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THE TENSOR ANALYZING POWER IN BACKWARD pd ELASTIC SCATTERING AND ITS RELATION TO pp → πd AT INTERMEDIATE ENERGIES

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Abstract: The tensor analyzing power for pd backward elastic scattering is calculated in the energy range 150 ≤ T_p ≤ 800 MeV. Two main contributions are considered: the one-nucleon-exchange and the so-called triangle graph, including pp → πd as a subprocess. The pd backward elastic cross section and tensor analyzing power are fairly well reproduced by the model.

1. Introduction

Backward pd elastic scattering has been the subject of extensive work, experimental and theoretical, for nearly fifteen years. The excitation function at 180° is now rather well known up to incident proton kinetic energies of T_p = 2.7 GeV. It exhibits a monotonous decrease with increasing energy, broken by two structures: the first is centered around T_p = 0.5 GeV, and the second, not yet completely explored, begins at T_p = 2.3 GeV [refs. 1–5)].

The mechanisms involved in the interpretation of the first of these structures can be roughly classified into three categories:

(i) a one-pion exchange, with virtual Δ(1232) production in the intermediate state 5–9);

(ii) a shoulder of the deuteron form factor in a Glauber-type model 10);

(iii) a three-nucleon resonance in the s-channel 11).

All these models reproduce the gross features of the backward cross section. A method to discriminate among them is to look at the tensor analyzing power, whose energy dependence, as pointed out by Vasan 12), should be sensitive to the microscopic structure of the model.

The T_{20} measurements at backward angles and for energies 0.15 ≤ T_p ≤ 1.2 GeV recently published 18), show that the analyzing tensor attains large negative values,

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and its energy trend is richly structured. These measurements are in marked disagreement with earlier ones \(^{13}\), where \(T_{20}\) is compatible with zero in the same energy range. Theoretical predictions \(^{14}\) do not adequately describe these new results.

In this paper we propose an interpretation of the backward tensor analyzing power and cross section in terms of a model in category (i).

We shall assume that the elastic proton–deuteron backward scattering amplitude is made up of two contributions: (i) the one-nucleon-exchange mechanism (ONE), fig. 1; (ii) the triangle diagram including \(pp \rightarrow \pi d\) as a subprocess, fig. 2.

The ONE is the most elementary process for the backward scattering. At high energies, however, the momentum transfer in the \(u\)-channel is far from the one-nucleon pole and we should take into account the other contributions which are represented by the diagram in fig. 3. In the rest frame of the initial deuteron, the exchanged nucleon is "slow" and the following picture emerges: the incident proton hits one of the nucleons inside the target deuteron; together they form a deuteron moving forward with the emission of a pion which is absorbed by the other nucleon inside the target deuteron.

This triangle mechanism was proposed, as far as we know, by Craigie and Wilkin \(^6\). Later Kolybasov and Smorodinskaya \(^7\) pointed out that an approximation employed in the paper by Craigie and Wilkin is too rough, and leads to a wrong result.

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Fig. 1. The one-nucleon-exchange graph for \(pd \rightarrow dp\).

Fig. 2. The one-pion-exchange (triangle) diagram for \(dp \rightarrow dp\).

Fig. 3. \(N\pi\) exchange in the \(u\)-channel. The triangle graph is a special case of this diagram.
Unfortunately, their calculation includes several mistakes and they introduced a certain factor for the ONE diagram to get the best agreement with experiment.

Starting from these pioneer papers, we will formulate the calculation of amplitudes corresponding to figs. 1 and 2, and analyze the experimental data for the cross section and the tensor analyzing power.

2. Scattering amplitudes

As discussed in sect. 1, we include the triangle diagram as well as the ONE. Spin amplitudes are, therefore, written as

\[ T_{m_{1}n_{1}}^{m_{2}n_{2}} = M_{m_{1}n_{1}}^{m_{2}n_{2}} + T_{m_{1}n_{1}}^{m_{2}n_{2}} \]

where the \( M \)'s are ONE amplitudes and the \( T \)'s triangle mechanism contributions. Spin states of the incident proton, the outgoing proton, the initial deuteron and the final deuteron are denoted by \( m_{1}, m_{2}, l_{1}, \) and \( l_{2} \). We quantize all spins along the direction of the initial momentum. These amplitudes are normalized as

\[ \frac{d\sigma}{d\Omega_{c.m.}} = \frac{(2m)^2}{64\pi^2S} \sum_{\text{spin}} |F|^2, \]

i.e. they are invariant amplitudes. Here \( m \) is the proton mass.

The thirty-six spin amplitudes are not independent. For the backward scattering \((\theta = \pi)\) which we will investigate in this paper, they are related to each other by parity conservation as

\[ T_{11}^{11} = a, \quad T_{-11}^{-11} = -a, \quad T_{10}^{10} = b, \quad T_{-10}^{-10} = -b, \]
\[ T_{11}^{00} = c, \quad T_{-11}^{-00} = -c, \quad T_{10}^{00} = c', \quad T_{-10}^{-00} = -c', \]
\[ T_{11}^{-11} = d, \quad T_{-11}^{11} = -d. \]

Furthermore time-reversal invariance requires

\[ c = c'. \]

2.1. ONE-NUCLEON EXCHANGE

We follow the procedure by Vasan\(^{12}\) and Anjos et al.\(^{13}\) to construct a ONE amplitude, but in order to clarify their contribution to the reaction we will give here the full expression. We use the notation of Bjorken and Drell\(^{14}\).

The Feynman amplitude corresponding to fig. 1 is

\[ M = \bar{u}(p_{2}) \frac{I_{1}^{1} \epsilon_{1}}{\sqrt{2}} \frac{i}{-\bar{q} - m + i \epsilon} \frac{I_{2}^{1} \epsilon_{2}}{\sqrt{2}} u(p_{1}), \]

where \( \epsilon_{1} (\epsilon_{2}) \) is the initial (final) polarization vector of the deuteron vertex function \( I_{1} (I_{2}) \). The four-momenta of the incident proton, the outgoing proton, the initial deuteron, final deuteron and exchanged nucleon are \( p_{1}, p_{2}, d_{1}, d_{2} \) and \( q \), respectively.
Replacing the vertex parts by the nonrelativistic deuteron wave function in the c.m. system, we get

\[
M_{l,m_1}^{L,m_2} = \frac{-i 2m(q^2 - m^2)}{2m} \left[ \frac{1}{2} \frac{(32\pi^3 m)^{1/2}}{m} \right]^2 \sum_{\alpha} \left\{ C_{m_2}^{1/2} l_{-m_2,-\alpha} \frac{1}{l_{1}+1-m_2-\alpha} \right\} \times \left[ \sum_{L=0,2} R_L(Q) Y_{L,l_2+m_2-l_2-m_2+\alpha}^{L} \left( \Theta \right) C_{l_2+m_2-l_2-m_2+\alpha}^{L} l_{1}+m_1-m_2-\alpha \frac{1}{l_1+1-m_1-m_2-\alpha} \right] \times C_{m_2}^{1/2} l_{1-m_2-\alpha} \frac{1}{l_{1-\alpha}} \left[ \sum_{L=0,2} R_L(Q) Y_{L,\alpha} \left( \Theta \right) C_{\alpha}^{L} l_{1-\alpha} \frac{1}{l_1+1-m_1-m_2-\alpha} \right],
\]

where \( C_{m_2}^{1/2} l_{-m_2,-\alpha} \) are Clebsch–Gordan coefficients, the \( Y_{LM} \) are spherical harmonic functions, \( Q = \frac{1}{2}(q - p_1) \) and \( Q' = \frac{1}{2}(q - p_2) \) are the relative momenta of nucleons inside the initial and final deuteron, and \( \Theta \) and \( \Theta' \) are their angles. \( \alpha \) is the spin of the internal nucleon. The deuteron wave functions in momentum space, \( R_L \), are related to the D and S wave functions, \( u \) and \( w \), as

\[
R_0(Q) = \frac{2}{\pi} \int u(r) j_0(rQ) r \, dr,
\]

\[
R_2(Q) = -\frac{2}{\pi} \int w(r) j_2(rQ) r \, dr.
\]

(7)

It is well known that the existence of the D-wave is essential to the tensor analyzing power in the ONE model,

\[
T_{22}^{\text{ONE}} = \frac{1}{\sqrt{2}} \frac{(2\sqrt{2} R_0 - R_2) R_2}{R_0^2 + R_2^2}.
\]

(8)

2.2. TRIANGLE DIAGRAM

The Feynman amplitude corresponding to fig. 2 is

\[
T = \bar{u}(p_2) \int \frac{d^4p}{(2\pi)^4} \frac{i}{K^2 - \mu^2 + i\epsilon} \sqrt{2} G_{\gamma\gamma} \frac{i}{n - m + i\epsilon} \frac{e_1 \Gamma}{\sqrt{2}} \frac{i}{-\mu - m + i\epsilon} \epsilon_\mu^+ \Delta u_1(p_1),
\]

(9)

where \( G \) denotes the pion–nucleon vertex function; \( p, n, \) and \( k \) are the four-momenta of the internal proton, neutron and pion, and \( \mu \) is the pion mass. The vertex function \( A \) is related to the amplitude for the process \( pp \to \pi^+ d \) by

\[
j_{\Delta+}^{j_3}(pp \to \pi d) = \bar{u}^{(\alpha)}(p)(e_2^{(j_2)^+} \cdot A) u^{(j_3)}(p_1).
\]

(10)

There is another diagram which is obtained by interchanging the internal proton and neutron and replacing \( \pi^+ \) by \( \pi^0 \) in fig. 2. This amplitude is related to that of fig. 2 by isospin and is one-half of eq. (9). Therefore the total amplitude is three-halves of eq. (9).
Following Kolybasov and Smorodinskaya, we evaluate eq. (9) in the rest system of the target deuteron and use the nonrelativistic propagator (see appendix (B))

\[
\int d^4p \frac{i}{-p - m + ie} = \int d^3p \frac{2m}{2m T_p} \frac{-p + m}{p^2 + ie},
\]

where we take the pole term in the integration over the kinetic energy \(T_p\). With the help of the formulae in the appendix (C, D, E) we obtain

\[
T_{\alpha i i_\beta}^{1/2} = \frac{1}{2\sqrt{2} \pi^3} \frac{\sqrt{E_2 + m}}{E_2} \sum_{\alpha, \beta} \chi^{(m_3)} \phi^{(l)} \int \frac{d^3p}{(p + \hat{p}_2)^2 + \delta^2 - ie} \frac{(p_2 - \hat{p}_2)(\chi^{(3)} G^{1/2 1/2})}{\chi^{(0)} G^{1/2 1/2}} \times \left[ \sum_{l} R_L(Q) Y_{LM}(\Omega) C_{LM}^{1/2 1/2} \frac{f_{\alpha,i \beta}^{1/2}}{f_{\alpha,i \beta}^{1/2}} \right],
\]

where

\[
p_2 = \frac{E_2 + m}{E_2 + m} p_2 = \frac{2m}{E_2 + m} p_2, \quad \hat{p}_2 = \frac{m}{E_2} p_2,
\]

and \(\chi\) are Pauli spinors. We take \(f\) and \(G\) out of the integral. To a first approximation we evaluate the energy of \(f(pp \rightarrow \pi d)\) in the system where the target proton is at rest. The \(\pi NN\) vertex must be evaluated at the pole of the propagator in the integrand. Using a trick by Kolybasov and Smorodinskaya (see appendix (F)), the triangle amplitude may be written as

\[
T_{\alpha i i_\beta}^{1/2} = \frac{1}{2\sqrt{2} \pi^3} \frac{\sqrt{E_2 + m}}{E_2} \frac{G}{2} \sum_{i \beta} f_{\alpha,i \beta}^{1/2} \left( \sum_{l} C_{LM}^{1/2 1/2} \chi^{(0)} \right) B_{i \beta},
\]

where

\[
B_{i \beta} = \frac{\sqrt{2}}{2} \int dr \frac{e^{i \phi}}{r} \frac{d^2}{d\cos \theta d\phi} \left\{ \frac{r^2 - 2 \hat{r} \delta}{r^2} \right\} \times \sum_{L} Y_{LM}(\phi) Y_{LM}(0, \phi) \frac{C_{LM}^{1/2 1/2} \chi^{(0)}}{C_{LM}^{1/2 1/2} \chi^{(0)}} \times \sum_{l} \psi_L(r) Y_{LM}(\theta, \phi) C_{LM}^{1/2 1/2} \frac{f_{\alpha,i \beta}^{1/2}}{f_{\alpha,i \beta}^{1/2}}.
\]

Here \(\psi_L\) is the deuteron wave function in \(r\)-space,

\[
\psi_L(r) = \begin{cases} \frac{u(r)}{r} & (L = 0) \\ \frac{w(r)}{r} & (L = 2). \end{cases}
\]

The angular integration in eq. (14) can be performed explicitly with the help of the formulae in the appendix (G). After a lengthy calculation we finally obtain the
following expression for the triangle diagram amplitudes for the backward pd scattering:

\[ a = -C y D_1, \quad c = C y D_1, \]
\[ b = -C y D_0, \quad c' = -C y D_0, \]
\[ d = 0, \]  

(15)

where

\[ C = \frac{1}{2 \sqrt{2} \pi} \frac{\sqrt{E_2 + m}}{E_2} G, \]
\[ y = -f_{1/2}^0 (pp \rightarrow \pi d), \quad v = f_{1/2}^1 (pp \rightarrow \pi d), \]
\[ D_0 = 2 B_{11}^{(x)} - \sqrt{2} B_{10}^{(x)}, \quad D_1 = \sqrt{2} B_{11}^{(x)} + B_{10}^{(x)}. \]

The amplitudes (15) do not satisfy the constraint from \( T \)-invariance, i.e. eq. (4). This is because there is no time-invariance relation in the subprocess \( pp \rightarrow \pi d \). We escape this difficulty as follows. We repeat the above calculation of the triangle diagram in the same way but in the rest frame of the final deuteron. In this case we get the same expression as eq. (15), but \( c \) and \( c' \) are interchanged. There is no reason to give a preference to one frame or the other, and therefore we average the results obtained in the two final frames:

\[ c = c' = \frac{1}{2} C (v D_1 - y D_0). \]  

(16)

3. Results and comparison with the data

For the numerical evaluation of the backward cross section and tensor analyzing power we employed the Reid hard-core wave functions \(^{16}\) for the S- and D-state wave functions of the deuteron. The \( pp \rightarrow \pi d \) amplitude \( f(pp \rightarrow \pi d) \) was constructed using the partial-wave analysis between threshold and 810 MeV of the Osaka City University group \(^{17}\).

The pion–nucleon coupling constant has been well determined in other reactions, and its numerical value is \( G^2/4\pi = 14.6 \). The results for the backward excitation function are compared with data in fig. 4. The two curves correspond to the two solutions of the partial-wave analysis, called S and D by the Osaka City University group \(^{17}\). Both solutions account rather well for the general shape of the bump in the cross section, although the quantitative agreement of the D-solution seems better. The curves stop at \( T_p = 810 \text{ MeV} \), where the input data for constructing \( f(pp \rightarrow \pi d) \) terminate. At the high-energy end of the theoretical calculation both solutions fall with energy more steeply than the experimental data. This fact might suggest that a different subprocess begins to contribute to the backward pd elastic scattering.

The calculation for \( T_{20} \) is shown in fig. 5 where data from refs. \(^{14,18}\) are reported. Here the S- and D-solutions account for the general trend of the measurements of
Fig. 4. $p_d \to dp$ cross section as a function of incident proton kinetic energy. *Solid line*: one-nucleon exchange; *dashed line*: $S$-solution of ref. 13); *dotted line*: $D$-solution of ref. 13). Data points are from refs. 1-3).

Fig. 5. $T_{20}$ measurements from refs. 14,18). *Solid line*: one-nucleon exchange; *dashed line*: $S$-solution of ref. 13); *dotted line*: $D$-solution of ref. 13).
ref.\textsuperscript{18}, but the D-solution is clearly favoured. Shape and position of the dip centered at \( T_p = 300 \) MeV are rather well reproduced. Also the second structure, positioned at \( T_p = 700 \) MeV, which is stressed by the authors of ref.\textsuperscript{18}, is hinted at by the D-solution. It should be pointed out that the rise in \( T_{20} \) shown by both solutions towards \( T_p = 800 \) MeV may be due to the lack of some high angular momentum wave in the partial-wave analysis of ref.\textsuperscript{17}).

\textbf{4. Concluding remarks}

We have observed much better agreement between the experimental data and the theory than we had expected. The essential difference between the present calculation and previous ones consists in imposing \( T \)-invariance on the triangle amplitudes. Besides this point we have simply tried to write down the spin amplitudes corresponding to figs. 1 and 2. There are, however, still ambiguities in treating the internal momenta. The procedure in the appendix (E) cannot be justified by the Feynman rules.

No parameters appear in the model. In fact we do not need the normalization factor \( \zeta \) of Barry\textsuperscript{5}) which comes from the ambiguity in the internal integral. The only uncertainty comes from two possible solutions of the \( pp \to \pi d \) amplitudes. We may use the \( pd \to dp \) process to supply new information to the \( pp \to \pi d \) partial-wave analysis. The \( T_{20} \) of \( pd \to dp \) clearly favors the D-solution. This solution is considered to include two dibaryons \( B^5(\text{mass}, J^P) = B^5(2.17, 2^+) \) and \( B^5(2.22, 3^-) \) as intermediate states (fig. 6).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig6}
\caption{s-channel dibaryon exchange in the \( pp \to \pi d \) reaction.}
\end{figure}

We have used the \( pp \to \pi d \) partial-wave amplitudes. They are expected to be very similar to the real amplitudes and to include many possible mechanisms like the \( \Delta N \) intermediate state. Therefore our triangle amplitudes automatically include diagrams such as that of fig. 7. Note that the diagram in fig. 7 satisfies \( T \)-invariance.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig7}
\caption{A possible intermediate mechanism already included in the partial-wave amplitudes employed in the present calculation.}
\end{figure}
At these energies, however, the ΔN intermediate state is not enough to describe the pp → πd amplitude \(^{17,19}\). Recently there has been some progress in the analysis of pp → πd [refs. \(^{19,20}\)]. The new analyses include deuteron polarization data, and we are planning to analyze pd → dp using these new inputs.

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Appendix

In this appendix some formulae helpful in the evaluation of the amplitudes have been assembled.

A

\[ p + m = 2m \sum_{a=1}^2 \bar{u}^{(a)}(p) \bar{u}^{(a)}(p), \]

\[ -p + m = -2m \sum_{a=1}^2 v^{(a)}(p) \bar{v}^{(a)}(p). \]

B

In the rest system of the deuteron (see fig. 1), we have

\[ d = (m, \Omega), \quad p = (m + T_p, p), \quad q = (m + T_q, -p), \]

\[ T_p = p^2 / 2m, \quad T_q = m_q - 2m - T_p, \]

\[ p^2 - m^2 = 2mT_p - p^2, \]

\[ Q^2 - m^2 = 2mT_q - p^2 = -2(k^2 + p^2), \]

where

\[ k = \sqrt{m(2m - m_q)}. \]

C

If the deuteron moves very slowly, its vertex function is related to the nonrelativistic wave function as

\[ \bar{u}^{(β)}(q) \frac{ε_1^{[1]}(1)}{\sqrt{2(q^2 - m^2)}} v^{(α)}(p) = \frac{(32\pi^3 m)^{1/2}}{2m} C_α^{1/2} C_β^{1/2} \sum_{L} R_L(Q) Y_{LM}(Ω) C_{m_a+1}^{L} C_{m_b+1}^{L}, \]

where \( α \) and \( β \) in the Clebsch-Gordan coefficients represent the spins of the states \( v^{(α)} \) and \( u^{(β)} \), respectively, and \( Q = p - \frac{1}{2}d = p. \)
\[\bar{u}^{(\|)}(p_2) \gamma_\mu u^{(\alpha)}(n) = -\frac{1}{2m} \sqrt{\frac{E_z + m}{E_n + m}} \phi^{(\|)} \sigma \cdot (p + p_2') \phi^{(\alpha)},\]

where \(\phi\) are Pauli spinors, and

\[p'_2 = \frac{E_n + m}{E_z + m} p_2.\]

In the rest system of the deuteron we may employ the approximation \(E_n \approx m\) in the above formulae.

\[p_2 = (m + T, p_2), \quad n = (m + T, -p).\]

In the rest system of the target deuteron we may replace \(T_n\) by \(T_p\). Then we get

\[
k^2 - \mu^2 = (T_z - T_p)^2 - (p_2 + p)^2 - \mu^2
= \left(1 + \frac{T_z}{m}\right) (p_2^2 - 2p_2 \cdot p - 2mT_z - \mu^2)
= \left(1 + \frac{T_z}{m}\right)((p + p_2)^2 + \delta^2).\]

Here \(\delta_2\) and \(\delta\) are defined in the text.

\[
a = \frac{a}{a^2 + k^2} = -\frac{i}{4\pi} \int \frac{r}{r^2 + \frac{\delta^2}{4}} \, e^{i\alpha r - i\phi} \, d^3r,
\]

\[
\frac{1}{a^2 + k^2} = \frac{1}{4\pi} \int \frac{1}{r} \, e^{i\alpha r - i\phi} \, d^3r.
\]

\[e^{i\phi} = 4\pi \sum_l \sum_i i^l_j (p r) \, Y_{lm} (\theta, \phi) Y_{lm}^*(\alpha, \beta).
\]

\[
\int d^3p \, e^{i\phi} R_L(p) \, Y_{LM} (\alpha, \beta) = \begin{cases} 
\sqrt{4\pi \sqrt{\frac{1}{2} \pi}} \frac{u(r)}{r} & (L = 0) \\
4\pi \sqrt{\frac{1}{2} \pi} \frac{w(r)}{r} \, Y_{2M} (\alpha, \beta) & (L = 2),
\end{cases}
\]

where we use the formula in part G.
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