Vector Branes

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based on work with Fabio Riccioni and Luca Romano

and related to work with

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In this talk I will review some recent advances in the classification of branes and how this includes a classification of vector branes, i.e. the branes underlying susy Born-Infeld

"Vector Branes" $\equiv$ branes whose worldvolume dynamics is described by a vector multiplet

the brane tension $\mathrm{T_V}$ of vector branes does not necessarily scale as

$$\mathrm{T_V} \sim 1/g_s$$

Outline

Branes, Kappa-symmetry and Born-Infeld

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Branes, Kappa-symmetry and Born-Infeld

Branes and weights

Branes and half-maximal supergravity

Wrapping Rules

Conclusions

Question

- Branes are massive objects with a number of worldvolume and transverse directions; they play an important role in string theory

- Question : What can we learn about branes using supergravity as a low-energy approximation to string theory ?

- we first consider the branes of maximal supergravity

"Standard" Branes

- "standard" branes are branes with three or more transverse directions

- these branes couple to p-form potentials with $0 \leq p \leq D-3$

- all p-form potentials and the dual $(D-p-2)$-form potentials describe physical states of the supergravity multiplet

- to each p-form potential corresponds a half-supersymmetric $(p+1)$-brane

Kappa-symmetry

branes couple to the metric via a Nambu-Goto term and to the p-form potentials via a Wess-Zumino term

these two terms together are needed for kappa-symmetry

kappa-symmetry, after gauge-fixing, leads to world-volume supersymmetry

Example

$$\mathcal{L}_{\mathrm{F1}}(D=10) \;=\; \mathrm{T}_{\mathrm{F1}}\sqrt{-g} \;+\; B_2$$

$$\mathrm{T}_{\mathrm{F1}} \;\sim (g_s)^0\,, \qquad\qquad g_s = < e^{\phi} >$$

The Wess-Zumino Term of Fundamental Strings

For every compactified direction we have two fundamental 0-branes coming from a wrapped fundamental string and a pp-wave

The corresponding 1-forms transform as a vector $B_{1,A}$ under the T-duality group $\mathrm{SO}(10-\mathrm{D}, 10-\mathrm{D})$

$$\mathcal{L}_{\mathrm{WZ}}(D < 10) = B_2 + \eta^{AB}\mathcal{F}_{1,A}B_{1,B}$$

contains "extra scalars" $b_{0,A}$ via $\mathcal{F}_{1,A} = d\,b_{0,A} + B_{1,A}$

Counting Worldvolume Degrees of Freedom

$$D = 10 \quad : \qquad (10-2) = 8\,,$$

$$D < 10 \quad : \qquad (D-2) + 2(10-D) \neq 8\,!$$

Twice too many "extra scalars" $b_{0,A}$ $\rightarrow$ "doubled geometry"

Hull, Reid-Edwards (2006-2008)

Self-duality conditions on the extra scalars $b_{0,A}$ give correct counting

Dirichlet branes and Born-Infeld

D-branes have two properties:

- their worldvolume dynamics is described by a vector multiplet
- their tension scales as T $\sim 1/g_s$

The worldvolume action contains a DBI-VA action:

$$\mathcal{L} \sim e^{-\phi}\sqrt{g+\mathcal{F}_2} + \mathcal{L}_{\rm WZ} \qquad\qquad \mathcal{F}_2 = d\,b_1 - B_2$$

Counting D-branes

D-branes transform as spinors under T-duality

$$\left[\mathcal{L}_{\mathrm{WZ}}(D \leq 10)\right]_\alpha = \left[e^{\mathcal{F}_2}\, e^{\mathcal{F}_{1,A}\Gamma^A}\, C\right]_\alpha$$

Riccioni + E.B. (2010)

$$D = 10 \quad : \qquad (p-1) + (10-p-1) = 8$$

$$D < 10 \quad : \qquad (p-1) + (D-p-1) + 2(10-D) \neq 8\,!$$

only half of the $\mathcal{F}_{1,A}\Gamma^A$ contributes to a particular spinor component !

“Wess-Zumino term requirement”

the construction of a gauge-invariant WZ term requires the introduction of a number of worldvolume p-form potentials

worldvolume supersymmetry requires that these worldvolume fields fit into a 8+8 vector (scalar) or tensor supermultiplet

for standard branes this leads to the following counting rules:

potentials = # dual potentials = # half-supersymmetric branes

"Defect Branes"

"defect branes" are branes with two transverse directions

Greene, Shapere, Vafa, Yau (1990); Gibbons, Green, Perry (1996), Vafa (1996)

- defect branes are not asymptotically flat
- they require orientifolds
- number of half-supersymmetric defect branes $\neq$ number of $(D-2)$-form potentials $\neq$ number of dual scalars
- follows from "Wess-Zumino term requirement"

Example: IIB supergravity

IIB supergravity contains:

- two scalars (axion and dilaton)
- three 8-form potentials $A_8^{\alpha\beta} = A_8^{\beta\alpha}$ $(\alpha = 1, 2)$ that transform as the **3** of SL(2,$\mathbb{R}$)
- two half-supersymmetric branes: the D7-brane and its S-dual

"Wess-Zumino term requirement"

$$\mathcal{L}_{\mathrm{WZ}} \sim \mathrm{T}_{\alpha\beta}\{A_8^{\alpha\beta} + A_6^{(\alpha}\mathcal{F}_2^{\beta)} + \cdots\} \qquad \alpha = 1,2$$

$\mathcal{F}_2^\alpha = d\,b_1^\alpha - B_2^\alpha$: two worldvolume vectors

only two out of three branes, with charges T_{11} and T_{22}, contain a single worldvolume vector.

the third brane, with charge T_{12}, contains two worldvolume vectors that cannot be part of a worldvolume multiplet with 16 supercharges

counting defect branes

Consider a maximal supergravity with U-duality group G and maximal compact subgroup H

- # scalars = dim G - dim H
- # dual potentials = dim G
- # half-supersymmetric branes = dim G - rank G

"Domain Walls" and "Space-filling Branes"

- "domain walls" are branes with one transverse direction
- they couple to $(D-1)$-form potentials which are dual to an integration constant
- "space-filling branes" are branes with zero transverse directions
- they couple to D-form potentials that do not even describe a constant
- both domain walls and space-filling branes require orientifolds

New Supergravity development

- A full classification of the $(D-1)$-form, $(D-1)$-form and D-form potentials in $3 \leq D \leq 11$ maximal supergravity has been obtained using three different techniques:

  - closure of the supersymmetry algebra

    de Roo, Hartong, Howe, Kerstan, Ortín, Riccioni + E.B. (2005-2010)

  - using the very extended Kac-Moody algebra E_{11}

    Riccioni, West (2007); Nutma + E.B. (2007)

    a similar analysis can be done for E_{10}, see e.g. Nicolai, Fischbacher (2002)

  - using the embedding tensor technique

    for a review, see de Wit, Nicolai, Samtleben (2008)

Question

given a $(p+1)$-form which components of its U-duality representation couple to a half-supersymmetric brane ?

standard branes: each component

defect branes: $\dim G - \operatorname{rank} G$ components

domain walls and space-filling branes : ??

Answer

There is a simple group-theoretical characterization of which components of the U-duality representation couple to a half-supersymmetric brane

for a derivation based on the WZ term requirement, see Riccioni + E.B (2010) and other papers

for an alternative E_{11} derivation, see Kleinschmidt (2011)

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Example: 8D domain walls

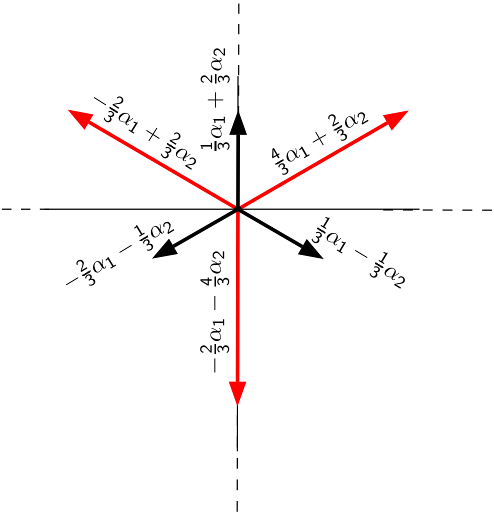
- The U-duality group G is SL(3,$\mathbb{R}$)$\times$ SL(2,$\mathbb{R}$)

- 8D supergravity contains 7-forms $A_{7,MNa} = A_{7,NMa}$ ($M = 1, 2, 3$; $a = 1, 2$) in the $(\mathbf{6}, \mathbf{2})$ of SL(3,$\mathbb{R}$)$\times$ SL(2,$\mathbb{R}$)

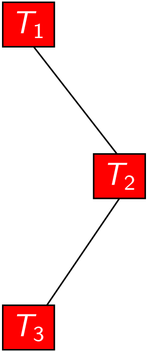
- how many components couple to a half-supersymmetric domain wall?

- The **6** of SL(3,$\mathbb{R}$) has *two* dominant weights:
    - the first dominant weight is the highest weight of the **6**
    - the second dominant weight is the highest weight of a $\overline{\mathbf{3}}$

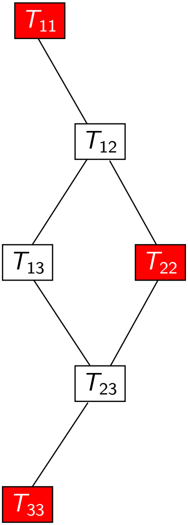
weights of different length



tensor components corresponding to the **3**



tensor components corresponding to the **6**



The “longest weight rule”

Given a supergravity theory with a $(p+1)$-form potential in a specific U-duality representation, then # of half-supersymmetric branes correspond to the longest weights of that U-duality representation

If, for given p, there are several U-duality representations for a given type of brane the half-supersymmetric branes belong to the highest-dimensional representation

The 8D non-standard branes

The U-duality group G is SL(3,$\mathbb{R}$)$\times$ SL(2,$\mathbb{R}$)

branes	repr.	highest dominant weights	weights of highest length
defect branes	$(\mathbf{8},\mathbf{1})$	$\boxed{1\ 1} \times \boxed{0}$	6
	$(\mathbf{1},\mathbf{3})$	$\boxed{0\ 0} \times \boxed{2}$	2_T
domain walls	$(\mathbf{6},\mathbf{2})$	$\boxed{2\ 0} \times \boxed{1}$	3×2
space-filling branes	$(\mathbf{15},\mathbf{1})$	$\boxed{2\ 1} \times \boxed{0}$	6

we have classified all half-supersymmetric branes, both the standard ones and the non-standard ones, and they all satisfy the same longest weight rule

The branes of maximal supergravity

we find that all branes of maximal supergravity are either vector branes (this includes the Dirichlet branes with $T_V \sim 1/g_s$) or tensor 5-branes

Next Question

what are the vector branes of half-maximal supergravity ?

- upon Type I truncation 8+8 vector multiplet of D5 brane reduces to a 4+4 hypermultiplet

Outline

Heterotic Supergravity

the field content of D=10-d dim. heterotic sugra is given by

$$\{e_\mu{}^a\,, B_2\,, d\times B_1\,, \phi\} \quad \text{plus} \quad 16+d\times\{B_1\,, d\times\phi\}$$

The T-duality symmetry group is SO(d,16+d)

- the vectors are in the fundamental of SO(d,16+d)
- the scalars parametrize the coset

$$SO(d,16+d)/[SO(d)\times SO(16+d)]$$

Heterotic Branes

heterotic branes only occur with tensions

$$T_H \sim (g_s)^\alpha \quad \text{with} \quad \alpha = 0, -2, -4, \ldots$$

we find the following fundamental and solitonic heterotic branes:

α	fields
0	$B_{1,A}$ B_2
-2	D_{D-4} $D_{D-3,A}$ D_{D-2,A_1A_2} $D_{D-1,A_1A_2A_3}$ $D_{D,A_1A_2A_3A_4}$

we find only worldvolume hypermultiplets with 8 supercharges

Heterotic Vector Branes

In 6D we find the following $\alpha = -4$ F-potentials:

$$F_{5,A} \qquad \text{and} \qquad F_{6,AB} = F_{6,BA}$$

they couple to V4-branes and V5-branes:

$$\mathcal{L}_{\rm WZ} \sim F_{5,A} + \mathcal{H}_2\, D_{3,A}\,, \qquad \mathcal{H}_2 = d\, d_1 - D_2$$

$$\mathcal{L}_{\rm WZ} \sim F_{6,AB} + \mathcal{F}_{1(A}\, F_{5,B)}\,, \qquad \mathcal{F}_{1A} = d\, b_{1A} - B_{1,A}$$

$\mathcal{N} = (0, 2)$ iib supergravity

the field content of matter-coupled iib supergravity is given by:

$$\{e_\mu{}^a\,, 5 \times B_2^+\} \qquad \text{plus} \qquad 21 \times \{B_2^-\,, 5 \times \phi\}$$

The T-duality symmetry group is SO(5,21)

- the 2-forms are are in the fundamental of SO(5,21)
- the scalar parametrize the coset

$$\mathrm{SO}(5,21)/[\mathrm{SO}(5) \times \mathrm{SO}(21)]$$

iib Vector Branes

we have the following potentials:

$$A_{2,A}\,, \qquad A_{4,AB}\,, \qquad A_{6,A,BC}$$

the 4-form couples to a V3-brane :

$$\mathcal{L}_{\mathrm{WZ}} \sim A_{4,AB} + \mathcal{F}_{2,[A}A_{2,B]}\,, \qquad \mathcal{F}_{2,A} = d\,v_{1,A} - A_{2,A}$$

the two vectors are related via a duality relation

Question

how are all these branes related via wrapping ?

Outline

T-duality

We consider branes whose tension has a fixed dependence on the string coupling constant g_s

$$\mathrm{T_F} \sim 1\,, \qquad \mathrm{T_D} \sim 1/g_s\,, \qquad \mathrm{T_S} \sim 1/g_s^2\,, \ldots$$

- we only consider branes of maximal supergravity

An Issue

supergravity is not complete!

"Brane Geometry"

the wrapping rules of standard geometry

$$\text{any brane} \quad \begin{cases} \text{wrapped} & \rightarrow \quad \text{undoubled} \\ \text{unwrapped} & \rightarrow \quad \text{undoubled} \end{cases}$$

only work for D-branes !

Fundamental Branes

the wrapping rules of fundamental branes are given by

$$\mathrm{T_F} \sim 1 \ : \quad \begin{cases} \text{wrapped} & \rightarrow \quad \text{doubled} \\ \text{unwrapped} & \rightarrow \quad \text{undoubled} \end{cases}$$

the extra input comes form pp-waves (generalized geometry?)

Solitonic Branes

the wrapping rules of solitonic branes are given by the dual rules

$$\mathrm{T}_S \sim 1/g_s^2 \; : \quad \begin{cases} \text{wrapped} & \rightarrow \quad \text{undoubled} \\ \text{unwrapped} & \rightarrow \quad \text{doubled} \end{cases}$$

the extra input requires Kaluza-Klein monopoles for the standard branes and <u>something new</u> for the non-standard branes:

- new objects such as "generalized" KK monopoles which couple to "mixed-symmetry" tensors ?

  Lozano-Tellechea, Ortín (2001); Ortín , Riccioni + E.B. (2011)

- new geometry ?

New Wrapping Rules

$$\mathrm{T}_E \sim 1/g_s^3 \ : \quad \begin{cases} \text{wrapped} & \rightarrow \quad \text{doubled} \\ \text{unwrapped} & \rightarrow \quad \text{doubled} \end{cases}$$

Example: the S-dual of the D7-brane

$$\mathrm{T}_{\text{space-filling}} \sim 1/g_s^4 \ : \quad \text{wrapped} \quad \rightarrow \quad \text{doubled}$$

Example: the S-dual of the D9-brane

there are also branes in lower dimensions that do not follow from the wrapping of any known brane in ten dimensions

Outline

Summary

- In this talk I discussed the half-supersymmetric branes of maximal and half-maximal supergravity

- I showed that both theories contain V-branes and I identified the scaling of the V-brane tensions

- I showed how the branes in different dimensions are related via wrapping rules whose interpretation is unclear yet (new objects or new geometry ?)

I did not discuss:

- BPS conditions, central charges and degeneracies

for earlier work, see Ferrara and Maldacena (1997)

- orbits and multi-charge configurations

for earlier work, see Ferrara, Gunaydin (1997); Lu, Pope, Stelle (1997)

Open Issues

- what are the precise properties of the V-branes ?

- what is the role of the non-standard branes ?

cp. to recent work of de Boer, Shigemori (2010-2012); Hassler, Lust (2013)

- what is the meaning of the wrapping rules ?
  - "generalized extended objects" or "brane geometry" ?

- how does the counting of branes work for supergravities with less supersymmetry ?