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ROLE OF RADIATIVE CORRECTIONS FOR PRECISION TESTS
OF THE STANDARD MODEL

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ROLE OF RADIATIVE CORRECTIONS FOR PRECISION TESTS
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Precision tests of the standard electro-weak model at LEP/SLC energies need the precise evaluation of radiative corrections, beyond the one-loop approximation. The theoretical situation after the S$p\bar{p}$S collider results is discussed in detail, for the main processes of interest at the $e^+e^-$ colliders operating in the next future.

The abundance of data\(^{(1)}\) on the $W^\pm$ and $Z^0$ weak bosons accumulated so far at the CERN S$p\bar{p}$S collider has allowed detailed tests of the Standard Model\(^{(2)}\) confirming that fundamental interactions – electro-weak-strong – are described by gauge theories, in the presently explored range of energies and momentum transfers. Yet the full experimental proof of the theory is far from being completed. Furthermore a large number of fundamental parameters is not determined by theory, including the mass of the Higgs boson. Indeed the Higgs mechanism of spontaneous symmetry breaking is the most peculiar aspect of the model which has to be tested, particularly in connection with the high energy behaviour of the theory. Many theoretical ideas have been developed\(^{(3)}\) to extend the standard model to distances much shorter than those we currently explore. In spite of the fact that none of the proposed extensions provides an overall solution to the various problems of the standard model, there is a common belief that new physics is to be expected in the range of energy (0.1-1) TeV.

Next operation of the new colliders LEP and SLC will provide a unique opportunity for testing the properties of the electroweak model beyond the tree-level\(^{(4)}\). Indeed the measurement
of various asymmetries in $e^+e^- \rightarrow \mu^+\mu^-, \tau^+\tau^-, \ldots$, in particular in the vicinity of the $Z_0$ where statistics are expected to be good, and/or the observation of longitudinal polarization—either in initial-state electron beams or in final state $\tau$ polarization—will test the standard model at the one loop level and possibly reveal the existence of a new level of physics. The latter would give observable effects via radiative corrections if the associated new particles are too heavy to be produced directly.

Experimental investigation of these effects will provide, however, precision tests of the theory only if QED radiative corrections, associated with radiation of real or virtual photons and dependent upon the details of the experimental arrangements, are under control at the level of $< 1\%$. In fact this is the order of magnitude of the weak effects which are expected to be measured. Indeed the most relevant higher order corrections can be directly reabsorbed into a redefinition of the basic parameters (M$_Z$, $\Gamma_\nu$, sin$^2$ $\theta_W$) and the renormalization of the fine structure constant. The leftover corrections lead then to observable effects of order 1%. On the other hand pure e.m. corrections, which add no additional theoretical information, are expected to be rather important, especially in the vicinity of the $Z_0$ boson and change sizably the naive expectations.

In the present talk I will briefly review the main results on radiative corrections which are relevant for precision tests of the standard model at LEP/SLC energies.

Let us consider first those predictions of the electro-weak theory which are of interest for the $\bar{p}p$ collider results. Assuming $1 - M^2_w/M^2_Z$ as a definition of $s^2 = \sin^2 \theta_W$, then it is customary write the weak boson masses as follows:

$$M_w = \cos \theta_w M_Z = (\mu/s) \left(1/(1 - \Delta r)\right)^{1/2},$$  \hspace{1cm} (1)

where

$$\mu = [\pi\alpha / \sqrt{2} G_F]^{1/2} = 37.281 \text{ GeV},$$  \hspace{1cm} (2)

and the correction factor $\Delta r$ is predicted $\Delta r = (7.0 \pm 0.5) 10^{-2}$

for $N_{\text{families}} = 3$ and a reasonable range of values

$$10 \text{ GeV} \leq m_H \leq 1 \text{ TeV}$$

$$20 \text{ GeV} \leq m_l \leq 60 \text{ TeV}$$ \hspace{1cm} (3)

As well known, the largest contribution to $\Delta r$ comes from running the e.m. coupling constant to the weak bosons mass, namely
\[ \alpha^{-1}(M) = \alpha^{-1} - \frac{(2/3\pi)}{\sum_f Q_f^2 \ln \left( \frac{M}{m_f} + \frac{1}{6\pi} \right)} + \ldots \]
\[ = 127.70 \pm 0.30 + \frac{(8/9\pi)}{\ln \left( \frac{m_t}{36 \text{ GeV}} \right)} , \]

where the summation is taken over all charged fermions with \( m_f \leq M \). The variation of the Higgs masse in the range indicated in eq. (3) has a very tiny effect on \( \Delta r \sim O(1\%) \).

The results from the \( \bar{p}p \bar{p} \) collider, which confirm the expected increase of the weak boson masses, agree with the \( \sin^2\theta_w \) determinations at low momentum transfers - deep inelastic \( eN \) scattering, \( eD \) scattering, \( eV \) elastic scattering, \ldots, after inclusion of radiative corrections - and can be used to set a limit on the top mass, due to the characteristic dependence of \( \Delta r \) on \( m_t \). One finds (5) \( m_t \leq 250 \text{ GeV} \). A better test of the theory at the one-loop level, or possible informations on heavy Higgs’s or heavy fermion doublets, can be gained only with measurements at the level \( \delta s^2/s^2 < 1\% \), which is the accuracy goal to be pursued at LEP/SLC.

It has to be stressed that the accuracy achieved in testing the electro-weak at the \( \bar{p}p \) collider also depends upon our clear understanding of QCD radiative corrections to all order. Indeed the \( q_T \) distributions of the \( W \) is sensitive to large terms of order

\[ \left( \frac{1}{q_T^2} \right) \alpha_s^n \left( \frac{q_T}{s^2} \right) \ln^m \left( \frac{Q^2}{q_T^2} \right) \quad (m \leq 2n-1) \]

which are characteristic of a theory with massless vector gluons and can be realiably resummed (8) for \( \Lambda \ll q_T \ll Q \) (Q - \( M_w \)).

Let us consider now the electro-weak processes which are of interest at LEP/SLC energies, beyond the Born approximation.

The higher order weak effects which can be simply taken into account are those which can be reabsorbed into a redefinition of the basic parameters (\( M_Z, \Gamma_Z, \sin^2\theta_w \)) and the renormalization of the fine structure constant. Other purely weak effects are contained in the full one-loop amplitude through the exchange of the heavy bosons of the theory. An additional dependence on unknown parameters, such as the Higgs boson mass \( m_H \), the top quark mass \( m_t \), etc., is then introduced into the various quantities which could provide a genuine test of the theory. Let us consider, for example, the role played by the corrections to the forward-backward and left-right asymmetries. At the resonance (\( s \approx M_Z^2 \)) the lowest order expression for \( e^+e^- \rightarrow \mu^+\mu^- \) is

\[ A_{FB} \approx 3 \left( \frac{v^2 a^2}{(v^2 + a^2)^2} \right) \approx 3 \left( 1 - 4 \sin^2 \theta_w \right)^2 , \]

which is very small and quite sensitive to higher order effects. Then expressing \( \sin^2\theta_w \) in terms of \( M_Z \), the accurately known quantities \( \alpha \) and \( G_F \) and the radiative correction \( \Delta r \), as
\[
\sin^2 \theta_w = (1/2) \left[ 1 - \left(1 - 4 \mu^2 / (M_Z^2 (1-\Delta r)) \right)^{1/2} \right], \quad (6)
\]

It follows\(^{(7)}\), for \(M_Z = 94 \text{ GeV}\),
\[
A_{FB}(s = M_Z^2) = 0.13 \quad , \quad \Delta r = 0 \quad (\sin^2 \theta_w = 0.195) \quad (7)
\]

and
\[
A_{FB}(s = M_Z^2) = 0.055 \quad , \quad \Delta r = 0.07 \quad (\sin^2 \theta_w = 0.216) \quad (7')
\]

This shows the relevance of the effects of the standard model prediction \(\Delta r = 0.07\) on the forward-backward asymmetry. Furthermore, the left over weak corrections appearing in the full one-loop amplitude affect of an additional \(-10\%\) the result \((7')\). In fact, one obtains for \(M_Z = 94 \text{ GeV}\), \(10 \text{ GeV} \leq m_H \leq 1000 \text{ GeV}\) and \(30 \text{ GeV} \leq m_t \leq 90 \text{ GeV}\)
\[
0.0619 \leq A_{FB} (s = M_Z^2) \leq 0.0532 \quad (8)
\]

The reason for such a sizeable effect can be traced to the fact that the zeroth order expression for \(A_{FB}\) is suppressed by the very small factor \(\nu^2 - (1 - 4 \sin^2 \theta_w)^2\) (eq. 5), whilst the radiative corrections contain contributions of \(O(\alpha \sin^2 \theta_w / \pi)\) which is not inhibited by \(\nu^2\).

A similar but smaller effect is found for the left-right asymmetry. At \(s \approx M_Z^2\) the lowest order expression is
\[
A_{LR} \approx 2 \left( \nu / (\nu^2 + a^2) \right) P_e \approx 2 (1 - 4 \sin^2 \theta_w)^2 P_e \quad (9)
\]

where \(P_e\) is the electron longitudinal polarization. With the help of eq. (6) and for \(P_e = 1\), it leads to
\[
A_{LR}(s = M_Z^2) = 0.42 \quad , \quad \Delta r = 0 \quad (10)
\]
\[
A_{LR}(s = M_Z^2) = 0.27 \quad , \quad \Delta r = 0.07
\]

in analogy to eqs\((7' - 7')\)

The effect of the leftover one-loop weak corrections on \(A_{LR}\) is less important than in the case of \(A_{FB}\), due to the only linear dependence on \(\nu\) of the zeroth-order term. Indeed, for the same range of values assumed above for \(m_H\) and \(M_Z\) one finds\(^{(7)}\)
\[
0.26 \leq A_{LR}(s = M_Z^2) \leq 0.28 \quad (11)
\]

From the above discussion it follows that an accurate test of the weak corrections and of the related dependence on \(m_H\), the number of families, etc., or the detection of possible effects of
new physics beyond the standard model require a precision of \(\lesssim 1\%\) and consequently an even better control of pure e.m. effects.

Indeed first-order corrections (6), for example, reduce the \(Z_\alpha\) peak cross section by more than 50\%, or shift the zero in the forward-backward asymmetry by about (±300) MeV, for an energy resolution of \((10^{-1}-10^{-2})\).

In addition, one must evaluate the corrections due to soft photon emission, which become increasingly important as the energy increases, to all orders in \(\alpha\). Usually, i.e., for non-resonant cross sections, this leads to the exponentiated form \(\propto \exp\left(\frac{4\alpha}{\pi} \ln \left(\frac{2E}{m}\right) \ln \left(\Delta \omega / E\right)\right)\), where \(m\) is the electron mass, \(2E\) is the c.m. energy and \(\Delta \omega\) is the resolution of the experiment. In the case of resonance production, the above factor is modified, and for a very narrow resonance-like \(J/\psi\) -the correction becomes(9) \(\exp\left(\frac{4\alpha}{\pi} \ln(2E/m) \ln(\Gamma/M)\right)\). Physically, this is understood by saying that the width \(\Gamma\) provides a natural cut-off in damping the energy loss in the initial state. For the case of the \(Z_\alpha\) boson, where neither of the preceding cases applies \((\Gamma/M-\Delta \omega /E)\), the soft correction is(10) a complicated function of to \(E, M, \Delta \omega\) and \(\Gamma\) raised to the power \(\left(\frac{4\alpha}{\pi}\right) \ln(2E/m)\), of the order of 50\%.

Nowaday charged particles are often detected in electromagnetic calorimeters which do not discriminate between the particle and the accompanying collinear photons. The effect of emission of hard and collinear photons becomes increasingly important at high energies and its contribution has to be included in the observable cross sections(11). In perturbation theory, as well known, this corresponds to large logarithms associated to the mass singularities of the emitting charged particles. Then introducing the angular resolution \(\delta\) of the calorimeter, the final correction includes powers of logarithmic terms of the type \((\alpha/\pi) \ln \delta^2 \ln (\Delta \omega / E)\) or \((\alpha/\pi) \ln \delta^2\). In Bhabha scattering, for example, the inclusion of such terms is crucial to obtain a high precision monitor of the beam luminosity(12,13).

Let me briefly discuss the main features of the two effects mentioned above, without going into the details of the derivation, which can be found elsewhere(14).

The method of coherent states is the most appropriate one to resum the full series of leading and next-to leading logarithms originated from multiple photon emission for the general process \(e\bar{e} \rightarrow \gamma Z \rightarrow f \bar{f}\), including interference effects. It is based on the generalization of the concept of classic currents associated to the initial and final particles in presence of a resonance. For QED processes, where the basic cross section \(d\sigma_o\) is not a rapidly varying function of the energy, or the momentum transfer, one has the well known result

\[
d\sigma^{QED} = \frac{d\sigma_o}{\gamma^\beta \Gamma(1+\beta)} (\Delta \omega / E)\beta + \ldots\ldots
\]

(12)

where \(\ln \gamma=0.5772\) is Euler's constant and \(\beta=\beta_e + 2\beta_{\text{int}} + \beta_f\). We have defined
\[ \beta_e = \frac{2\alpha}{\pi} \left[ \ln \left( \frac{s}{m_e^2} \right) - 1 \right] \]

\[ \beta_f = \frac{2\alpha}{\pi} Q_f^2 \left[ \ln \left( \frac{s}{m_f^2} \right) - 1 \right] \]

\[ \beta_{\text{int}} = -Q_f \left( \frac{4\alpha}{\pi} \right) \ln \tan \left( \theta/2 \right) \]

and \( Q_f \) is the electric charge of the particle \( f \).

On the other hand, for the pure resonant case for \( ee \rightarrow R \rightarrow ff \) one finds\(^{9,10}\)

\[ d\sigma^{\text{RES}}_{\text{O}} = d\sigma^{\text{RES}}_{\text{O}} \left( (\Delta\omega/E)\beta_f \left| \frac{2\Delta\omega}{\sqrt{s}} \frac{M_R^2 - s}{M_R^2 - s + 2\sqrt{s} \Delta\omega} \right| \beta_e \right. \]

\[ \cdot \left. \left| \frac{2\sqrt{s} \Delta\omega}{M_R^2 - s + 2\sqrt{s} \Delta\omega} \right| \beta_{\text{int}} \left[ 1+ \frac{s - M^2}{M} \beta_e \delta(s, \Delta\omega) \right] + C_F^{\text{RES}} \right) \]  

(14)

where \( M_R^2 = M^2 - iM\Gamma \),

\[ \delta(s, \Delta\omega) = \arctg \frac{2\sqrt{s} \Delta\omega - (s - M^2)}{M\Gamma} + \arctg \frac{s - M^2}{M\Gamma}, \]  

(15)

and \( C_F^{\text{RES}} \sim O(\alpha) \) accounts for the rest of the finite corrections and has to be calculated perturbatively. We have not included in eq. (14) factors of the type \( \gamma^\beta \Gamma^{-1}(1+\beta) \), as in eq. (12), which contribute to order \( \beta^2 \) only. First order expansion of eq. (14) clearly coincides with the usual soft term of \( O(\alpha) \). \( C_F^{\text{RES}} \) is then determined by comparison with the exact \( O(\alpha) \) result.

Let us briefly discuss this result. First, it is easily seen that the infrared factors appearing in eq. (14) reduce to the standard one \( (\Delta\omega/E)\beta_e + \beta_f + 2\beta_{\text{int}} \) in the limit \( \Delta\omega \ll (M_R^2 - s)/2\sqrt{s} \), as they should. On the other hand, in the case of a narrow resonance like the \( J/\Psi \), for which in a typical experiment \( \Delta\omega \ll (M_R^2 - s)/2\sqrt{s} \), the \( \Delta\omega \) dependence drops completely out, namely the width of the resonance provides a natural cut-off in damping the energy loss in the initial state. Furthermore, the \( \beta_{\text{int}} \) dependence eq. (14) also cancels out, giving no interference between the soft emission from the initial and final states. Finally the term proportional to \( \delta(s, \Delta\omega) \) gives the radiative tail of the resonance. In fact for narrow resonances \( \delta(s, \Delta\omega) \) reduces to \( \delta_R(s) \), the usual phase shift of the Breit-Wigner resonances.

Similar effects are found\(^{10}\) in the interference of a resonant term with a pure QED term.
From the above formulae it is clear that, in experiments studying forward-backward asymmetries, dips and similar subtle effects, the $\Gamma$ and $\Delta\omega$ dependence in the infrared factors has to be taken into account properly.

A particularly simple case is provided by the measurement of the line shape of the $Z_0$ in the total cross section $e^+e^- \to X$. Then one can take the limit $\Delta\omega \to \frac{1}{2\sqrt{s}}$ in eq. (14) and obtains

$$d\sigma^{RES} = d\sigma_0 \left[ \frac{(s-M^2)^2 + M^2\Gamma^2}{M^4} \right] \left[ 1 + \beta_e \frac{s-M^2}{M\Gamma} \delta_R \right] + \ldots,$$

(16)

the dots indicating additional non-leading terms, in exact analogy to the case of production of a very narrow resonance. Physically, this is understood by saying that the width $\Gamma$ provides a natural cut-off in damping the energy loss in the initial state.

The resummation of the leading and next-to-leading logarithms from initial state bremsstrahlung has been also studied more recently by extending to QED the formalism used in the context of the QCD-improved parton model. In this case one defines electron or position densities $e(x,s)$ at the scale $s \lesssim M^2$. The corrected cross section is then obtained by a convolution of the $e^-$ and $e^+$ densities with a reduced cross section $\tilde{\sigma}$

$$\sigma(s) = \int_1^1 dx_1 dx_2 \theta(x_1 x_2 s) e(x_1, s) e(x_2, s) \tilde{\sigma}(x_1 x_2 s)$$

(17)

in exact analogy to Drell-Yan processes in QCD.

The electron density $e(x,s)$ and photon density $\gamma(x,s)$ in an electron satisfy evolution equations, as in QCD, with the strong coupling constant replaced by the QED running coupling constant ($\alpha = \alpha(m^2_e)$)

$$\alpha/\alpha(s) \simeq 1 - (1/3\pi) \alpha \sum_f Q_f^2 \ln (s/m^2_f)$$

(18)

and the sum extends over quarks and leptons with masses $m^2_f < s$. Clearly $e(x,m^2_e) = \delta(1-x)$ and $\gamma(x,m^2_e) = 0$.

In the soft limit ($\Delta\omega/E \ll 1$), the contribution to (17) comes from the non-singlet part of the distributions $e(x,s)$ only, corresponding to the annihilation of the initial electrons after emission of the soft radiation. Then the solution to the evolution equations can be computed analytically if $\tilde{\sigma}(s)$ is a smooth function of $s$, and takes the form

$$\tilde{\sigma}(s) = \sigma(s) R (1 - \Delta\omega/E, s)$$

(19)
with \((19)\)

\[
R(x,s) = \frac{(1-x)^\eta}{\gamma^\eta \Gamma(1+\eta)} e^{3/4 \eta}
\]

(20)

and

\[
\eta = \int \frac{2\alpha(Q^2)}{m^2} \frac{dQ^2}{\pi Q^2} = -6 \ln \left[ 1 - (\alpha/3\pi) \ln \left( s/m^2 \right) \right]
\]

(21)

To leading order \(\eta = \beta e\) and eq. (20) coincides with the previous result (12) a part the term \(\exp (3/4\eta)\), arising from the improvement of simple soft bremsstrahlung spectrum \(-2/x\) by \([1+(1-x)^2]/x\).

It has to be emphasized that this factorized result is not valid when the cross section \(\tilde{\sigma}(s)\) is a rapidly varying function of \(s\), as in the case of resonance production with \(\Gamma \lesssim \Delta \omega\). Furthermore it does not include the emission from the final state and interference effects, and therefore can only be applied to the observation of the line shape of the resonance, after taking appropriately into account the \(s\) dependence of \(\tilde{\sigma}(s)\).

The detection of charged particles in electromagnetic calorimeters does not allow to discriminate a particle from the accompanying hard collinear radiation. One has to include then the corresponding contribution in the observed cross section.

In perturbation theory this effect becomes increasingly important at high energies because of the large logarithms associated to the mass singularities of the emitting particle. Kinoshita, Lee and Nauenberg\((11)\) have observed that if one sums over all degenerate states the mass singularities cancel out to all orders of perturbation theory. Then by introducing a small but finite angular resolution \(\delta(\delta \times 1)\) of the calorimeter, one is finally led to an observable cross section which contains, to leading order, logarithms of order \((\alpha/\pi) \ln \delta^2\)^n. This result is well known and rather commonly used in QCD jet analyses\((11)\).

The formulae\((20)\) reported below apply to a typical experiment in which the following requirements are satisfied: (i) the final state consists of a particle and an antiparticle \((f \equiv e, \mu, \ldots)\) detected within a certain acollinearity angle \(J\) of a few degrees \((J \lesssim 5^\circ)\). The energy resolution \(\Delta \omega\) then depends upon \(J\). Muon pairs production is the most typical example of such a process. In the following we will take \(f = \mu\). (ii) An electromagnetic calorimeter of finite and small angular resolution \(\delta\) is centred along the muon direction. In principle it does not discriminate between a charged particle and the accompanying collinear photons.

Then using (i) and (ii) one would be sure that all but a fraction \(\Delta = \Delta \omega/E\) of the beam energy is taken by the muons and the accompanying hard photons. For small \(\delta\) and \(\Delta\), fully analytic expressions can be used, neglecting hard-photon effects of order \([\alpha/\pi] \Delta, (\alpha/\pi) \delta\]. On the contrary, all double logarithmic terms of the form
\[(\alpha/\pi) \ln (s/m_\circ^2) \ln (\Delta, \Gamma/M) , (\alpha/\pi) \ln \delta^2 \ln \Delta,\]

or simple logs such as

\[(\alpha/\pi) \ln (s/m_\circ^2) , (\alpha/\pi) \ln (\Delta, \Gamma/M, \ln \delta^2)\]

can be resummed to all orders, using known results on the exponentiation of the infrared and mass singularities.

To first order in \(\alpha\), the contribution from collinear hard radiation (\(k \geq \Delta\omega\)) from the final particles, when detected within a small cone of half opening angle \(\delta\), consists of the following correction factor\(^{(11,12)}\): (\(j = \text{QED, INT, RES}\))

\[
\delta_j^{\text{coll}} = d \sigma_j^0 \left[ (\ln (E/\Delta\omega) - 3/4) \ln (E\delta/m_\mu) - 1/2 \ln (E/\Delta\omega) + 1/2 (9/4 - \pi^2/3) \right]
\]

(22)

Then in agreement with the Kinoshita-Lee-Nauenberg theorem on the mass singularities\(^{(11)}\), the \(m_\mu\)-dependence disappears after adding eq. (22) to the virtual and soft contributions. This result can be generalized\(^{(20)}\) to all orders using the known results on the exponentiation of soft and collinear divergences.

All the above considerations are crucial for the reaction \(e^+e^- \rightarrow e^+e^-\), which because of its large cross section could provide a high precision monitor of the beam luminosity. Indeed an accurate description of QED radiative effects for Bhabha scattering to 1% level should include\(^{(13)}\) exact expressions for all one-loop diagrams, and soft and collinear hard photon effects resummed to all orders.

So far we have considered radiative corrections to various \(e^+e^-\) processes due to virtual and soft-photon effects or the emission of hard bremsstrahlung collinear to the final charged particles. The level of accuracy of the analytical formulae is of order \((\alpha/\pi)\Delta, (\alpha/\pi)\delta\) with \(\Delta\) and \(\delta\) are the energy and angular resolutions. This is certainly sufficient as long as the final \((e^+e^-, \mu^+\mu^-, \ldots)\) pair is detected to a good collinearity and, if it is the case, the electromagnetic shower cones are small. The results described above include the largest contributions from double and single logarithms to first and higher orders in \(\alpha\), the latter being often larger than simple \(O(\alpha/\pi)\) terms.

When however the above criteria are not satisfied, namely the kinematical domain allowed in the experiment is a full three body phase space, then an additional term has to be added to the previous formulae, corresponding to hard photon bremsstrahlung\(^{(21,22)}\). This effects is normally included in a way which is suitable for a Monte Carlo simulation of events. This also allows to account for all features of experimental detection.

It has to be stressed however that the numerical treatments usually available\(^{(23)}\) are based on
first order cross sections only, and should not be used as a full description of e.m. radiative effects, due to the relevance of higher-order terms. Furthermore numerical integration of the O(α) cross section in the infrared and collinear regions requires a careful analysis and is computer time consuming, while simple analytic expressions can be used alternatively. Therefore the numerical study of hard photon effects has to be appropriately performed in the large angle regions, and if it is the case, the corresponding term added to those discussed above.

As an example of the relevance of higher order effects, the muon asymmetry $A_{FB}$ is shown in Fig. 1, for $\sqrt{s} \sim M_Z$ and a calorimetric-type experiment with an angular resolution $\delta \simeq 1^\circ$ and $\Delta=\Delta\omega/E=10^{-1}$ and, $\Delta=10^{-2}$. The electro-weak parameters are $\sin^2\theta_w=0.23$, $M_Z=92$ GeV and $T=2.9$ GeV. The dashed curve shows the Born cross section and the dot-dashed ones the first order corrections.

**FIG. 1 - Integrated $\mu$ forward-backward asymmetry for $\sqrt{s} \sim M_Z$.**

The comparison with the full curves corresponding to the all orders corrections, shows the relevance of the effect of higher order.

The QED radiative corrections are almost negligible for $A_{LR}= (\sigma_L + \sigma_R)/ (\sigma_L + \sigma_R)$, which is accessible with longitudinally polarized beams only. As well known, $A_{LR}$ is very sensitive to $\sin^2\theta_w$ and, being measurable in an inclusive mode, can lead to test the e-w theory with great precision.

To conclude, precision tests of the standard model demand a very careful treatment of e.m. radiative corrections, well beyond the one loop level. In addition to complete O(α) formulae for various processes of interest at LEP/SLC energies, a detailed treatment of higher order effects is required, which sums up the full series of double leading logarithms associated to multiple soft and collinear emission, as well as some classes of single logarithms. A full account of e.m. effects below the (1%) level would require complete calculations to two-loops accuracy.
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