CONSTRUCTION OF EFFECTIVE ACTION FOR $\mathcal{N}=2$, $d=3$ SUPERSIMMETRIC FIELD THEORIES

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Aims

- Development of background field method for three dimensional extended supersymmetric gauge models formulated in terms of $\mathcal{N}=2$ superfields.
- Developing a general procedure for calculating the effective action for such theories preserving manifest gauge invariance and manifest $\mathcal{N}=2$ supersymmetry.
- Study of the one-loop superfield effective action for super Yang-Mills and super Chern-Simons theories.

I.L.B, N.G. Pletnev, I.B. Samsonov, JHEP 1004, 124 (2010);
I.L.B, N.G. Pletnev and I.B. Samsonov, JHEP 1101, 121 (2011);
I.L.B, N.G. Pletnev, I.B. Samsonov, JHEP 1111, 085 (2011).

- Studying a quantum structure of Bagger-Lambert-Gustavsson (BLG) theory ($d = 3, \mathcal{N} = 8$) (J. Bagger, N. Lambert, Phys. Rev. 2007; 2008; JHEP 2008; A. Gustavsson, JHEP 2008) and Aharony-Bergman-Jefferis-Maldacena (ABJM) theory ($d = 3, \mathcal{N} = 6$) (O. Aharony, O. Bergman, D.L. Jafferis, J. Maldacena, JHEP 2008). In both theories the vector fields dynamics is described by Chern-Simons action.
- Quantization of the BLG and ABJM theories can shed some light on quantum dynamics of multiple M2 branes. If one of the scalars in BLG or ABJM theory develops non-vanishing vev, the M2 brane turns into D2 brane where the vector field dynamics is described by Yang-Mills action.
- M2 brane can be considered in general as the infrared limit (at strong coupling) of D2 brane. One can hope that study of effective action in supergauge models related to D2 branes should help to understand some quantum aspects of M2 branes.
- Low-energy quantum dynamics of $d = 3$ extended supersymmetric Yang-Mills and Chern-Simons theories can be interesting itself.

Why $\mathcal{N}=2$ superfield formulation

- The SUSY models can be formulated in terms of bosonic and fermionic component fields. Supersymmetry is hidden. It is not very convenient in quantum field theory: very many Feynman diagrams, miraculous cancelations.
- Superfield formulation: manifest SUSY, comparatively small number of supergraphs, origin of miraculous cancelations. Problem: how to formulate the of $\mathcal{N}$-extended SUSY models in terms of unconstrained $\mathcal{N}$-extended superfields. General solution for all d and $\mathcal{N}$ is unknown.
- It is possible to formulate the $d=3$ extended SUSY models in terms of $\mathcal{N}=1$ superfields, in terms of $\mathcal{N}=2$ superfields and in terms of $\mathcal{N}=3$ harmonic superfields.
- Quantum aspects of $\mathcal{N}=3$, $d=3$ harmonic superspace formulation are not well developed. $\mathcal{N}=2$ superfield formulation of $d=3$ SUSY theories is analogous to $\mathcal{N}=1$ formulation of $d=4$ SUSY theories, which is very well developed.

Contents

$d = 3, \mathcal{N} = 2$ superspace coordinates z^M:

$$\{x^m, \theta^\alpha, \bar{\theta}^\alpha\}, \qquad m = 0, 1, 2; \alpha = 1, 2$$

Supergauge covariant derivatives ∇_M:

$$\nabla_\alpha = D_\alpha + A_\alpha \,, \qquad \bar{\nabla}_\alpha = \bar{D}_\alpha + \bar{A}_\alpha, \qquad \nabla_m = \partial_m + A_m$$

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\bar{\theta}^\beta \partial_{\alpha\beta}, \qquad \bar{D}_\alpha = -\frac{\partial}{\partial\bar{\theta}_\alpha} - i\theta^\beta \partial_{\alpha\beta}$$

Notations:

$$\gamma^0 = -i\sigma_2, \gamma^1 = \sigma_3, \gamma^3 = \sigma_1$$

$$\partial_{\alpha\beta} = (\gamma^m)_{\alpha\beta}\partial_m$$

Basic superfield strengths $G, W_\alpha, \bar{W}_\alpha, \mathcal{F}_{mn}$:

$$\{\nabla_\alpha, \bar{\nabla}_\beta\} = -2i(\gamma^m)_{\alpha\beta}\nabla_m + 2i\epsilon_{\alpha\beta}G$$

$$[\nabla_\alpha, \nabla_m] = -(\gamma^m)_{\alpha\beta}\bar{W}^\beta, \qquad [\bar{\nabla}_\alpha, \nabla_m] = (\gamma^m)_{\alpha\beta}W^\beta$$

$$[\nabla_m, \nabla_n] = i\mathcal{F}_{mn}$$

Constraints:

$$\nabla^\alpha\nabla_\alpha G = 0, \qquad \bar{\nabla}^\alpha\bar{\nabla}_\alpha G = 0, \qquad \nabla_\alpha\bar{W}_\beta = 0, \qquad \bar{\nabla}_\alpha W_\beta = 0$$

$$\nabla^\alpha W_\alpha = \bar{\nabla}^\alpha\bar{W}_\alpha$$

Solution to the constraints (in chiral representation):

$$\nabla_\alpha = e^{-2V} D_\alpha e^{2V}, \qquad \bar{\nabla}_\alpha = \bar{D}_\alpha, \qquad V^\dagger = V$$

Superfield potential V takes the value in the Lie algebra of a gauge group.

$$G = \frac{i}{4}\bar{D}^\alpha(e^{-2V} D_\alpha e^{2V}), \bar{W}_\alpha = \frac{i}{4}\nabla_\alpha \bar{D}^\beta(e^{-2V} D_\beta e^{2V}), W_\alpha = -\frac{i}{4}\bar{D}^2(e^{-2V} D_\alpha e^{2V})$$

Gauge transformations:

$$e^{2V'} = e^{i\bar{\lambda}} e^{2V} e^{-\lambda}$$

λ and $\bar{\lambda}$ are the chiral and antichiral superfield gauge parameters

- Action of $d = 3, \mathcal{N} = 2$ super Yang-Mills theory

$$S_{SYM}[V] = \frac{1}{g^2} \int d^3x d^4\theta \text{tr} G^2 = -\frac{1}{g^2} \int d^3x d^2\theta \text{tr} W^\alpha W_\alpha$$

g is a dimensional coupling constant, $[g] = \frac{1}{2}$
On-shell field contents: real vector, real scalar, complex spinor.

- Action of $\mathcal{N} = 2$ super Chern-Simons theory (E.A. Ivanov, 1991)

$$S_{\text{SCS}} = \frac{ik}{8\pi} \text{tr} \int_0^1 dt \int d^3x d^4\theta \, \bar{D}^\alpha (e^{-2tV} D_\alpha e^{2tV}) e^{-2tV} \partial_t e^{2tV} \,.$$

t is an auxiliary real parameter and k is an integer (Chern-Simons level).
Variation of super Chern-Simoins action does not depend on t.
Sum of SYM and SCS action describes the topologically massive supersymmetric gauge model. In Abelian case the equations of motion have the form in terms of superfield G

$$(\Box + m^2) G = 0 \,, \qquad m^2 = \frac{k^2 g^4}{4\pi^2} \,.$$

- Action of $d = 3, \mathcal{N} = 2$ matter

$$S_{Matter} = -\frac{1}{2}\int d^3x d^4\theta \bar{\Phi} e^{2V}\Phi$$

$\Phi, \bar{\Phi}$ are the chiral and antichiral $\mathcal{N} = 2$ superfelds.
On-shell field contents: complex scalar, complex spinor

- Gauge transformations

$$\Phi' = e^{i\lambda}\Phi, \bar{\Phi}' = \bar{\Phi}e^{-i\bar{\lambda}}, e^{2V'} = e^{i\bar{\lambda}}e^{2V}e^{-i\lambda}$$

The superfield gauge parameters take the values in the Lie algebra of the gauge group.

Background field method

Aim: construction of quantum effective action which is gauge invariant under the classical gauge transformations (B.S. DeWitt).

Motivations

- Quantization of gauge theories assumes imposing the gauge fixing conditions. As a result, the basic object of quantum field theory - effective action is not gauge invariant. The attractive features of gauge theories related with gauge invariance are hidden.
- Background field method is a special quantization procedure preserving the gauge invariance of quantum effective action under the classical gauge transformations.
- Generic idea of background field method: separating the initial gauge field into classical and quantum fields (background-quantum splitting) and imposing the special background field dependent gauge fixing conditions only on quantum field.
- Realization of background field method in each concrete theory is some art.

- Background quantum splitting
  Initial field V is spitted into background field V and quantum field v by the rule

$$e^{2V} \to e^{2V} e^{2gv}$$

- Background gauge transformations

$$e^{2V} \to e^{i\bar{\lambda}} e^{2V} e^{-i\lambda}, \qquad e^{2gv} \to e^{i\tau} e^{2gv} e^{-i\tau}$$

  τ is a real superfield parameter

- Quantum gauge transformations

$$e^{2V} \to e^{2V}, \qquad e^{2gv} \to e^{i\bar{\lambda}} e^{2gv} e^{-i\lambda}$$

- Imposing the gauge fixing only on quantum fields
  Gauge fixing functions:

$$f = i\bar{\mathcal{D}}^2 v, \qquad \bar{f} = i\mathcal{D}^2 v$$

$$\mathcal{D}_\alpha = e^{-2V} D_\alpha e^{2V}, \qquad \bar{\mathcal{D}}_\alpha = \bar{D}_\alpha$$

Effective action is constructed on the base of Faddeev-Popov anzatz
One-loop effective action $\Gamma^{(1)}_{SYM}$

$$e^{i\Gamma^{(1)}_{SYM}[V]} = e^{iS_{SYM}[V]} \int \mathcal{D}v\mathcal{D}b\mathcal{D}c\mathcal{D}\phi e^{iS_v[v,V]+iS_{FP}[b,c,V]+iS_{NK}[\phi,V]}$$

$$\Gamma^{(1)}_{SYM}[V] = S_{SYM}[V] + \tilde{\Gamma}^{(1)}[V]$$

$$\tilde{\Gamma}^{(1)}[V] = \Gamma^{(1)}_{\mathrm{v}}[V] + \Gamma^{(1)}_{\mathrm{ghosts}}[V]$$

$$\Gamma^{(1)}_{\mathrm{v}}[V] = \frac{i}{2}\mathrm{Tr}_v \ln\Box_v, \qquad \Gamma^{(1)}_{\mathrm{ghosts}}[V] = -\frac{3i}{2}\mathrm{Tr}_+ \ln\Box_+$$

$\Box_v$ is the covariant d'Alambertian operator in space of real superfields

$$\Box_v = \mathcal{D}^m\mathcal{D}_m + G^2 + iW^\alpha\mathcal{D}_\alpha - i\bar{W}^\alpha\bar{\mathcal{D}}_\alpha$$

$\Box_+$ is the covariant d'Alambertian operator in space of covariantly chiral superfields

$$\Box_+ = \mathcal{D}^m\mathcal{D}_m + G^2 + \frac{i}{2}(\mathcal{D}^\alpha W_\alpha) + \frac{i}{2}W^\alpha\mathcal{D}^\alpha$$

Calculating the low-energy effective action for the theory with gauge group $SU(N)$ spontaneously broken down to maximal Abelian subgroup $U(1)^{N-1}$

- Lie algebra $su(N)$ consists of Hermitian traceless matrices
- Any element v of $su(N)$ can be represented by the decomposition over the Cartan-Weyl basis in $gl(N)$ algebra

$$(e_{IJ})_{LK} = \delta_{IL}\delta_{JK}, \qquad v = \sum_{I<J}^{N}(v_{IJ}e_{IJ} + \bar{v}_{IJ}e_{JI}) + \sum_{I=1}^{N} v_I e_{II},$$

$$\bar{v}_I = v_I\,, \quad \sum_{I=1}^{N} v_I = 0.$$

- Background superfield V belongs to the Cartan subalgebra spanned by the basis elements e_{II}

$$V = \sum_{I=1}^{N} \mathbf{V}_I e_{II} = \mathrm{diag}(\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_N), \quad \bar{\mathbf{V}}_I = \mathbf{V}_I\,, \quad \sum_{I=1}^{N} \mathbf{V}_I = 0.$$

Specification of background field

- Slowly varying background: all space-time derivatives of background superfield are neglected
- Background superfield satisfies the classical equations of motion

Effective action is computed with help of superfield proper-time technique in terms of kernel $K(z,z'|s)$

Proper-time technique (V.A. Fock, 1938; J. Schwinger, 1951; B.S. DeWitt, 1964; superfield formulaiton I.L.B, S.M. Kuzenko, Ideas and Methods of Supersymmetry and Supergravity, 1995, 1998)

$$\Gamma^{(1)} \sim \mathrm{Tr}\ln\mathcal{H} \sim \int_0^\infty \frac{ds}{s}\int d^7z \mathrm{tr} K(z,z'|s)|_{z'=z}$$

$$i\frac{\partial K}{\partial s} = -\mathcal{H}K, \qquad K(z,z's)|_{s=0} = \delta$$

The kernels K_v and K_{ghosts}, corresponding to the effective actions $\Gamma^{(1)}_v[V]$ and $\Gamma^{(1)}_{ghosts}[V]$ are exactly found for the background under consideration

Final result for the effective action

$$\tilde{\Gamma}^{(1)}[V] = \Gamma^{(1)}_{\rm v}[V] + \Gamma^{(1)}_{\rm ghosts}[V]$$

$$\Gamma^{(1)}_{\rm v} = -\frac{1}{\pi}\sum_{I<J}^{N}\int d^3x d^4\theta \int_0^\infty \frac{ds}{s\sqrt{i\pi s}} \frac{\mathbf{W}^2_{IJ}\bar{\mathbf{W}}^2_{IJ}}{\mathbf{B}^3_{IJ}} e^{is\mathbf{G}^2_{IJ}} \tanh\frac{s\mathbf{B}_{IJ}}{2}\sinh^2\frac{s\mathbf{B}_{IJ}}{2}$$

$$\Gamma^{(1)}_{\rm ghosts} = -\frac{3}{2\pi}\sum_{I<J}^{N}\int d^3x d^4\theta \Big[\mathbf{G}_{IJ}\ln\mathbf{G}_{IJ}$$

$$+\frac{1}{4}\int_0^\infty \frac{ds}{\sqrt{i\pi s}} e^{is\mathbf{G}^2_{IJ}} \frac{\mathbf{W}^2_{IJ}\bar{\mathbf{W}}^2_{IJ}}{\mathbf{B}^2_{IJ}}\left(\frac{\tanh(s\mathbf{B}_{IJ}/2)}{s\mathbf{B}_{IJ}/2} - 1\right)\Big],$$

Notations:

$$\mathbf{B}_{IJ} = \frac{1}{2}D_\alpha \mathbf{W}^\beta_{IJ} D_\beta \mathbf{W}^\alpha_{IJ}, \qquad \mathbf{G}_{IJ} = \mathbf{G}_I - \mathbf{G}_J, \qquad \mathbf{W}^\alpha_{IJ} = \mathbf{W}^\alpha_I - \mathbf{W}^\alpha_J$$

$$\mathbf{G}_I = \frac{i}{2}\bar{D}^\alpha D_\alpha \mathbf{V}_I, \qquad \mathbf{W}^\alpha_I = \bar{D}^\alpha \bar{G}_I$$

Only leading term $\mathbf{G}\ln\mathbf{G}$ was calculated before (J. De Boer et al, 1997).

Calculating the low-energy effective action for the theory with minimal gauge symmetry breaking $SU(N) \to SU(N-1) \times U(1)$

Background superfield

$$V = \frac{1}{N}\mathrm{diag}\left((N-1)\mathbf{V}, \underbrace{-\mathbf{V}, \dots, -\mathbf{V}}_{N-1}\right)$$

The computations of effective action are analogous to previous.

The leading low-energy terms in effective action:

$$\tilde{\Gamma}^{(1)}[V] = -\frac{3(N-1)}{2\pi}\int d^3x d^4\theta\,\mathbf{G}\ln\mathbf{G} + \frac{9(N-1)}{128\pi}\int d^3x d^4\theta \frac{\mathbf{W}^2\bar{\mathbf{W}}^2}{\mathbf{G}^5} + \dots.$$

- First term is manifestly $\mathcal{N}=2$ supersymmetric (and superconformal) generalization of Maxwell F^2 term. In bosonic sector it gives $\frac{F^2}{\phi}$.
- Second term is manifestly $\mathcal{N}=2$ supersymmetric generalization of F^4 term. In bosonic sector it gives $\frac{F^4}{\phi^5}$.
- The dots stand for higher order terms in F

Classical action of $\mathcal{N}=4$ SYM theory in terms of $d=3, \mathcal{N}=2$ superfields

$$S_{\mathcal{N}=4} = \frac{1}{g^2}\int d^3x d^4\theta\,\mathrm{tr}[G^2 - \frac{1}{2}e^{-2V}\bar{\Phi}e^{2V}\Phi]$$

Φ is the chiral superfield in the adjoint representation.
Gauge transformations:

$$\Phi \to e^{i\lambda}\Phi e^{-i\lambda}\,, \quad \bar{\Phi} \to e^{i\bar{\lambda}}\bar{\Phi}e^{-i\bar{\lambda}}\,, \quad e^{2V} \to e^{i\bar{\lambda}}e^{2V}e^{-i\lambda}$$

Hidden $\mathcal{N}=2$ supersymmetry

$$e^{2V}\delta_\epsilon e^{2V} = \theta^\alpha\epsilon_\alpha\bar{\Phi}_c - \bar{\theta}^\alpha\bar{\epsilon}_\alpha\Phi_c\,, \quad \delta_\epsilon\Phi_c = -i\epsilon^\alpha\bar{\nabla}_\alpha G\,, \quad \delta_\epsilon\bar{\Phi}_c = -i\bar{\epsilon}^\alpha\nabla_\alpha G$$

in terms of covariantly (anti))chiral superfields:

$$\bar{\Phi}_c = e^{-2V}\bar{\Phi}e^{2V}\,, \quad \Phi_c = \Phi\,, \quad \nabla_\alpha\bar{\Phi}_c = 0\,, \quad \bar{\nabla}_\alpha\Phi_c = 0$$

Background-quantum splitting (label "c" is omitted, $\bar{\Phi}_c \to \bar{\Phi}$, $\Phi_c \to \Phi$)

$$\Phi \to \Phi + g\phi\,, \qquad \bar{\Phi} \to \bar{\Phi} + g\bar{\phi}$$

Effective action for $\mathcal{N}=4$ theory

Specification of gauge fixing functions to remove the mixing terms between quantum v and $\phi, \bar{\phi}$ fields

Background field method yields one-loop effective action

$$\Gamma^{(1)}_{\mathcal{N}=4}[V,\Phi,\bar{\Phi}] = S_{\mathcal{N}=4}[V,\Phi,\bar{\Phi}] + \tilde{\Gamma}^{(1)}_{\mathcal{N}=4}[V,\Phi,\bar{\Phi}]$$

$$\tilde{\Gamma}^{(1)}_{\mathcal{N}=4} = \frac{i}{2}\mathrm{Tr}_{\mathrm{v}}\ln(\Box_{\mathrm{v}} + \bar{\Phi}\Phi) - i\mathrm{Tr}_{+}\ln(\Box_{+} + \bar{\Phi}\Phi)$$

Specification of the background field

- Gauge group $S(N)$ is broken down to $U(1)^{N-1}$

$$V = \sum_{I=1}^{N} \mathbf{V}_I e_{II} = \mathrm{diag}(\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_N), \quad \bar{\mathbf{V}}_I = \mathbf{V}_I\,, \quad \sum_{I=1}^{N} \mathbf{V}_I = 0$$

$$\Phi = \mathrm{diag}(\mathbf{\Phi}_1, \mathbf{\Phi}_2, \ldots, \mathbf{\Phi}_N)\,, \qquad \sum_{I=1}^{N} \mathbf{\Phi}_I = 0$$

- Background fields satisfy the classical equations of motion.
- Background fields are space-time independent, $\mathcal{D}_\alpha \Phi = 0, \bar{\mathcal{D}}_\alpha \bar{\Phi} = 0$

Exact result for one-loop effective action in the given background field

$$\tilde{\Gamma}^{(1)}_{\mathcal{N}=4}[V,\Phi,\bar{\phi}] =$$

$$-\frac{1}{\pi}\sum_{I<J}^{N}\int d^3x d^4\theta \int_0^\infty \frac{ds}{s\sqrt{i\pi s}} \frac{\mathbf{W}^2_{IJ}\bar{\mathbf{W}}^2_{IJ}}{\mathbf{B}^3_{IJ}} e^{is(\mathbf{G}^2_{IJ}+\bar{\mathbf{\Phi}}_{IJ}\mathbf{\Phi}_{IJ})} \tanh\frac{s\mathbf{B}_{IJ}}{2}\sinh^2\frac{s\mathbf{B}_{IJ}}{2}$$

$$-\frac{2}{\pi}\sum_{I<J}^{N}\int d^3x d^4\theta \Big[\mathbf{G}_{IJ}\ln(\mathbf{G}_{IJ}+\sqrt{\mathbf{G}^2_{IJ}+\bar{\mathbf{\Phi}}_{IJ}\mathbf{\Phi}_{IJ}}) - \sqrt{\mathbf{G}^2_{IJ}+\bar{\mathbf{\Phi}}_{IJ}\mathbf{\Phi}_{IJ}}$$

$$+\frac{1}{4}\int_0^\infty \frac{ds}{\sqrt{i\pi s}} e^{is(\mathbf{G}^2_{IJ}+\bar{\mathbf{\Phi}}_{IJ}\mathbf{\Phi}_{IJ})} \frac{\mathbf{W}^2_{IJ}\bar{\mathbf{W}}^2_{IJ}}{\mathbf{B}^2_{IJ}} \left(\frac{\tanh(s\mathbf{B}_{IJ}/2)}{s\mathbf{B}_{IJ}/2} - 1\right)\Big].$$

Notations:

$$\mathbf{\Phi}_{IJ} = \mathbf{\Phi}_I - \mathbf{\Phi}_J$$

Only leading low-energy term
$\mathbf{G}_{IJ}\ln(\mathbf{G}_{IJ}+\sqrt{\mathbf{G}^2_{IJ}+\bar{\mathbf{\Phi}}_{IJ}\mathbf{\Phi}_{IJ}}) - \sqrt{\mathbf{G}^2_{IJ}+\bar{\mathbf{\Phi}}_{IJ}\mathbf{\Phi}_{IJ}}$
$\sim F^2$ was calculated before by indirect method (J. de Boer, K. Hori, Y. Oz, 1997; J. de Boer, K. Hori, Y. Oz, Z. Yin, 1997).

Effective action for $\mathcal{N} = 4$ theory

Gauge group $S(N)$ is broken down to $U(N-1) \times U(1)$

Background superfields

$$V = \frac{1}{N}\mathrm{diag}\left((N-1)\mathbf{V}, \underbrace{-\mathbf{V}, \ldots, -\mathbf{V}}_{N-1}\right),$$

$$\Phi = \frac{1}{N}\mathrm{diag}\left((N-1)\mathbf{\Phi}, \underbrace{-\mathbf{\Phi}, \ldots, -\mathbf{\Phi}}_{N-1}\right)$$

Effective action is computed on the base of superfield proper-time technique.
Exact one-loop result for the given background field can be obtained.
The leading low-energy terms:

$$\tilde{\Gamma}^{(1)}_{\mathcal{N}=4} = \frac{2(N-1)}{\pi}\int d^3x d^4\theta \Big[\sqrt{\mathbf{G}^2 + \bar{\mathbf{\Phi}}\mathbf{\Phi}} - \mathbf{G}\ln(\mathbf{G} + \sqrt{\mathbf{G}^2 + \bar{\mathbf{\Phi}}\mathbf{\Phi}})$$

$$+ \frac{1}{32}\frac{\mathbf{W}^2\bar{\mathbf{W}}^2}{(\mathbf{G}^2 + \bar{\mathbf{\Phi}}\mathbf{\Phi})^{5/2}} + \ldots\Big].$$

Comments on the leading low-energy term

$$\int d^3x d^4\theta \left[\sqrt{\mathbf{G}^2 + \bar{\mathbf{\Phi}}\mathbf{\Phi}} - \mathbf{G}\ln(\mathbf{G} + \sqrt{\mathbf{G}^2 + \bar{\mathbf{\Phi}}\mathbf{\Phi}} \right]$$

- $\mathcal{N}=2, d=3$ superspace action of improved tensor multiplet (N.J. Hitchin, A. Karlhede, U. Lindstróm, M. Roček, 1987)
- $\mathcal{N}=1, d=4$ superspace action for improved tensor multiplet (U. Lindstróm, M. Roček, 1983)
- One-loop exact (N. Seiberg, 1996; J. de Boer, K. Hori, Y. Oz, 1997; J. de Boer, K. Hori, Y. Oz, Z. Yin, 1997; J. de Boer, K. Hori, H. Oohguri, Y. Oz, 1997)
- Dual representation classical action of the Abelian Gaiotto-Witten model (E.Koh, S. Lee, S. Lee, 2009; D. Gaiotto, E. Witten, 2008)
  Classical action of the Abelian Gaiotto-Witten model in the dual representation arises as the leading term in $\mathcal{N}=4$ one-loop effective action

Classical action of $\mathcal{N}=8$ SYM theory in terms of $d=3,\mathcal{N}=2$ superfields

$$S_{\mathcal{N}=8} = \frac{1}{g^2}\mathrm{tr}\int d^3x d^4\theta\,[G^2 - \frac{1}{2}e^{-2V}\bar{\Phi}^i e^{2V}\Phi_i] +$$

$$\frac{1}{12g^2}\left(\mathrm{tr}\int d^3x d^2\theta\,\varepsilon^{ijk}\Phi_i[\Phi_j,\Phi_k] + c.c.\right)$$

Φ_i, $i=1,2,3$, is a triplet of chiral superfields.
Hidden $\mathcal{N}=6$ supersymmetry with triplet of complex parameters $\epsilon_{\alpha\, i}$

$$e^{-2V}\delta_\epsilon e^{2V} = \theta^\alpha\epsilon_{\alpha\, i}\bar{\Phi}^i_c - \bar{\theta}^\alpha\bar{\epsilon}^i_\alpha\Phi_{c\, i}\,,$$

$$\delta_\epsilon\Phi_{c\, i} = -i\epsilon^\alpha_i\bar{\nabla}_\alpha G + \frac{1}{4}\varepsilon_{ijk}\bar{\nabla}^2(\bar{\theta}^\alpha\bar{\epsilon}^j_\alpha\bar{\Phi}^k_c)\,,$$

$$\delta_\epsilon\bar{\Phi}^i_c = -i\bar{\epsilon}^{\alpha\, i}\nabla_\alpha G + \frac{1}{4}\varepsilon^{ijk}\nabla^2(\theta^\alpha\epsilon_{\alpha\, j}\Phi_{c\, k})$$

in terms of covariantly (anti)chiral superfields $\bar{\Phi}_c = e^{-2V}\bar{\Phi}e^{2V}\,,\quad \Phi_c = \Phi$
Background-quantum splitting (label "c" is omitted, $\bar{\Phi}_c\to\bar{\Phi}$, $\Phi_c\to\Phi$)
$\Phi\to\Phi+g\phi\,,\bar{\Phi}\to\bar{\Phi}+g\bar{\phi}$

Specification of gauge fixing functions to remove the mixing terms between quantum v and $\phi, \bar{\phi}$ fields

$$f = i\bar{\mathcal{D}}^2 v - \frac{i}{2}[\Phi_i, \bar{\mathcal{D}}^2 \Box_-^{-1} \bar{\phi}^i], \qquad \bar{f} = i\mathcal{D}^2 v + \frac{i}{2}[\bar{\Phi}^i, \mathcal{D}^2 \Box_+^{-1} \phi_i].$$

Specification of background fields: constant on-shell Φ and constant on-shell strengths.

Background field method yields one-loop effective action:

$$\Gamma^{(1)}_{\mathcal{N}=8}[V, \Phi, \bar{\Phi}] = S_{\mathcal{N}=8}[V, \Phi, \bar{\Phi}] + \tilde{\Gamma}^{(1)}_{\mathcal{N}=8}[V, \Phi, \bar{\Phi}]$$

$$\tilde{\Gamma}^{(1)}_{\mathcal{N}=8} = \frac{i}{2}\mathrm{Tr}_{\mathrm{v}} \ln(\Box_{\mathrm{v}} + \bar{\Phi}^i \Phi_i)$$

The contributions from ghosts and chiral superfields cancel each other at one-loop (like in $\mathcal{N}=4, d=4$ SYM theory).

Effective action for $\mathcal{N} = 8$ theory

Gauge group $SU(N)$ is spontaneously broken down to $U(1)^{N-1}$
Exact result for one-loop effective action in the given background

$$\tilde{\Gamma}^{(1)}_{\mathcal{N}=8}[V, \Phi, \bar{\Phi}] =$$

$$-\frac{1}{\pi}\sum_{I<J}^{N}\int d^3x d^4\theta \int_0^\infty \frac{ds}{s\sqrt{i\pi s}} \frac{\mathbf{W}^2_{IJ}\bar{\mathbf{W}}^2_{IJ}}{\mathbf{B}^3_{IJ}} e^{is(\mathbf{G}^2_{IJ}+\bar{\mathbf{\Phi}}^i{}_{IJ}\mathbf{\Phi}_{i\,IJ})} \tanh\frac{s\mathbf{B}_{IJ}}{2}\sinh^2\frac{s\mathbf{B}_{IJ}}{2}$$

Gauge group $SU(N)$ is spontaneously broken down to $S(N-1)\times U(1)$
Leading contributions to the one-loop effective action

$$\Gamma_{\mathcal{N}=8} = \frac{3(N-1)}{32\pi}\int d^3x d^4\theta \frac{\mathbf{W}^2\bar{\mathbf{W}}^2}{(\mathbf{G}^2+\bar{\mathbf{\Phi}}^i\mathbf{\Phi}_i)^{5/2}} + \ldots \sim \int d^3x \frac{(F^{mn}F_{mn})^2}{(f^r f^r)^{5/2}} + \ldots$$

$f^r, (r = 1, 2, .., 7)$ are the seven real scalars in the $\mathcal{N} = 8, d = 3$ SYM theory.
Leading term is F^4

Everything is analogous to super Yang-Mills theory

- Background quantum splitting
  Initial field V is spitted into background field V and quantum field v by the rule
  $$e^{2V} \to e^{2V} e^{2gv}$$
- Background gauge transformations
  $$e^{2V} \to e^{i\bar{\lambda}} e^{2V} e^{-i\lambda}, \qquad e^{2gv} \to e^{i\tau} e^{2gv} e^{-i\tau}$$
  τ is a real superfield parameter
- Quantum gauge transformations
  $$e^{2V} \to e^{2V}, \qquad e^{2gv} \to e^{i\bar{\lambda}} e^{2gv} e^{-i\lambda}$$
- Imposing the gauge fixing only on quantum fields
  Gauge fixing functions:
  $$f = i\bar{\mathcal{D}}^2 v, \qquad \bar{f} = i\mathcal{D}^2 v$$
  $$\mathcal{D}_\alpha = e^{-2V} D_\alpha e^{2V}, \qquad \bar{\mathcal{D}}_\alpha = \bar{D}_\alpha$$

Background field formulation for SCS theory

Effective action is constructed on the base of Faddeev-Popov anzatz
One-loop effective action $\Gamma^{(1)}_{SCS}[V]$

$$e^{i\Gamma^{(1)}_{SCS}[V]} = e^{iS_{SCS}[V]} \int \mathcal{D}v\mathcal{D}b\mathcal{D}c e^{iS_v[v,V]+iS_{FP}[b,c,V]}$$

$S_v[v, V]$ is background field dependent action of quantum superfields v

$$S_v[v, V] = S_2 + S_{gf} = \frac{1}{2}\text{tr} \int d^7 z v\mathcal{H}v$$

$$\mathcal{H} = \frac{1}{4}(\mathcal{D}^\alpha\bar{\mathcal{D}}_\alpha + \bar{\mathcal{D}}^\alpha\mathcal{D}_\alpha + \bar{\mathcal{D}}^2 + \mathcal{D}^2)$$

$S_{FP}[b, c, V]$ is background field dependent ghost superfield action

$$S_{FP}[b, c, V] = \text{tr} \int d^7 z(\bar{b}c - b\bar{c})$$

$c, \bar{c}, b, \bar{b}$ are the covariantly chiral and antichiral superfields.

One-loop effective action $\Gamma^{(1)}_{SCS}[V]$

$$\Gamma^{(1)}_{SCS}[V] = S_{SCS}[V] + \tilde{\Gamma}^{(1)}[V]$$

$$\tilde{\Gamma}^{(1)}[V] = \Gamma^{(1)}_{\rm v}[V] + \Gamma^{(1)}_{\rm ghosts}[V]$$

Gauge superfield contribution

$$\Gamma^{(1)}_{\rm v}[V] = \frac{i}{2}{\rm Trln}(\mathcal{H})$$

Ghost contribution

$$\Gamma^{(1)}_{\rm ghosts}[V] = -i{\rm Trln}(\frac{1}{16}\mathcal{D}^2\bar{\mathcal{D}}^2) - i{\rm Trln}(\frac{1}{16}\bar{\mathcal{D}}^2\mathcal{D}^2) = -i{\rm Tr}_+{\rm ln}(\Box_+) - i{\rm Tr}_-{\rm ln}(\Box_-)$$

$$\Box_+ = \mathcal{D}^m\mathcal{D}_m + G^2 + \frac{i}{2}(\mathcal{D}^\alpha W_\alpha) + iW^\alpha\mathcal{D}_\alpha$$

$$\Box_- = \mathcal{D}^m\mathcal{D}_m + G^2 - \frac{i}{2}(\bar{\mathcal{D}}^\alpha \bar{W}_\alpha) - \bar{W}^\alpha\bar{\mathcal{D}}_\alpha$$

Problem of $\mathrm{Det}(\mathcal{H})$

Operator $\mathcal{H}$ has first order in power of ∂_m. Standard method for computing the $\mathrm{Det}(\mathcal{H})$ in this case is squaring

$$\mathrm{Det}(\mathcal{H}) = \mathrm{Det}^{\frac{1}{2}}(\mathcal{H}^2)$$

$$\mathcal{H}^2 = \mathcal{D}^m\mathcal{D}_m + \frac{i}{2}(W^\alpha - \bar{W}^\alpha)(\mathcal{D}_\alpha - \bar{\mathcal{D}}_\alpha)$$

Operator $\mathcal{H}^2$ has a standard form for application of proper-time technique

Contribution from gauge superfield

Proper-time representation of one-loop effective action $\Gamma^{(1)}_{\rm v}[V]$

$$\Gamma^{(1)}_{\rm v}[V] = \frac{i}{4}{\rm Tr}_{\rm v}\int_0^\infty \frac{ds}{s}e^{-m^2 s}e^{-s\mathcal{H}^2}$$

m is infrared regulator.

$${\rm Tr}_{\rm v}\mathcal{O} = {\rm tr}\int d^7z\ \mathcal{O}\delta^7(z-z')|_{z=z'}$$

Here $\delta^7(z-z') = \delta^3(x-x')\delta^4(\theta-\theta')$. To get non-zero result we should use exactly two derivatives $\mathcal{D}_\alpha$ and exactly two derivatives $\bar{\mathcal{D}}_\alpha$ from the expansion of $e^{-s\mathcal{H}^2}$. Leading contribution

$$\Gamma^{(1)}_{\rm v}[V] = -\frac{1}{256\pi m^5}\int d^7z\left(\mathcal{W}^{a\alpha}\mathcal{W}^b_\alpha\mathcal{W}^{c\beta}\mathcal{W}^d_\beta - \frac{1}{2}\mathcal{W}^{a\alpha}\mathcal{W}^{b\beta}\mathcal{W}^c_\alpha\mathcal{W}^d_\beta\right)\mathbf{f}_{abcd}$$

where $\mathcal{W}^{a\alpha} \equiv (W^{a\alpha} - \bar{W}^{a\alpha})T_a$, $[T_a, T_b] = f_{abc}T_c$,
$\mathbf{f}_{a_1a_2a_3a_4} = f_{b_1a_1b_2}f_{b_2a_2b_3}f_{b_3a_3b_4}f_{b_4a_4b_1}$. Zero in Abelian case.

Contribution from ghost superfields

Proper-time representation for ghost contribution $\Gamma^{(1)}_{\rm ghost}[V]$

$$i\Gamma^{(1)}_{\rm ghost}[V] = {\rm tr}\int d^5z \int_0^\infty \frac{ds}{s} e^{-sm^2}(K_+(z|s) + K_-(z|s))$$

Here $K_+(z|s), K_-(z|s)$ are chiral and antichiral superfield kernels and m is infrared regulator,

$$K_+(z|s) = {\rm tr} e^{-s\Box_+}\delta_+(z,z')|_{z=z'}\mathbf{1}$$

Leading contribution to effective action

$$\Gamma^{(1)}_{\rm gh} = \frac{1}{8\pi^2 m}{\rm tr}\int d^7z\, G^2 + O(m^{-2}) = -\frac{1}{16\pi^2 m}{\rm tr}\int d^5z\, W^\alpha W_\alpha + O(m^{-2})$$

Generation of SYM action as quantum correction in SCS theory.

Basic results

- Formulation of the background field method for $\mathcal{N} = 2, d = 3$ SYM theories coupled to a matter. Application to problem of effective action in $\mathcal{N} = 4$ and $\mathcal{N} = 8$ SYM theories
- Construction of superfield operators defying the one-loop effective action
- Development of $\mathcal{N} = 2, d = 3$ superfield proper-time technique
- Computing the exact one-loop effective actions for constant backgrounds and various gauge symmetry violations. Finding the dependence effective action on all powers of Maxwell strength
- Formulation of background field method for $\mathcal{N} = 2$ SCS theory
- One-loop effective action for pure SCS theory. Inducing the SYM action.
- New superfield differential operators associated with $\mathcal{N} = 2$, $d = 3$ SYM and SCS theories
- Ingredients for computing the effective action in ABJM and BLG theories.

Open problems

- Background field method and effective action in $\mathcal{N}=2, d=3$ Chern-Simons theories coupled to matter
- Effective action in ABJM ($\mathcal{N}=6, d=3$ gauge model) and BLG ($\mathcal{N}=8, d=3$ gauge model) theories

THANK YOU VERY MUCH