The Beauty of the Brown Muck -- News from the Heavy Quark Expansion

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(0) Status '04

$m_b(1\ \mathrm{GeV}) = (4.61 \pm 0.068)\ \mathrm{GeV}$ ⇐ 1.5 %

$m_c(1\ \mathrm{GeV}) = (1.18 \pm 0.092)\ \mathrm{GeV}$ ⇐ 7.8 %

$m_b(1\ \mathrm{GeV}) - 0.74\ m_c(1\ \mathrm{GeV}) = (3.74 \pm 0.017)\ \mathrm{GeV}$ ⇐ 0.5 %

$|V(cb)| = (41.390 \pm 0.870)\times 10^{-3}$ ⇐ 2.1 %

vs.

$|V(us)|_{KTeV} = 0.2252 \pm 0.0022$ ⇐ 1.1 %

lessons form a paradigm (the tale behind these numbers):

- robust theory coupled with
- high quality data
  - precision, i.e. small defensible uncertainties

Outline

(I) Heavy Quark Expansions

(II) Master Formulae for SL Width

(III) Heavy Quark Parameters

(IV) Theoretical Uncertainties

(V) Cut-induced `Biases'

(VI) |V(cb)| & |V(ub)| Lessons & Outlook

emphasis on the whole program

-- its elements, its self consistency and cross checks -- rather than numbers

You can ignore recent PDG `review' on V(cb)!

(I) H(eavy) Q(uark) E(xpansions)

hopes & goals (largely realized!)

- deepen understanding of QCD
  - new conceptual insights (quark masses, duality, …)
  - previously unanticipated precision in describing hadronic processes

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- refine & sharpen SM predictions for ~~C~~P in B decays, rates, distributions
- \`inverse theoret. engineering': interprete deviations

  from SM predictions in terms of a specific NP scenario

☺ the hope: $m_b \gg \Lambda_{QCD}$

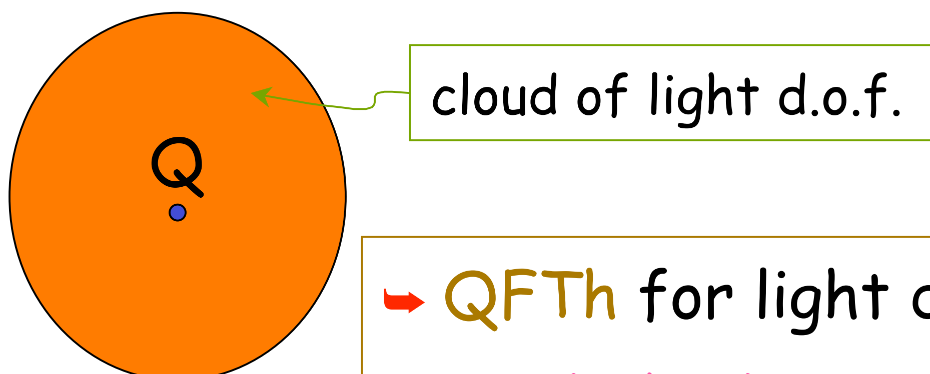
Methodology

symmetry principle asymptotic limit	dynamical approach pre-asymptotia

- chiral invariance: $m_q=0,\ m_\pi=0$ + chiral pert.theory: $(m_q/4\pi f_\pi)^{2n}$
- Heavy Quark Symmetry + Heavy Quark Expans.

$H_Q=[Qq]$

energy-X with soft medium ~ Λ ~ 1/2 GeV ~ no QQ fluctuations



Q static
→ QM for Q

→ QFTh for light d.o.f.: $(\Lambda/m_Q)^n$

Heavy Quark Symmetry ⊕ Heavy Quark Expans.

$\sim H_{Pauli} = - A_0 + (i\partial - \mathbf{A})^2/2m_Q + \sigma\mathbf{B}/2m_Q \quad \rightarrow - A_0 \quad$ as $\; m_Q \rightarrow \infty$

i.e.,

infinitely heavy static quark, without spin dynamics,

only colour Coulomb potential!

- hadrons H_Q labeled by total spin S and by $j_q = l_q + s_q$:
  - ground states: $[S|l_q|j_q] = [0,1|0|1/2]$: P & V
  - 1st excit. states: $[0,1|1|1/2]$ & $[1,2\;|1|3/2]$

- conjecture

$$\text{rate}(H_Q \to f_{incl}) = K(CKM, m_Q)\Sigma_{i=0}\, c_i(\alpha_S)(\Lambda/m_Q)^i$$

complication in weak decays:

$$\square(H_Q)\ \mu\ m_Q^{5}(\square)\ [\, z_3(m,\square,\square_S) + \ldots]$$

tool chest to implement idea:

- operator product expansion (OPE)
- dispersion relations
- sum rules

when is a quark heavy -- when light?

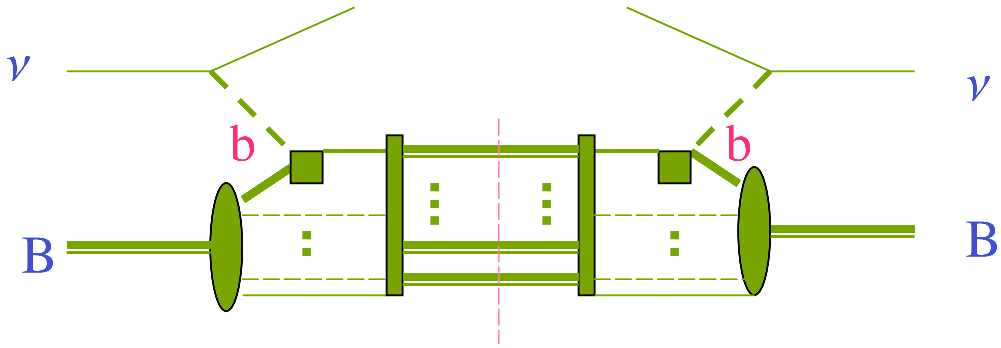
u & d quarks clearly light, b quarks heavy!

need a yard stick

- 1st guess: $\Lambda_{NonPD} \sim \Lambda_{QCD} \sim 200$ MeV
  - c quarks clearly heavy, yet s quarks ??
- 2nd guess: $\Lambda_{NonPD} \sim 700$ MeV ($\sim N_C \times \Lambda_{QCD}$?)
  - s quarks light by default;

    c quarks: iffy -- need case-by-case study!

semileptonic transitions -- like ~ DIS



$$h_{\mu\nu} = \frac{i}{2M_{H_Q}} \int d^4x\, e^{-ikx} \left\langle H_Q \left| \{ J_\mu(x) J_\nu^*(0) \}_T \right| H_Q \right\rangle$$

$$= -h_1 g_{\mu\nu} + h_2 v_\mu v_\nu - i h_3 \epsilon_{\mu\nu\rho\sigma} v^\rho k^\sigma + h_4 k_\mu k_\nu + h_5 (k_\mu v_\nu + v_\mu k_\nu)$$

"structure functions" $w_i = 2\,\mathrm{Im}\, h_i$

O(perator)P(roduct)E(xpansion)

$$\Gamma(H_Q \to f) = \Sigma_i \, c_i^{(f)} (KM, M_W, m_Q, \alpha_S, \mu) \langle H_Q \mid O_i \mid H_Q \rangle_{(\mu)}$$

- short distance dynamics → coeff. $c_i^{(f)}$
- universal cast of local operators O_i
- $\langle H_Q|O_i|H_Q\rangle$ inferred from other observables or lattice QCD!
- expansion parameter

$$1/E_{release} \sim \begin{cases} 1/(m_b - m_c) & \text{for } b \to c \\ 1/m_b & \text{for } b \to u \end{cases}$$

- Wilson: auxiliary scale μ s.t.

  short distance < μ^{-1} < long distance

  - c_i ⇔ short distance dynamics
  - O_i active fields - long distance dynamics

: longer dist. d.o.f. short dist. d.o.f.

➥ both c_i and O_i change!

☞ choose judiciously!

Scylla & Charybdis: $\Lambda_{QCD} \ll \mu \ll m_Q$

➻ matrix elements calculable

➻ $\alpha_S \ll 1$

$\mu \sim 1$ GeV √ for Q = b! yet: Q = c?

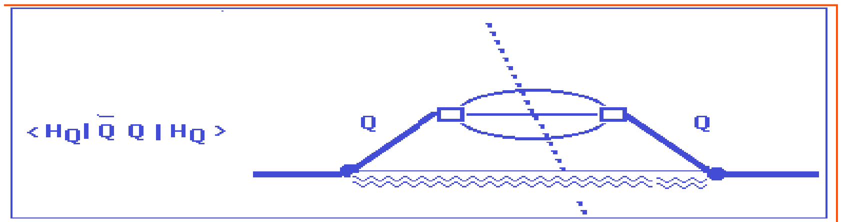
leads to \`smart' pert.treatment

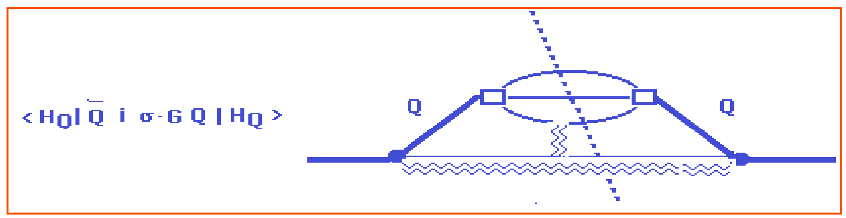
☞ treat as physical parameter (s. sum rules)

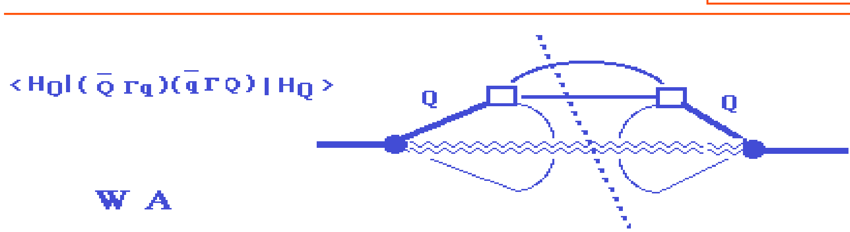
Caveat: now everyone invokes OPE -- but often without using Wilson prescription

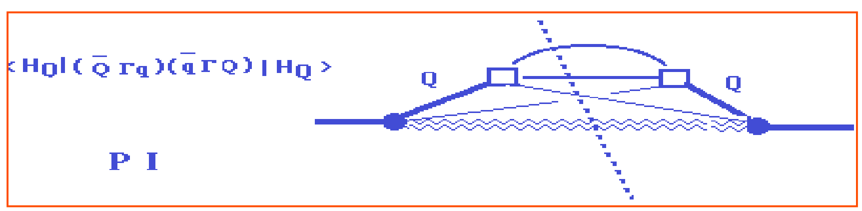
$$\square\,(H_Q \;\square\; f_{incl}) = \frac{G_F^2\,|KM|^2\,m_Q^5}{192\,\square^3}\;\square$$

$$\square\left\{ c_3(f)\,\boxed{\langle H_Q|\bar{Q}\,Q|H_Q\rangle} + c_5(f)\,\frac{\boxed{\langle H_Q|\bar{Q}i\square\cdot G\,Q|H_Q\rangle}}{m_Q^2} + c_6(f)\,\frac{\boxed{\langle H_Q|(\bar{Q}\square q)\cdot(\bar{q}\square Q)|H_Q\rangle}}{m_Q^3} + \ldots \right\}$$



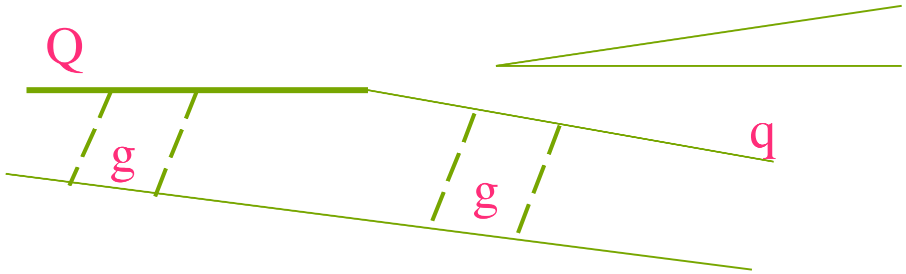






❶ term of order $1/m_Q$ anathema!

- *no independent* dim-4 operator! $\boxed{\bar{Q}\,\gamma\cdot\partial\,Q \simeq m_Q\,\bar{Q}Q}$



- phase space correct. ~ $1/m_Q$ in initial & final state
- yet cancel since colour charge of Q and q identical

- local colour gauge symmetry essential:

  single operator $\bar{Q}\gamma_\mu D_\mu Q$, D=covariant derivative

  global colour symmetry would allow $1/m_Q$ term:

  $\bar{Q}\gamma_\mu \partial_\mu Q$ and $g\,\bar{Q}\gamma_\mu A_\mu Q$ disconnected

☛ need for $1/m_Q$ contrib. =violation of quark-hadron duality!

➡ replacing m_Q by $M(H_Q)$ = duality violation!

❷ leading nonperturb. contrib. ~ $O(1/m_Q^2)$:

~ O(5 %) for Q = b

weight of nonpert. effects greatly reduced in beauty decays

➥ perturb. contributions numerically important

➼ with `smart' pert. treatment $□_{parton}$ good estimate

❸ often find $O(1/m_Q^2) \sim O(1/m_Q^3)$:

what about $O(1/m_Q^4)$ -- no convergence?

No!

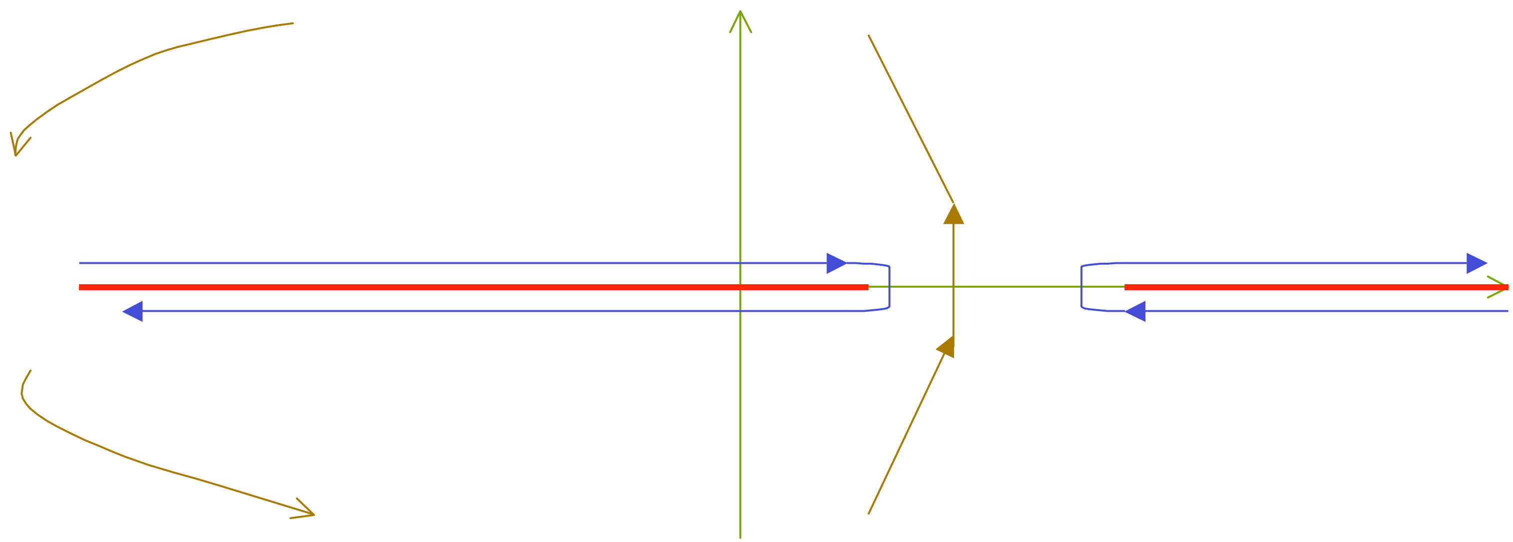
$$\left.\begin{array}{l} \text{❑ complete cancellation in } O(1/m_Q); \\ \text{partial cancellation in } O(1/m_Q^2); \\ \text{no cancellation in } O(1/m_Q^3) \end{array}\right\} O(1/m_Q^2) \text{ reduced}$$

❑ one-time enhancem. of $O(1/m_Q^3)$ by "16□^{2}"

Dispersion relations

OPE defined and constructed in Euklidean regime

➥ extrapolate to Minkowskian regime via dispersion relations



$\Gamma(H_Q)|_{OPE} \leftrightarrow \Gamma(H_Q)|_{observ}$

dispersion integral

- assume: QCD creates no unphysical singularity
- intrinsic limitation of algorithm
  - ➥ limitations to duality!

Sum rules

not to be confused with `*the* SVZ QCD sum rules

- different regimes, observables …
- those are an art rather than a technology
  - ☛ their irreducible theor. uncert. ~ 20 - 30 %

sum rules to be sketched here

- lead to novel conceptual insights
- imply rigorous inequalities
- allow translation of experim. info into constraints on HQP
  - ☛ impact from charm spectroscopy (s. later)
- with an intrinsic accuracy not limited to 20 %

  harnessing additional theoret. tool: HQE!

➼ SV sum rules, Spin/Uraltsev sum rules, D'Orsay sum rules

☞ $\rho^2(\mu) - 1/4 = \Sigma_n |\tau_{1/2}{}^{(n)}|^2 + 2\, \Sigma_m |\tau_{3/2}{}^{(m)}|^2$

☞ $\quad 1/2 \quad = -\,2\, \Sigma_n |\tau_{1/2}{}^{(n)}|^2 + \Sigma_m |\tau_{3/2}{}^{(m)}|^2$

☞ $\overline{\Lambda}(\mu) = 2\,(\Sigma_n \epsilon_n |\tau_{1/2}{}^{(n)}|^2 + 2\, \Sigma_m \epsilon_m |\tau_{3/2}{}^{(m)}|^2)$

☞ $\mu^2{}_\pi(\mu)/3 = \Sigma_n \epsilon_n{}^2 |\tau_{1/2}{}^{(n)}|^2 + 2\, \Sigma_m \epsilon_m{}^2 |\tau_{3/2}{}^{(m)}|^2$

☞ $\mu^2{}_G(\mu)/3 = -2\, \Sigma_n \epsilon_n{}^2 |\tau_{1/2}{}^{(n)}|^2 + 2\Sigma_m \epsilon_m{}^2 |\tau_{3/2}{}^{(m)}|^2$

☞ ......

where: $\tau_{1/2}$ & $\tau_{3/2}$ denote transition amplitudes for

B→l□D(s_q=1/2[3/2]) with excit. energy ϵ_k = M(B)-M(D_k), $\epsilon_k \leq \mu$

➥ rigorous inequalities + experim. constraints: e.g.,

- $\Sigma_n \epsilon_n{}^2 |\tau_{1/2}{}^{(n)}|^2 \leq \Sigma_m \epsilon_m{}^2 |\tau_{3/2}{}^{(m)}|^2$
- $\mu^2{}_\pi(\mu) - \mu^2{}_G(\mu) = 9\, \Sigma_n \epsilon_n{}^2 |\tau_{1/2}{}^{(n)}|^2$

2 step procedure

❶ express observable in terms of HQP

❷ determine HQP from independent observable
both with commensurate accuracy & reliability!

- m_b, m_c: external to QCD
- $\mu^2_\pi, \mu^2_G, \rho^3_D, \rho^3_{LS}$: intrinsic to QCD

➥ sensitive tests for lattice QCD

will focus on semileptonic (& radiative) B decays

(II) Master Formulae for Γ_{SL}(B)

$$\Gamma_{SL}(B) = \Gamma_0(b)$$

$$\{f(z)[1-(2/3)(\alpha_S/\pi)(g(z)/f(z))+c_2\alpha_S^2+c_3\alpha_S^3+..]\times[1-(\mu_\pi^2(\mu)-\mu_G^2(\mu))/2m_b^2]$$
$$-2(1-z)^4\mu_G^2(\mu)/m_b^2$$
$$+[d(z)\rho_D^3(\mu)+l(z)\rho_{LS}^3(\mu)]/m_b^3+O(1/m_b^4)\}$$

$\Gamma_0(b) = G_F^2 m_b^5(\mu)\,|V(cb)|^2/192\pi^3$

f(z), g(z),d(z),l(z): phase space function of $z=m_c^2/m_b^2$

c_2: BLM $\alpha_S^2\beta_0$+ estimate for non-BLM, $\beta_0 = 11N_C/3 - 2N_f/3 = 9$

c_3: BLM $\alpha_S^3\beta_0^2$, [BLM known to all orders $\alpha_S^n\beta_0^{n-1}$]

$\mu_\pi^2(\mu) = \langle B|b(i\mathbf{D})^2 b|B\rangle|_\mu/2M_B$ kinetic energy

$\mu_G^2(\mu) = \langle B|b(i/2)\sigma_{\mu\nu}G^{\mu\nu}b|B\rangle|_\mu/2M_B$ chromomagn. moment

$\rho_D^3(\mu) = \langle B|b(-1/2\mathbf{D}\bullet\mathbf{E})b|B\rangle|_\mu/2M_B$ Darwin term

$\rho_{LS}^3(\mu) = \langle B|b(\boldsymbol{\sigma}\bullet\mathbf{E}\times\boldsymbol{\pi})b|B\rangle|_\mu/2M_B$ LS term analysis by Benson et al., Nucl.Phys.B665('03)367

- perturb. BLM correct. to all orders for realistic values of quark masses with Wilsonian cut-off μ:

  $$\alpha_S \{1+\Sigma_{n=1} c_n(\beta_0\alpha_S)^n\},\ \beta_0 = 11 - 2n_f/3 = 9 >> 1$$

  $c_1\beta_0\alpha_S$ $[c_2(\beta_0\alpha_S)^2]$ second[third] BLM correct.
- estimates for non-BLM α_S^2 term included
- detailed [some] consideration of $1/m_Q^3[1/m_Q^4]$
- keeping track of μ dependence
- only local operators employed
- expansions in $1/(m_b-m_c)$ and $1/m_b$, not in $1/m_c$
  - do not impose constraint a priori

    $m_b-m_c=\langle M_B\rangle - \langle M_D\rangle+\mu_\pi^2(1/2m_c-1/2m_b)$ +nonlocal operat.
  - include $\langle B|(\overline{b}\ldots c)(\overline{c}\ldots b)|B\rangle$ --
    otherwise terms $1/m_b^3 m_c^{n-3}$, n>2, arise

unknown higher order contributions:

- perturbative
  - partonic term: non-BLM ~ α_S^n, $n \geq 3$
  - $O(\alpha_S)$ correct. to nonpert. contrib. ~ $1/m_b^{3,4}$
- nonperturbative ~ $1/m_b^4$ [$(\sim 1/5)^4 \sim 10^{-3}$]
- duality violations: < 0.5 % [see later]
- limiting factor: size of perturb. corrections!

➥ $\Gamma_{SL}(B \to l\nu X_c)$ =

$$F(HQP) \pm 1\%|_{pert} \pm 2.4\%|_{hWc} \pm 0.8\%|_{hpc} \pm 1.4\%|_{IC} =$$

$$F(HQP) \pm 3\%|_{th}$$

$$|V(cb)|/0.0417 = (1+\delta_{th}) \times [1+0.30(\alpha_S(m_b) - 0.22)] \times$$

$$[1-0.66\times(m_b(1\ \mathrm{GeV}) - 4.6\ \mathrm{GeV}) + 0.39\times(m_c(1\ \mathrm{GeV}) - 1.15\ \mathrm{GeV}) +$$

$$+0.013\times(\mu_\pi^2 - 0.4\ \mathrm{GeV}^2) + 0.05\times(\mu_G^2 - 0.35\ \mathrm{GeV}^2) +$$

$$+0.09\times(\rho_D^3 - 0.2\ \mathrm{GeV}^3) + 0.01\times(\rho_{LS}^3 + 0.15\ \mathrm{GeV}^3)];$$

$$\delta_{th} = \pm\ 0.5\ \%|_{pert} \pm 1.2\%|_{hWc} \pm 0.4\%|_{hpc} \pm 0.7\%|_{IC}$$

$$|V(cb)|/0.0417 = (1+\delta_{th}) \times$$

$$[1-0.14\times(\langle M_X^2\rangle - 4.54\ \mathrm{GeV}^2) + 0.03\times(m_c(1\ \mathrm{GeV}) - 1.15\ \mathrm{GeV}) +$$

$$+0.1\times(\mu_\pi^2 - 0.4\ \mathrm{GeV}^2) + 0.01\times(\mu_G^2 - 0.35\ \mathrm{GeV}^2) +$$

$$+0.1\times(\rho_D^3 - 0.2\ \mathrm{GeV}^3) + 0.06\times(\rho_{LS}^3 + 0.15\ \mathrm{GeV}^3)];$$

Γ $(B \rightarrow l\,\nu\, X_u)$: V(cb) $\rightarrow$ V(ub)

☞ (even) better theoretical control

- $z = 0$: $f, g, d, l = 1$
- full □$_S^2$ known
- $1/m_b < 1/(m_b - m_c)$
- HQP known from $B \rightarrow l\,\nu\, X_c$

Yet experimentally …

(III) Heavy Quark Parameters (HQP)

Second task --

determine HQP without compromising advantages of OPE:

- expansions in terms of local operators
- with expansion parameter at least $1/(m_b - m_c)$, not $1/m_c$

☞ primary goal:

determine V(cb) rather than m_b, m_c, …

☞ since $\Gamma_{SL} = |V(cb)|^2$ F(HQP) $\pm (\leq 3\%)$

want to determine HQP s.t.

value of F(HQP) induces at most $\pm (\leq 1\text{-}2\%)$

2 different classes of HQP

- m_b, m_c -- external to QCD, i.e. can never be calculated by LQCD without experimental input
- $\square_{\square}^2$, $\square_D^3$, … internal to QCD, i.e. can be calculated by LQCD without experimental input

first need definitions of HQP that can pass muster by quantum field theory!

(3.1) Quark masses

unlike for QED due to confinement no `natural' definin. of quark mass

- pole mass (position of pole in perturb. Green function) IR unstable in complete theory

$$\Delta_{\mathrm{intrinsic}}\, \mathrm{m}_{\mathrm{pole}} \sim O(\Lambda_{\mathrm{QCD}})$$ `renormalons'

$$\mathrm{m}^5_{Q,\mathrm{pole}} = (\mathrm{m}_{Q,\mathrm{pert}} + \Delta_{\mathrm{intrinsic}}\mathrm{m}_{\mathrm{pole}})^5 = \mathrm{m}^5_{Q,\mathrm{pert}}(1 + \underbrace{\Delta_{\mathrm{intrinsic}}\mathrm{m}_{\mathrm{pole}} / \mathrm{m}_{Q,\mathrm{pert}})}_{\boxed{1/m_Q \text{ contribution!}}}$$

- must use running mass with IR cut-off μ to `freeze out' renormalons (pole mass: μ → 0) intuitive picture:

$m_Q \;\propto$ energy stored in chromoelectric field in a sphere of radius $R \gg 1/\, m_Q$

$$E_{CCoul}\,(R) \propto \int_{1/mb\, \leq |x| < R} d^3x\; |\mathbf{E}_{CCoul}|^2 \propto \text{const.} - \alpha_S(R)/\Box R$$

pole mass $\cong R \rightarrow \infty$; yet $\alpha_S(R)$ strong at $R \sim 1/\Box_{QCD}$

➾ $\Box_{IR}\, m_Q^{pole} = \mathcal{O}(\Box_{QCD})$!

- `kinetic mass'

$$dm_Q(\mu)/d\mu = -\ 16\alpha_S(\mu)/3\pi\ -\ 4\alpha_S(\mu)/3\pi(\mu/m_Q) + \ldots$$

i.e., linear scale dependence in IR

□$_{HQET}$ ≈ □$_{kin}$(1 GeV) - 0.255 GeV   to one-loop

- $\overline{\text{MS}}$ mass not a parameter in the effect. Lagrangian, rather an ad-hoc combination convenient in perturb. calc.

$$m_Q(\mu) = m_Q(m_Q)[1+2(\alpha_S/\pi)\log(m_Q/\mu)\ \to \infty \quad \text{as} \quad \mu/m_Q \to 0\ !$$

  - ☞ appropriate when relevant scale □ > m_Q production process, e.g.: $Z \to b\,\bar{b}$

  - ☞ inadequate for decays where □ < m_Q

acceptable as reference point

before 2002

$\Upsilon(4S) \to b\bar{b}$:

$$m_{b,kin}(1\ \text{GeV}) = \begin{cases} 4.56 \pm 0.06\ \text{GeV} & \text{MeYe} \\ 4.57 \pm 0.05\ \text{GeV} & \text{Ho} \\ 4.59 \pm 0.06\ \text{GeV} & \text{BeSi} \\ 4.58 \pm 0.05\ \text{GeV} & \text{KuSt} \end{cases}$$

➥ $\langle m_{b,kin}(1\ \text{GeV})\rangle|_{bb} = 4.57 \pm 0.08\ \text{GeV}$

m_b - m_c

$$m_b - m_c = \langle M_B\rangle - \langle M_D\rangle + \mu_\pi^2(1/2m_c - 1/2m_b) + \ldots$$
$$= 3.50\ \text{GeV} + 40\ \text{MeV}(\mu_\pi^2 - 0.5\ \text{GeV}^2)/0.1\ \text{GeV}^2$$

vulnerable relation since

- ☛ expansion in $1/m_c$
- ☛ nonlocal correlators at $1/m_c^2$

➥ do not impose this relation on m_b - m_c!

(3.2) Hadronic expectation values

- *dim 3*

$$\langle H_Q | \bar{Q}\, Q | H_Q \rangle = 1 - \frac{\langle H_Q | \bar{Q}\, (i\vec{D})^2 Q | H_Q \rangle}{2m_Q^2} + \frac{\langle H_Q | \bar{Q}\, \frac{i}{2}\, \sigma \cdot G\, Q | H_Q \rangle}{2m_Q^2} + \ldots$$

- *dim 5*

chromomagnetic moment μ_G^2

$$\mu_G^2 = \langle H_Q | Q i/2 \sigma_{\mu\nu} G_{\mu\nu} Q | H_Q \rangle / 2M(H_Q) = (3/2)\,[M^2(V_Q) - M^2(P_Q)]$$

for b = Q: $\mu_G^2 \approx 0.35^{+0.03}_{-0.02}$ GeV^2

kinetic energy μ_π^2

$$\mu_\pi^2 = \langle H_Q | Q \pi^2 Q | H_Q \rangle / 2M(H_Q) \approx -\lambda_1 + 0.18 \text{ GeV}^2$$ to one-loop

SV SR: $\mu_\pi^2 > \mu_G^2$;

`QCD' SR: $\mu_\pi^2 = 0.45 \pm 0.1$ GeV^2

- *dim 6*

Darwin term ρ_D^3

$$\rho_D^3(\mu) = \langle B|b(-1/2\mathbf{D}\cdot\mathbf{E})b|B\rangle|_\mu/2M_B$$

$$\rho_D^3(\mu) \sim 0.1 \ \mathrm{GeV}^3$$

LS term ρ_{LS}^3

$$\rho_{LS}^3(\mu) = \langle B|b(\boldsymbol{\sigma}\cdot\mathbf{E}\times\boldsymbol{\pi})b|B\rangle|_\mu/2M_B$$

$$-\rho_{LS}^3(\mu) \leq \rho_D^3(\mu)$$

hardly relevant, since very reduced contribution

(3.3) Summary on Defining & Constructing the OPE

$$\Gamma(H_Q) \propto m_Q^5(\mu)[\, z_3(m,\mu,\alpha_S) + z_5(m,\mu,\alpha_S) \langle H_Q | Q\sigma \bullet G Q | H_Q \rangle_{(\mu)} + \ldots]$$

tasks at hand:

- pick a mass well-defined perturb. & nonperturb.
- decide at which scale □ to evaluate it
- employ same scale □ in other perturbative corrections and hadronic expectation values

Our theoretical framework a la Wilson very robust with smallish higher order pert. & nonpert. contributions --

not true for other implementation of the OPE (~HQET) e.g.:

using the pole mass:

$$A_{pert}^{pole} = 1 - 1.78\,\alpha_S(m_b)/\pi - 15.8[\alpha_S(m_b)/\pi]^2 - 230[\alpha_S(m_b)/\pi]^3 - 3640[\alpha_S(m_b)/\pi]^4 \ldots$$

using kinetic mass at μ=1 GeV

$$A_{pert}^{kin} = 1 - 0.94\,\alpha_S(m_b)/\pi - 5.1[\alpha_S(m_b)/\pi]^2 - 17[\alpha_S(m_b)/\pi]^3 - 63[\alpha_S(m_b)/\pi]^4 \ldots$$

$\boxed{a_2^{BLM} = -4.1,\ a_2^{nonBLM} = -1.0}$

(3.4) Extracting Values of HQP from Moments

V(cb) & HQP $\Longrightarrow$ $\Gamma_{SL}(B \to l\nu X_c)$, i.e. integrated spectrum

~~V(cb) &~~ HQP $\Longrightarrow$ shape of (E_l&M_X) spectrum

normalized moments $\Longleftrightarrow$ shape of spectrum

➥ normalized moments $\Longrightarrow$ HQP

Lepton energy and hadronic mass moments:

$$M_1(E_l) = \Gamma^{-1} \int dE_l \, E_l \, d\Gamma/dE_l$$

$$M_n(E_l) = \Gamma^{-1} \int dE_l \, [E_l - \overline{M}_1(E_l)]^n d\Gamma/dE_l \ , \ n > 1$$

$$M_1(M_X) = \Gamma^{-1} \int dM_X^2 (M_X^2 - M_D^2) d\Gamma/dM_X^2$$

$$M_n(M_X) = \Gamma^{-1} \int dM_X^2 (M_X^2 - \langle M_X^2 \rangle)^n d\Gamma/dM_X^2 \ , \ n > 1$$

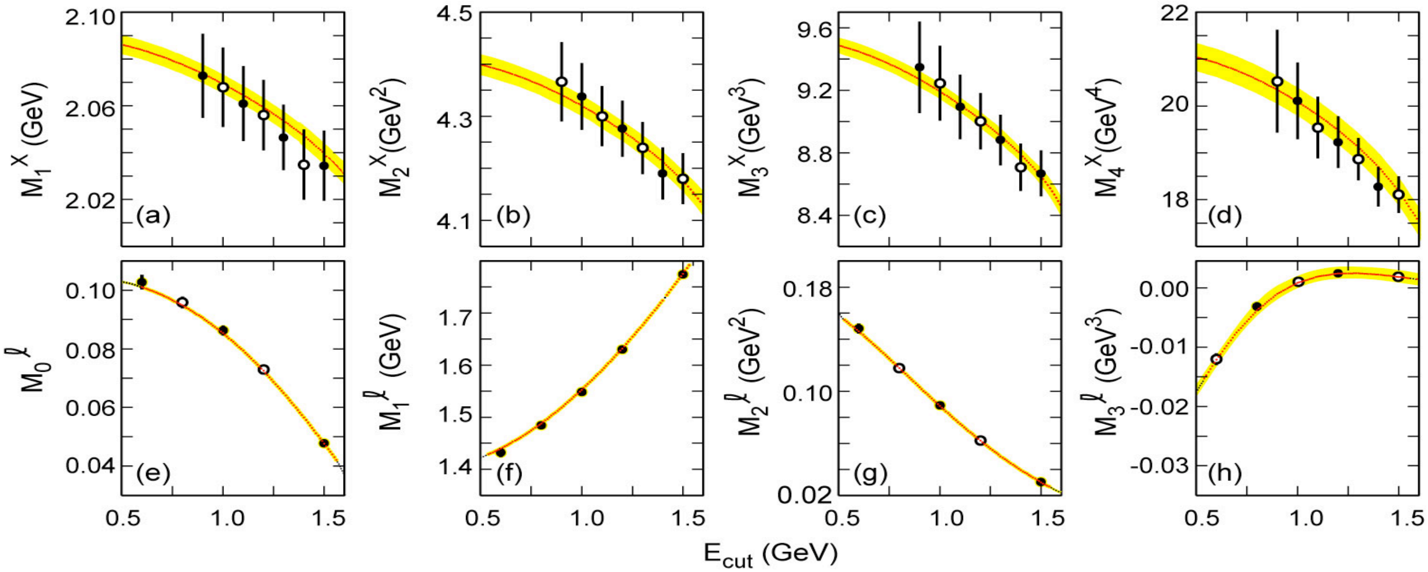
general considerations

- moments can/must be calculated in terms of same HQP & to same order (at least) in α_S & $1/m_Q$
- higher moments more sensitive to higher order contrib.
- need more moments to extract more HQP
  - ☞ 1 more moment does not always yield 1 more HQP correlations -- dependance on same combination of HQP
- aim for overconstraints
- optimize total error, not just experimental error
  - ➥ lower cut on lepton energy as low as possible --

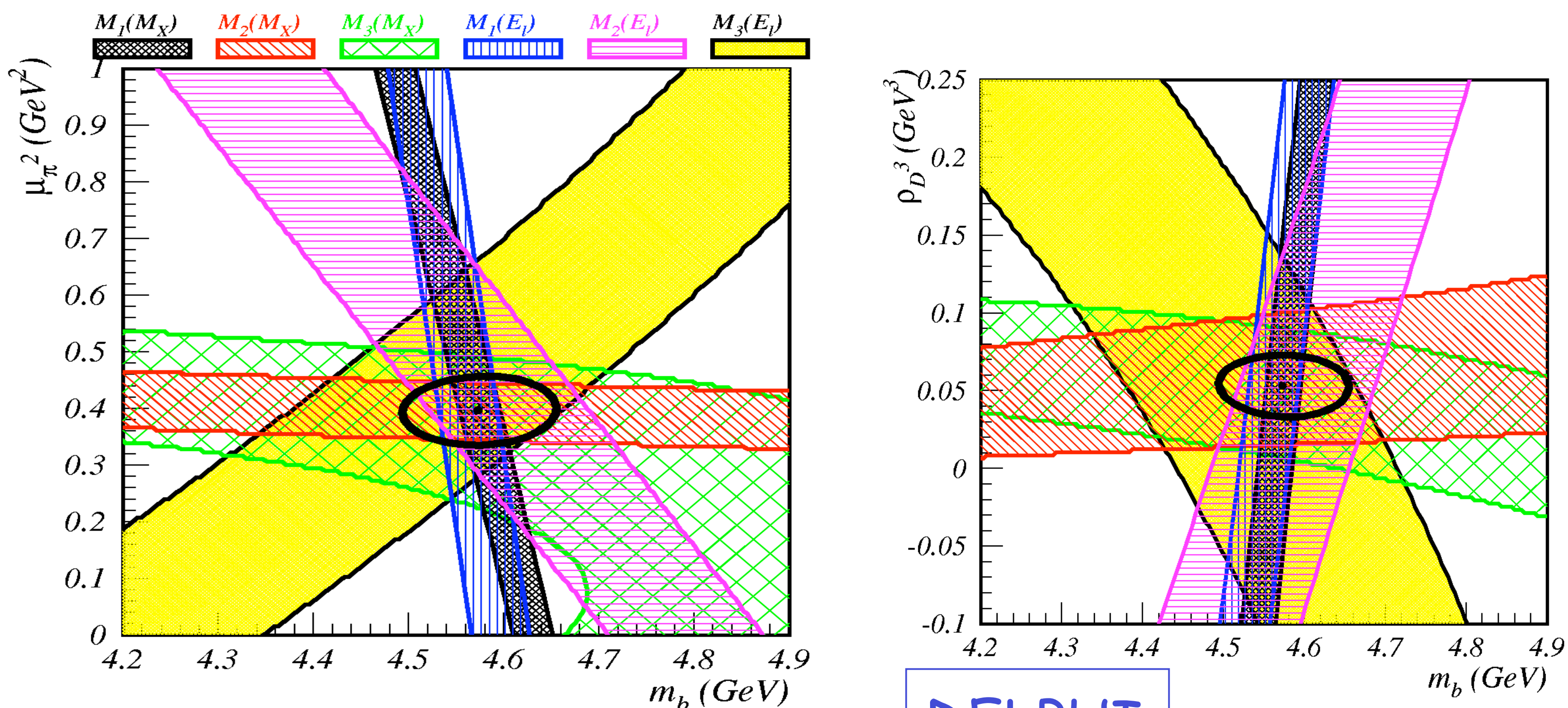
    otherwise: biases in theoret. interpretation (s. later) special considerations

- $M_{1\text{-}3}(E_l)$ & $M_1(M_X)$ depend on similar combination of HQP
  - $M_{2,3}(M_X)$ essential
- treat m_b - m_c as free parameters --

  do not impose constraint on m_b - m_c a priori



BABAR



DELPHI

excellent description of large set of data points in terms of 6 or even merely 4 parameters: m_b, m_c, μ_π^2, ρ_D^3, (μ_G^2, ρ_{LS}^3)

$m_b(1\ \mathrm{GeV}) = (4.61 \pm 0.068)$ GeV $\quad m_{b,kin}(1\ \mathrm{GeV})|_{bb}=4.57 \pm 0.08$ GeV

$m_c(1\ \mathrm{GeV}) = (1.18 \pm 0.092)$ GeV

$m_b(1\ \mathrm{GeV}) - m_c(1\ \mathrm{GeV}) = (3.436 \pm 0.032)$ GeV

$m_b(1\ \mathrm{GeV}) - m_c(1\ \mathrm{GeV})|_{MB-MD} = (3.48 \pm 0.02 \pm\ ?)$ GeV

$m_b(1\ \mathrm{GeV}) - 0.74 m_c(1\ \mathrm{GeV}) = (3.737 \pm 0.017)$ GeV

$\mu_G^2(1\ \mathrm{GeV}) = (0.267 \pm 0.067)\ \mathrm{GeV}^2 \quad \mu_G^2|_{HF} \approx 0.35 \pm 0.03\ \mathrm{GeV}^2$

$\mu_\pi^2(1\ \mathrm{GeV}) = (0.447 \pm 0.053)\ \mathrm{GeV}^2 \quad \mu_\pi^2|_{QCDSR} = 0.45 \pm 0.1\ \mathrm{GeV}^2$

$\rho_D^3(1\ \mathrm{GeV}) = (0.195 \pm 0.029)\ \mathrm{GeV}^3 \quad \rho_D^3(1\ \mathrm{GeV}) \sim 0.1\ \mathrm{GeV}^3$

(IV) Theoretical Uncertainties

3 classes of theoret. uncertainties

❶ uncertainties in input parameters: $\square_S$, ...

❷ " due to truncation in expansions

❸ limitations to quark-hadron duality

---

❶ straightforward

❷ uncalculated terms of higher order in α_S, m_Q, ...

- mostly systematic in nature
- numerically largest uncertainty in general
- encounter all the time

❸ "fear of the unfamiliar"

critics of duality in $\square_{SL}(B)$ often silent about $\square\square(B_s)$

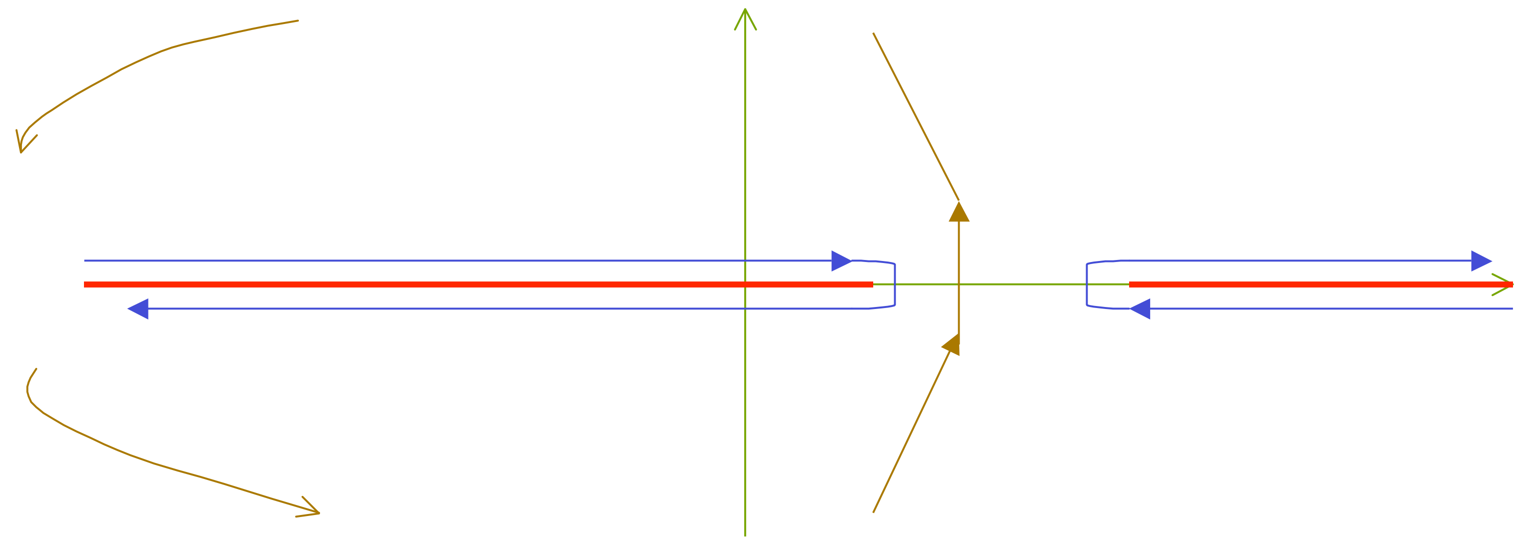
ancient concept, yet long period of stagnation

- pre-requisite for analyz. duality & its limitations:
  control over nonperturb. effects often not achieved -- yet in general can be evaded by going to higher energy scales: Q^2, p_t, etc.
- this option does not exist for B decays -- yet for a long time data insufficiently precise to push theory!

new paradigm:

- QCD is the theory of strong interactions
- hadronic observables = quark-gluon results -- provided all possible sources of corrections to the parton picture are properly accounted for!

Quark-hadron duality not an `ab initio' assumption!



$\Gamma(H_Q)|_{OPE} \leftrightarrow \Gamma(H_Q)|_{observ}$

dispersion integral

assume: QCD creates no unphysical singularity

duality violation = correction not accounted for due to

- truncation in expansion
- limitation in algorithm employed!

- no complete theory yet for duality and its limitations -- but we have moved beyond the folkloric stage in the last few years

- need OPE formulation to talk about duality & its limitations
  - we understand physical origins
    - hadronic thresholds
    - $1/m_c$ expansions
  - we have identified mathematical portals

Euclidean $\exp\{-m_Q/\Lambda\}$ $\longrightarrow$ Minkowskian $\sin(m_Q/\Lambda)$

- findings
  - fundamental question:

    is there an OPE -- or not!

  - duality cannot be exact at finite scales; limitations to duality will depend on the process: $\sim \sin(m_Q/\Lambda)/m_Q^k, \quad k > 1$
  - violations of local duality have the form of an oscillating function of the scale; duality violation cannot be blamed for a systematic excess or deficit in rates
  - duality violations larger in NL than SL decays, but no fundamental difference!
  - difference between local and other versions of duality quantitative rather than qualitative
  - a proliferation of decay channels not necessary for the onset of duality: in SV limit 1 or 2 channels will saturate inclusive width.
  - duality violation in $\Gamma_{SL}(B)$ < 0.5 %!

final arbiter ⇒ redundant determinations!

checks systematics -- whether higher orders or duality

determine m_b, $m_b - m_c$, μ_π^2 in different ways

(V) Cut-induced `Biases'

(5.1) Experimental cuts & `hardness'

Experimental cuts on energy etc. applied for practical reasons yet they degrade `hardness' $\mathcal{Q}$ of transition

 `exponential' contributions $\exp[-c\mathcal{Q}/\mu_{had}]$ missed in usual OPE expressions

- quite irrelevant for $\mathcal{Q} \gg \mu_{had}$
- yet relevant for $\mathcal{Q} \sim \mu_{had}$!

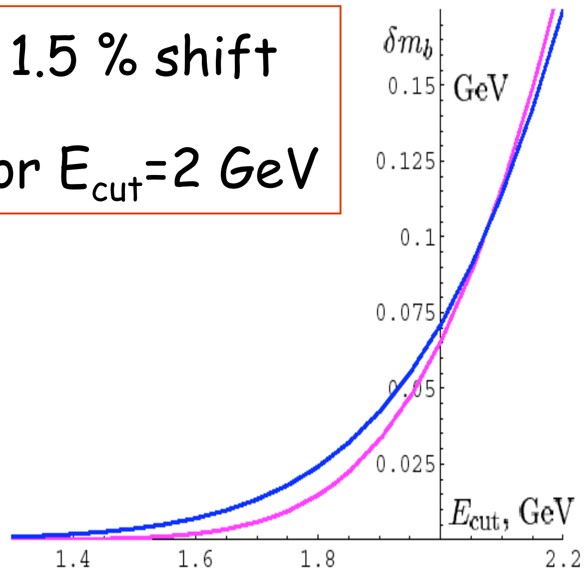
for $B \to \gamma X_q$: $\mathcal{Q} = m_b - 2 E_{cut}$

e.g.: for $E_{cut} \sim 2$ GeV, $\mathcal{Q} \sim 1$ GeV !

Pilot study (Uraltsev, IB)

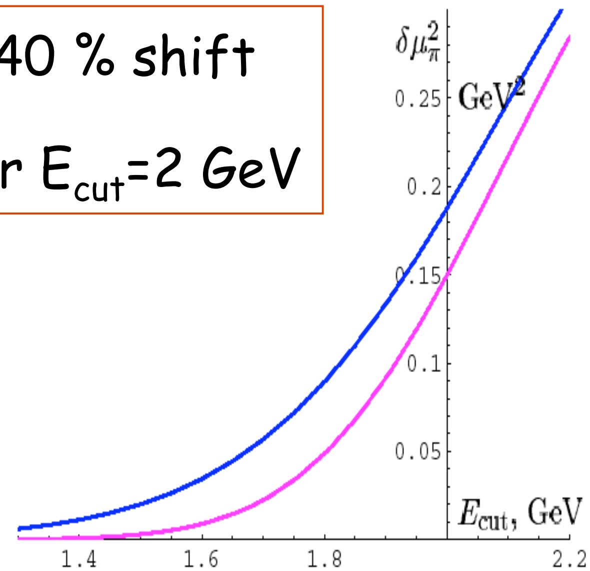
~ 1.5 % shift

for E_{cut}=2 GeV



~ 40 % shift

for E_{cut}=2 GeV



absolute bias due to experim. cut

2 different ansaetze for distribution function

[curves shown for m_b=4.6 GeV, μ_π^2 = 0.45 GeV^2; bias depends on HQP]

Lessons:

- keep the cuts as low as possible
- bias in the meas. moments induced by cuts
  - ☞ can be corrected for
  - ☞ not a pretext for inflating theor. uncert.
- moments meas. as fction of cuts: important cross check!

(VI) |V(cb)| & |V(ub)| Lessons & Outlook

(6.1) Status

extract CKM parameters with accuracy seemingly unrealistic less than 10 years ago:

detailed error budgets from (some) theorists!

... CLEO ... DELPHI ...

BABAR '04:

$V(cb)|_{incl}=(41.390 \pm 0.870) \times 10^{-3}=41.390\times(1 \pm 0.021) \times 10^{-3}$

DELPHI '04 preliminary

$V(cb)|_{incl}=(42.1 \pm 1.1) \times 10^{-3} =42.1\times(1 \pm 0.025) \times 10^{-3}$

amazing success in quantitative control over nonperturbative dynamics

essential ingredients

- robust & multilayered theoret. framework
- broad, high-quality data base
- overconstraints:
  - certified small errors

(6.2) Outlook

|V(ub)|

Same HQP -- m_b, μ_π^2, μ_G^2, ρ_D^3 -- control $B \to l\nu X_u$

likewise for $B \to \gamma X$

|V(td)/V(ts)|

from $B \to \gamma X_d$ vs. $B \to \gamma X_s$?

(6.3) General perspective

Heavy-flavour transitions

high sensitivity + high accuracy $\Longrightarrow$ precision probes for NP

comprehensive high quality $\left\{ \begin{array}{l} \text{data base} \\ \text{theoret. framwork} \end{array} \right.$