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KAON PHYSICS AT A \( \phi \) FACTORY
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ABSTRACT

Various items are reviewed concerning kaon physics at a powerful Φ factory, namely: Quantum Mechanics tests on a large scale, statistical accuracy on the measurement of $(\xi_\pi)$ and detection of CP violation in $3\pi$ decay, $K\bar{K}$ molecules production and the kaon form factor.

Quantum Mechanics tests on a large scale

The Φ decay into a neutral kaon pair is a very suitable process to test Quantum Mechanics (QM) on a large scale. Many authors have emphasized paradoxes related to this process (1), which are a good illustration of the celebrated Einstein, Podolsky, Rosen arguments (2).

In the following two particularly non intuitive QM expectations are pointed out, which may be exploited at a powerful Φ factory.
Actually paradoxes arise because the $\Phi$ and the neutral kaons are both superposition of states. Namely the $\Phi$ is

$$|\Phi| = \frac{1}{\sqrt{2}} [|K^0_+(p) > - |K^0_-(p) >]$$

to achieve a $C = -1$ eigenstate, and the strongly interacting neutral kaons are superposition of the short-living and long-living mass eigenstates

$$|K_S| = a|K^0_0 > + b|K^0_0 >$$

$$|K_L| = c|K^0_0 > + d|K^0_0 >.$$

If CPT is conserved: $a = c$, $b = -d$. If CP is also conserved: $a = c = b = -d = \frac{1}{\sqrt{2}}$. Therefore, if neutral kaons decay without interacting, it is also

$$|\Phi| = \frac{1}{\sqrt{2}} [|K_S(p) > - |K_L(-p) > - |K_L(p) > + |K_S(-p) >]$$

and only events $K_S(p), K_L(-p)$ will be detected.

If a thin regenerator is introduced on one side coherent regeneration takes place and a fraction of the events, decaying downstream the regenerator, will be either $K_S(p)K_S(-p)$ or $K_L(p)K_L(-p)$.

Yet coherent regeneration cannot arise for a spherical regenerator (Fig. 1). Things are as if a neutral kaon, crossing a regenerator, knows what is occurring simultaneously to the other neutral kaon, no matter how far one kaon is from the other. This paradox is predicted by QM because the $\Phi$ decomposition is invariant under the simultaneous transformations

$$|K^0_0(\pm p) > = |K^0_0(\pm p) > + \alpha |K^0_0(\pm p) >$$

$$|K^0_0(\pm p) > = |K^0_0(\pm p) > + \beta |K^0_0(\pm p) >$$

where $\alpha$ and $\beta$ are the coherent regeneration amplitudes.
Fig. 1  Illustration of the regenerator Q.M. paradox.

The doughnut of the storage ring is an appropriate regenerator to verify this prediction. By the way this argument supports the suitability of a $\Phi$ factory to study CP violation in neutral kaon decay. In fact this effect reduces in practice $K_S$ regeneration in the apparatus, which may simulate a CP violating decay.

Detection of CP violation in neutral kaon decay provides another striking QM paradox. Namely if the two final states are available to both $K_S$ and $K_L$ decays, the time evolution is:

$$<f_1 f_2|\Psi> \propto \eta_2 e^{i(\Gamma_S t_1 + \Gamma_L t_2)} - \eta_1 e^{i(\Gamma_S t_2 + \Gamma_L t_1)}$$

where

$$\eta_i = \frac{<f_i|K_L>}{<f_i|K_S>}, \quad \Gamma = m + \frac{i}{2r}.$$  

If CPT holds, it is expected:

$$\text{Rate} \propto \eta_2^2 e^{-\frac{t_1}{T_S} - \frac{t_2}{T_L}} + \eta_1^2 e^{-\frac{t_1}{T_L} - \frac{t_2}{T_S}} - 2\eta_1 \eta_2 \cos(\Delta m \Delta t) e^{-\left(\frac{t_1}{T_S} + \frac{t_2}{T_L}\right)\frac{1 + i}{2}}$$

The interference term introduces a correlation between the two decay times, again no matter how far one kaon is from the other. In particular if the final states are the same
or with the same number of pions the two kaons cannot decay at the same time, as shown by the dip at the origin in Fig. 2. Detection of such a dip is well within the capabilities of the suggested detector and storage ring.

![Graph showing correlation between two decay times](image)

**Fig. 2** Correlation between the two decay times $t_1$, $t_2$, as caused by the interference term; one kaon decays to $\pi^+\pi^-$ and the other to $\pi^0\pi^0$ ($\xi/\epsilon \sim 3 \cdot 10^{-2}$).

Statistical accuracy in a measurement of $Re(\xi/\epsilon)$ and detection of CP violation in 3π decay

A new measurement of $(\xi/\epsilon)$ is the most important result to be achieved at a $\Phi$ factory. It is relevant to do a correct evaluation of statistical and systematical errors in such a measurement.
Proposals have been done for suitable variables and number of $\Phi$ to be produced, to reach a given statistical error $^{(3,4)}$. In the following different proposals are reviewed emphasizing the possibility, available at a $\Phi$ factory, to perform internal checks and to reduce systematical errors. 

The exemplifying case of a $(\xi^0)$ vanishing phase is considered, as required if CPT invariance holds. Concerning time integrated rates $N_i$, the interference term between $K_S$ and $K_L$ amplitudes may be neglected in evaluating $(\xi^0)$, as far as higher order terms in $\frac{\tau_k}{\tau_L} = 1.72 \cdot 10^{-3}$ are negligible. Therefore, in this approximation, the first decaying particle is identified as $K_S$ and the time integrated rates are:

$$\frac{N(K_S \to \pi^+\pi^-) N(K_L \to \pi^0\pi^0)}{N(\Phi \to K_SK_L)} \sim \frac{\epsilon^2}{2} (1 - 4\epsilon')$$

$$\frac{N(K_S \to \pi^0\pi^0) N(K_L \to \pi^+\pi^-)}{N(\Phi \to K_SK_L)} \sim \frac{\epsilon^2}{2} (1 + 2\epsilon')$$

$$\frac{N(K_S \to \pi^+\pi^-) N(K_L \to \pi^+\pi^-)}{N(\Phi \to K_SK_L)} \sim \epsilon^2 (1 + 2\epsilon')$$

$$\frac{N(K_S \to \pi^0\pi^0) N(K_L \to \pi^0\pi^0)}{N(\Phi \to K_SK_L)} \sim \frac{\epsilon^2}{4} (1 - 4\epsilon')$$

Any of these rates is suitable to evaluate $(\xi^0)$, the latter may be too difficult to handle from an experimental point of view. Contributions proportional to $(\xi^0)$ have different signs in different rates, and overall coherence reduces statistical and systematical errors. 

A weighted mean has a statistical error 

$$\sigma_N(\frac{\epsilon'}{\epsilon}) = \frac{1}{6\sqrt{N_1}},$$

where $N_1 = N(K_S \to \pi^+\pi^-)N(K_L \to \pi^0\pi^0)$. Therefore to achieve $\sigma(\frac{\epsilon'}{\epsilon}) \sim 2 \cdot 10^{-4}$, with an ideal fully efficient apparatus, in 100 typical running days, a mean luminosity $\bar{L} = 0.8 \cdot 10^{32}cm^{-2}sec^{-1}$ is required.
The asymmetry
\[ A = \frac{N_2 - N_1}{N_2 + N_1} \sim 3 \frac{\epsilon'}{\epsilon} \]
is a suitable quantity to evaluate \( \frac{\epsilon'}{\epsilon} \); only CP violating events are selected and systematical errors would be reduced.

The corresponding statistical error is
\[ \sigma_A(\frac{\epsilon'}{\epsilon}) = \frac{1}{3\sqrt{2}\sqrt{N_1}}. \]

Therefore to achieve \( \sigma(\frac{\epsilon'}{\epsilon}) \sim 2 \cdot 10^{-4} \) in 100 days from the asymmetry measurement, a mean luminosity \( \bar{L} = 1.5 \cdot 10^{32} \text{cm}^{-2} \text{sec}^{-1} \) is required.

Of course finite dimensions of the experimental apparatus must also be taken into account. On the other hand non integrated rates carry more informations. Hence a simulation has been done for a spherical fiducial volume with a radius \( R = 1.5 \text{m} \), taking into account a finite resolution \( \sigma = \pm 3 \text{mm} \) in the vertex reconstruction. No background has been simulated. The \( t_1, t_2 \) distribution has been fitted with the expected distribution. To achieve \( \sigma(\frac{\epsilon'}{\epsilon}) = 2 \cdot 10^{-4} \) in 100 running days the required mean luminosity is \( \bar{L} = 2.5 \cdot 10^{32} \text{cm}^{-2} \text{sec}^{-1} \).

The observation of the decay \( K_S \to 3\pi \) (up to now undetected) is also in the capabilities of an experiment at a powerful \( \Phi \) factory. In particular \( K_S \to 3\pi^0 \) and isotropic \( K_S \to \pi^+\pi^-\pi^0 \) are CP violating decays. The decay \( K_S \to \pi^+\pi^-\pi^0 \) is allowed, but strongly reduced, if orbital angular momenta are involved. Concerning the CP violating amplitude it is predicted

\[ B_{CP_viol}(K_S \to 3\pi) \sim B(K_L \to 2\pi) \cdot B(K_L \to 3\pi) \cdot \left( \frac{T_S}{T_L} \right)^2 \sim 3 \cdot 10^{-9} \]

if CP violation is mainly due to the mass mixing. The CP allowed decay is predicted \(^5\), yet never detected:

\[ B_{CP_{cons}}(K_S \to \pi^+\pi^-\pi^0) \sim (2 \pm 1) \cdot 10^{-7}. \]

In the \( \pi^+\pi^-\pi^0 \) final state the two amplitudes may interfere, increasing the possibility to detect CP violation in this \( K_S \) decay. The \( \pi^+\pi^-\pi^0 \) Dalitz plot must be uniform for the CP
violating decay. On the contrary the CP allowed decay must have a strong radial dependence: the simplest distribution \(^6\), taking into account the spherical harmonics involved, is like \([(T_1 - T_2)(T_2 - T_3)(T_3 - T_1)]^2\). CP violation manifests itself as an interference with alternate signs in the six sectors of the Dalitz plot. A factor 2 is gained in sensitivity by looking at the interference in the \(\pi^+\pi^-\pi^0\) final state, with respect to the \(3\pi^0\) final state, taking into account the different branching ratios \(^7\).

CP violation in the decay \(K_L \rightarrow \pi^+\pi^-\pi^0\), with non zero orbital angular momenta, might be detected as well. The relative width is related to the above mentioned widths, if CP violation is mainly due to the mass mixing. Namely:

\[
B_{CP_{\text{viol}}}(K_L \rightarrow \pi^+\pi^-\pi^0) \sim B_{CP_{\text{viol}}}(K_S \rightarrow \pi^+\pi^-\pi^0) \frac{T_L}{T_S} \sim 0.6 \cdot 10^{-9}.
\]

Yet this amplitude must interfere with the large, CP allowed, \(K_L\) decay amplitude. Finally, \(K_L\) regeneration could be implemented in principle, to add informations on the phases of these amplitudes.

Other physics in \(\Phi\) decays into kaon pairs.

There are interesting features also in other \(\Phi\) decays into kaon pairs, for instance in the radiative \(\Phi\) decay into kaon and pion pairs. By the way the decay \(\Phi \rightarrow K^0\bar{K}^0\gamma\) as been considered \(^8\) as a possible source of background in the measurements of \((\xi)\). However it has been demonstrated \(^9\) that such a background may be eliminated on the basis of relative decay times distributions. This radiative decay must occur via the production of the rather narrow resonances \(S^+(970)\) and \(\delta(980)\) [now called \(f_0(975)\) and \(a_0(980)\)], whose widths are 34 and 57 Mev.

Therefore it is very likely that the energy-momentum resolution of the proposed detector will isolate almost all these events (even if the radiative photon is not detected) and a measurement of the radiative \(\Phi\) decay into neutral and charged kaons and pions is interesting per se. In fact the nature of these resonances is far from being established. There is now a considerable body of evidence that the \(S^+\) and \(\delta\) are not simple \(q\bar{q}~^3P_0\) mesons and the fascinating hypothesis has been put forward that they are \(K\bar{K}\) molecules \(^{10}\), roughly analogous to the deuteron.
An anomalous feature of these resonances is their width, much narrower than expected. The close values of masses and widths between $S^*$ and $\delta$ strongly suggests an ideally mixed pair of mesons, like $\rho$ and $\omega$, hence the $2\pi$ decay width should be much larger than observed. On the contrary, in the $K\bar{K}$ molecule interpretation masses, widths and decay modes are naturally explained.

The radiative $\Phi$ decay width into these resonances is relevant because of the controlled environment, especially if compared to the $\Phi \to \eta'\gamma$ width. In particular

$$\Gamma(\Phi \to S^*\gamma) \sim 10^{-3} MeV$$

is expected (8) if the $S^*$ is also a $s\bar{s}$ state, yet a much smaller width is expected if a molecular state must be produced in the radiative decay.

A similar discrepancy has been already observed (11) in another e.m. process, namely the $S^*$ and $\delta$ widths into $\gamma\gamma$ much narrower than expected (12) (Tab.1).

<table>
<thead>
<tr>
<th>meson</th>
<th>$\Gamma_{\gamma\gamma}$ (KeV)</th>
<th>$\Gamma_{\gamma\gamma}^0$ (KeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S^*$</td>
<td>$0.27 \pm 0.12$</td>
<td>4.5</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$0.23 \pm 0.09$</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Finally a more accurate determination of the kaon form factor is also an interesting byproduct at a $\Phi$ factory. An accurate $\Phi$ excitation curve measurement would allow to disentangle the $\rho$ and $\omega$ contributions (13), by means of their interference with the $\Phi$.

A measurement at somewhat higher energies than the $\Phi$ peak would allow the detection of predicted, yet still undetected, spectacular interference patterns in the neutral kaon form factor (Fig. 3).
Fig. 3  $K^0$ squared form factor [dashed curve = $\rho\omega\Phi$ tail + $\rho'(1600)$, solid curve = $\rho\omega\Phi$ tail + many $\rho', \Phi'$ states].
REFERENCES


