$\mathcal{N}=2$ *Supersymmetry and* $U(1)$*-Duality*

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Breaking of supersymmetry and ultraviolet divergences in extended supergravities

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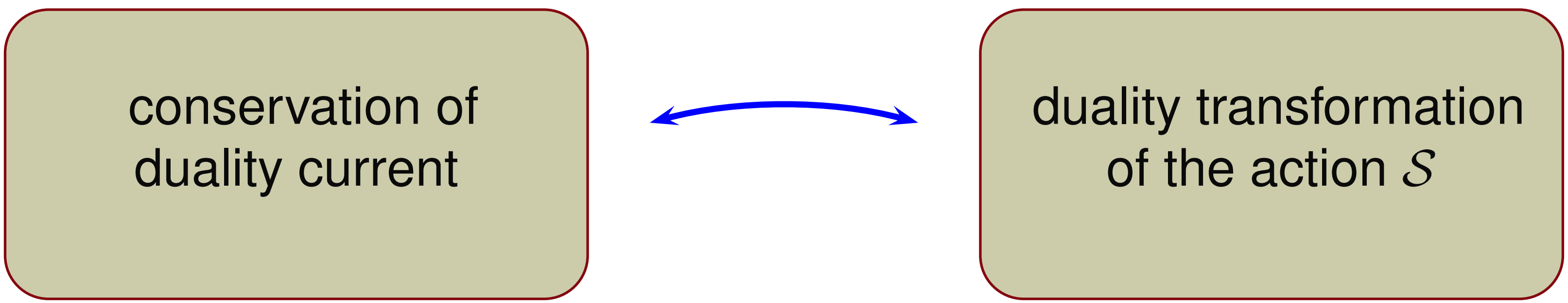
Motivation

- How to deform an action $\mathcal{S}$ without breaking its symmetries?
- Which deformations can be added in a way, such that duality symmetry can be restored order by order in a deformation parameter?
- Why: duality symmetries might play a role in explaining UV-properties of supergravities
- main tool:



This talk:

- formalize the way to obtain duality-invariant theories starting from an initial deformation
- apply the formalized method to abelian $\mathcal{N} = 2$ supersymmetric gauge theory

Outline

From Maxwell to Born-Infeld

- duality symmetry in $U(1)$ gauge theory
- twisted self-duality constraint
- how to obtain a duality invariant action starting from a deformation?
- Born-Infeld example

$\mathcal{N} = 2$ *gauge theory and* $U(1)$*-duality*

- $\mathcal{N} = 2$ theories
- different sources of deformation
- $\mathcal{N} = 2$ Born-Infeld action

Duality symmetry in $U(1)$ gauge theory

Rotating the electric and magnetic fields

$$\delta \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{B} \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{B} \end{pmatrix}$$

in the source-free *Maxwell equations*

$$\begin{aligned} \partial_t \boldsymbol{B} &= -\nabla \times \boldsymbol{E} \quad , \quad \nabla \cdot \boldsymbol{B} = 0 \\ \partial_t \boldsymbol{D} &= \nabla \times \boldsymbol{H} \quad , \quad \nabla \cdot \boldsymbol{D} = 0 \end{aligned}$$

with

$$\boldsymbol{D} = \boldsymbol{E}\,, \qquad \boldsymbol{H} = \boldsymbol{B}$$

leaves the Hamiltonian

$$\mathcal{H} = \frac{1}{2}(\boldsymbol{E}^2 + \boldsymbol{B}^2)$$

unchanged, but alters the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\boldsymbol{E}^2 - \boldsymbol{B}^2)\,.$$

Duality symmetry in $U(1)$ gauge theory

In the presence of matter, the equations are still valid

$$\begin{aligned} \partial_t \boldsymbol{B} &= -\nabla \times \boldsymbol{E} \quad , \quad \nabla \cdot \boldsymbol{B} = 0 \\ \partial_t \boldsymbol{D} &= \nabla \times \boldsymbol{H} \quad , \quad \nabla \cdot \boldsymbol{D} = 0 \end{aligned}$$

however,

$$\boldsymbol{D} = \boldsymbol{D}(\boldsymbol{E}, \boldsymbol{B}) \,, \quad \text{and} \quad \boldsymbol{H} = \boldsymbol{H}(\boldsymbol{B}, \boldsymbol{E})$$

are *non-linear*.

- which (duality) transformations leave the above non-linear system invariant?
- how can one generalize to different (supersymmetric) theories with more fields?

Duality symmetry in $U(1)$ gauge theory

More precisely: search for theories admitting a Lagrangian formulation

varying the usual Maxwell Lagrangian for matter with respect to the gauge potential leads to

$$\boldsymbol{D} = \frac{\partial \mathcal{L}(\boldsymbol{E},\boldsymbol{B})}{\partial \boldsymbol{E}}, \quad \text{and} \quad \boldsymbol{H} = -\frac{\partial \mathcal{L}(\boldsymbol{E},\boldsymbol{B})}{\partial \boldsymbol{B}}.$$

Prepare for treating more general theories: switch to four-component notation:

$$\{\boldsymbol{E},\boldsymbol{B}\} \quad \rightarrow \quad \{F,\tilde{F},G,\tilde{G}\}$$

leads to the (constitutive) relation:

$$\tilde{G}^{\mu\nu} = 2\frac{\partial \mathcal{L}(F)}{\partial F_{\mu\nu}}$$

where $\tilde{G}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}G_{\rho\sigma}$.

Describe duality tranformation as *Legendre-transformation*:

$$\tilde{\mathcal{L}}(F,G) = \mathcal{L}(F) - \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}\partial_\rho \tilde{A}_\sigma \quad \text{where} \quad G_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu$$

The above Legendre transformation is a duality rotation *if and only if* the symmetry condition is met. Otherwise one obtains just a different formulation of the theory.

Duality symmetry in $U(1)$ gauge theory

Shall be preserved under duality:

$$\begin{aligned} \partial_\mu \tilde{F}^{\mu\nu} &= 0 \quad \text{Bianchi identities} \\ \partial_\mu \tilde{G}^{\mu\nu} &= 0 \quad \text{equations of motion} \end{aligned}$$

General duality rotations

$$\delta \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix}$$

exchanges the role of Bianchi identity and the equation of motion.

Too general:

- the functional form of shall not change, i.e.

$$\tilde{G}'^{\mu\nu} = 2\frac{\partial \mathcal{L}(F')}{\partial F'_{\mu\nu}} \quad \text{where} \quad F' = F + \delta F, \ \ G' = G + \delta G$$

- deformed theory shall reduce to Maxwell in the weak field limit ($F^4 \ll F^2$)

Under those conditions: $GL(2,\mathbb{R}) \rightarrow SO(2,\mathbb{R})$, maximal connected Lie group of duality rotations in pure non-linear electromagnetism [Aschieri, Ferrara Zumino]

NGZ current conservation

Thus, the infinitesimal duality transformations to consider read:

$$\delta \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} 0 & B \\ -B & 0 \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix}$$

Equivalent formulation: Noether-Gaillard-Zumino (NGZ) current conservation:

$$F\tilde{F} + G\tilde{G} = 0$$

[Gibbons Rasheed]

[Gaillard Zumino]

Formulation is not symmetric in field F and G: find a more general language.
Define:

$$T = F - \mathrm{i}G \quad \overline{T} = F + \mathrm{i}G$$

and their self-dual and anti-self-dual components

$$T^{\pm} = \frac{1}{2}(T \pm \mathrm{i}\tilde{T}) \quad \overline{T}^{\pm} = \frac{1}{2}(\overline{T} \pm \mathrm{i}\tilde{\overline{T}})$$

Maxwell theory in vacuum:

$T^{+} = F^{+} - \mathrm{i}G^{+} = 0$ is equivalent to $F\tilde{F} + G\tilde{G} = 0$

Born-Infeld theory

Best known deformation of Maxwell ***Born-Infeld theory***: [Born Infeld]

$$\mathcal{L}_{\mathsf{BI}} = g^{-2}\left(1-\sqrt{\Delta}\right) \quad \text{where} \quad \Delta = 1 + 2g^2\left(\frac{F^2}{4}\right) - g^4\left(\frac{F\tilde{F}}{4}\right)$$

with dual field $\tilde{G}$

$$G_{\mu\nu} = -\varepsilon_{\mu\nu\rho\sigma}\frac{\partial\mathcal{L}(F)}{\partial F_{\rho\sigma}} = \frac{1}{\sqrt{\Delta}}\left(\tilde{F}_{\mu\nu} + \frac{g^2}{4}(F\tilde{F})F_{\mu\nu}\right).$$

Short calculation: $F\tilde{F} + G\tilde{G} = 0$.

However, there is a different formulation in form of the *non-linear constraint*: [Schrödinger]

$$T^+ + \frac{g^2}{16}\frac{\overline{T}^+(T\tilde{T})^2}{(T^-)^2} = 0 \qquad \mathcal{L}_{\mathrm{Sch}}(T) = 4\frac{T^2}{(T\tilde{T})}, \qquad \mathcal{L}_{\mathrm{Sch}} = -\mathcal{L}^*_{\mathrm{Sch}}.$$

- one can readily recover the Born-Infeld action from this constraint.
- constraint contains complete information about the theory.

Deformation

How can one construct an action from an initial deformation? [Bossard, Nicolai][Carrasco, Kallosh, Roiban]

$$T^+ = 0 \quad \text{Maxwell}$$
$$T^+ + \frac{g^2}{16}\frac{\overline{T}^+ (T\tilde{T})^2}{(T^-)^2} = 0 \quad \text{Born-Infeld}$$

Information about the deformation is contained in the constraint.

How to obtain the constraint and the corresponding action?

- start from a deformation $\mathcal{I}(T^-, \overline{T}^+, g)$ invariant under the classical (unperturbed) equations of motion
- set
$$T^+ = \frac{\delta \mathcal{I}(T^-, \overline{T}^+, \lambda)}{\delta \overline{T}^+}\,.$$
- solve the above equation iteratively (e.g. in terms of the dual field). Start from the classical solution. Ensure validity of the NGZ constraint in every step.
- reconstruct the action using $\tilde{G}^{\mu\nu} = 2\frac{\partial \mathcal{L}(F)}{\partial F_{\mu\nu}}$.

Reconsider Born-Infeld theory

Consider the following ansatz for a deformation: [Carrasco, Kallosh Roiban]

$$\mathcal{I}(T^-,\overline{T}^+) = \sum_{n=0}^{\infty} \frac{a_n}{8\,g^2} \left(\tfrac{1}{4}\,g^4(\overline{T}^-)^2(T^-)^2\right)^{n+1}$$

$$T^+_{\mu\nu} = \frac{g^2}{16}\overline{T}^+_{\mu\nu}(T^-)^2\left[1+\sum_{n=0}^{\infty} a_n\left(\tfrac{1}{4}\,g^4(\overline{T}^+)^2(T^-)^2\right)^n\right],$$

where $a_n = \frac{d_n}{n+1}$ and $a_0 = 1 + d_0$.

Constraining the the coefficients to yield the Born-Infeld action leads to

$$\mathcal{I}(T^-,\overline{T}^+,g) = \frac{6}{g^2}\Big(1 - {}_3F_2(-\tfrac{1}{2},-\tfrac{1}{4},\tfrac{1}{4};\tfrac{1}{3},\tfrac{2}{3};-\tfrac{1}{27}\,g^4\,(\overline{T}^+)^2\,(T^-)^2).\Big)$$

- infinite number of deformations necessary to reproduce Born-Infeld
- method allows application to other sources and in other theories
- the hypergeometric function leading to the Born-Infeld theory satisfies a hidden fourth-order constraint [Aschieri Ferrara]

⇒ apply to $\mathcal{N}=2$ abelian gauge theory

$\mathcal{N}=2$ supersymmetric theory

$\mathcal{N}=2$ superspace: $\mathcal{Z}^A=(x^a,\theta^\alpha_i,\bar\theta^i_{\dot\alpha})$
Chiral and antichiral superfield strength $\mathcal{W}$ and $\overline{\mathcal{W}}$ satisfy Bianchi identities

$$\mathcal{D}^{ij}\mathcal{W}=\overline{\mathcal{D}}^{ij}\,\overline{\mathcal{W}} \qquad \text{where} \quad \begin{aligned}\mathcal{W} &= \overline{\mathcal{D}}^4\mathcal{D}^{ij}V_{ij}\\ \overline{\mathcal{W}} &= \mathcal{D}^4\overline{\mathcal{D}}^{ij}V_{ij}\end{aligned}$$

in terms of the unconstrained prepotential V_{ij}.
Write duality transformation as a Legendre transformation

$$\mathcal{S}_{\text{inv}}=\mathcal{S}[\mathcal{W},\overline{\mathcal{W}}]-\frac{\mathrm{i}}{8}\int d^8\mathcal{Z}\,\mathcal{W}\mathcal{M}+\frac{\mathrm{i}}{8}\int d^8\,\overline{\mathcal{Z}}\,\overline{\mathcal{W}}\,\overline{\mathcal{M}}\,,$$

which is only valid if

$$\mathrm{i}\mathcal{M}=4\frac{\delta}{\delta\mathcal{W}}\mathcal{S}[\mathcal{W},\overline{\mathcal{W}}] \qquad \text{and} \qquad \mathrm{i}\overline{\mathcal{M}}=4\frac{\delta}{\delta\overline{\mathcal{W}}}\mathcal{S}[\mathcal{W},\overline{\mathcal{W}}].$$

Duality transformation: $\quad \delta\mathcal{W}=B\mathcal{M} \qquad \delta\overline{\mathcal{W}}=-B\mathcal{W}$

$\mathcal{N}=2$ *Noether-Gaillard-Zumino-condition:*

$$\int \mathrm{d}^8\mathcal{Z}\left(\mathcal{W}^2+\mathcal{M}^2\right)=\int \mathrm{d}^8\overline{\mathcal{Z}}\left(\overline{\mathcal{W}}^2+\overline{\mathcal{M}}^2\right),$$

Construction of duality-compatible action for $\mathcal{N} = 2$-theories

Define objetcs similar to the Maxwell case:

$$T^+ = \mathcal{W} - \mathrm{i}\mathcal{M} \qquad \overline{T}^+ = \mathcal{W} + \mathrm{i}\mathcal{M}$$
$$T^- = \overline{\mathcal{W}} - \mathrm{i}\overline{\mathcal{M}} \qquad \overline{T}^- = \overline{\mathcal{W}} + \mathrm{i}\overline{\mathcal{M}}$$

with infinitesimal rotations

$$\delta \begin{pmatrix} T^+ \\ \overline{T}^+ \end{pmatrix} = \begin{pmatrix} \mathrm{i}B & 0 \\ 0 & -\mathrm{i}B \end{pmatrix} \begin{pmatrix} T^+ \\ \overline{T}^+ \end{pmatrix} \qquad \delta \begin{pmatrix} T^- \\ \overline{T}^- \end{pmatrix} = \begin{pmatrix} \mathrm{i}B & 0 \\ 0 & -\mathrm{i}B \end{pmatrix} \begin{pmatrix} T^- \\ \overline{T}^- \end{pmatrix}$$

$\mathcal{N} = 2$ Noether-Gaillard-Zumino-condition:

$$\int \mathrm{d}^8\overline{\mathcal{Z}}\,\overline{T}^+ T^+ - \int \mathrm{d}^8\mathcal{Z}\,\overline{T}^- T^- = 0$$

$\mathcal{N} = 2$ supersymmetric theory

In analogy to the Maxwell case, consider deformation sources: [Broedel, Carrasco, Ferrara Kallosh, Roiban]

$$T^+ = \frac{\delta \mathcal{I}(T^-, \overline{T}^+)}{\delta \overline{T}^+} \qquad \overline{T}^- = \frac{\delta \mathcal{I}(T^-, \overline{T}^+)}{\delta T^-}$$

Thus, the NGZ constraint reads:

$$0 = \int \mathrm{d}\overline{\mathcal{Z}}\, \overline{T}^+ \frac{\delta}{\delta \overline{T}^+} \mathcal{I}(T^-, \overline{T}^+) - \int \mathrm{d}\mathcal{Z} T^- \frac{\delta}{\delta T^-} \mathcal{I}(T^-, \overline{T}^+)$$

which translates into

$$\left(\overline{T}^+ \frac{\delta}{\delta \overline{T}^+} - T^- \frac{\delta}{\delta T^-} \right) \mathcal{I}(T^-, \overline{T}^+).$$

$\Rightarrow$ *measures the charge under a duality transformation.*

$\mathcal{N} = 2$ supersymmetric theory

How to efficiently solve [Bossard Nicolai][Broedel, Carrasco, Ferrara Kallosh, Roiban]

$$T^{+} = \frac{\delta \mathcal{I}(T^{-}, \overline{T}^{+})}{\delta \overline{T}^{+}} \qquad \overline{T}^{-} = \frac{\delta \mathcal{I}(T^{-}, \overline{T}^{+})}{\delta T^{-}}$$

?

Ansatz:

$$\mathcal{M} = \mathcal{M}^{(0)} + \sum_{n \geq 1} \lambda^{n} \mathcal{M}^{(n)}(\mathcal{W}, \overline{\mathcal{W}})$$

Any higher orders can be obtained recursively ($\mathcal{I}$ is of order λ):

$$\mathcal{M}^{(n)} \equiv \lambda^{-n} \left(\frac{\delta}{\delta \overline{T}^{+}} \mathcal{I}\left[T^{-}(\mathcal{W}, \mathcal{M}^{(n-1)}), \overline{T}^{+}(\overline{\mathcal{W}}, \overline{\mathcal{M}}^{(n-1)}) \right] - \sum_{j=1}^{n-1} \lambda^{j} \mathcal{M}^{(j)} \right)$$

with $\lambda^{m>n} \to 0$.

From a solution to the above equation one can reconstruct a duality invariant action:

$$\mathcal{S} = \mathrm{i} \int \mathrm{d}^{8}\mathcal{Z}\, \mathcal{W} \sum_{n=0} \frac{\lambda^{n}}{8(n+1)} \mathcal{M}^{(n)}[\mathcal{W}, \overline{\mathcal{W}}] + \text{h.c.}$$

(follows from integrating the $\mathcal{N} = 2$-analogue of $\tilde{G}^{\mu\nu} = 2\frac{\partial \mathcal{L}(F)}{\partial F_{\mu\nu}}$)

$\mathcal{N}=2$ BI solution from Kuzenko/Theisen

Various actions of Born-Infeld type for $\mathcal{N}=2$ theories have been constructed.

[Ketov][Bellucci, Ivanov Krivonos][Kuzenko Theisen]

Here: Kuzenko/Theisen proposal with an additional condition beyond NGZ:

$$\mathcal{W}(\mathcal{Z}) \;\to\; \mathcal{W}(\mathcal{Z}) + \sigma + \mathcal{O}(\mathcal{W},\overline{\mathcal{W}})$$

Susy analogue of the $D3$-brane shift symmetry of the transverse coordinates:

$$\begin{aligned}
\mathcal{S}_{\mathrm{BI}} &= \mathcal{S}_{\mathrm{free}} + \mathcal{S}_{\mathrm{int}} \\
\mathcal{S}_{\mathrm{int}} &= \frac{1}{8}\int \mathrm{d}^{12}\mathcal{Z}\, \Bigg\{ \mathcal{W}^2\,\overline{\mathcal{W}}^2 \Bigg[\lambda + \frac{\lambda^2}{2}\left(\mathcal{D}^4\mathcal{W}^2 + \overline{\mathcal{D}}^4\overline{\mathcal{W}}^2\right) \\
&\qquad + \frac{\lambda^3}{4}\left((\mathcal{D}^4\mathcal{W}^2)^2 + (\overline{\mathcal{D}}^4\overline{\mathcal{W}}^2)^2\right) + 3(\mathcal{D}^4\mathcal{W}^2)(\overline{\mathcal{D}}^4\overline{\mathcal{W}}^2) \Bigg] \\
&\quad + \frac{\lambda^2}{9}\mathcal{W}^3\Box\overline{\mathcal{W}}^3 + \frac{\lambda^3}{6}\left((\mathcal{W}^3\Box\overline{\mathcal{W}}^3)\overline{\mathcal{D}}^4\overline{\mathcal{W}}^2 + (\overline{\mathcal{W}}^3\Box\mathcal{W}^3)\mathcal{D}^4\mathcal{W}^2\right) \\
&\quad + \frac{\lambda^3}{144}\mathcal{W}^4\Box^2\overline{\mathcal{W}}^4 + \mathcal{O}(\mathcal{W}^{10}) \Bigg\}
\end{aligned}$$

What are the sources reproducing this particular action up to the given order?

Example 1

Let us have look to a first deformation:

$$\mathcal{I}_1 \;=\; a\,\lambda \int \mathrm{d}^{12}\mathcal{Z}\,(T^{-})^2(\overline{T}^{+})^2$$

leads to the equation

$$\mathcal{M} \;=\; -\mathrm{i}\,\mathcal{W} \;+\; 2a\,\lambda\mathrm{i}\left(\overline{\mathcal{D}}^4(\overline{\mathcal{W}} - \mathrm{i}\overline{\mathcal{M}})^2\right)(\mathcal{W} + \mathrm{i}\mathcal{M})$$

Solving iteratively yields:

$$\begin{aligned}
\mathcal{M}^{(0)} &= -\mathrm{i}\,\mathcal{W}\,,\\
\mathcal{M}^{(n)}|_{n>0} &= (-2)^{5-n} a \sum_{l=0}^{n-1}\sum_{q=0}^{n-(1+l)} \alpha(l,q;n)\overline{\mathcal{D}}^4[\,\overline{\mathcal{M}}^{(n-(1+q+l))}\overline{\mathcal{M}}^{(q)}\mathcal{M}^{(l)}\,]
\end{aligned}$$

with

$$\begin{aligned}
\alpha(q,l;n) &\equiv \xi_2(q)\xi_2(l)\xi_2(n-l-q-1)\,.\\
\xi_2(x)|_{x>0} &\equiv (-2)^x/2\;,\\
\xi_2(x)|_{x=0} &\equiv 1\;.
\end{aligned}$$

Example 1 - continued

$$\mathcal{M} \;=\; -\mathrm{i}\,\mathcal{W} + 16a\,\mathrm{i}\,\lambda\,\mathcal{W}\,\overline{\mathcal{D}}^4(\overline{\mathcal{W}}^2) - \frac{\mathrm{i}}{2}(16a)^2\lambda^2\mathcal{W}\left(\left(\overline{\mathcal{D}}^4(\overline{\mathcal{W}}^2)\right)^2 + 2\,\mathcal{D}^4\left(\mathcal{W}^2\overline{\mathcal{D}}^4\overline{\mathcal{W}}^2\right)\right)$$

which results in the action

$$\begin{aligned}\mathcal{S}_1^{\mathrm{int}} = \int \mathrm{d}^{12}\mathcal{Z}\mathcal{W}^2\overline{\mathcal{W}}^2\Big\{ & -2a\lambda + 16a^2\lambda^2(\mathcal{D}^4(\mathcal{W}^2) + \overline{\mathcal{D}}^4(\overline{\mathcal{W}}^2)) \\ & -128a^3\lambda^3\left((\mathcal{D}^4(\mathcal{W}^2))^2 + 2\mathcal{D}^4\left(\mathcal{W}^2\overline{\mathcal{D}}^4\overline{\mathcal{W}}^2\right) + (\overline{\mathcal{D}}^4(\overline{\mathcal{W}}^2))^2 + 2\overline{\mathcal{D}}^4\left(\overline{\mathcal{W}}^2\mathcal{D}^4\mathcal{W}^2\right)\right) \\ & + \mathcal{O}(\lambda^4)\end{aligned}$$

Examples 2,3,4

Deformation	Action
$b\,\lambda^2\int \mathrm{d}^{12}\mathcal{Z}(T^-)^3\Box(\overline{T}^+)^3$	$\int \mathrm{d}^{12}\mathcal{Z}\Big\{-4b\lambda^2\Big(\mathcal{W}^3\Box\Big(\overline{\mathcal{W}}^3\Big)+\overline{\mathcal{W}}^3\Box\left(\mathcal{W}^3\right)\Big)+\ldots\Big\}$
$c\,\lambda^3\int \mathrm{d}^{12}\mathcal{Z}(T^-)^4\Box^2(\overline{T}^+)^4$	$\int \mathrm{d}^{12}\mathcal{Z}\Big\{-16c\lambda^3\Big(\mathcal{W}^4\Box\Big(\Box(\overline{\mathcal{W}}^4)\Big)+\overline{\mathcal{W}}^4\Box\left(\Box(\mathcal{W}^4)\right)\Big)\ldots\Big\}$
$d\,\lambda^2\int \mathrm{d}^{12}\mathcal{Z}(T^-)^2(\overline{T}^+)^2\times$ $\overline{\mathcal{D}}^4((T^-)^2)\mathcal{D}^4((\overline{T}^+)^2)$	$\int \mathrm{d}^{12}\mathcal{Z}\Big\{-16d\lambda^3\Big(\mathcal{D}^4[\mathcal{W}^2\overline{\mathcal{D}}^4(\overline{\mathcal{W}})^2]+\overline{\mathcal{D}}^4[\overline{\mathcal{W}}^2\mathcal{D}^4(\mathcal{W})^2]\Big)+\ldots\Big\}$

- choosing $a=-2^{-4}$, $b=-2^{-6}3^{-2}$, $c=-2^{-12}3^{-2}$ and $d=2^{-10}$ one recovers the first terms in the BI action with $D3$-brane condition.
- deformations and coefficients for the next order $\mathcal{O}(\mathcal{W}^{10})$ reproducing the action of Bellucci, Ivanov and Krivonos have been published today [Bellucci, Ivanov Krivonos][Carrasco Kallosh]

Conclusions

- for an initial deformation $\mathcal{I}$ one can iteratively solve for higher order deformations necessary to maintain duality invariance
- there is an infinite space of possible duality compatible deformations of $\mathcal{N} = 2$ supersymmetric gauge theory
- a plethora of valid deformations exists, necessary e.g. for finding the $\mathcal{N} = 2$ Born-Infeld action at higher orders
- method can be applied to a variety of other (supergravity) theories [Chemissany, Ferrara Kallosh, Shabazi]

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THANKS !