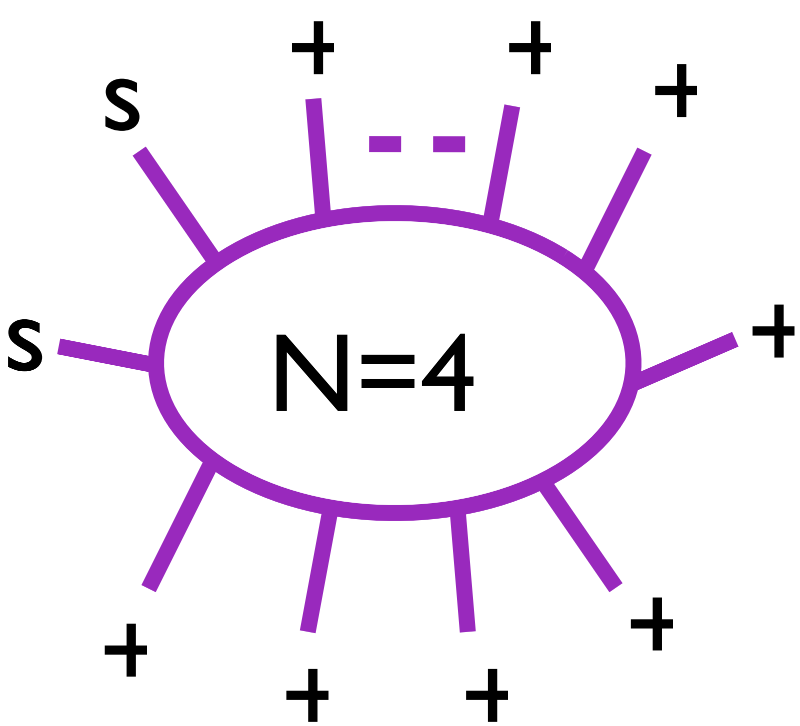
New stuff in N=4 SG



Is there a way to line this up and identify / clarify known U(1) anomaly in N=4 SG?



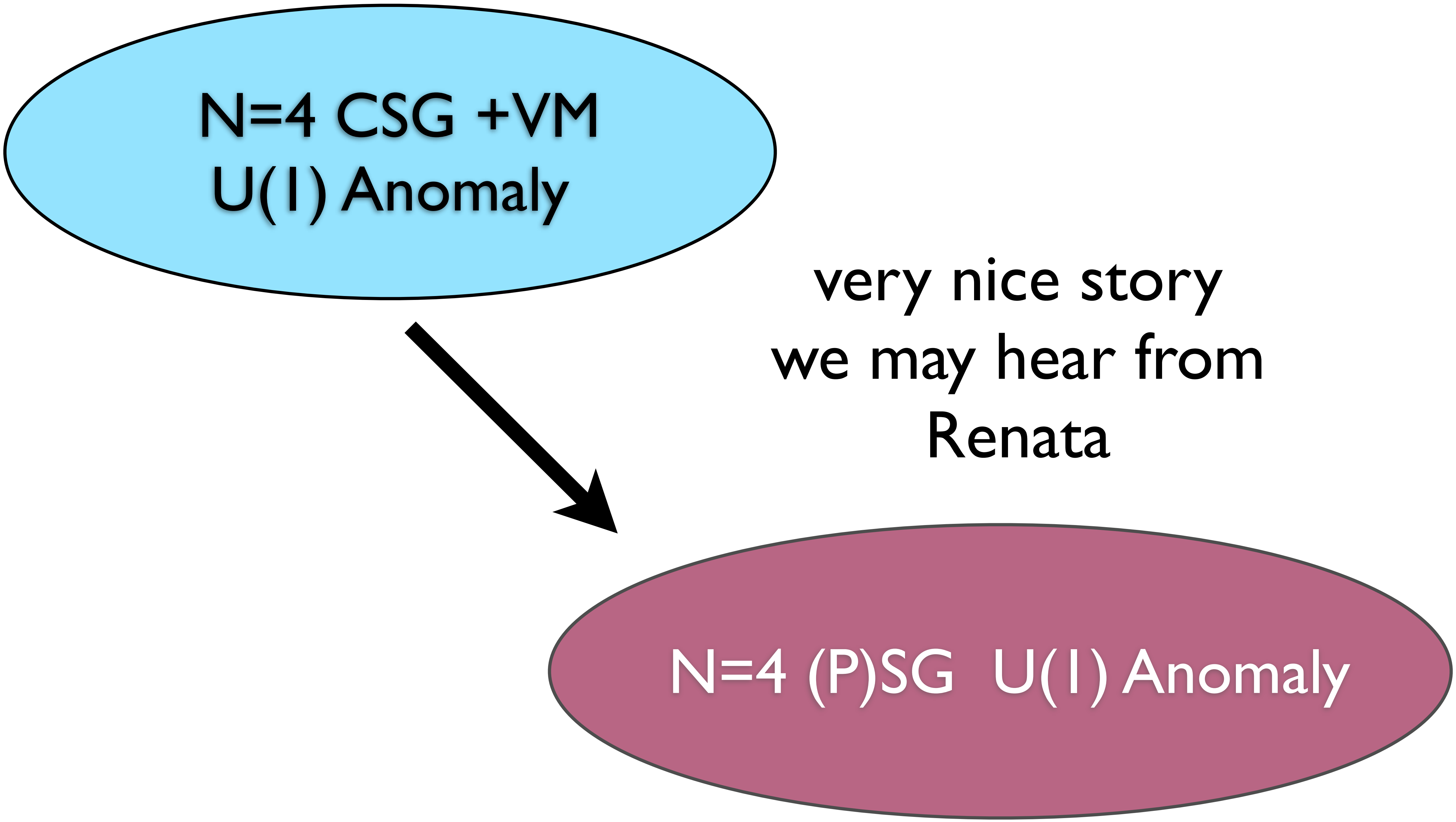
New stuff in N=4 SG



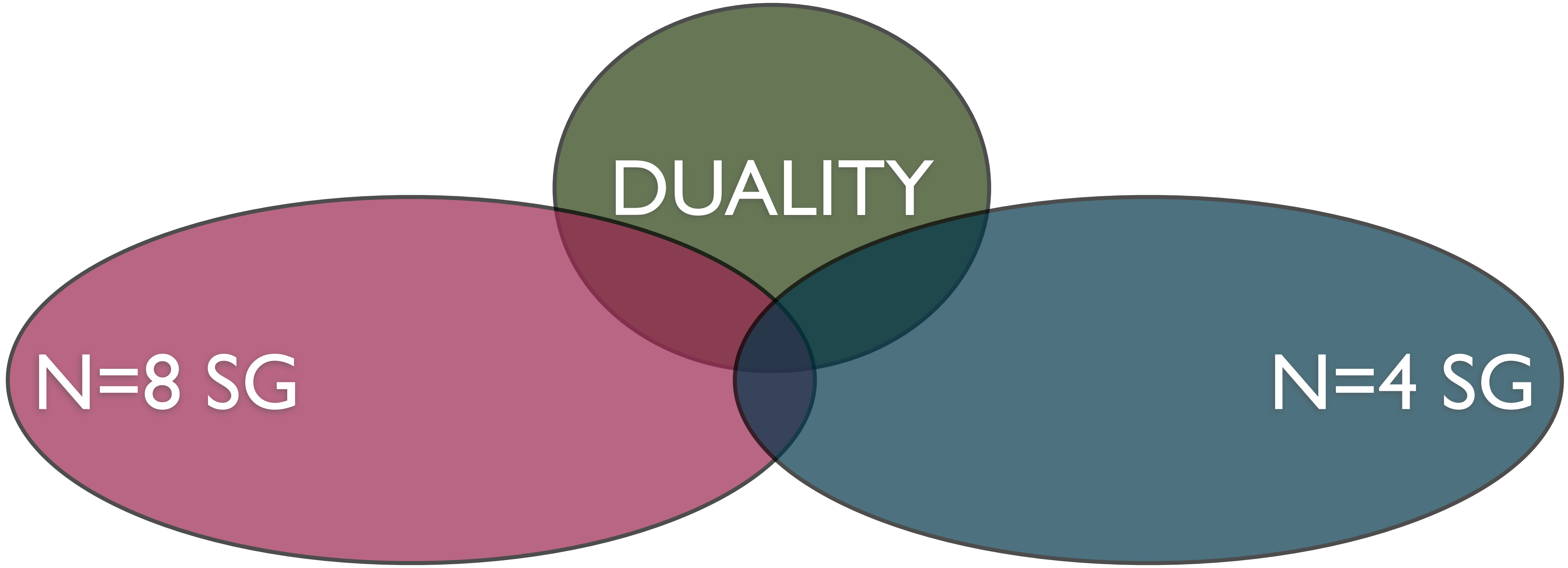
Is there a way to line this up and identify / clarify known U(1) anomaly in N=4 SG?

YES

JJMC, Kallosh, Roiban, Tseytlin



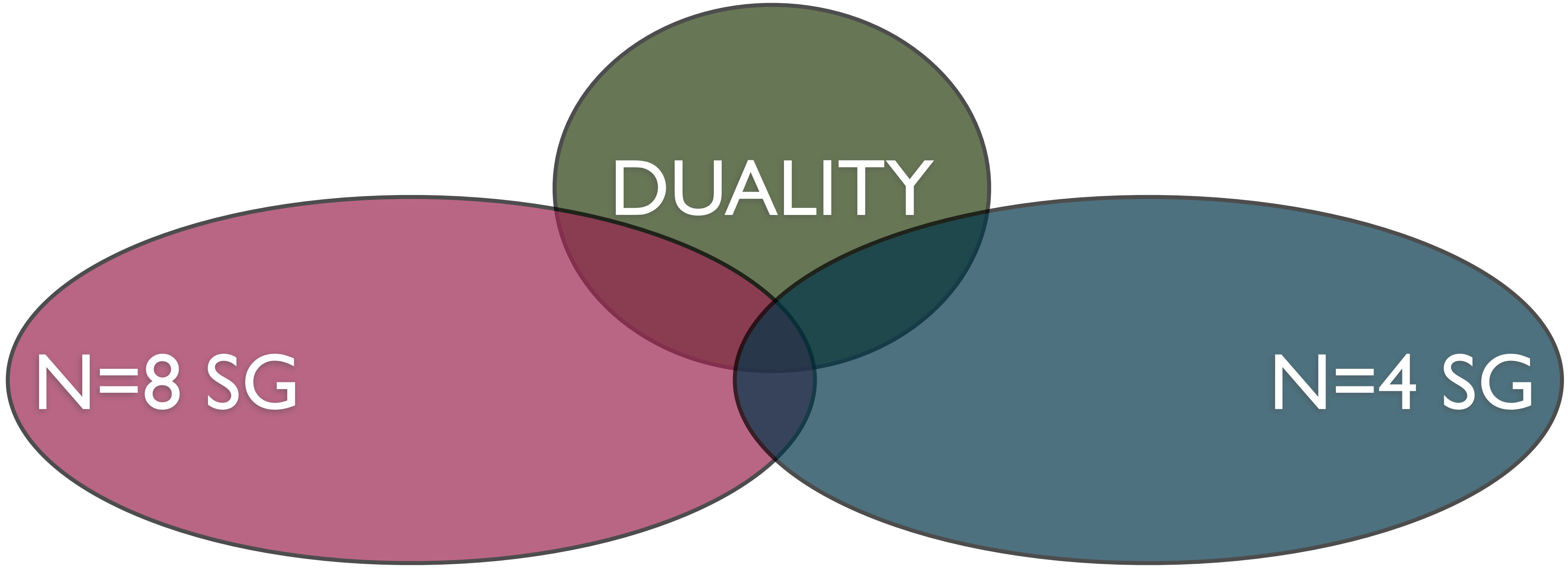
JJMC, Kallosh, Roiban, Tseytlin



N=8: $SU(8) \supset SU(4) \times SU(4) \times U(1)$.

$$
\begin{aligned}
\mathbf{1} &= (\mathbf{1},\mathbf{1})^{0} \\
\mathbf{8} &= (\mathbf{4},\mathbf{1})^{q} \oplus (\mathbf{1},\mathbf{4})^{-q} \\
\mathbf{28} &= (\mathbf{6},\mathbf{1})^{2q} \oplus (\mathbf{1},\mathbf{6})^{-2q} \oplus (\mathbf{4},\mathbf{4})^{0} \\
\mathbf{56} &= (\bar{\mathbf{4}},\mathbf{1})^{3q} \oplus (\mathbf{1},\bar{\mathbf{4}})^{-3q} \oplus (\mathbf{6},\mathbf{4})^{q} \oplus (\mathbf{4},\mathbf{6})^{-q} \\
\mathbf{70} &= (\mathbf{1},\mathbf{1})^{4q} \oplus (\mathbf{1},\mathbf{1})^{-4q} \oplus (\bar{\mathbf{4}},\mathbf{4})^{2q} \oplus (\mathbf{4},\bar{\mathbf{4}})^{-2q} \oplus (\mathbf{6},\mathbf{6})^{0} \ .
\end{aligned}
$$

JJMC, Kallosh, Roiban, Tseytlin



N=8: $SU(8) \supset SU(4) \times SU(4) \times U(1)\ .$

$$\begin{aligned}
\mathbf{1} &= (\mathbf{1},\mathbf{1})^{0} \\
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\mathbf{28} &= (\mathbf{6},\mathbf{1})^{2q} \oplus (\mathbf{1},\mathbf{6})^{-2q} \oplus (\mathbf{4},\mathbf{4})^{0} \\
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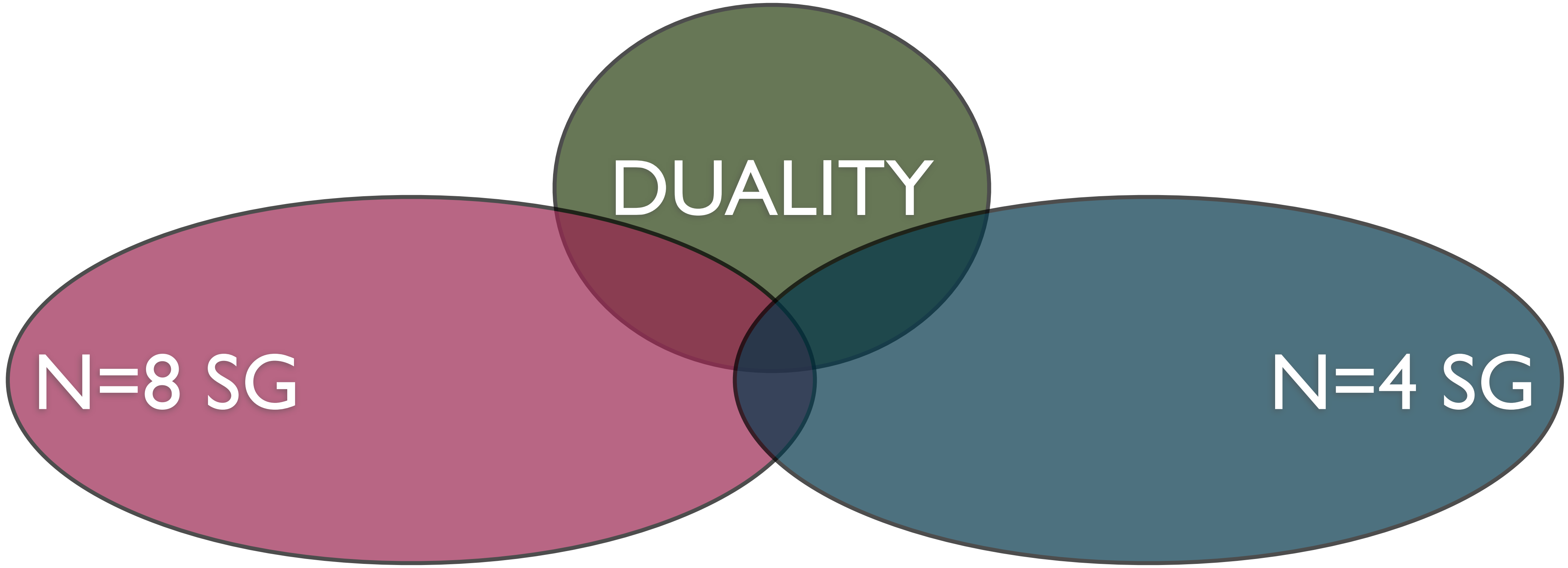
\end{aligned}$$

N=4 SG mult.

JJMC, Kallosh, Roiban, Tseytlin



N=8: $SU(8) \supset SU(4) \times SU(4) \times \boxed{U(1)}\,.$

U(1) identified w/ U(1) symmetry of N=4 SG

$$
\begin{aligned}
\mathbf{1} &= (\mathbf{1},\mathbf{1})^{0}\\
\mathbf{8} &= (\mathbf{4},\mathbf{1})^{q} \oplus (\mathbf{1},\mathbf{4})^{-q}\\
\mathbf{28} &= (\mathbf{6},\mathbf{1})^{2q} \oplus (\mathbf{1},\mathbf{6})^{-2q} \oplus (\mathbf{4},\mathbf{4})^{0}\\
\mathbf{56} &= (\bar{\mathbf{4}},\mathbf{1})^{3q} \oplus (\mathbf{1},\bar{\mathbf{4}})^{-3q} \oplus (\mathbf{6},\mathbf{4})^{q} \oplus (\mathbf{4},\mathbf{6})^{-q}\\
\mathbf{70} &= (\mathbf{1},\mathbf{1})^{4q} \oplus \boxed{(\mathbf{1},\mathbf{1})^{-4q}} \oplus (\bar{\mathbf{4}},\mathbf{4})^{2q} \oplus (\mathbf{4},\bar{\mathbf{4}})^{-2q} \oplus (\mathbf{6},\mathbf{6})^{0}\ .
\end{aligned}
$$

N=4 SG mult.

JJMC, Kallosh, Roiban, Tseytlin

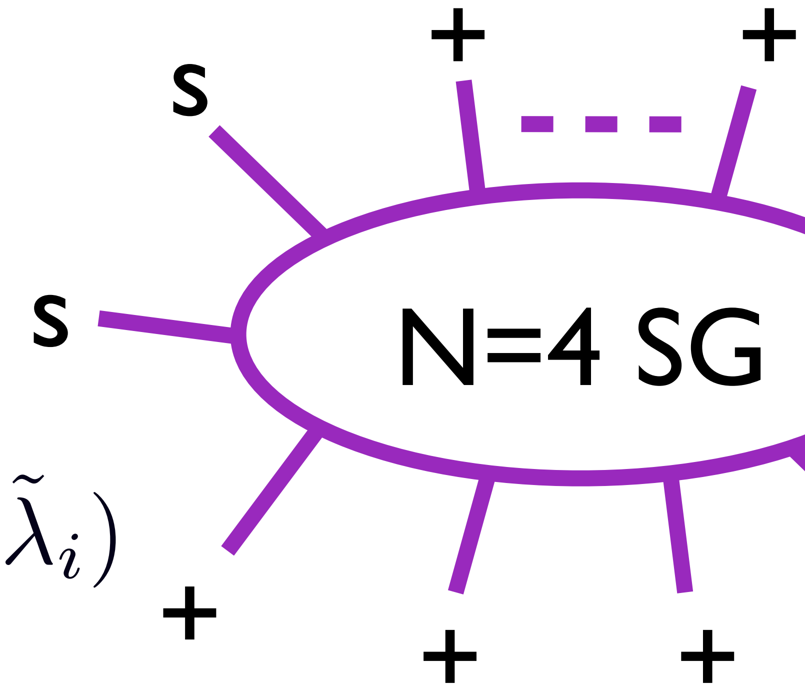
Correct choice of charge q under U(1) leads to anomalous part of effective action

$$\Gamma_{\rm an}^{\mathcal{N}=4,n_{\rm v}} = \frac{1}{2}(2+n_{\rm v})\Gamma_{\rm an}^{\mathcal{N}=4,n_{\rm v}=0} = \frac{2+n_{\rm v}}{4(4\pi)^2}\int RR^*\nabla^{-2}\nabla_\mu a^\mu$$

Amplitude considerations reproduces this exactly (n_v corresponding to including n_v scalars in N=0 YM)

JJMC, Kallosh, Roiban, Tseytlin

Preview (anomalous amplitudes):



$$\mathcal{M}_3^{(1);\mathcal{N}=4}(1,2,3) = \frac{i}{(4\pi)^2}\left(\frac{\kappa}{2}\right)^3 \delta^{(8)}(\sum_{i=1}^{3}\tilde{\eta}_i^A\tilde{\lambda}_i)$$

$$\mathcal{M}_4^{(1);\mathcal{N}=4}(1,2,3,4) = \frac{i}{(4\pi)^2}\left(\frac{\kappa}{2}\right)^4 \delta^{(8)}(\sum_{i=1}^{4}\tilde{\eta}_i^A\tilde{\lambda}_i)$$

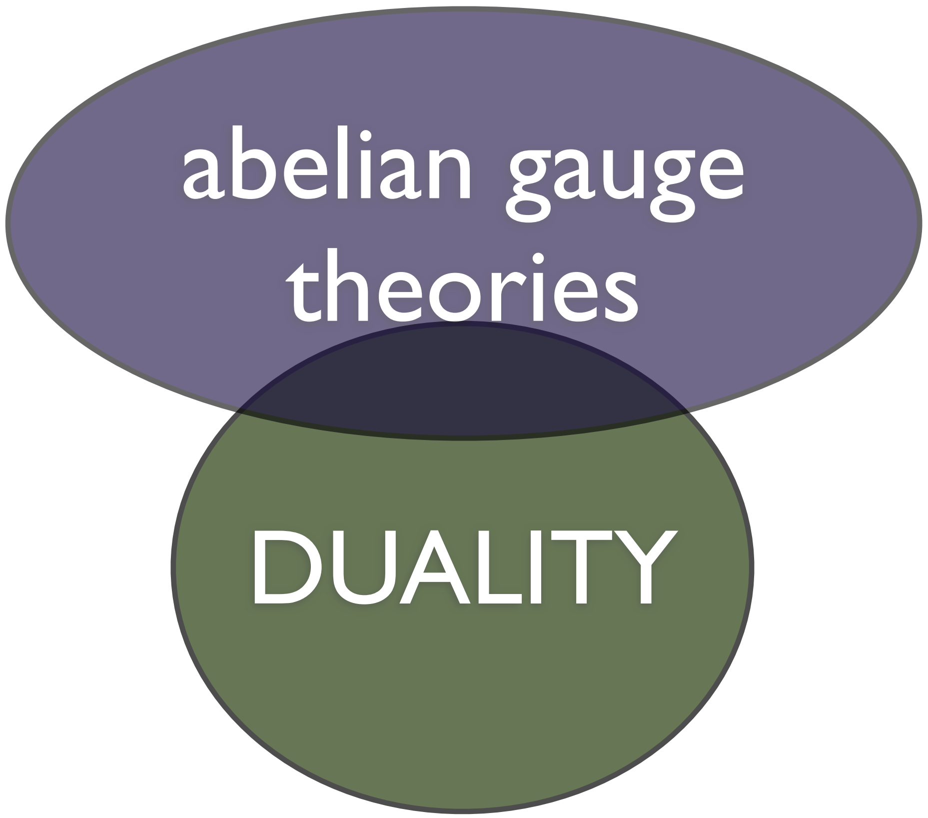
$$\mathcal{M}_5^{(1);\mathcal{N}=4}(1,2,3,4,5) = \frac{2i}{(4\pi)^2}\left(\frac{\kappa}{2}\right)^5 \delta^{(8)}(\sum_{i=1}^{5}\tilde{\eta}_i^A\tilde{\lambda}_i)$$

related by soft-limits

$$\mathcal{M}_n^{(L)}(1,2,\ldots n-1,n) \xrightarrow{k_n\to 0} \frac{\kappa}{2}\mathcal{S}_n^{(0)}\,\mathcal{M}_{n-1}^{(L)}(1,2,\ldots n-1)$$

...more very soon....

JJMC, Kallosh, Roiban, Tseytlin



2001 BIK derives nice recursive formula for N=2 "Born-Infeld" action gives to order W^10 open question if this last term satisfied duality...

JJMC, Kallosh '13

Bellucci, Ivanov, and Krivonos '01

$$\mathcal{S}_{\rm BI} = \mathcal{S}_{\rm free} + \mathcal{S}_{\rm int,10}$$

$$\begin{aligned}
\mathcal{S}_{\rm int,10} = \int \mathrm{d}^{12}\mathcal{Z}\Bigg\{ & \frac{1}{8}\mathcal{W}^2\overline{\mathcal{W}}^2\lambda + \lambda^2\left(\frac{1}{72}\mathcal{W}^3\Box\left[\overline{\mathcal{W}}^3\right] + \frac{1}{16}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right] + \frac{1}{16}\mathcal{W}^2\overline{\mathcal{W}}^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]\right) \\
& + \lambda^3\Bigg(\frac{\mathcal{W}^4\Box\left[\Box\left[\overline{\mathcal{W}}^4\right]\right]}{1152} + \frac{1}{48}\mathcal{W}^3\Box\left[\overline{\mathcal{W}}^3\mathcal{D}^4\left[\mathcal{W}^2\right]\right] + \frac{1}{48}\overline{\mathcal{W}}^3\Box\left[\mathcal{W}^3\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]\right] + \frac{1}{32}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]^2 \\
& \qquad + \frac{3}{32}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right] + \frac{1}{32}\mathcal{W}^2\overline{\mathcal{W}}^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]^2\Bigg) \\
& + \lambda^4\Bigg(-\frac{\mathcal{W}^5\Box\left[\Box\left[\Box\left[\overline{\mathcal{W}}^5\right]\right]\right]}{28800} + \frac{1}{576}\overline{\mathcal{W}}^4\Box\left[\Box\left[\mathcal{W}^4\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]\right]\right] + \frac{1}{576}\Box\left[\mathcal{W}^4\right]\Box\left[\overline{\mathcal{W}}^4\mathcal{D}^4\left[\mathcal{W}^2\right]\right] + \frac{1}{48}\mathcal{W}^3\Box\left[\overline{\mathcal{W}}^3\mathcal{D}^4\left[\mathcal{W}^2\right]^2\right] \\
& + \frac{1}{48}\overline{\mathcal{W}}^3\Box\left[\mathcal{W}^3\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]^2\right] + \frac{1}{36}\overline{\mathcal{W}}^3\Box\left[\mathcal{W}^3\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]\right]\mathcal{D}^4\left[\mathcal{W}^2\right] + \frac{1}{64}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]^3 + \frac{1}{576}\overline{\mathcal{W}}^4\Box\left[\mathcal{W}^3\right]\mathcal{D}^4\left[\Box\left[\mathcal{W}^3\right]\right] \\
& + \frac{1}{16}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right] + \frac{1}{32}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]\right]\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right] + \frac{1}{16}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]^2 \\
& + \frac{1}{64}\mathcal{W}^2\overline{\mathcal{W}}^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]^3 + \frac{1}{576}\mathcal{W}^4\Box\left[\overline{\mathcal{W}}^3\right]\overline{\mathcal{D}}^4\left[\Box\left[\overline{\mathcal{W}}^3\right]\right] + \frac{1}{32}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\right]\Bigg)\Bigg\}
\end{aligned} \tag{1.2}$$

Bellucci, Ivanov, and Krivonos '01

$\mathcal{S}_{\mathrm{BI}} = \mathcal{S}_{\mathrm{free}} + \mathcal{S}_{\mathrm{int},10}$

$$
\begin{aligned}
\mathcal{S}_{\mathrm{int},10} = \int \mathrm{d}^{12}\mathcal{Z} \Big\{ & \frac{1}{8}\mathcal{W}^2\overline{\mathcal{W}}^2\lambda + \lambda^2 \left(\frac{1}{72}\mathcal{W}^3\Box\left[\overline{\mathcal{W}}^3\right] + \frac{1}{16}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right] + \frac{1}{16}\mathcal{W}^2\overline{\mathcal{W}}^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right] \right) \\
& + \lambda^3 \left(\frac{\mathcal{W}^4\Box\left[\Box\left[\overline{\mathcal{W}}^4\right]\right]}{1152} + \frac{1}{48}\mathcal{W}^3\Box\left[\overline{\mathcal{W}}^3\mathcal{D}^4\left[\mathcal{W}^2\right]\right] + \frac{1}{48}\overline{\mathcal{W}}^3\Box\left[\mathcal{W}^3\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]\right] + \frac{1}{32}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]^2 \right. \\
& \left. + \frac{3}{32}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right] + \frac{1}{32}\mathcal{W}^2\overline{\mathcal{W}}^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]^2 \right) \\
& + \lambda^4 \left(-\frac{\mathcal{W}^5\Box\left[\Box\left[\Box\left[\overline{\mathcal{W}}^5\right]\right]\right]}{28800} + \frac{1}{576}\overline{\mathcal{W}}^4\Box\left[\Box\left[\mathcal{W}^4\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]\right]\right] + \frac{1}{576}\Box\left[\mathcal{W}^4\right]\Box\left[\overline{\mathcal{W}}^4\mathcal{D}^4\left[\mathcal{W}^2\right]\right] + \frac{1}{48}\mathcal{W}^3\Box\left[\overline{\mathcal{W}}^3\mathcal{D}^4\left[\mathcal{W}^2\right]^2\right] \right. \\
& + \frac{1}{48}\overline{\mathcal{W}}^3\Box\left[\mathcal{W}^3\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]^2\right] + \frac{1}{36}\overline{\mathcal{W}}^3\Box\left[\mathcal{W}^3\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]\right]\mathcal{D}^4\left[\mathcal{W}^2\right] + \frac{1}{64}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]^3 + \frac{1}{576}\overline{\mathcal{W}}^4\Box\left[\mathcal{W}^3\right]\mathcal{D}^4\left[\Box\left[\mathcal{W}^3\right]\right] \\
& + \frac{1}{16}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right] + \frac{1}{32}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]\right]\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right] + \frac{1}{16}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]^2 \\
& \left. + \frac{1}{64}\mathcal{W}^2\overline{\mathcal{W}}^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]^3 + \frac{1}{576}\mathcal{W}^4\Box\left[\overline{\mathcal{W}}^3\right]\overline{\mathcal{D}}^4\left[\Box\left[\overline{\mathcal{W}}^3\right]\right] + \frac{1}{32}\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\right] \right) \Big\} \qquad (1.2)
\end{aligned}
$$

Broedel, JJMC, Ferrara, Kallosh, Roiban '12

duality satisfied if

$$\int \mathrm{d}^8\mathcal{Z}\left(\mathcal{W}^2+\mathcal{M}^2\right)-\int \mathrm{d}^8\overline{\mathcal{Z}}\left(\overline{\mathcal{W}}^2+\overline{\mathcal{M}}^2\right)=0$$

$$\mathrm{i}\,\mathcal{M}\;\equiv\;4\,\frac{\mathrm{d}}{\mathrm{d}\mathcal{W}}\,\mathcal{S}[\mathcal{W},\overline{\mathcal{W}}].$$

$$0=\frac{1}{3}I_2+\frac{1}{36}I_{3a}+\frac{1}{2}I_{3b}+2I_{3c}+\frac{1}{720}I_{4a}+\frac{1}{12}I_{4b}+\frac{1}{12}I_{4c}$$
$$+\frac{1}{6}I_{4d}+\frac{1}{6}I_{4e}+\frac{1}{18}I_{4f}+\frac{1}{18}I_{4g}+\frac{1}{18}I_{4h}+\frac{1}{2}I_{4i}+\frac{1}{2}I_{4j}$$

JJMC, Kallosh '13

somewhat tedious, but straightforward integration by parts identities shows all terms vanishes....

$$I_{4i} = \lambda^4 \int d^{12}\mathcal{Z} \Big(+7\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right] - 4\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\mathcal{D}^4\left[\mathcal{W}^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\right]\right]$$

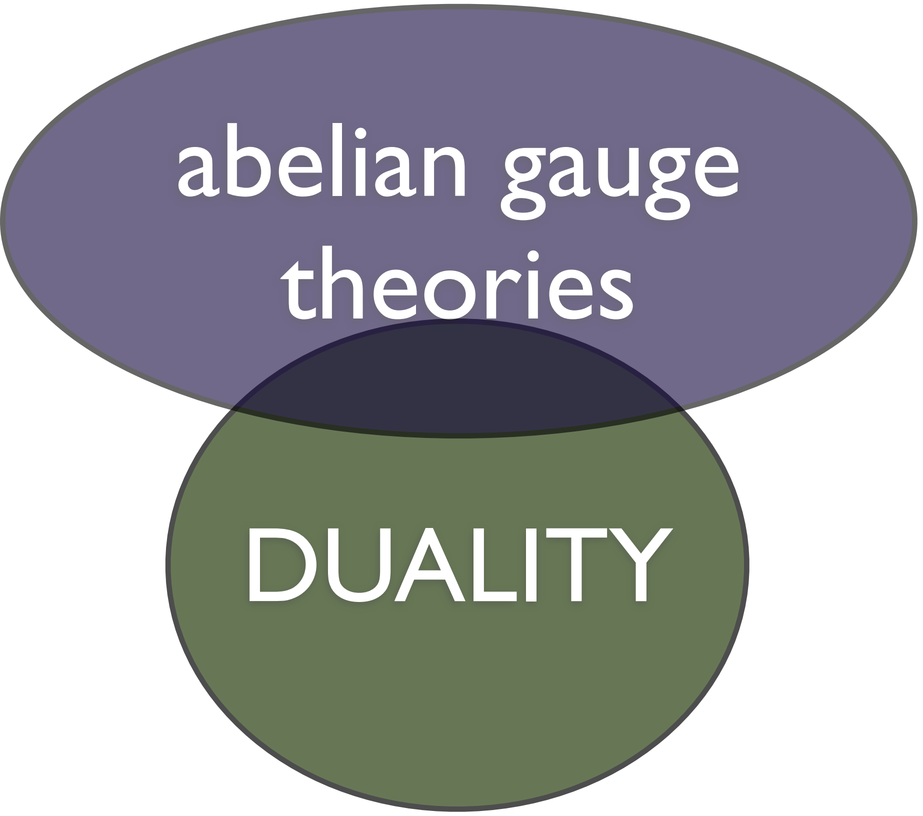
$$+3\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\right]\right] - 3\mathcal{W}^2\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]\right]$$

$$-3\mathcal{W}^2\overline{\mathcal{W}}^2\overline{\mathcal{D}}^4\left[\overline{\mathcal{W}}^2\mathcal{D}^4\left[\mathcal{W}^2\right]^2\right]\Big)$$

+ 13 others

exists much more satisfying constructive proof...

JJMC, Kallosh '13



find duality-conserving sources of deformation...

JJMC, Kallosh '13

through W^8

abelian gauge theories

DUALITY

$$\mathcal{I}(T^-,\overline{T}^+) = \int \mathrm{d}^{12}\mathcal{Z}\Big(\lambda\, a_0\, (T^-)^2(\overline{T}^+)^2 + \lambda^2\, a_1\, (T^-)^3\Box(\overline{T}^+)^3 + \lambda^3\, a_2 (T^-)^4\Box^2(\overline{T}^+)^4 + \lambda^3\, a_3\, (T^-)^2(\overline{T}^+)^2\overline{\mathcal{D}}^4((T^-)^2)\mathcal{D}^4((\overline{T}^+)^2) + \mathcal{O}(\lambda^4)\Big)$$

Broedel, JJMC, Ferrara, Kallosh, Roiban '12

$$\mathcal{I}_{\lambda^4}(T^-,\overline{T}^+) = \lambda^4 \int \mathrm{d}^{12}\mathcal{Z}\Big(a_4\, (T^-)^5\Box^3(\overline{T}^+)^5 + a_5\, (T^-)^3\mathcal{D}^4((\overline{T}^+)^2)\Box(\overline{T}^+)^3\overline{\mathcal{D}}^4((T^-)^2)\Big)$$

JJMC, Kallosh '13

ASK THE RIGHT QUESTIONS

when in doubt, calculate

beauty that trivializes calculations is very special