A. Rindi and F. Celani:
MEASURING VERY LOW NEUTRON FLUX UNDERGROUND. SOME STATISTICS
REMINDERS AND PRACTICAL CONSIDERATIONS
MEASURING VERY LOW NEUTRON FLUX UNDERGROUND. Some statistics reminders and practical considerations

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1. - INTRODUCTION

Measuring very low fluxes of neutrons, of the order of $10^{-7}$ n cm$^{-2}$ s$^{-1}$ or less, like those expected inside the Gran Sasso underground laboratory, is a rather challenging problem.

The usual most sensitive neutron detectors, like the $^3$He or BF$_3$ filled proportional counters, for volumes of about 500 cm$^3$ and at pressure higher than 760 mm of Hg, show typical sensitivity to thermal neutrons of about 150 c n$^{-1}$ cm$^2$. When introduced into moderators for detecting neutrons of energy larger than 1 MeV, their sensitivity is reduced to about 30% of that to thermal neutrons.

In order to achieve a better signal-to-noise ratio, the counters are connected to a multichannel analyzer which records the spectrum of the energy deposited by the charged products of the (n, $^3$He) or (n, $^{10}$B) reactions, rather than use the more common threshold discrimination method. In the case, for instance, of $^3$He filled counters, the reaction $^3$He + n $\rightarrow$ $^3$H + p, which has a Q value of 765 keV, leads to a p of 574 keV and a $^3$H nucleus of 191 keV. However, the energy spectrum at the MCA shows a peak at about 765 keV (with a typical FWHM of about 4%) plus a large plateau covering the energy range between about 200 keV and 800 keV, due to wall effects in the counter and reactions with slow and fast neutrons. The sensitivity of the counter
as given by the manufacturer is intended for the whole plateau range. The ratio of the area in the peak to that in the whole energy range is typically 1:3 for 2.54 cm diameter counters.

![Graph showing channel number vs counts](image)

**Fig. 1**

Detecting $10^{-7} \text{n cm}^{-2}\text{s}^{-1}$ (or even $10^{-6} \text{n cm}^{-2}\text{s}^{-1}$) with a $^3\text{He}$ filled counter having 150 c $\text{n}^{-1}\text{cm}^2$ sensitivity means to count $5.4 \times 10^{-2}$ count per hour (or, respectively, 0.54 cph). It is not easy to build the electronics to associate to the detector, i.e. a fast charge preamplifier, a $\times 100$ shaping amplifier, a very stable HV Supply etc. with a noise in that range of energy much lower than the signal to be detected, capable to steady operate in a adverse climatic environment like that in an underground laboratory (about 100% humidity). In addition, the noise pulses due to the radioactivity of the materials composing the counter itself are not negligible. These problems will, however, be presented and discussed in details in a next report.

We shall present in the following some feasibility problems related just to counting statistical considerations; the considerations will be applied to some practical cases.

2.- STATISTICS CONSIDERATIONS

Let us indicate with $S$ the net counting rate from neutrons

$$S = \Phi \varepsilon$$
where $\phi$ = neutron flux (in n cm$^{-2}$s$^{-1}$), and

$S$ = sensitivity of the counter (in c n$^{-1}$cm$^2$)

and with $B$ the total background counting rate (due to electronic noise, counting noise, internal radioactivity, etc.). It is rather difficult to precisely evaluate $B$. An upper approximation value can be obtained by a measurement with the counter shielded with Cd and introduced into the moderator; however, some neutrons may still penetrate till the counter.

The measurement of $S$ can only be performed by counting the source plus background rate ($S + B$).

The first optimization is made on the choice of the counting time allocated to the source-plus-background measurement ($T_{S+B}$) and the counting time allocated to the background measurement ($T_B$). A simple application of error propagation(1) gives the optimum division of time which is obtained by meeting the condition

$$\frac{T_{S+B}}{T_B} = \sqrt{\frac{S+B}{B}}.$$  

In our case, one expects $B$ to be not very different from $S$, perhaps even larger than $S$. For $S=B$, the optimization of counting times gives

$$T_{S+B} = 1.4 \ T_B.$$  

For $S<B$, $T_{S+B} \rightarrow T_B$. It is acceptable to assume $T_{S+B} = T_B = T$.

A first condition that we may impose on time $T$ is that the total net count is higher than the background standard deviation

$$ST > \sqrt{BT}$$

i.e.

$$T > \frac{B}{S^2}.$$  

The source net value with error will be

\[(A-F) \pm \sqrt{A+F}\]

and the net counting rate

\[S \pm dS = \frac{A-F}{T} \pm \frac{\sqrt{A+F}}{T} = S \pm \frac{\sqrt{2BT+ST}}{T}. \quad (2)\]

The relative error \(\sigma_s\) in the evaluation of the net counting rate will be given by:

\[\sigma_s = \frac{\sqrt{2BT+ST}}{ST} = \frac{\sqrt{2B+S}}{S} \frac{1}{\sqrt{T}}. \quad (3)\]

From this expression, given an expected flux and background rate, one can calculate the time needed for achieving a certain precision for a given counter efficiency.

**Case 1**

\[\phi = 10^{-7} \text{ n cm}^{-2} \text{s}^{-1}; \quad \varepsilon = 150 \text{ c n}^{-1} \text{ cm}^{2}; \quad B = 10^{-1} \text{ ch}^{-1} \]

\[S = \phi \varepsilon = 5.4 \times 10^{-2} \text{ ch}^{-1}.\]

The first constraint on \(T\) imposes:

\[T > 3.43 \text{ h}.\]

The following table shows the counting time needed for a given relative error:

<table>
<thead>
<tr>
<th>(\sigma_s)</th>
<th>(T) (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (100%)</td>
<td>87.1</td>
</tr>
<tr>
<td>0.5</td>
<td>348</td>
</tr>
<tr>
<td>0.2</td>
<td>2180</td>
</tr>
<tr>
<td>0.1</td>
<td>8700</td>
</tr>
</tbody>
</table>

It is rather difficult to achieve such counting times. The sensitivity of the counter has to be improved by at least an order of magnitude.

**Case 2**

\[\phi = 10^{-6} \text{ n cm}^{-2} \text{s}^{-1}; \quad \varepsilon = 150 \text{ c n}^{-1} \text{ cm}^{2}; \quad B = 10^{-1} \text{ ch}^{-1}; \]

\[S = 5.4 \times 10^{-1} \text{ ch}^{-1}.\]

The first constraint imposes:

\[T > 0.34 \text{ h}.\]

and the table:
\[ \sigma_s \quad T \ (h) \]
\begin{align*}
1 & \quad 2.54 \\
0.5 & \quad 10.2 \\
0.2 & \quad 63.5 \\
0.1 & \quad 254 \\
\end{align*}

Case 3)

\[ \phi = 3.5 \times 10^{-6} \text{ n cm}^{-2} \text{s}^{-1}; \quad \varepsilon = 150 \text{ c n}^{-1} \text{cm}^2; \quad B = 3 \times 10^{-1} \text{ ch}^{-1} \]

\[ S = 1.9 \text{ ch}^{-1}. \]

The first constraint imposes:

\[ T > 8.3 \times 10^{-2} \text{ h}. \]

and the table:

\[ \sigma_s \quad T \ (h) \]
\begin{align*}
1 & \quad 0.7 \\
0.5 & \quad 2.8 \\
0.2 & \quad 17.3 \\
0.1 & \quad 69.3 \\
\end{align*}

3.- CONCLUSIONS

Some counter manufacturers offer \( BF_3 \) or \(^3\)He filled proportional counters with sensitivities to thermal neutrons up to \( 2000 \text{ c n}^{-1} \text{cm}^2 \). However, the dimensions and internal pressure required for achieving such a sensitivity degrade the shape of the pulses and of the neutron spectrum, increasing markedly the noise background. One has to compromise between sensitivity of the counter and shape of the pulses for achieving the best signal-to-noise ratio.

Our experience up to now shows that it is not practical to use commercial counters with sensitivity higher than about \( 300 \text{ c n}^{-1} \text{cm}^2 \).

With such sensitivity one has to foresee counting times for each measurement of the order of 50 hours or more for achieving the statistical accuracy requested for our experiment. It is not easy, under the present climatic conditions inside the Gran Sasso underground laboratory, to maintain a stable counting system for such long times.

We are presently studying methods to improve the counter sensitivity without degrading the pulse shape.