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Published on "Weak Superconductivity"
Series of "Advances in the Physics of Condensed Matter"
Ed. by A. Barone (World Scientific 1987)
INTRINSIC RESOLUTION OF SUPERCONDUCTING TUNNEL JUNCTIONS USED AS IONIZING PARTICLE DETECTORS*

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1. - INTRODUCTION

It is known that superconducting tunnel junctions (STJ) used as ionizing particle detectors could have a theoretical energetic resolution 30-50 times better than the best semiconductor detectors available today (1). In this way such devices could give large improvements in the sensitivity of measurements in some fundamental experiments of particle physics. However, until now, the real interest grown about this application of STJ (2-7) has been limited mainly because the experimental values of the energetic resolution have been disappointing worse than the expected one. We believe that it will be possible to overcome the previous situation only by a deep analysis of all the phenomena involved both in the two films under the excitation of quasi particles by ionizing particles, and in the whole junction during the collection process.

* Invited paper at the "II Soviet-Italian Symposium on Weak Superconductivity" (Naples 5-7 May, 1987)
As far as the excited state of the films is concerned, it can be described by a non-equilibrium state or in terms of a local heating. The usual non-equilibrium theory has been developed for systems near the equilibrium, homogeneous in the space and non explicitly time dependent. The experimental condition of a STJ under the excitation of a nuclear particle is quite different and just opposite to the three previous items: a state very far from the equilibrium is generated in a very limited spatial region. Later on, the quasiparticles diffuse in the surrounding film generating a smaller perturbation into a larger area. Other excitations such as surface plasmons could play some role in the diffusion process, but, until now, they have not been considered. In particular it should be analysed:

a) the recombination time for a superconducting state far from the equilibrium,
b) quasiparticles diffusion dynamics,
c) effects of space inhomogeneity on the effective quasiparticles recombination time.

Moreover, at the end of the relaxation processes an heating of the films occurs. If the non-equilibrium relaxation-time is shorter than the heat-diffusion time, a local heating model can describe the whole behaviour of the junction, and the usual heat diffusion equation will describe the time evolution of the junction area influenced by the excitation process. In the opposite condition, the non-equilibrium effects will dominate the time evolution of the perturbation.

As far as the actual detection of the perturbed state is concerned, the process of signals collection in tunnel junctions is quite different from the usual collection process in conventional detectors. Some misunderstandings in the world of high energy physics arise because in the "traditional" particle detectors the applied electrical field acts on the free charges, generated in the active region by the ionization. On the contrary, in tunnel junctions the applied voltage lies only in the barrier and no electrical fields are present in the two films where the excited quasiparticles are generated. The tunnel current can affect only indirectly the diffusion process. Moreover, in usual detectors, the collection time is much shorter than the recombination time (\(\tau_p\)), while in superconducting tunnel junctions this condition is difficult to obtain since the quasiparticle collection process is strictly related to the tunnel phenomenon. The tunnel time (\(\tau_t\)) is inversely proportional to the tunnel probability \(p\) (\(\tau_t = 1/p\)) and it is related to the normal resistance \(R_{NN}\) (that is the resistance of the STJ at a voltage \(V>(\Delta_1+\Delta_2)/e\)) by the equation:

\[
\tau_t = R_{NN} \frac{e^2}{N_0} A d
\]

where \(e\) is the electron charge, \(N_0\) is the density of states at Fermi energy, \(A\) is the area of the junction, \(\Delta_{1,2}\) are the gaps of the two electrodes of the STJ, and \(d\) is the thickness of the film. The typical order of magnitude is: \(\tau_t = 2(\text{ns}) R_{NN} (\Omega) A (\mu m^2) d (\mu m)\). On the contrary, the recombination time in a quasiequilibrium state is given by (7):

\[
\tau_r = \tau_o \exp \left( \frac{\Delta}{K_B T} \right) \pi^{-1/2} \left( \frac{2\Delta}{K_B T_c} \right)^{-5/2} \left( \frac{T}{T_c} \right)^{-1/2}
\]
where $\tau_0$ and $T_c$ are respectively the characteristic quasiparticle time and the transition temperature, which depend on the superconducting materials as reported in Table I.

**TABLE I - Transition temperatures and quasiparticle characteristic times for some superconducting materials.**

<table>
<thead>
<tr>
<th></th>
<th>Zn</th>
<th>Al</th>
<th>In</th>
<th>Sn</th>
<th>Pb</th>
<th>Nb</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_c$(K)</td>
<td>0.87</td>
<td>1.19</td>
<td>3.4</td>
<td>3.75</td>
<td>7.18</td>
<td>9.2</td>
</tr>
<tr>
<td>$\tau_0$(ns)</td>
<td>780</td>
<td>438</td>
<td>0.8</td>
<td>2.3</td>
<td>0.2</td>
<td>0.15</td>
</tr>
</tbody>
</table>

If the condition $\tau_t \ll \tau_r$ is reached, other problems can arise. In fact, in the usual lowest order perturbative analysis of the tunnel current, the tunnel process doesn’t change the equilibrium state of the two films. On the contrary, if $\tau_t \gg \tau_r$ the tunnel itself perturbs the thermal equilibrium in the superconducting films. In this limit some anomalies in the I-V characteristics should be observed and a higher order perturbative analysis should be performed, so that the usual semiconducting model could not be still valid. Neglecting these problems, it should be taken into account that at the arrival of the ionizing particle only a small portion of the junction is strongly excited and the influence of such variation on the global current-voltage (I-V) characteristic should be studied in more details.

In this paper the previously mentioned problems are not analysed. We discuss the signal to noise ratio as a function of both the temperature $T$ and the tunnel probability $p_t$, by assuming the complete junction uniformly perturbed. For this reason in section 2 we briefly analyse the general features of the I-V characteristic both in the unperturbed state and under the excitation generated by ionizing particles, and the usual expression of the current noise of a STJ is recalled. In both cases ideal junctions without true leakage currents are supposed. The general features of a STJ detection system is outlined in section 3. In sections 4 and 5 respectively ideal voltage and charge amplifiers coupled to the junction are considered. The dependance of the signal to noise ratio ($S/N$) on both temperature and tunnel probability is analysed. Conclusions are drawn in section 6.

2. - PERTURBED AND UNPERTURBED I-V CHARACTERISTICS

In the limit of low tunnel probability $p_t$, that is $\tau_t \gg \tau_r$, the tunnel current, in the unperturbed state at the temperature $T$, has the following expression (8):

$$I = f(V,T) / R_{NN}$$ (3)
where $R_{NN}$ and both the static resistance $R_S = V/I$, and the dynamic resistance $R_D = dV/dI$ are inversely proportional to $p$, and the function $f(V,T)$ depends on the junction electrodes. In Eq. (3) the polarization point is characterized by the voltage value more than by the current value, which is proportional to $p$. In actual junctions, for high values of $p$ and voltages lower than the sum of gaps, excess currents are observed which are usually ascribed to leakage currents, but in our opinion further investigations are necessary.

When the ionizing particle hits the junction the quasi-static I-V characteristic can be written by the following expression:

$$I' = f(V,T,n'(E,t)) / R_{NN}$$

(4)

where $n'(E,t)$ describes the time dependence of the energy distribution of the excess quasiparticles with energy $E$. In a first approximation we can neglect the details of the energy distribution, and the current $I'$ is only a function of the total number of quasiparticles $n'(t) = \int_o^\infty n'(E,t) dE$. If the thermal quasiparticle current is independent of the excess quasiparticle current (i.e. for a voltage bias), in some intermediate voltage range between zero and the gap sum $\Delta_1 + \Delta_2$, the signal current $\Delta I = I' - I$ is given by:

$$\Delta I = p \ n'(0) \ exp(-t/\tau_R) = \Delta I_o \ exp(-t/\tau_R)$$

(5)

where $\tau_R$ is the effective quasiparticles relaxation time. As usually, out of the low $p$ limit, $\tau_R$ is given by:

$$\frac{1}{\tau_R} = \frac{1}{\tau_t} + \frac{1}{\tau_i}$$

(6)

We note that in an usual ionization detector there is no connection between the detector resistance and the amplitude of the current signal; on the contrary in a STJ, following Eq. (5), the tunnel current is proportional to $p$, i.e. to the inverse of the resistance. In Eq. 5 we have neglected that for voltages close to the sum of the gaps we would take into account the gap reduction caused by the excess quasiparticles. This phenomenon and others that can enable multiplication or "avalanche like" processes are neglected in the present paper. The spectral distribution of the current noise in a STJ is given by (8):

$$I_n^2(\omega) = \frac{(e/\pi)}{I(V) \ coth(eV/2K_B T)}$$

(7)

which can be written as:

$$I_n^2(\omega) = \frac{F(V,T)}{(\pi R_S)}$$

(8)

where $F(V,T)$ in the two limits $eV/K_B T \gg 1$ and $eV/K_B T \ll 1$ is respectively equal to $eV$ and
2KBbT, and is respectively equivalent to a shotky noise and to a Johnson noise generated by the static resistance Rs. We point out that the background noise depends on the unperturbed state of the STJ, while the signal amplitude indetermination depends on the noise in the perturbed state.

3. - STJ DETECTION SYSTEM

We analyse a superconducting junction connected to a bias network and to a preamplifier followed by an optional low pass filter as shown in Fig. 1a. The equivalent circuit is shown in Fig. 1b where the junction is described by a noise source, a capacitor Cd, and a resistance R(V), which is changed by the ionizing particle and generates the signal.

![Diagram](image)

**FIG 1** - (a) Block diagram of the STJ detection system, and (b) equivalent circuit: Cde is a decoupling capacity, Z1 is the input impedance of the amplifier, I_n and e_n are the usual noise sources.

As shown in Fig. 2, due to the presence of the bias network, the effective current or voltage signals are strictly related to the intersection of the load line with the perturbed and the unperturbed I-V characteristics.

![Graph](image)

**FIG. 2** - I-V characteristics of a STJ in the unperturbed (full line) and in the perturbed (dashed line) states; dotted and dotted-dashed lines are respectively the static and the dynamic load lines.
In dynamic conditions the true dynamic load line must be considered, taking into account the presence of the cables, of the input impedance of the amplifier, and of possible matching impedance network. Moreover the junction capacity $C_D$ cannot be neglected because its typical value is of the order of $30 \, \mu F/cm^2$. Consequently the characteristic detector time $\tau_D = C_D R_D$ can be comparable with the signal times $\tau_r$ and $\tau_f$, so that, at the arrival time of the ionizing particle, the working point $O$ jumps to $O''$ at a fixed voltage from the unperturbed characteristic to the perturbed one and afterwards moves on the $I'(V)$ curve with the characteristic detector time $\tau_D$.

The equivalent circuit of usual ionizing detectors is given by a current pulse generator, so that charge amplifiers are the best choice for the S/N ratio optimization. On the contrary, superconducting tunnel junctions can be described by the equivalent circuit shown in Fig. 3, so that we analyze not only current-charge amplifiers but also voltage amplifiers.

![FIG. 3 - Equivalent circuit of the STJ.](image)

In order to investigate the intrinsic signal to noise ratio, the external noise sources are neglected and both ideal bias network and amplifiers are assumed. The possibility of using pulse transformers will be not considered here.

As far as the filter is concerned, the optimum filter conditions should be analyzed. In this paper we consider only a simple low pass filter with a characteristic time $\tau_F$ which determines the bandwidth (BW) of the amplifier. In this way our detection system shall be characterized by four characteristic times, which are:

1) the detector time $\tau_D = C_D R_D = \tau_D (p,T)$;
2) the tunnel time $\tau_t = 1/p$;
3) the recombination time $\tau_r (T)$;
4) the filter time $\tau_F$.

Following Eq. (6), $\tau_t$ and $\tau_r$ determine the effective relaxation time $\tau_R$, and in a first analysis $\tau_F$ can be taken equal to $\tau_R$. By changing the junction area, the tunnel probability, and the temperature, the relations among the four times can be modified, changing the value of the signal to noise ratio.
4. - VOLTAGE AMPLIFIER

We suppose that in the experimental set up it is possible to neglect the presence of cables and that the impedance matching can be avoided, so that, since the input impedance of the voltage amplifier is high, the dynamic load line coincides with the static one. The bias conditions which maximize the amplitude of the voltage variations are obtained by a current bias.

4.1. - S/N Dependence on the Tunnel Probability

In order to study the variations of the S/N ratio induced by the change of p, we must take into account the variations of \( \tau_D \), \( \tau_F \) and \( \tau_p \), and study the variations of the signal, of the spectral density of the noise and of the BW. From the model of Fig. 3 the voltage signal is given by:

\[
\Delta V = R_D \Delta I_0 \left[ \exp \left( -\tau_R / \tau_D \right) - \exp \left( -\tau_D / \tau_R \right) \right] \left( 1 - \tau_D / \tau_R \right) \tag{9}
\]

From Eq. (9), for very low values of the tunnel probability p, i.e. for \( R_D \) values high enough to get \( \tau_D > \tau_R \), the detector time \( \tau_D \) cuts the voltage signal. As p increases the dynamic resistance decreases; as we get \( \tau_D < \tau_R \), the capacitor effects can be neglected. In this condition and with a current bias, the amplitude of the voltage signal is:

\[
\Delta V = \Delta I \ R_D \tag{10}
\]

According to Eq. (10) and taking a fixed voltage value of the unperturbed working point, the voltage signal is independent of p. This effect is also clear from a graphical point of view: as shown in Fig. 2, different p values changes only the current scale leaving the voltage signal unchanged in the current bias condition.

On the contrary, the spectral density of the voltage noise is given by:

\[
V_n^2(\omega) = I_n^2(\omega) R_D^2 \tag{11}
\]

From Eqs. (8) and (11) \( V_n^2(\omega) \) is proportional to \( 1/p \).

In the very low p limit, taking into account the effect of \( \tau_D \), the ratio S/n between the signal and the spectral density of the noise will increase as \( (p)^{3/2} \), so that the optimum condition is out of this limit. In the case of negligible \( \tau_D \) and for \( eV/k_B T \gg 1 \), the S/n is:

\[
S/n = \Delta I (\pi/eI)^{1/2} = \Delta I (\pi R_S eV)^{1/2} \propto p^{1/2} \tag{12}
\]

In the voltage range where \( \Delta I \) is constant the S/n ratio increases with the decreasing of the
current value at the unperturbed working point.

In the limit \( eV/K_B T \ll 1 \) we obtain:

\[
S/n = \Delta I \left( \pi R_S / 2K_B T \right)^{1/2} \propto p^{1/2}
\]  

(13)

and the \( S/n \) will increase with the rise of the static resistance. In both cases the \( S/n \) ratio will increase as the root square of \( p \).

As far as the BW is concerned, in the low \( p \) limit (i.e. \( \tau_i > \tau_r \) ) \( \tau_F \equiv \tau_R \) and the exponential decay of the signal is \( p \) independent, so that the bandwidth \( 1/\tau_F = 1/\tau_r \) can be assumed constant. On the contrary, in the high \( p \) limit \( \tau_R = \tau_i \), so that the signal time-constant will decrease as \( p \) increases and a larger BW will be necessary. The total noise, equal to the spectral density times \( \text{(BW)}^{1/2} \), will be independent of \( p \). In this way the \( S/N \) ratio increases with the tunnel probability until \( 1/p = \tau_i \), and for higher \( p \) the ratio \( S/N \) becomes \( p \) independent.

In actual junctions the limit of \( S/N \) comes from true leakage currents, which increase as \( p \) increases. Anyway, the discussion of this last aspect is out of the purpose of this paper.

### 4.2. Dependence of \( S/N \) on the Temperature

In order to analyze the temperature behaviour of a STJ detection systems, two different voltage ranges \( V < (\Delta_1 + \Delta_2)/e \) and \( V = (\Delta_1 + \Delta_2)/e \) can be identified in the I-V characteristic. In the first region the dynamic and the static resistances increase almost exponentially with the temperature decrease, while in the second region the static resistance weakly increases and the dynamic one decreases. Neglecting the gap variations, by Eq. (10) at the decrease of the temperature, the signal increases or decreases respectively in the first or in the second voltage range. By Eq. (11) a similar behaviour is shown by the voltage spectral density. By Eqs. (10) and (11), the \( S/n \) ratio is:

\[
S/n = \Delta I \left[ \pi R_S / F(V,T) \right]^{1/2}
\]  

(14)

so that the \( S/n \) will increase with the temperature decrease, and it takes its maximum value at the working point where \( R_S \) is maximum.

At the same time, with the decrease of \( T \), the BW = \( 1/\tau_F = 1/\tau_R \) at the beginning will decrease because of the increasing of the recombination time. Further temperature decreases lead the junction in the high \( p \) limit and to a saturation value of the BW for \( \tau_F = \tau_i \). Limits to the \( S/N \) arise because of the junction intrinsic capacitance, so that the cutoff of the signal generated by large values of \( C_D \) must be taken into account. In the limit \( \tau_D > 1/p \) and \( \tau_i < \tau_r \), we get:

\[
\Delta V \equiv \Delta I_o \exp(-t/\tau_D) / (C_D p)
\]  

(15)
In this way, for further decrease of $T$, the signal will be temperature independent, the spectral density of the noise will increase and the BW will be constant, so that the S/N will decrease. Following the above discussion the optimum condition is reached for $\tau_D \approx \tau_r$.

5. - CURRENT AND CHARGE AMPLIFIER

In the case of a current or charge amplifier coupled to the junction, the dynamic load line is determined by the low value of the input impedance of the amplifier. The current signal will be optimized under the condition of a voltage dynamic polarization, i.e. by a dynamic load $R_L^D \ll R_D$. Moreover the detector plus the bias network must be a current source in comparison with the amplifier. As a consequence it holds: $R_B > R_S (V) \gg |Z|$.

5.1. - S/N Dependence on the Tunnel Probability

Following Eq. (5) the amplitude of the current signal is proportional to $p$. By simple integration the charge signal is equal to:

$$\Delta Q = p \, n'(0) \, \tau_R$$ (16)

Then, as $p$ increases, $\Delta Q$ at the beginning is proportional to $p$, but as $\tau_r < \tau$, $\Delta Q$ saturates to the limit value $\Delta Q = n'(0)$. On the other hand, by Eq. (8) the spectral density of the current noise increases indefinitely with the increase of $p$. The S/N ratio has the following expression:

$$S/N = pn'(0) \, \tau_R \, \left[ \pi R_S / (F(V,T)BW) \right]^{1/2} = pn'(0) \left[ \pi R_S / (F(V,T)) \right]^{1/2} \tau_R^{3/2}$$ (17)

which shows its maximum value for $p=1/2 \, \tau_r$. By substitution in Eq. (16) for $eV/K_B T \gg 1$, we obtain:

$$(S/N)_{\text{max}} = n'(0) \left[ 2 \tau_r \pi / (eI) \right]^{1/2} / 3^{3/2}$$ (18)

In the limit $eV/K_B T \ll 1$ we get:

$$(S/N)_{\text{max}} = n'(0) \left[ \tau_r \pi R_S / (K_B T) \right]^{1/2} / 3^{3/2}$$ (19)

so that in the two limits the S/N ratio increases respectively by decreasing the current bias value and by increasing the value of the static resistance of the working point.
5.2. - S/N Dependance on the Temperature

In the case of a charge amplifier coupled to the junction the characteristic detector time $\tau_d$ doesn't play any role. In this way with the decrease of the temperature we get:

1) an increase of the signal, which saturates to its limit value $\Delta Q = n'(0)$,
2) a decrease of the spectral density of the current noise,
3) a decrease of the BW, which saturates to its limit value $1/\tau_r$.

In this way, following Eq. (17), the S/N increases indefinitely with the decrease of the temperature.

6. - CONCLUSIONS

In conclusions we have supposed ideal STJ, without true leakage currents, perturbed in a uniform way by ionizing particles, coupled to a noiseless bias network and to noiseless ideal voltage or charge amplifiers. By using a voltage amplifier the signal to noise ratio has the following behaviour:

1) the S/N increases as the tunnel probability increases but it saturates as $\tau_t = \tau_r$,
2) the S/N shows a maximum with the decrease of the temperature.

In the case of a charge amplifier coupled to the junction we have:

1) the S/N shows a maximum as a function of $p$,
2) the S/N increases indefinitely as the temperature decreases.

We believe that our analysis in ideal conditions is only the starting point to optimize the measurements conditions in practical experimental set up. In order to obtain more significative results, the effects of non ideal amplifiers, of the localization of the perturbation in the junction, of the disuniformities of the STJ, and of the true leakage currents in STJ should be taken into account.
REFERENCES