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CONTRIBUTION OF QUARK-ANTIQUARK ANNIHILATION TO THE PRODUCTION OF VIRTUAL $Z_0$-PAIRS IN HRADRON COLLISIONS
Contribution of Quark-antiquark Annihilation
to the Production of Virtual $Z_0$-pairs in Hadron Collisions

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Abstract

We calculate the contribution of quark-antiquark annihilation to the production of virtual $Z_0$-pairs in hadron colliders like LHC or SSC. This process can constitute a potentially important background to the detection of an intermediate Higgs boson, i.e. $M_Z \leq M_H \leq 2M_Z$. We show that this cross-section is always one or more orders of magnitude smaller than the signal, provided a resolution of at least 10 GeV in the invariant mass of the $Z_0$ pairs is available. The signal for different values of the top mass is compared with the background cross-section at two c.m. energies, 16, and 40 TeV.
Recent measurements in $e^+e^-$ annihilation\cite{1} have established that the Higgs boson mass cannot be less than 24 GeV. This limit is expected to be raised to as much as 70-80 GeV in the years to come, when LEP-200 will start operating\cite{2}. The search for Higgs in the higher mass range will then be left to the hadron colliders. This range can be roughly divided into two main regimes: one in which the Higgs boson is lighter than two Vector Bosons (IVB’s) and one in which it is heavier. The limits recently imposed on the top mass by the CDF collaboration \cite{3}, have now established that the heaviest quark below the W-mass is the bottom quark, so that the two regions differ because the dominant Higgs decay mode in one case will be into $b\bar{b}$ pairs and in the other into a pair of IVB’s. The hadronic backgrounds, however, are so overwhelming that the search for the Higgs boson, in either regions, may have to focus on the leptonic decay channels\cite{4}, i.e. $H \rightarrow \gamma \gamma$ or

$$H \rightarrow Z_0 Z_0 \rightarrow 2l^+2l^-$$  \hspace{1cm} (1)

where $l^\pm$ stands for either electrons or $\mu$’s. Notice that, in what follows, we shall consider $l = \mu$: if both electrons and muons can be detected, all the relevant quantities can be multiplied by a factor 4.

One of the possible background processes is constituted by $Z_0$-pairs of QCD origin. In the region above the two IVB threshold this background has been extensively studied and shown to be well below the signal \cite{5} until $M_H \leq 700 GeV$ at the LHC/SSC. On the other hand, the studies of this background in the lower region, the so called ”intermediate Higgs” regime, are not yet complete. In this region, we have background contributions from the following processes:

$$q\bar{q} \rightarrow Z_0^* Z_0^* \rightarrow (\mu^+\mu^-)(\mu^+\mu^-)$$ \hspace{1cm} (2)

$$q\bar{q} \rightarrow Z_0^* \gamma^* \rightarrow (\mu^+\mu^-)(\mu^+\mu^-)$$ \hspace{1cm} (3)

and

$$gg \rightarrow Z_0^* Z_0^* \rightarrow (\mu^+\mu^-)(\mu^+\mu^-)$$ \hspace{1cm} (4)

or

$$gg \rightarrow Z_0^* \gamma^* \rightarrow (\mu^+\mu^-)(\mu^+\mu^-)$$ \hspace{1cm} (5)

Although processes (4) and (5) are of higher order in the strong coupling constant $\alpha_s$, the large gluon densities in the x-values of interest make these processes comparable in magnitude to $q\bar{q}$ annihilation. In other words, the logarithmic growth in the gluon densities compensates for the (inverse) logarithmic behaviour in $\alpha_s$. Recent calculations by Glover et al. for the case in which the two $Z_0$’s are real have shown that the gluon-gluon processes contribute as much as 50-70% of the $q\bar{q}$ processes, in the collider energy region of interest. One can expect this to remain approximately true also for production of $Z$-pairs below threshold, i.e. for processes
(2) and (4), although as the Higgs mass decreases, the production of gluons over that of quarks is bound to increase further. While work is in progress to estimate the contribution of processes (4) and (5) to production of 4\mu's from real and virtual \(Z_0\)'s, in what follows we shall study the cross section for the process

\[ pp \rightarrow q\bar{q} + X \rightarrow Z_0^*Z_0^* + X \rightarrow (\mu^+\mu^-)(\mu^+\mu^-) + X \]  

(6)

The parton-parton subprocess receives contribution from the two graphs of fig.1. The cross-section can be written as

\[
\begin{align*}
    d\hat{\sigma} &= \frac{1}{2\hat{s}} \frac{1}{(2\pi)^8} \sum |M|^2 dPS(ab \rightarrow p_1 + p_2 + p_3 + p_4) = \\
    &= \frac{1}{24} \frac{1}{2\hat{s}} \frac{1}{(2\pi)^8} \sum |M|^2 d^2PS (a + b \rightarrow (12) + (34)) \\
    dQ_1^2dQ_2^2d^2PS(Q_1 \rightarrow p_1 + p_2)d^2PS(Q_2 \rightarrow p_3 + p_4)\theta(\sqrt{\hat{s}} - Q_1 - Q_2)
\end{align*}
\]  

(7)

where the factor \(\frac{1}{24}\) comes from averaging over initial color and spin and symmetrizing over the two \(Z_0^*\) in the final state. In the above expression, the four-body phase space has been factorized into a sequence of two-body phase space factors while the \(\theta\) function ensures energy conservation for any given parton-parton c.m. energy \(\sqrt{\hat{s}}\). The squared matrix element \(|M|^2\) is given by

\[
|M|^2 = \frac{g^4}{\cos^4\theta_W} \frac{\Tr [\hat{\lambda}(V - A\gamma_5)\gamma^\mu \hat{\lambda} \gamma^\nu (V + A\gamma_5)] \Tr [G_{\mu\nu}(12)] \Tr [G_{\lambda\nu}(34)]}{D_1D_2}
\]

where

\[
V = (g_{Vf})^2 + (g_{Af})^2 \quad A = 2g_A g_V
\]

with \(g_V\) and \(g_A\) the usual coupling of the \(Z_0\)-boson to the fermions. For massless fermions, we also have

\[
t_{\mu\lambda} = \gamma_\mu \frac{\hat{p}}{q^2} \gamma_\lambda + \gamma_\lambda \frac{\hat{p}}{p^2} \gamma_\mu
\]

and

\[
G_{\mu\sigma}(12) = \hat{p}_1 \gamma_\lambda \Gamma_f \hat{p}_2 \Gamma^+ f \gamma_\sigma
\]

with

\[
\Gamma_f = -i \frac{g}{\cos\theta_W} (g_{Vf}^2 + g_{Af}^2 \gamma_5)
\]

with the index \(f\) to indicate the particular fermion pair into which each real or virtual \(Z_0\) will decay. The denominators \(D_1\) and \(D_2\) represent the \(Z_0\) propagators, i.e.

\[
D_i = (Q_i^2 - m_{Z_0}^2)^2 + m_{Z_0}^2 \Gamma_{Z_0}^2
\]
The calculation is straightforward and similar to the one in which the $Z_0$ are real, except that one must keep track of the different masses $Q_1^2$ and $Q_2^2$. Upon performing phase space integration, and redefining

$$g_V = \frac{1}{2}(g_R + g_L), \quad g_A = \frac{1}{2}(g_R - g_L)$$

the cross section becomes

$$d^3\hat{s} = \int_{\text{bodyLIPS}} d\hat{s} =$$

$$\frac{\pi p^* d\hat{z}^*}{\hat{s}\sqrt{\hat{s}}} 6 \left( \frac{m_Z\Gamma_Z}{\pi} \right)^2 \frac{dQ_1^2}{(Q_1^2 - m_1^2)^2 + m_1^2\Gamma_1^2} \frac{dQ_2^2}{(Q_2^2 - m_2^2)^2 + m_2^2\Gamma_2^2}$$

$$\left( \frac{\Gamma_{Z-Z_-^\mu^+\mu^-}}{\Gamma_Z} \right)^2 \frac{a^2}{x_W^2(1-x_W)^2} (g_L^4 + g_R^4) \frac{Q_1^2 Q_2^2}{m_2^4} B(\hat{s}, \hat{t}, \hat{u})$$

with

$$B(\hat{s}, \hat{t}, \hat{u}) = -Q_1^2 Q_2^2 \left( \frac{1}{\hat{t}^2 + \frac{1}{\hat{u}^2}} + \frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + \frac{2\hat{s}(Q_1^2 + Q_2^2)}{\hat{u}\hat{t}} \right)$$

where the invariants of the parton-parton subprocess are given by

$$\hat{t} = Q_1^2 - \sqrt{\hat{s}} E_1 + p^* \sqrt{\hat{s}} z^*$$

$$\hat{u} = Q_1^2 - \sqrt{\hat{s}} E_1 - p^* \sqrt{\hat{s}} z^*$$

and $z^* = \cos\theta^*$ with $\theta^*$ the parton-parton center of mass scattering angle. In the above equation, $E_1$ is the energy of one of the two $Z_0$, which is given by

$$E_1 = \frac{\hat{s} + Q_1^2 - Q_2^2}{2\sqrt{\hat{s}}}$$

and $p^*$ is the center of mass momentum of the final IVB's. It is easy to see that, in the narrow width approximation, the above expression reduces to the known one for production of two real $Z_0$'s. One can also check that if one of two $Z_0$'s is on the mass shell, our expression concides with the one obtained by Gunion, Kalyniak, Soldate and Galison[6]. To check it is necessary to integrate the expression obtained by these authors over the invariant phase space of the two fermions into which the virtual $Z_0$ decays. The result of this integration shows that the function $B(\hat{s}, \hat{t}, \hat{u})$ is replaced by
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Eq. (10) should be substituted by:

\[ \int_{-z_1}^{z_0} B(\hat{s}, \hat{t}, \hat{u}) = -2Q_1^2Q_2^2 \left[ \frac{z_0}{(\sqrt{\hat{s}}E_1 - Q_1^2)^2 - \hat{s}p^*z_0^2} + \frac{z_1}{(\sqrt{\hat{s}}E_1 - Q_2^2)^2 - \hat{s}p^*z_1^2} \right] - 2(z_0 + z_1) + \frac{1}{\sqrt{\hat{s}}p^*} \left[ 2(\sqrt{\hat{s}}E_1 - Q_1^2) + \frac{\hat{s}(Q_1^2 + Q_2^2)}{\sqrt{\hat{s}}E_1 - Q_1^2} \right] \cdot \left[ \frac{\ln \sqrt{\hat{s}}E_1 - Q_1^2 + \sqrt{\hat{s}}p^*z_1}{\sqrt{\hat{s}}E_1 - Q_1^2} + \frac{\ln \sqrt{\hat{s}}E_1 - Q_2^2 + \sqrt{\hat{s}}p^*z_0}{\sqrt{\hat{s}}E_1 - Q_2^2} \right] \]

where

\[ z_1 = \min \left[ 1, \frac{1}{\beta_2^{-1}} \tanh(y_{cut} - |y|), \frac{1}{\beta_1^{-1}} \tanh(y_{cut} + |y|) \right], \]

\[ z_0 = \min \left[ 1, \frac{1}{\beta_1^{-1}} \tanh(y_{cut} - |y|) \right], \quad \text{for} \ |y_Z| \text{ or} \ |y_H| \leq y_{cut} \]

and

\[ \beta_1 = \frac{p^*}{E_1}, \quad \beta_2 = \frac{p^*}{E_2} \]

The numerical results remain unchanged.
\[ B'(\hat{s}, \hat{t}, \hat{u}) = -m_Z^2 s_H \left( \frac{1}{\hat{t}_2} + \frac{1}{\hat{u}_2} \right) + \hat{u} + \hat{t} + \frac{2\hat{s}(m_Z^2 + s_H)}{\hat{u}\hat{t}} \]

which is precisely what one would obtain, by performing the narrow width approximation for one of the two \( Z_0 \)'s.

We can obtain the hadronic cross-section for process (1) by folding the parton-parton cross-section of eq.(8) with the parton densities. The cross-section then becomes

\[
\frac{d\sigma(pp \rightarrow q\bar{q} + X \rightarrow Z_0^* Z_0^* + X \rightarrow 4\mu + X)}{dM_dQ_1dQ_2dy} = \sum_i (g_{L_i}^4 + g_{R_i}^4) F_i(\sqrt{\tau e^y}, M^2) F_i(\sqrt{\tau e^{-y}}, M^2)
\]

\[
\frac{4\pi^2}{3 M^4} \frac{\alpha^2}{x_W^2 (1 - x_W)^2} (B_{Z^- \mu^+ \mu^-})^2 \frac{Q_1^3 Q_2^3}{m_Z^4} \left( \frac{m_Z \Gamma_Z}{\pi} \right)^2 \frac{1}{D_1 D_2} \int_{-z_0}^{z_0} B(\hat{s}, \hat{t}, \hat{u})
\]

The integration over the scattering angle gives

\[
\int_{-z_0}^{z_0} B(\hat{s}, \hat{t}, \hat{u}) = -4z_0 + \frac{2}{\sqrt{\hat{s}p^*}} \left[ 2 \left( \frac{\sqrt{\hat{s}} E_1 - Q_1^2}{\sqrt{\hat{s}} E_1 - Q_1^2} \right) + \frac{\hat{s}(Q_1^2 + Q_2^2)}{\sqrt{\hat{s}} E_1 - Q_1^2} \right]
\]

\[
\log \frac{\sqrt{\hat{s}} E_1 - Q_1^2 + \sqrt{\hat{s}p^*} z_0}{\sqrt{\hat{s}} E_1 - Q_1^2 - \sqrt{\hat{s}p^*} z_0}
\]

\[
\frac{4Q_1^2 Q_2^2}{(\sqrt{\hat{s}} E_1 - Q_1^2)^2 - \hat{s} p^* z_0^2}
\]

This expression should be directly compared with the one which describes the same final state as obtained from gluon-gluon fusion into a Higgs boson. Before discussing the phenomenological applications of our expression, let us in fact list the one to be used for the signal. We shall then examine these two, first separately, and then together.

The cross section for the process of fig.2 can be written as

\[
\frac{d\sigma(pp \rightarrow gg + X \rightarrow H + X \rightarrow 4\mu + X)}{dM_dQ_1dQ_2dy} = MG(\sqrt{\tau e^y}, M_H^2)G(\sqrt{\tau e^{-y}}, M_H^2) \frac{G_F \pi}{16\sqrt{2}}
\]

\[
\left( \frac{\alpha_s}{\pi} \right)^2 |\eta_{gg}|^2 \frac{M_H \Gamma_H}{\pi D_H} \frac{p^*}{\Gamma_H} \frac{G_F}{8\pi \sqrt{2}} \frac{Q_1^3 Q_2^3}{M_H^2 D_1 D_2} \left( \frac{m_Z}{\pi} \right)^2 (B_{Z^- \mu^+ \mu^-})^2 C(Q_1^2, Q_2^2, M^2)
\]

(11)
where
\[ C(Q_1^2, Q_2^2, M^2) = 4 \frac{Q_1^2 Q_2^2 + (Q_1 \cdot Q_2)^2}{Q_1^2 Q_2^2} \]

In eq. (11) the factor \( C(Q_1^2, Q_2^2, M^2) \) is the analogue of factor \( B(\hat{s}, \hat{t}, \hat{u}) \) of eq. (10), \( |\eta_{gg}|^2 \) is the usual integral over the triangle diagram, i.e. such that
\[ \Gamma(H \rightarrow \text{gluon gluon}) = \frac{G_F M_H^3}{4\pi \sqrt{s}} \left[ \frac{\alpha_s(M_H^2)}{\pi} \right]^2 |\eta_{gg}|^2 \]

Also
\[ D_H = (M^2 - M_H^2)^2 + M_H^2 \Gamma_H^2 \]

In the region where the \( Z_0 \)'s are off the mass shell (generally only one of them, if \( M_H \geq m_Z \)), the Higgs boson is extremely narrow, and one can very well use the narrow width approximation to estimate the cross-section. On the other hand, above the two IVB threshold, the Higgs has an increasingly large width and it is necessary to keep the propagator. In this region however the two \( Z_0 \)'s are on the mass shell and one can use the delta function approximation for the \( Z_0 \) propagators. We show in Fig. 3 the Higgs boson production cross section and subsequent decay into \( 4\mu \)'s in the region below and around the two \( Z_0 \)'s threshold, for different values of the top mass, at the energy \( \sqrt{s} = 16 TeV \). We notice in this figure two interesting features: a drop in the cross section at \( M_H = 2m_W \) due to the opening of the \( W^+W^- \)-threshold and the peaking of the cross-section for \( \frac{4m_{\mu \tau}^2}{M_H^2} \approx 0.7 \).

We are now in a position to compare the cross-section for Higgs production as given by eq. (11) with that for the background of process (2) as given by eq. (9). Because the Higgs boson is very narrow below threshold, a meaningful comparison of the signal to the background in this region must take into account the experimental resolution. For conservativeness, we take a resolution \( \Delta E = 10 GeV \). In Figs. 4a-d, we show the differential cross section \( \frac{d\sigma}{dM} \) as from eq. (9) together with \( \frac{\sigma_{\text{Higgs}}^{\Delta E}}{\Delta E} \), at the proposed LHC and SSC energy, for two different top mass values, and for both \( Z_0 \)'s produced with rapidity \( |y| \leq 2.5 \). We see that the signal is well visible above this particular background. In the figures we have also indicated, with a dashed line, the level expected for the \( Z_0^*Z_0^* \) background if process (4) is also included, the latter being estimate to contribute about as much as process (2). This is certainly an overestimate at the LHC, but perhaps an underestimate at the SSC. As for the background due to processes like (3) and (5), they can eliminated [7] by suitable cuts on the photon invariant mass.

In conclusion, we have shown that the production cross section of the Higgs boson in the \( 4\mu \) channel for mass values \( m_Z \leq M_H \leq 2m_Z \) is well above the background from process (2). Estimates of the background from process (4) and
the use of proper cuts on the mass of the mu-pairs produced through processes (3) and (5) indicate therefore that detection of the Higgs boson depend on the expected experimental resolution and machine luminosity, but not from overwhelming physics backgounds.

References

Fig. 1 Graphs contributing to the process
\[ q\bar{q} \rightarrow Z^*_0 Z^*_0 \rightarrow (\mu^+\mu^-)(\mu^+\mu^-). \]

Fig. 2 Graph for the process \( H \rightarrow Z^*_0 Z^*_0 \rightarrow (\mu^+\mu^-)(\mu^+\mu^-). \)
\[ \sqrt{s}=16 \text{ TeV}, \; pp \rightarrow H+X \rightarrow 4\mu +X \]

![Graph showing production cross-section for various Higgs masses and top quark masses.](image)

Fig. 3 Production cross-section for

\[ pp \rightarrow H + X \rightarrow Z_0^* Z_0^* + X \rightarrow (\mu^+ \mu^-)(\mu^+ \mu^-) + X \]

at \( \sqrt{s} = 16 \text{TeV} \). Plots are for \( m_{top} = 90 \text{ GeV} \) (full line), \( m_{top} = 120 \text{ GeV} \) (dashes), \( m_{top} = 160 \text{ GeV} \) (dot-dashes) and \( m_{top} = 200 \text{ GeV} \) (dots). The Higgs branching ratio has been calculated for \( m_{top} = 90 \text{ GeV} \) for all the curves, since the difference is very small.
Fig. 4a  Differential cross section (full line) for the process

\[ pp \rightarrow q\bar{q} + X \rightarrow Z^*Z^* + X \rightarrow (\mu^+\mu^-)(\mu^+\mu^-) + X \]

and the total cross-section per 10 GeV for the process

\[ pp \rightarrow H + X \rightarrow Z^*Z^* + X \rightarrow (\mu^+\mu^-)(\mu^+\mu^-) + X \]

Curves are for \( \sqrt{s} = 16\text{ TeV} \), \( m_{\text{top}} = 90 \text{ GeV} \) (full line) and 200 GeV (dots), \( |y| \leq 2.5 \).
Fig. 4b  As in Fig. 4a with $\sqrt{s} = 40$ TeV.