e^+e^- versus τ data:
The muon anomalous magnetic moment

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Outline

1. A few definitions
   - Muon anomalous magnetic moment
   - e^+e^- annihilation, τ decay
2. The problem
   - e^+e^- versus τ
3. A solution (?)
4. The model
   - HLS
   - Pion form factor from e^+e^- and τ data
   - One loop corrections

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5. Fit/results
   - Fit
   - Pion form factor
   - Branching ratios
   - $\rho^0 - \omega - \phi$ mixing
   - Δm for $\rho^0 - \rho^\pm$
   - Isospin symmetry breaking ρ^0 vs $\rho^\pm$
6. Conclusion
   - About the model
   - About the muon anomalous magnetic moment

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   - About the muon anomalous magnetic moment

The muon magnetic moment

- Definition: $\vec{\mu}_{\text{muon}} = g_{\text{muon}} \left(\frac{q}{2m}\right) \vec{s}$
- Anomalous magnetic moment: couplings between spin and virtual fields

$$a_{\text{muon}} = \frac{1}{2}(g_{\text{muon}} - 2)$$
$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{weak}}$$

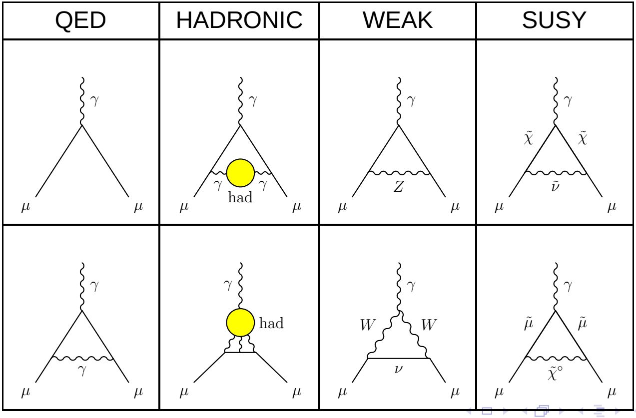
$$a_\mu^{\text{QED}} = (116584716 \pm 9) \times 10^{-11}$$
$$a_\mu^{\text{had}} = a_\mu^{\text{had,L0}} + a_\mu^{\text{had,H0}} + a_\mu^{\text{had,LBL}} = (6913 \pm 44) \times 10^{-11}$$
$$a_\mu^{\text{weak}} = (154 \pm 3) \times 10^{-11}$$

- Experience versus theory

$$a_\mu^{\text{exp}} = 116592080(63) \times 10^{-11} \,, \quad a_\mu^{\text{th}} = 116591785(61) \times 10^{-11}$$

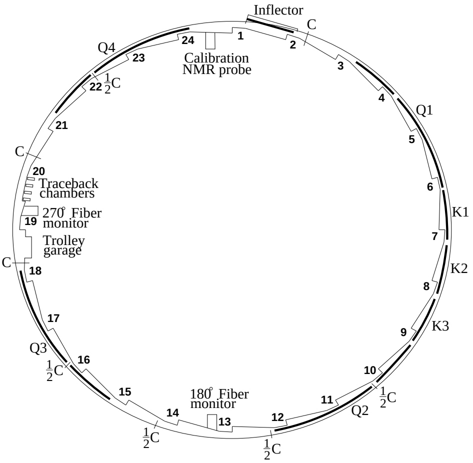
The muon anomalous magnetic moment

Diagrams contributing to the anomalous magnetic moment



The muon anomalous magnetic moment measurement

The muon $g-2$ collaboration: PRD **73**: 072003, 2006



- BNL E821 experiment
- 0.46 ppm statistic
- 0.28 ppm systematic
- magnetic field: 1.45T

The most accurate measurement in quantum world

Experimental Progress: from CERN to BNL

Experiment	Beam	Measurement	$\delta a_\mu / a_\mu$	Required th. terms
Columbia-Nevis(1957)	μ^+	g=2.00(0.10)		$g = 2$
Columbia-Nevis(1959)	μ^+	0.001 13(+16)(-12)	12.4%	α/π
CERN 1 (1961)	μ^+	0.001 145(22)	1.9%	α/π
CERN 1 (1962)	μ^+	0.001 162(5)	0.43%	$(\alpha/\pi)^2$
CERN 2 (1968)	μ^+	0.001 166 16(31)	265 ppm	$(\alpha/\pi)^3$
CERN 3 (1975)	$\mu^\pm$	0.001 165 895(27)	23 ppm	$(\alpha/\pi)^3$+ had
CERN 3 (1979)	$\mu^\pm$	0.001 165 911(11)	7.3 ppm	$(\alpha/\pi)^3$+ had
BNL E821 (2000)	μ^+	0.001 165 919(59)	5 ppm	$(\alpha/\pi)^3$+ had
BNL E821 (2001)	μ^+	0.001 165 920 2(16)	1.3 ppm	$(\alpha/\pi)^4$ + had + weak
BNL E821 (2002)	μ^+	0.001 165 920 3(8)	0.7 ppm	$(\alpha/\pi)^4$ + had + weak + ?
BNL E821 (2004)	μ^-	0.001 165 921 4(8)(3)	0.7 ppm	$(\alpha/\pi)^4$ + had + weak + ?

From $(g-2)$ to e^+e^-/τ data

- From a_μ to e^+e^-:

$$a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{had}}$$

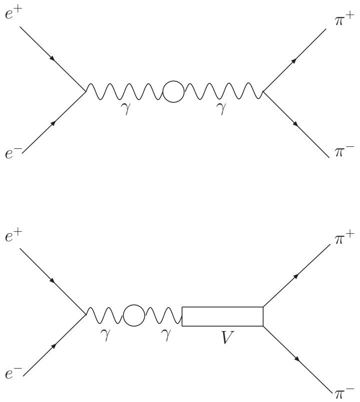
$$a_\mu^{\text{had}} \rightarrow \begin{array}{rcl} \text{LO} & : & \text{vacuum polarization loop : DR} \\ \text{HO} & : & \text{DR + Kernel: loop++} \\ \text{LBL} & : & \text{QCD model dependent with our exp.data} \end{array}$$

$$a_\mu^{\text{had,LO}} = \left(\frac{2m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} K(s)R(s) \text{ with } \quad R(s) \simeq \frac{\sigma_{tot}(e^+e^- \rightarrow \text{ hadrons})}{\sigma_{tot}(e^+e^- \rightarrow \mu^+\mu^-)}$$

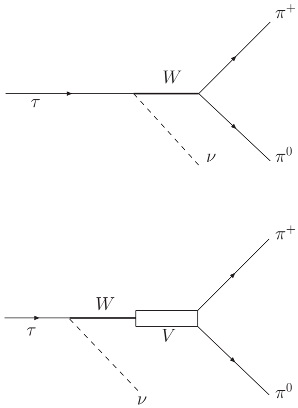
- From e^+e^- to τ:

$$\sigma^{I=1}[e^+e^- \rightarrow \pi^+\pi^-] \simeq \frac{4\pi\alpha^2}{s}\,\sigma[\tau^- \rightarrow \pi^-\pi^0\nu_\tau]$$

e^+e^- annihilation, τ decay

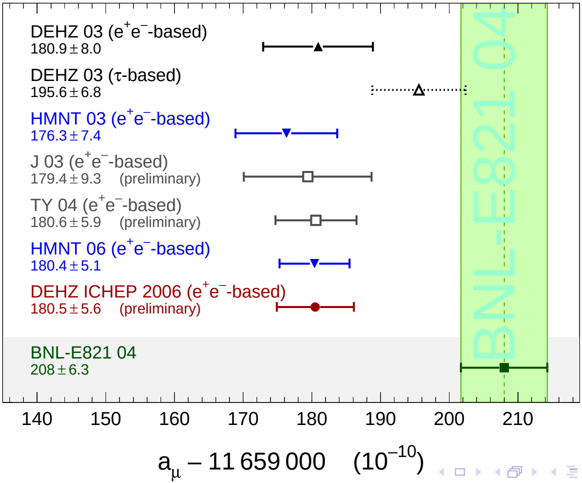


- Data from CMD, CMD2, DM1, SND, OLYA



- Data from CLEO, ALEPH, OPAL

BNL a_μ versus e^+e^- and τ data



The problem

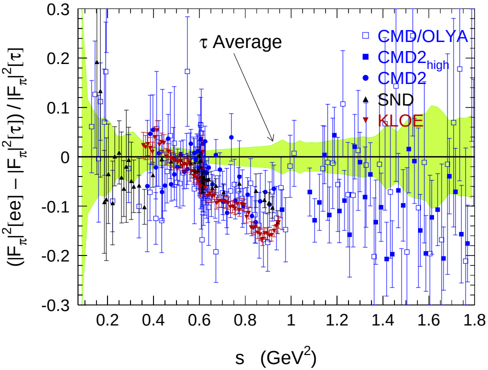
- Observation: Disagreement between e^+e^- and τ data
- Action: Large activity to identify isospin symmetry breaking in both e^+e^- annihilation and τ decay
- Result: Disagreement survives

So...

Is there a missing piece, a systematic effect (in e^+e^- or τ) or new physics?

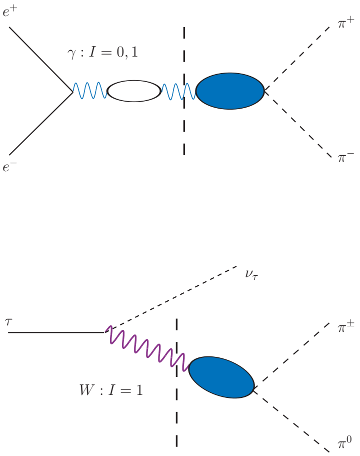
Interest: test of the Standard Model extension

The latest account



- Invariant mass dependent missing effect!
- Is it isospin breaking effect?

The pion form factor



- Not a pointlike coupling
- e^+e^- $\rho(770), \omega(782), \phi(1020), \rho(1450), \rho(1700)$
- τ : $\rho(770), \rho(1450), \rho(1700)$
- CVC: current vector conservation

The Hidden Local Symmetry model

- Description of nonet for mesons: (π, k, η, η') and $(\rho, k^*, \omega, \phi)$
- Vector mesons: gauge bosons of a HL symmetry
- Define $\xi_{L/R} = \exp[\mp iP/f_\pi]$
  Define covariant derivatives $D_\mu \xi_L \quad D_\mu \xi_R$

$$L/R = D_\mu \xi_{L/R} \xi^+_{L/R} \Rightarrow \mathcal{L}_{A/V} = -\frac{f_\pi^2}{4} \mathrm{Tr}[L \mp R]^2$$

- The HLS Lagrangian: $\mathcal{L}_{HLS} = \mathcal{L}_A + a\mathcal{L}_V$
  $\mathcal{L}_{HLS} = \mathcal{L}_{VMD} + \mathcal{L}_\tau + \mathcal{L}_{\text{anomalous}} + \mathcal{L}_{YM}$
  a: phenomenological parameter $\sim 2, 4$

e^+e^- Lagrangian

$$
\begin{aligned}
\mathcal{L}_{VMD} &= ie(1-\frac{a}{2})A\cdot\pi^- \overleftrightarrow{\partial}\pi^+ + i\frac{e}{z_A}(z_A-\frac{a}{2}-b)A\cdot K^- \overleftrightarrow{\partial} K^+ + i\frac{e}{z_A}bA\cdot K^0 \overleftrightarrow{\partial}\overline{K}^0 \\
&+\frac{iag}{2}\rho_I^0\cdot\pi^-\overleftrightarrow{\partial}\pi^+ + \frac{iag}{4z_A}(\rho_I^0+\omega_I-\sqrt{2}z_V\phi_I)K^-\overleftrightarrow{\partial}K^+ + \frac{iag}{4z_A}(\rho_I^0-\omega_I+\sqrt{2}z_V\phi_I)K^0\overleftrightarrow{\partial}\overline{K}^0 \\
&-eagf_\pi^2\left[\rho_I^0+\frac{1}{3}\omega_I-\frac{\sqrt{2}}{3}z_V\phi_I\right]\cdot A+\frac{1}{9}af_\pi^2e^2(5+z_V)A^2+\frac{af_\pi^2g^2}{2}\left[(\rho_I^0)^2+\omega_I^2+z_V\phi_I^2\right]
\end{aligned}
$$

τ Lagrangian

$$\mathcal{L}_\tau = -\frac{ig_2}{2} V_{ud} W^+ \cdot \left[(1-\tfrac{a}{2})\pi^- \overleftrightarrow{\partial} \pi^0 + (z_A - \frac{a}{2}) \frac{1}{z_A\sqrt{2}} K^0 \overleftrightarrow{\partial} K^- \right]$$

$$-\frac{af_\pi^2 g g_2}{2} V_{ud} W^+ \cdot \rho^- - \frac{iag}{2}\rho^- \left[\pi^0 \overleftrightarrow{\partial} \pi^+ - \frac{1}{z_A\sqrt{2}} \overline{K}^0 \overleftrightarrow{\partial} K^+ \right]$$

$$+f_\pi^2 g_2^2 \left\{ \frac{1+a}{4} \left[z_A |V_{us}|^2 + |V_{ud}|^2 \right] + \frac{a}{4} [\sqrt{z_V} - z_A] |V_{us}|^2 \right\} W^+ \cdot W^- + af_\pi^2 g^2 \rho^+ \rho^-$$

$SU(3) \bigotimes U(3) \bigotimes SU(2)$ symmetry breakings

- SU(3) symmetry:

$$\Rightarrow \mathcal{L}_{A(v)} = -(f_\pi^2/4)\mathrm{Tr}\left[(L_\mu \mp R_\mu)X_{A(v)}\right]^2$$

- Nonet symmetry:

$$\Rightarrow P_8' + xP_0' = X_A^{1/2}(P_8 + P_0)X_A^{1/2}$$

- Isospin symmetry:

$$\Rightarrow \rho^0 - \omega - \phi \text{ mixing}$$

The pion form factor in e^+e^- and τ physics

Without symmetry breaking

- In e^+e^-:

$$F_\pi^e(s) = \left[\left(1 - \frac{a}{2}\right) - \frac{F_\rho^\gamma(s) g_{\rho\pi\pi}}{D(s)}\right]$$

- In τ:

$$F_\pi^\tau(s) = \left[\left(1 - \frac{a}{2}\right) - \frac{F_\rho^W(s) g_{\rho\pi\pi}}{D(s)}\right]$$

so one gets:

$$\begin{cases} \sigma(e^+e^- \to \pi^+\pi^-) & = \dfrac{8\pi\alpha^2}{3s^{5/2}} |F_\pi(s)|^2 q_\pi^3 \\ \dfrac{d\Gamma(s)}{ds} & = \dfrac{|V_{ud}|^2 G_F^2}{64\pi^3 m_\tau^3} |F_\pi(s)|^2 [G_0(s) + \epsilon^2 G_2(s)] \end{cases}$$

The pion form factor in e^+e^- and τ physics

Without symmetry breaking/with symmetry breaking

- In e^+e^-:

$$F_\pi^e(s) = \left[\left(1 - \frac{a}{2}\right) - \frac{F_\rho^\gamma(s) g_{\rho\pi\pi}}{D(s)} - \frac{F_\rho^\omega(s) g_{\omega\pi\pi}}{D_\omega(s)} - \frac{F_\phi^\gamma(s) g_{\phi\pi\pi}}{D_\phi(s)}\right]$$

- In τ:

$$F_\pi^\tau(s) = \left[\left(1 - \frac{a}{2}\right) - \frac{F_\rho^W(s) g_{\rho\pi\pi}}{D(s)}\right] S_{ew} G_{EM}(s)$$

so one gets:

$$\begin{cases} \sigma(e^+e^- \to \pi^+\pi^-) & = \dfrac{8\pi\alpha^2}{3s^{5/2}} |F_\pi(s)|^2 q_\pi^3 \\ \dfrac{d\Gamma(s)}{ds} & = \dfrac{|V_{ud}|^2 G_F^2}{64\pi^3 m_\tau^3} |F_\pi(s)|^2 [G_0(s) + \epsilon^2 G_2(s)] \end{cases}$$

The effective Lagrangian

Take into account self masses and transitions terms at one-loop order

$$\mathcal{L}_{mass} = \left\{ \begin{array}{l} \frac{1}{2}\left\{[m^2 + \Pi_{\rho\rho}(s)]\rho_I^2 + [m^2 + \Pi_{\omega\omega}(s)]\omega_I^2 + [z_V m^2 + \Pi_{\phi\phi}(s)]\phi_I^2 \right. \\ \left. +2\Pi_{\rho\omega}(s)\rho_I\omega_I + 2\Pi_{\rho\phi}(s)\rho_I\phi_I + 2\Pi_{\omega\phi}(s)\omega_I\phi_I\right\} + [m^2 + \Pi'_{\rho\rho}(s)]\rho^+\rho^- \end{array} \right.$$

The modified vector mass matrix

$$M^2(s) = \begin{bmatrix} m^2 + \Pi_{\pi\pi}(s) + \epsilon_2(s) & \epsilon_1(s) & -\mu\epsilon_1(s) \\ \epsilon_1(s) & m^2 + \epsilon_2(s) & -\mu\epsilon_2(s) \\ -\mu\epsilon_1(s) & -\mu\epsilon_2(s) & z_V m^2 + \mu^2\epsilon_2(s) \end{bmatrix}$$

with $\mu = \sqrt{2} z_V$

Taking into account:

- loop effects : $\omega - \phi$ mixing
- $SU(3)$ breaking
- isospin breaking : $\rho^0 - \omega - \phi$ mixing

The mass matrix eigen system

- Expecting $(m^2, \Pi_{\pi\pi}(s)) \gg \epsilon_2(s) \gg \epsilon_1(s)$
- Solving perturbatively the eigensystem: $M^2(s) = M_0^2(s) + \delta M^2(s)$

$$M_0^2(s) = \begin{pmatrix} m^2 + \Pi_{\pi\pi}(s) & 0 & 0 \\ 0 & m^2 + \epsilon_2(s) & 0 \\ 0 & 0 & z_V m^2 + \mu^2 \epsilon_2(s) \end{pmatrix}$$

$$\delta M^2(s) = \begin{pmatrix} 0 & \epsilon_1(s) & -\mu\epsilon_1(s) \\ \epsilon_1(s) & 0 & -\mu\epsilon_2(s) \\ -\mu\epsilon_1(s) & -\mu\epsilon_2(s) & 0 \end{pmatrix}$$

From ideal to physical fields

$$\begin{pmatrix} \rho^0 \\ \omega \\ \phi \end{pmatrix} = R(s) \begin{pmatrix} \rho_I^0 \\ \omega_I \\ \phi_I \end{pmatrix}$$

$R(s)$: real analytic matrix function

: fulfill Unitarity Condition

: $R(s+i\epsilon)\tilde{R}(s+i\epsilon) = 1$

$$R(s) = \begin{pmatrix} 1 & \dfrac{\epsilon_1(s)}{\Pi_{\pi\pi}(s) - \epsilon_2(s)} & \dfrac{-\mu\epsilon_1(s)}{z'm^2 + \Pi_{\pi\pi}(s) - \mu^2\epsilon_2(s)} \\ \dfrac{-\epsilon_1(s)}{\Pi_{\pi\pi}(s) - \epsilon_2(s)} & 1 & \dfrac{-\mu\epsilon_2(s)}{z'm^2 + (1-\mu^2)\epsilon_2(s)} \\ \dfrac{\mu\epsilon_1(s)}{z'm^2 + \Pi_{\pi\pi}(s) - \mu^2\epsilon_2(s)} & \dfrac{\mu\epsilon_2(s)}{z'm^2 + (1-\mu^2)\epsilon_2(s)} & 1 \end{pmatrix}$$

with $z' = (1 - z_V)$

Transitions among vector fields at one loop

- At tree level: ideal fields $\equiv$ mass eigenstates
- At one loop:

$$(\rho_I + \omega_I - \sqrt{2}\, z_V \phi_I) K^- \overleftrightarrow{\partial} K^+ + (\rho_I - \omega_I + \sqrt{2}\, z_V \phi_I) K^0 \overleftrightarrow{\partial} \bar{K}^0$$

$\Rightarrow$ ideal fields $\neq$ mass eigenstates

$\Rightarrow$ isospin symmetry breaking: $m_{K^\pm} \neq m_K^0$

$\Rightarrow$ dispersion relation define the loops: $\Pi_{\omega\phi}(s),\ \Pi_{\rho\omega}(s),\ \Pi_{\rho\phi}(s)$

Vector meson coupling to γ/W

- $(\gamma/W)V$ couplings: constant + (PP, VP ...) loops

where loop term: dispersion relation (substraction)

- $\rho^\pm - W$ coupling:

  $\pi^+\pi^0$ and $K^0K^\pm$ loops

  $f_\rho^W \simeq agf_\pi^2$

Vector meson coupling to γ

- From ideal fields:

$$-e\, agf_\pi^2 \left[\rho_I^0 + \frac{1}{3}\omega_I - \frac{\sqrt{2}}{3}\, z_V \phi_I\right]_\mu \cdot A^\mu$$

- To real fields:

$$-\, e \left[f_\rho^\gamma \rho^0 + f_\omega^\gamma\, \frac{1}{3}\omega - f_\phi^\gamma\, \frac{\sqrt{2}}{3}\, z_V \phi\right]_\mu \cdot A^\mu$$

with $f_V^\gamma(s) = agf_\pi^2[1 + O(\epsilon_1(s))]$

$V\pi\pi$ transition

- From ideal fields:

$$\frac{iag}{2}\,\rho_I^0\,\pi^-\overleftrightarrow{\partial}\,\pi^+$$

- To real fields:

$$\frac{iag}{2}\left[\rho^0-\frac{\epsilon_1(s)}{\Pi_{\pi\pi}(s)-\epsilon_2(s)}\omega+\frac{\mu\epsilon_1(s)}{(1-z_V)m^2+\Pi_{\pi\pi}(s)-\mu^2\epsilon_2(s)}\phi\right]\pi$$

Parameter freedom

- Basic HLS parameters:

$$a,\ g,\ z_A,\ \delta m^2$$

- Subtraction polynomials from DR:

$$\Pi^{\rho}_{\pi\pi}(s),\ \Pi^{\gamma/W}_{\pi\pi}(s),\ \epsilon_1(s),\ \epsilon_2(s)$$

too many parameters to fit the pion form factor from e^+e^-
$\Rightarrow$ extend the fitted data

The extended data sample

- Anomalous decay modes: $VP\gamma$
  - $\rho^0/\omega/\phi \to \pi^0\gamma/\eta\gamma$
  - $\phi \to \eta'\gamma$
  - $\eta' \to (\rho^0/\omega)\gamma$
  - $K^* \to K\gamma$
  - $\eta/\eta' \to \gamma\gamma$
  - $\rho^\pm \to \pi^\pm\gamma$

$\Rightarrow$ 18 decay modes + pion form factor in e^+e^- annihilation

$\Rightarrow$ 15 parameters and 344 data points

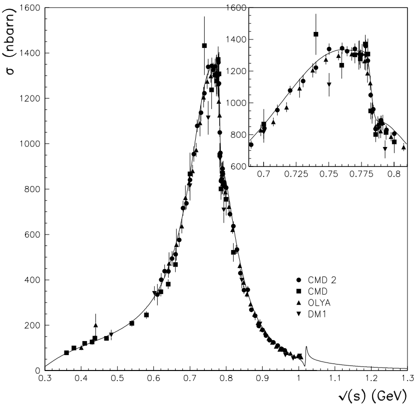
Global fit against data set

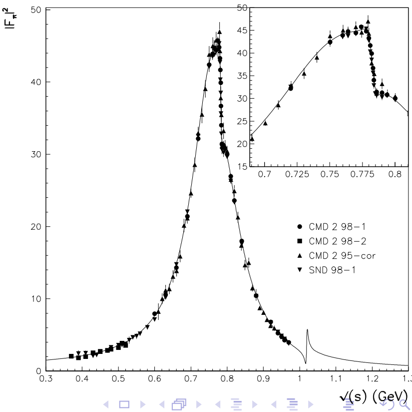
Data Set	Without VP	With Vacuum polarisation (VP)			
$\sharp$ (data + conditions)	Full Fit	Full Fit	No τ	No Spacelike	No ρ mass shift
Decays (18+1)	11.46	11.13	11.52	11.48	11.25
New Timelike (127+1)	132.81	128.10	122.02	125.76	132.23
Old Timelike (82+1)	62.22	59.05	54.68	55.20	60.15
Spacelike(59+2)	68.53	65.70	55.20	**89.82/(59)**	65.13
τ ALEPH (33)	27.06	23.86	**42.27/(33)**	20.80	24.48
τ CLEO (25+1)	25.53	26.06	**26.16/(25)**	29.72	28.55
χ^2/dof	327.40/331	313.83/331	257.73/274	238.81/272	321.75/332
Probability	54.6 %	74.3 %	75.2%	92.7%	64.7%

- Statistic and systematic errors: quadratic
- Global scale factor: from 0.4% to 0.9 %

Pion form factor in e^+e^- timelike: fit

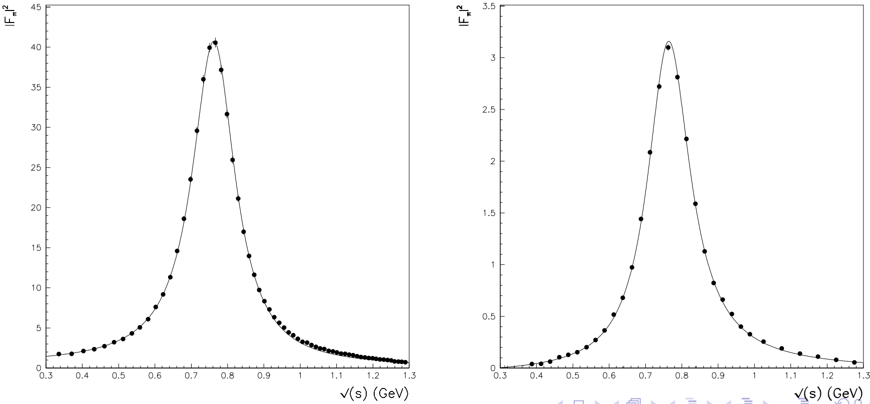
- Data from OLYA, CMO, DM1, CMD2





Pion form factor in τ decay: prediction

- Data from ALEPH and CLEO



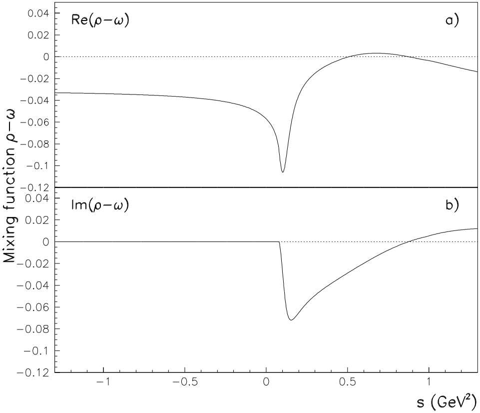
Fit against branching ratios

Decay Mode	Fit Value	PDG/Reference
$\rho \to \pi^0\gamma$ [$\times 10^4$]	5.17 ± 0.04	6.0 ± 0.8
$\rho \to \pi^\pm\gamma$ [$\times 10^4$]	5.03 ± 0.03	4.5 ± 0.5
$\rho \to \eta\gamma$ [$\times 10^4$]	3.05 ± 0.04	2.95 ± 0.30
$\eta' \to \rho\gamma$ [$\times 10^2$]	33.3 ± 1.0	29.4 ± 0.9
$K^{*\pm} \to K^\pm\gamma$[$\times 10^4$]	9.8 ± 0.9	9.9 ± 0.9
$K^{*0} \to K^0\gamma$[$\times 10^3$]	2.26 ± 0.02	2.31 ± 0.20
$\omega \to \pi^0\gamma$ [$\times 10^2$]	8.23 ± 0.04	$8.9^{+0.27}_{-0.23}$ $(*)$
$\omega \to \eta\gamma$ [$\times 10^4$]	6.60 ± 0.09	4.9 ± 0.5 $(*)$
$\eta' \to \omega\gamma$ [$\times 10^2$]	3.14 ± 0.10	3.03 ± 0.31
$\phi \to \pi^0\gamma$ [$\times 10^3$]	1.24 ± 0.07	1.25 ± 0.07
$\phi \to \eta\gamma$ [$\times 10^2$]	1.292 ± 0.025	1.301 ± 0.024
$\phi \to \eta'\gamma$ [$\times 10^4$]	0.60 ± 0.02	0.62 ± 0.07

Fit against branching ratios

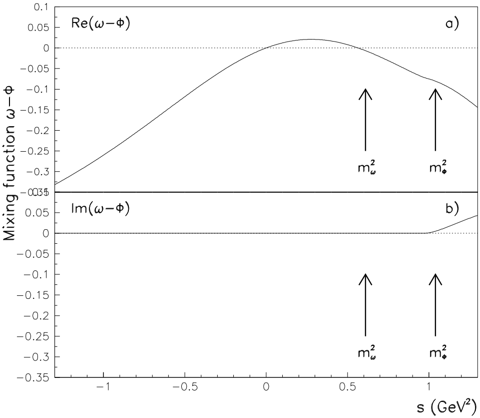
Decay Mode	Fit Value	PDG/Reference
$\eta \to \gamma\gamma$ [$\times 10^2$]	35.50 ± 0.56	39.38 ± 0.26
$\eta' \to \gamma\gamma$ [$\times 10^2$]	2.10 ± 0.06	2.12 ± 0.14
$\rho \to e^+e^-$ [$\times 10^5$]	5.56 ± 0.06	4.70 ± 0.08 $(**)$
$\omega \to e^+e^-$ [$\times 10^5$]	7.15 ± 0.13	7.18 ± 0.12
$\phi \to e^+e^-$ [$\times 10^4$]	2.98 ± 0.05	2.97 ± 0.04
$\omega \to \pi^+\pi^-$ [$\times 10^2$]	1.13 ± 0.08	1.70 ± 0.27 $(**)$
$g_{\omega\pi^+\pi^-}$ phase [degr]	101.2 ± 1.6	104.7 ± 4.1 $(**)$
$\phi \to \pi^+\pi^-$ [$\times 10^5$]	7.14 ± 1.7	7.3 ± 1.3
$g_{\phi\pi^+\pi^-}$ phase [degr]	-27.0 ± 0.5	-34 ± 5
$\phi \to K^+K^-$ [$\times 10^2$]	50.3 ± 1.0	49.2 ± 0.6 $(**)$
$\phi \to K^0_S K^0_L$ [$\times 10^2$]	33.0 ± 0.7	34.0 ± 0.5 $(**)$

$\rho^0 - \omega$ mixing: phase and modulus



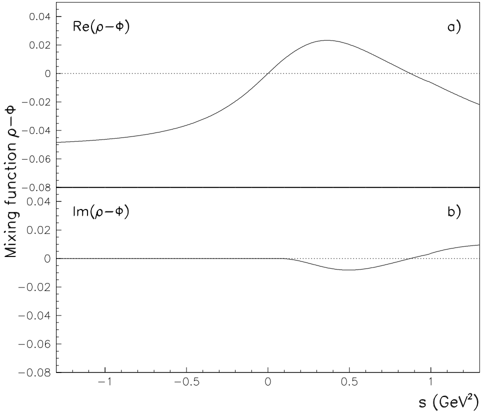
- $$\frac{\epsilon_1(s)}{\Pi_{\pi\pi}(s) - \epsilon_2(s)}$$

$\omega - \phi$ mixing: phase and modulus



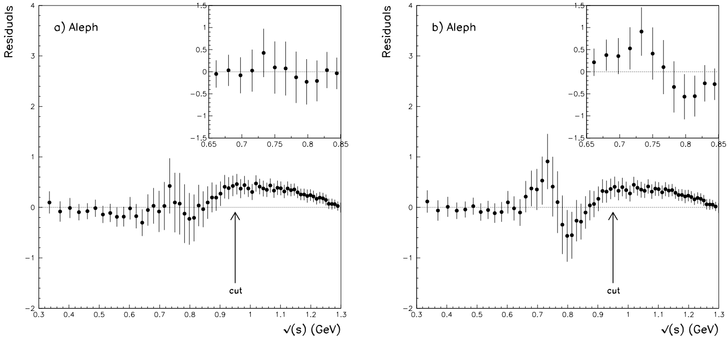
- $$\frac{-\mu\epsilon_2(s)}{(1-z_V)m^2+(1-\mu^2)\epsilon_2(s)}$$

$\rho^0 - \phi$ mixing: phase and modulus



- $$\frac{-\mu\epsilon_1(s)}{(1 - z_V)m^2 + \Pi_{\pi\pi}(s) - \mu^2\epsilon_2(s)}$$

The $\rho^0 - \rho^\pm$ mass difference

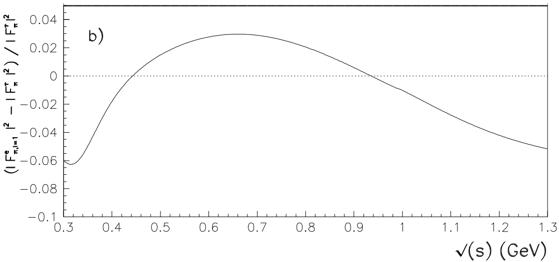


- $M_{\rho^0} - M_{\rho^\pm} \simeq 1.35 \pm 0.15 \pm 0.53$ MeV
  Only visible in ALEPH data

Isospin symmetry breaking: ρ^0 vs $\rho^\pm$

$$IS = \frac{\left(|F^e_{\pi\text{“}I=1\text{”}}(s)|^2 - |F^e_\pi(s)|^2\right)}{|F^e_\pi(s)|^2}$$

at ϕ mass: IS=-2%, at ρ peak: IS=+3%; at threshold: IS=-6%

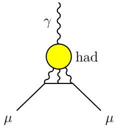


Conclusion in waiting for the upgrade E969

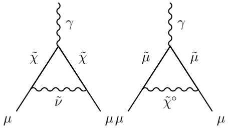
- About the model
  - No mismatch between e^+e^- and τ data: CVC OK
  - Radiative decays and pion form factor in e^+e^- annihilation predict the observed pion form factor in τ decay
  - Previously unaccounted for effects: $I = 0$ (ω_I, ϕ_I) components inside the ρ^0 meson
  - The 3.3σ discrepancy between prediction (from e^+e^- data) and the BNL measurement for a_μ is confirmed.

Conclusion in waiting for the upgrade E969

- About the muon anomalous magnetic moment:
  - Better calculation for $a_\mu^{\text{had, LBL}}$ is required



  - Missing piece from MSSM?



  - Missing piece from unknown particle of new physics?